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100% screening economic order quantity model under shortage and delay in payment

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Abstract. For many years, the Economic Order Quantity (EOQ) model has been successfully applied to inventory management. This paper studies a multiproduct EOQ problem in which the defective items will be screened out by 100% screening process, and will be sold after the screening period. Delay in payment is permissible, though payment should be made during the grace period, and the warehouse capacity is limited. If not, there will be an additional penalty cost for late payment and the retailer will not be able to buy products at discount prices. All-units and incremental discounts are considered for the products which depend on order quantity, just like the permissible delay in payment. The Genetic Algorithm (GA) and the Particle Swarm Optimization (PSO) algorithm are used to solve the proposed model, and numerical examples are provided for better illustration.

1. Introduction

The basic economic order quantity model is expanded by researchers using different assumptions. Some of these assumptions seem to be more realistic and can be observed in real market environments. Retailers do not usually receive perfect goods and, conceivably, some defective items are found in their orders. The defect may be caused during the delivery process or by bad production. Porteus [1] studied the effect of defective items on an EOQ model in which the production process goes out of control considering a hypothesized probability. Wu and Ouyang [2] assumed the number of defective items as a random variable in a (Q,r,L) inventory model. They developed an algorithm procedure to obtain the optimal order quantity, the order point and the lead time. Shortages and permissible delays in payment are practical assumptions that help reach a tangible model. Economic order quantity under permissible delays in payment was studied by Goyal [3] for the first time. Huang [4] studied a partial delay in payment, wherein the retailer pays the purchase cost at the end of the grace period in cases where the order quantity is more than the minimum amount of quantity, which leads to complete delay in payment. Otherwise, a part of the payment must be made as the order is filled. All-units discount and incremental discount are the policies that suppliers use to encourage retailers to increase their order size. Benton and Park [5] overviewed different purchase discounts and Weng [6] studied the all-units discount and incremental discount in inventory models, which were subsequently mentioned by many researchers.

Salameh and Jaber [7] studied the EOQ model in which all the products are screened and defective ones are sold in a single batch after the screening process. (There was an error in their paper that was corrected by Barron [8].) Wee et al. [9] extended the model

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of Salameh and Jaber [7] and gained optimal order and backorder quantities when shortage is permissible and completely backordered. Eroglu and Ozdemir [10] developed an EOQ model with shortages and defective items that are categorized as imperfect quality and scrap items. Chang and Ho [11] revisited the model by Wee et al. [9] and used a renewal-reward theorem to obtain the expected profit per unit time. Kevin Hsu and Yu [12] considered a one-time only discount for Salameh and Jaber [7] model. They obtained the optimal order size, which is placed at a time when a price decrease is effective for three possible situations. Khan and Mehmood [13] studied an EOQ model considering errors in inspections and sales returns. In their model, the amount of returns was added to actual demands and was equal to perfect screened out items at a maximum to avoid shortage during the sales period. Hsu and Hsu [14] showed that there is an error in the model of Wee et al. [9], wherein the units that were backordered were shipped to customers before the screening process. They corrected the model and obtained the optimal order and backorder quantities using a renewal-reward theorem. This model was extended by Tai [15] considering two warehouses and using a multi screening process. Moreover, Hsu and Hsu [16] developed the model of Khan et al. [13], where the shortage is allowed and backordered. This paper further developed the model of Hsu and Hsu [14] by adding some new assumptions and considerations. As a result, it changed to a multi-product model. The mathematical model is later described in Sections 2 and 3. The genetic algorithm and the particle swarm optimization algorithm are used to solve the proposed model in Section 4, and these algorithms are then compared in the different examples in Section 5.

2. Notations and assumptions

The following notations and assumptions are used throughout this paper.

