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Nonlocal conductance of topological insulator F/S/I/S/F junction

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Abstract. In this work the Non-Local Conductance (NLC) in the topological insulator Ferromagnetic/Superconductor/Insulator/Superconductor/Ferromagnetic (F/S/I/S/F) structure is numerically investigated. The insulator region is a thin barrier and the magnetization of the ferromagnetic regions is considered perpendicular to the surface of the topological insulator. Our results indicate, when the magnetization vectors in the ferromagnetic regions are parallel (anti-parallel), the crossed Andreev reflection (electron elastic cotunneling) process does not occur and also the NLC in terms of barrier strength shows a π periodic behaviour. In addition, the NLC as a function of the barrier strength shows an on-off quantum switching properties. We find also that this junction can be used as a quantum entangler devise which is of high important in quantum computing.

1. Introduction

Quantum mechanically entangled pairs of particles are a major building block of quantum computation and information processing. A natural source of entangled electrons is a BCS-type superconductor where the Cooper pairs form spin singlet states. The two electrons of a Cooper pair can be spatially separated into two different metallic (ferromagnetic) leads in a nonlocal process called Crossed Andreev Reflection (CAR) and is proposed as the basic of the solid-state entangler [1]. However, the signatures of CAR are often completely masked by a competing process known as the electron Elastic Cotunneling (EC) which occurs when an electron directly tunnels from the left electrode to right electrode without the formation of the Cooper pair [2]. The CAR and EC processes give opposite contributions to the nonlocal conductance. The realization of a system, where CAR can be observed, has been the aim of both experimental and theoretical works lately. This interest is due to the fact that CAR is an inherently mesoscopic phenomenon, with the prospect of creating entangled electrons [3].

Topological Insulators (TIs) offer a new state of matter topologically different from the conventional band insulator. Edge channels or surface states of the TIs are topologically protected and described by massless Dirac fermions at low energies [4]. Several works have investigated proximity induced superconducting and ferromagnetic order on the surface of a TI. When a magnetic insulator is deposited on the top of the topological insulator, the magnetic field in the TI induced by the magnetic insulator will act on the 2D Dirac electrons through the vector potential of field. The vector potential appearing along the z-direction would manifest itself as a magnet-induced mass for the 2D Dirac electrons [5]. Nonlocal transport properties in hybrid structures, consisting two normal or ferromagnetic metals separated by a superconductor, have attracted much attention [6-9]. Recently, the nonlocal transport properties of a F/S/F junction,

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formed on the surface of a three dimensional TI with in-plane magnetization, have been investigated [2]. In this paper, we investigate the nonlocal conductance through F/S/I/S/F junction deposited on the surface of a TI. In our model, the direction of magnetization in the ferromagnetic region is perpendicular to the surface. Furthermore, the phase of superconductor layers is considered to be identical. We find that both parallel and anti-parallel nonlocal conductance show a π periodic oscillatory behaviour when depicted as a function of the barrier strength of the insulator region. Furthermore, we find the suppression of CAR (EC) in parallel (anti-parallel) magnetization configuration which suggests that in anti-parallel configuration, this structure may be used as a solid state entangler.

2. Method

A typical structure of our model as F/S/I/S/F junction is deposited on the surface of a 3-dimensional Topological Insulator (TI) in the $x-y$ plane, as depicted in Figure 1. The insulator thin barrier is controlled by the gate voltage, U, and a gate voltage, V_0, is applied to the superconductor regions. The superconductor, ferromagnetic and insulator regions are in $-l<x<0$, $d<x<l+d$, $x<-l$, $x>l+d$, and $0<x<d$, respectively, with $\vec{m}=m\hat{z}\left[\theta(-x-l)\pm\theta(x-l-d)\right]$ as the magnetization vector which is considered to be perpendicular to the surface of the TI. The Bogoliubov-de Gennes equation for the electron and hole excitations is given as [10]:

$$\begin{pmatrix} H(P)-E_f-V(x) & \Delta(x) \\ -\Delta^*(x) & -H(-P)+E_f+V(x) \end{pmatrix}$$

$$\psi(x,y)=E\psi(x,y), \tag{1}$$

in which $\psi(x,y)=\psi(x)e^{\frac{iyP_y}{\hbar}}$ and $H(P)=V_f\vec{P}.\vec{\sigma}+e\vec{A}_{eff}.\vec{\sigma}$ is the Dirac Hamiltonian. The vector potential $e\vec{A}_{eff}$ is proportional to the magnetization $\vec{m}$ [5]. V_f, $\vec{\sigma}$ and $\vec{P}=\left\langle -i\frac{\partial}{\partial x}, -i\frac{\partial}{\partial y}\right\rangle$ are the Fermi velocity of quasi particles in TI, the Pauli spin matrix and momentum operator, respectively. Here, $\Delta(x)=\sigma_y\Delta_0 e^{i\varphi}$ where Δ_0 and φ (which is given by $e^{i\varphi}=\frac{\Delta(x)}{|\Delta(x)|}=1$ for s-wave superconductors) are, respectively, the amplitude and phase of the superconducting pair potential in superconductor regions. The potential profile $V(x)$ is given as:

$$V(x)=U\theta(x)\theta(d-x)+V_0[\theta(x+l)\theta(-x)+\theta(x-d)\theta(l+d-x)]. \tag{2}$$

To proceed, solving Eq. (1) for the five regions we obtain:

$$\psi_1=\psi_F^{e+}+r_A\psi_F^{h-}+r\psi_F^{e-},$$

$$\psi_i=a_i\psi_S^{e+}+b_i\psi_S^{e-}+c_i\psi_S^{h+}+d_i\psi_S^{h-},$$

$$\psi_3=m\psi_I^{e+}+n\psi_I^{e-}+p\psi_I^{h+}+q\psi_I^{h-},$$

$$\psi_5=t\psi_F^{e+}+t'\psi_F^{h+}, \tag{3}$$

in which $r_A(r)$ and $a_i,b_i,c_i,d_i(i=2,4)$ and m,n,p,q and $t(t')$ are the Andreev (normal) reflection amplitudes in the left ferromagnetic region, the amplitudes of the electron-like and hole-like quasiparticles in the superconductor regions, the amplitudes of electrons and holes in the insulator region and the amplitude of EC(CAR) process, respectively. Furthermore, in Eq. (3) the wave functions in left ferromagnetic region are:

$$\psi_F^{e\pm}=(1,\pm A_{1e}e^{\pm i\theta_{1e}},0,0)^T e^{\pm ik_{1e}x},$$

and:

$$\psi_F^{h-}=(0,0,1,-A_{1h}e^{-i\theta_{1h}})^T e^{-ik_{1h}x},$$

where:

$$A_{1e(h)}=\frac{E+(-)E_f-m}{\sqrt{(E+(-)E_f)^2-m^2}},$$

and:

$$k_{1e(h)}=\sqrt{(E+(-)E_f)^2-m^2}\cos\theta_{1e(h)}/\hbar V_f.$$

From the conservation of the y component of the wave vector, which is held due to translational symmetry invariant along y axis, we have:

$$\theta_{1h}=\arcsin\left(\frac{\sqrt{(E+E_f)^2-m^2}}{\sqrt{(E-E_f)^2-m^2}}\sin\theta_{1e}\right),$$

and:

$$\theta_{Se(h)}=\arcsin\left(\frac{\sqrt{(E+E_f)^2-m^2}}{(E_f+V_0+(-)\sqrt{E^2-\Delta^2})}\sin\theta_{1e}\right).$$

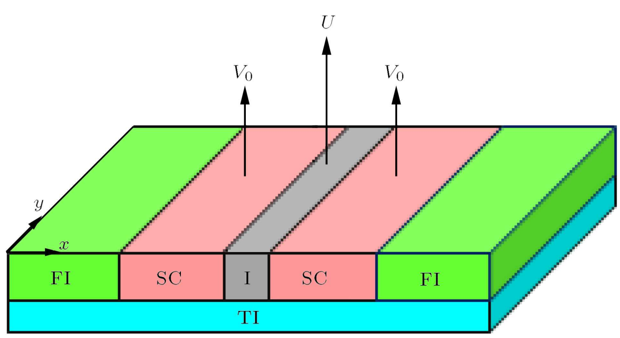

Figure 1. A schematic representation of the F/S/I/S/F structure deposited on the surface of a topological insulator.