2.1. Notation

Q_i : Order size of product i;

D_i : Demand rate of product i;

x_i : Screening rate for product i;

A_i : Ordering cost for product i;

p_i : Average fraction of an order quantity for product i, that is defective in Q_i;

ϑ_i : Selling price per unit for product i;

V_i : Salvage value per defective item for product i, $V_i < c_i$;

d_i : Screening cost per unit for product i;

B_i : Maximum backordering quantity in units for product i;

b_i : Backordering cost per each unit of product per unit of time for product i;

π_i : Backordering cost per unit for product i;

h_i : Holding cost per each unit of product per unit of time for product i;

H : Length of planning horizon (in this paper it is considered as one year), $H = 1$;

n : Number of products;

f_i : Capacity of product i;

F : Total warehouse available space;

γ_i : Delay cost per unit of time for product i;

T_i : Cycle time for product i;

N_i : Number of cycle times for product i, $N_i = H/T_i$;

t_{1i} : Length of cycle time for product i, in which there is an inventory;

t_{2i} : Length of cycle time for product i in which there is no inventory;

t_{3i} : Time taken to fill B_i for product i;

t_i : Length of cycle time for screening product i;

k : Number of products that benefit all-units discount;

M_i : Permissible delay period for paying the purchasing cost of product i to the supplier;

C_i : Unit purchasing cost without discount of product i;

$C_{i,j}$: Purchasing cost per unit of product i at the jth discount point; $j = 1, 2, ..., m + 1$;

$M_{i,j}$: Permissible delay time for product i at the jth discount point;

u : An infinite number;

TS_i : Ordering cost per cycle for product i;

TB_i : Shortage cost per cycle for product i;

TH_i : Holding cost per cycle for product i;

TM_i : Delay cost of product i;

TP'_i : Purchasing cost per cycle for product i considering discount;

TP''_i : Purchasing cost per cycle for product i without discount;

TP_i : Purchasing cost per cycle for product i;

TR_i : Revenue per cycle for product i;

TPV : TotalNetProfit value per cycle.

$$s_i = \begin{cases} 1; & \text{If product } i \text{ receives discount } (TM_i = 0) \\ 0; & \text{Otherwise } (TM_i > 0) \end{cases}$$

Decision variables:

Q_i : Order quantity of product i;

B_i : Maximum shortage (backorder) level of product i.

2.2. Assumptions

1. Replenishment is instantaneous.
2. Shortage is allowed and will be backordered in the next period.
3. 100% screening process is used and screening rate is greater than demand rate ($x_i > D_i$).
4. Defective items are sold at price V_i, subsequently, the screening process is finished.
5. The supplier demands the cost of each product batch in just one payment, and, since the retailer pays for his current costs, like holding costs, during the cycle time, the purchasing cost occurs at the end of the selling period when all the goods are sold (when the inventory level reaches zero). Therefore, the payment may occur during or after the grace period and hinges on the cycle time.
6. There are all-units discount and incremental discount policies for the products.
7. Permissible delay in payment hinges on order quantity. If the payment occurs in the permissible time, the retailer benefits from discount prices. Otherwise, not only are discount prices not considered for him, he will be charged a fee for lateness, as a delay cost.

3. Mathematical model

Figure 1 depicts the inventory model in which the shortage is satisfied in each cycle at a rate of $x_i(1 - p_i) - D_i$, after that the time period t_{3i}, the shortage is completely backordered. According to Hsu and Hsu [13], $B_i + t_{3i}D_i = x_i(1 - p_i)t_{3i} = \frac{B_i x_i(1-p_i)}{x_i(1-p_i)-D_i}$ and t_{1i}, t_{2i}, t_{3i} are parts of the cycle with no shortage, the part of the cycle with shortage, and the time taken to fill B_i instantaneously, which can be shown as:

$$t_{1i} = \frac{Q_i(1 - p_i) - B_i}{D_i}, \tag{1}$$

$$t_{2i} = \frac{B_i}{D_i}, \tag{2}$$

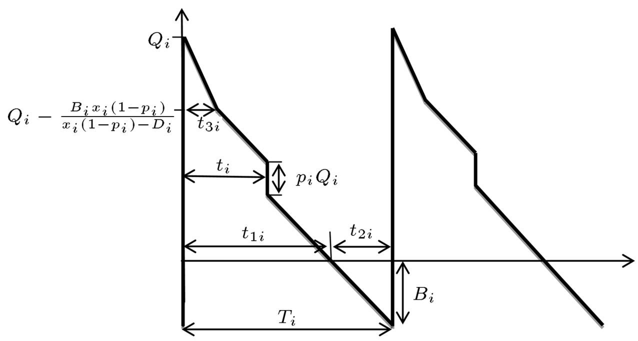

Figure 1. Behavior of the proposed inventory model.

Table 1. The amounts of M_i and C_i depend on the order quantity.

Q_i	M_i	C_i
$q_{i,0} < Q_i \leq q_{i,1}$	$M_{i,1}$	$C_{i,1}$
$q_{i,1} < Q_i \leq q_{i,2}$	$M_{i,2}$	$C_{i,2}$
...	...	...
$q_{i,m-1} < Q_i \leq q_{i,m}$	$M_{i,m}$	$C_{i,m}$
$q_{i,m} < Q_i$	$M_{i,m+1}$	$C_{i,m+1}$

$$t_{3i} = \frac{B_i}{x_i(1 - p_i) - D_i}. \tag{3}$$

Eqs. (4) and (5) calculate shortage cost and the holding cost per cycle:

$$TB_i = \frac{1}{2} b_i B_i^2 \left(\frac{1}{D_i} + \frac{1}{x_i(1 - p_i - \frac{D}{x_i})} \right) + B_i \pi_i, \tag{4}$$

$$\begin{aligned} TH_i =& h_i \Bigg(\frac{1}{2} \frac{Q_i B_i}{x_i} \left(\frac{(1 - p_i)}{(1 - p_i - \frac{D_i}{x_i})} \right) \\ &+ \frac{1}{2} \Bigg(\frac{Q_i^2 (1 - p_i)^2}{D_i} - \frac{Q_i B_i (1 - p_i)^2}{D_i (1 - p_i - \frac{D_i}{x_i})} \\ &- \frac{Q_i B_i (1 - p_i)}{D_i} + \frac{B_i^2 (1 - p_i)}{D_i (1 - p_i - \frac{D_i}{x_i})} \Bigg) + \frac{p_i Q_i^2}{x_i} \Bigg). \end{aligned} \tag{5}$$

Ordering cost per cycle is A_i and the price discount and permissible delay in payment depend on the order quantity. Table 1 shows the relationship between them. In this table:

$$q_{i0} = 0, \quad 0 < M_{i,1} < M_{i,2} < ... < M_{i,m+1},$$

and:

$$C_{i,1} > C_{i,2} > ... > C_{i,m+1}.$$

All the discount items for every product are the same. This situation may occur for many companies. For example, consider a company that produces TV sets in different models. The discount rate does not differ between TV set models and only depends on the price of the merchandise.

The retailer is not able to pay the supplier before selling all units in each period, and the payment may occur during or after the grace period. Therefore, there will be a delay cost if the inventory level reaches zero after the grace period, due to non-payment within the defined period and delay in payment. In this case, the retailer is not allowed to benefit from discount prices, and delay cost equals Eq. (6). When the inventory level reaches zero before finishing the grace period (all the products of type i are sold and the retailer can pay the purchasing cost before the grace period is finished), $TM_i = 0$ and the retailer can benefit from the discount price. Moreover, M_i and C_i can be gained from Table 1. They can be replaced by $\sum_{j=1}^{m+1} y_{i,j} M_{i,j}$, $\sum_{j=1}^{m+1} y_{i,j} C_{i,j}$ in which $y_{i,j}$ is a binary variable and $\sum_{j=1}^{m+1} y_{i,j} = 1$. Therefore, TM_i can be summarized to Eq. (7).

$$TM_i = \left(\frac{Q_i(1-p_i) - B_i}{D_i} - M_i \right) \gamma_i, \tag{6}$$

$$TM_i = \max\left\{0, \left(\frac{Q_i(1-p_i) - B_i}{D_i} - \sum_{j=1}^{m+1} y_{i,j} M_{i,j} \right) \gamma_i \right\}. \tag{7}$$

The first k products receive an all-units discount and the incremental discount is considered for the others. As mentioned before, the retailer can benefit from the discount only when the payment occurs within the grace period. Unit price can be obtained from Table 1. When $M_i < t_{1i}$, the retailer has to pay the purchasing cost with no discount:

$$TP''_i = Q_i C_{i,1}; \quad i = 1, 2, ..., n. \tag{8}$$

When $M_i \geq t_{1i}$, the retailer can use discount prices and the purchasing cost can be formulated as:

$$Tp'_i = \begin{cases} \sum\limits_{j=1}^{m+1} Q_i y_{i,j} C_{i,j}; & i = 1, 2, ..., k \\ \sum\limits_{j=2}^{m+1} \Big((Q_i - q_{i,j-1}) C_{i,j} & \\ \quad + \sum\limits_{f=1}^{j-1} (q_{i,j-f} - q_{i,j-f-1}) C_{i,j-f} \Big) y_{i,j} & \\ \quad + Q_i C_{i,1} y_{i,1}; & i = k+1, ..., n \end{cases} \tag{9}$$