The wave functions in the right ferromagnet region are:

$$\psi_F^{e+} = (1, A_{2e}e^{i\theta_{2e}}, 0, 0)^T e^{e^{ik_{2e}x}},$$

and:

$$\psi_F^{h+} = (0, 0, 1, A_{2h}e^{i\theta_{2h}})^T e^{ik_{2h}x},$$

where $k_{2e(h)} = k_{1e(h)}$ and $\theta_{1e(h)} = \theta_{2e(h)}$. For Parallel (P) magnetization configuration $A_{2e(h)} = A_{1e(h)}$ while, for Anti-Parallel (AP) magnetization configuration:

$$A_{2e(h)} = \frac{E + (-)E_f + m}{\sqrt{(E + (-)E_f)^2 - m^2}}.$$

Also in the superconductor regions the wave functions are given as:

$$\psi_S^{e\pm} = \left(1, \pm e^{\pm i\theta_{Se}}, e^{-i\beta}e^{-i\varphi}, \pm e^{\pm i\theta_{Se}}e^{-i\beta}e^{-i\varphi}\right)^T e^{\pm ik_{Se}x},$$

$$\psi_S^{h\pm} = \left(1, \pm e^{\pm i\theta_{Sh}}, e^{i\beta}e^{-i\varphi}, \pm e^{\pm i\theta_{Sh}}e^{i\beta}e^{-i\varphi}\right)^T e^{\pm ik_{Sh}x}, \quad (4)$$

in which:

$$k_{Se(h)} = \left(E_f + V_0 + (-)\sqrt{E^2 - \Delta^2}\right)\cos\theta_{Se(h)}/\hbar V_f,$$

and:

$$\beta = \cos^{-1}\left(\frac{E}{\Delta_0}\right), \text{ for } E < \Delta_0,$$

$$\beta = -i\cosh^{-1}\left(\frac{E}{\Delta_0}\right) \text{ for } E > \Delta_0.$$

In addition, in the insulator region the wave functions are:

$$\psi_I^{e\pm} = (1, \pm e^{\pm i\theta_{Ie}}, 0, 0)^T e^{\pm ik_{Ie}x},$$

and:

$$\psi_I^{h\pm} = (0, 0, 1, \mp e^{\mp i\theta_{Ih}})^T e^{\pm ik_{Ih}x}.$$

For a thin barrier, i.e. $d \to 0$ and $U \to \infty$, we set $\chi = \frac{Ud}{\hbar V_f}$ and $\theta_{Ie}, \theta_{Ih} \approx 0$, $k_{Ie}d, k_{Ih}d \approx \chi$. The incident angle to each region is determined from the conversation of the momentum components parallel to the interfaces. All the coefficients of the wave functions are calculated by the transfer matrix method from their continuity conditions on the boundaries. The NLC is defined as the conductance in the right ferromagnetic region when the superconducting and right ferromagnetic regions are grounded and a bias voltage V is applied to the left ferromagnetic region [2,11-13]. In the parallel (anti-parallel) case, it is given as $G_C^{\mathrm{P(AP)}} = G_{\mathrm{CAR}}^{\mathrm{P(AP)}} - G_{\mathrm{EC}}^{\mathrm{P(AP)}}$, where the EC and the CAR processes are as follows [2,13]:

$$T = \left|t^{\mathrm{P(AP)}}\right|^2, \qquad T' = \frac{k_{2h}}{k_{1e}}\left|t'^{\mathrm{P(AP)}}\right|^2,$$