Therefore, the purchasing cost per cycle can be obtained as:

$$TP_i = TP'_i s_i + TP''_i (1 - s_i); \quad i = 1, 2, ..., n.$$

The revenue per cycle is:

$$TR_i = (1 - p_i) Q_i \vartheta_i + p_i Q_i V_i; \quad i = 1, 2, ..., n.$$

The net profit value that the retailer earns per cycle is:

$$TPV = \sum_{i=1}^{n} \left[TR_i - (TS_i + TP_i + TM_i + TH_i + TB_i) \right]; \quad i = 1, 2, ..., n.$$

The objective is to maximize the net profit value per cycle, and the mathematical model becomes:

$$\text{Max } TC = \sum_{i=1}^{n} \Bigg[TR_i - \Bigg(A_i + \frac{1}{2} b_i B_i^2 \left(\frac{1}{D_i} + \frac{1}{x_i(1 - p_i - \frac{D_i}{x_i})} \right) + B_i \pi_i + TM_i + TH_i + TP'_i s_i + TP''_i (1 - s_i) \Bigg) \Bigg]. \tag{10}$$

S.t.:

$$TP'_i = \sum_{j=1}^{m+1} Q_i y_{i,j} C_{i,j}; \quad i = k+1, ..., n, \tag{11}$$

$$TP'_i = \sum_{j=2}^{m+1} \left((Q_i - q_{i,j-1}) C_{i,j} + \sum_{f=1}^{j-1} (q_{i,j-f} - q_{i,j-f-1}) C_{i,j-f} \right) y_{i,j} + Q_i C_{i,1} y_{i,1}; \quad i = k+1, ..., n, \tag{12}$$

$$TP''_i = Q_i C_{i,1}; \quad i = 1, 2, ..., n, \tag{13}$$

$$TR_i = (1 - p_i) Q_i \vartheta_i + p_i Q_i V_i; \quad i = 1, 2, ..., n, \tag{14}$$

$$TH_i = \frac{1}{2} h_i \left(\frac{Q_i B_i}{x_i} \left(\frac{(1-p_i)}{(1-p_i) - \frac{D_i}{x_i}} \right) + \frac{Q_i^2 (1-p_i)^2}{D_i} - \frac{Q_i B_i (1-p_i)^2}{D_i (1 - p_i - \frac{D_i}{x_i})} - \frac{Q_i B_i (1-p_i)}{D_i} + \frac{B_i^2 (1-p_i)}{D_i (1 - p_i - \frac{D_i}{x_i})} + \frac{p_i Q_i^2}{x_i} \right), \tag{15}$$

$$\sum_{j=0}^{m} q_{i,j} y_{i,j+1} \leq Q_i \leq \sum_{j=1}^{m} q_{i,j} y_{i,j} + u y_{i,m+1}; \quad i = 1, 2, ..., n, \tag{16}$$

$$\sum_{j=1}^{m+1} y_{i,j} = 1; \quad i = 1, 2, ..., n, \tag{17}$$

$$TM_i \geq \left(\frac{Q_i(1-p_i)-B_i}{D_i} - \sum_{j=1}^{m+1} y_{i,j} M_{i,j} \right) \gamma_i;$$

$$i=1,2,...,n, \tag{18}$$

$$TM_i \times s_i = 0; \quad i=1,2,...,n, \tag{19}$$

$$TM_i + s_i > 0; \quad i=1,2,...,n, \tag{20}$$

$$\sum_{i=1}^{n} Q_i f_i \leq F, \tag{21}$$

$$B_i \geq 0, \quad Q_i \geq 0, \quad x_{i,j} \geq 0, \quad y_{i,j} \geq 0, \quad s_i \geq 0,$$

$$TM_i > 0 \qquad i=1,2,...,n; \ j=1,2,...,m+1$$

Eqs. (11), (12) and (13): Calculation of the purchasing cost for an all-unit discount, an incremental discount and purchasing cost when $M_i \geq t_{1i}$:
Eq. (14): Calculation the revenue per cycle;
Eq. (15): Calculation the holding cost per cycle;
Eq. (16) and (17): Find the amounts of binary variable for defining M_i and C_i.
Eq. (18): $TM_i = \max\left\{0, \left(\frac{Q_i(1-p_i)-B_i}{D_i} - M_{i,j}\right)\gamma_i\right\}$ is replaced by:

$$TM_i \geq \left(\frac{Q_i(1-p_i)-B_i}{D_i} - \sum_{j=1}^{m+1} y_{i.j} M_{i,j} \right) \gamma_i,$$

and:

$$TM_i > 0.$$

Eqs. (19) and (20): These two constraints together define:

$$s_i = \begin{cases} 1; & \text{If product } i \text{ receives discount } (TM_i = 0) \\ 0; & \text{Otherwise } (TM_i > 0) \end{cases}$$