$$G_{\mathrm{EC}}^{\mathrm{P(AP)}} = G_0 \int_{-\pi/2}^{\pi/2} d\theta_{1e}\cos\theta_{1e}\frac{\sqrt{(E+E_f)^2 - m^2}}{2|E+E_f|}T,$$

$$G_{\mathrm{CAR}}^{\mathrm{P(AP)}} = G_0 \int_{-\pi/2}^{\pi/2} d\theta_{1e}\cos\theta_{1e}\frac{\sqrt{(E+E_f)^2 - m^2}}{2|E+E_f|}T', \quad (5)$$

where $G_0 = \frac{2e^2}{\hbar}N(E+E_f)$ and $N(eV) = \frac{W}{\pi\hbar V_f}|E+E_f|$ are the densities of the states with W as the width of the junction.

3. Numerical results and discussion

Based on the formalism presented in the previous section, here we investigate the effect of the barrier strength on the NLC through the considered structure. In Figure 2, for the parameters $E_F = 6\Delta_0$, $V_0 = 500\Delta_0$, $E = eV = 0.05\Delta_0$, $l = 2\xi$ and $m = 5.9\Delta_0$, we illustrate the variations of NLC as a function of barrier strength. Here, $\xi = \hbar v_F/\Delta_0$ is the superconducting coherence length. As shown, for parallel (anti-parallel)

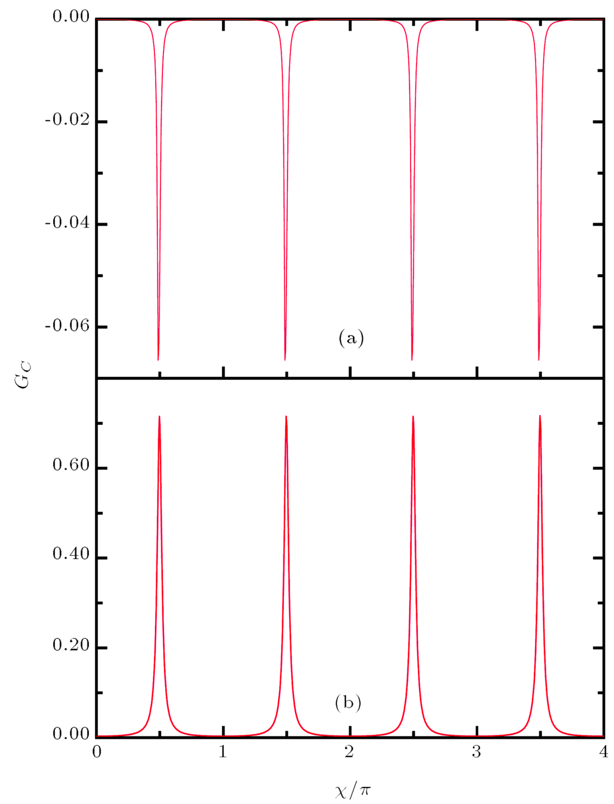

Figure 2. The nonlocal conductance G_c (in unit of G_0) as a function of barrier strength. (a) For parallel, and (b) for anti-parallel magnetization configuration.

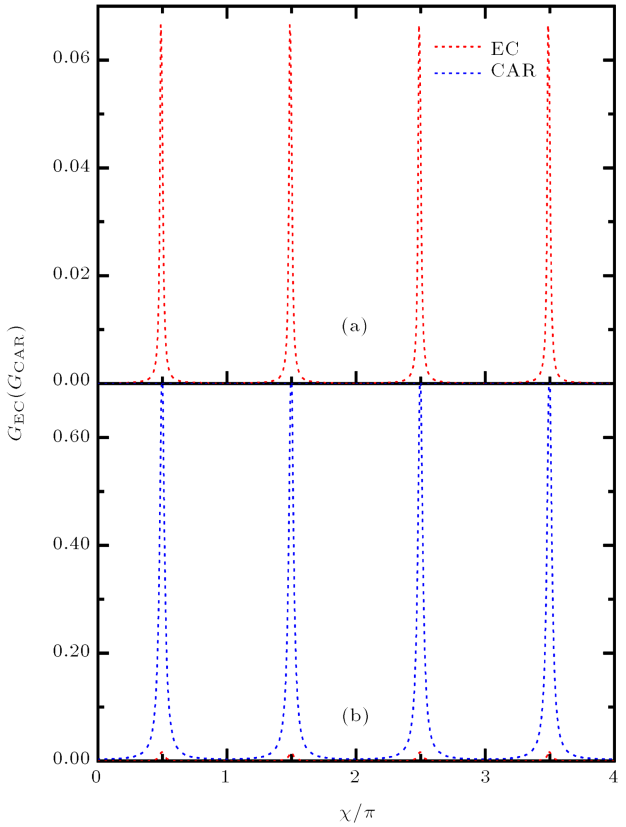

Figure 3. The nonlocal conductance for the EC and CAR processes (in unit of G_0) versus the barrier strength. (a) For parallel, and (b) for anti-parallel magnetization configuration.

configuration, G_C is negative (positive) which demonstrates that the EC (CAR) process dominates in the NLC process. In addition, as it is clear, the NLC shows a periodic behavior when the strength of the barrier is changed which is a consequence of the periodic variations of the EC and CAR conductances. For the parallel and anti-parallel configurations, the maxima of the NLC occur at $\chi = (n + 1/2)\pi$ with $n = 0, 1, 2, ...$, and for $\chi = n\pi$ the NLC is zeros, so that NLC shows a quantum on-off switching effect. Figure 3 shows the EC and CAR conductance as a function of barrier strength. From Figure 3(a), we see that EC process is favoured in parallel configuration of magnetization while Figure 3(b) shows that the CAR process is favoured in anti-parallel magnetization configuration. In fact, in the parallel (anti-parallel) magnetization configuration, the CAR (EC) conductance is zero. This can be understood as follows.

On the surface of the topological insulator, due to the fact that the magnetization acts as vector potential, and also due to the spin-momentum locking, the z-component of spin in left ferromagnetic region is parallel to $\hat{z}$ direction while for the right ferromagnetic region, there are two choices: it is parallel to $\hat{z}$ direction in parallel magnetization configuration, whereas it is parallel to $-\hat{z}$ direction in the case of anti-parallel magnetization configuration [12,14]. Thus, in parallel magnetization configuration, where the z-component of the spin in the left and right ferromagnetic regions is the same, the probability of the CAR process, which requires an electron from left ferromagnetic region forms a Cooper pair with another electron with opposite spin from right ferromagnetic region, is zero. On the other hand, in anti-parallel magnetization configuration, where the z-component of the spin in right ferromagnetic region is opposite to that of the left ferromagnetic region, the probability of the EC process, which consists of direct tunneling of an electron from left ferromagnetic region to the right ferromagnetic region, is zero.

From Figure 3, we can see also that for the parallel (anti-parallel) magnetization configuration, the EC (CAR) conductance shows an oscillatory behaviour with minima equal to zero at $\chi = n\pi$ leading to an on/off switching effect. In an anti-parallel magnetization configuration, the on-state conductance is merely due to CAR process, indicating that this device can be used as a solid state entangler. The oscillatory behaviour of EC and CAR conductance originates from the linear dispersion on the surface of the TI which reflects the fact that the wave functions of the quasi particles cannot be damped by any potential barrier. A similar oscillatory behaviour has also been observed in local conductance of topological insulator F/I/F/SC junction as a function of barrier strength of the insulator (I) layer [15,16].

4. Conclusions

In conclusion, we find that the G_C in topological insulator F/S/I/S/F junction, for both the parallel and anti-parallel magnetization configurations, exhibits an oscillatory behaviour with maxima at $\chi = (n + 1/2)\pi$ and minima equal to zero at $\chi = n\pi$, so that this junction presents a quantum on-off switching effect. The oscillatory behaviour of G_C basically stems from the oscillatory behaviour of the EC and CAR conductance. Furthermore, in anti-parallel magnetization configuration G_C is merely due to the CAR process, implying the potential use of this structure as a solid state entangler device appropriate for use in quantum information processing.

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