Eq. (21): The warehouse capacity is limited.

4. Solving algorithms

The mathematical model mentioned is a constrained nonlinear-programming model, and the number of constraints depends on the number of products. The greater the number of products, the more constraints are faced, which makes the problem more complicated, and more time is needed for solution. In this paper, the genetic algorithm and the Particle Swarm Optimization (PSO) algorithm are used to solve the proposed model and numerical examples are given to clarify their workability.

4.1. Genetic algorithm

The genetic algorithm was proposed by J. Holland [17]. This algorithm starts with a random population in which infeasible chromosomes are vanished and reproduced. Each chromosome has two rows; the first indicating order quantities and the second indicating shortages. Eq. (22) shows the proposed chromosome:

$$\text{Chromosome} = \begin{bmatrix} \mathbf{Q}_1 & \mathbf{Q}_2 & \dots & \mathbf{Q}_n \\ \mathbf{B}_1 & \mathbf{B}_2 & \dots & \mathbf{B}_n \end{bmatrix}. \tag{22}$$

Other populations are created via elitism, crossover and mutation. A specific number of generations are considered as the algorithm stopping criteria. To avoid producing infeasible siblings which contain columns in which the amount of shortage is greater than the order quantity, the crossover and mutation operations are performed on the columns of the chromosomes. Eqs. (23) to (28) show how these operations work. In Eqs. (23)-(25), the random-crossover-mask defines how to select columns from the parents. For example, if the second number is 0, then the son chromosome's second column is equal to the father chromosome's second column. In Eqs. (26)-(28), the mutation percentage is considered 0.1 and the columns, whose corresponding elements in the random matrix are less than 0.1, will be regenerated according to $B_i < Q_i$.

Eqs. (23)-(25) show the performance of the crossover operation:

$$\text{Parents}: \begin{bmatrix} \mathbf{Q}_1 & \mathbf{Q}_2 & \mathbf{Q}_3 \\ \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{B}_3 \end{bmatrix}, \quad \begin{bmatrix} \mathbf{Q'}_1 & \mathbf{Q'}_2 & \mathbf{Q'}_3 \\ \mathbf{B'}_1 & \mathbf{B'}_2 & \mathbf{B'}_3 \end{bmatrix}, \tag{23}$$

$$\text{Random-crossover-mask}: \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}, \tag{24}$$

$$\text{Siblings}: \begin{bmatrix} \mathbf{Q}_1 & \mathbf{Q'}_2 & \mathbf{Q}_3 \\ \mathbf{B}_1 & \mathbf{B'}_2 & \mathbf{B}_3 \end{bmatrix}, \quad \begin{bmatrix} \mathbf{Q'}_1 & \mathbf{Q}_2 & \mathbf{Q'}_3 \\ \mathbf{B'}_1 & \mathbf{B}_2 & \mathbf{B'}_3 \end{bmatrix}. \tag{25}$$

Eqs. (26)-(28) show the performance of the mutation operation:

$$\text{Selected chromosome}: \begin{bmatrix} \mathbf{Q}_1 & \mathbf{Q}_2 & \mathbf{Q}_3 \\ \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{B}_3 \end{bmatrix}, \tag{26}$$

$$\text{Random-matrix}: \begin{bmatrix} 0.8 & 0.03 & 0.66 \end{bmatrix}, \tag{27}$$

$$\text{Mutation-offspring}: \begin{bmatrix} \mathbf{Q}_1 & \mathbf{Q'}_2 & \mathbf{Q}_3 \\ \mathbf{B}_1 & \mathbf{B'}_2 & \mathbf{B}_3 \end{bmatrix}. \tag{28}$$

4.2. Particle swarm optimization

PSO was proposed by Kenedy and Eberhard [18] to find solutions for optimization problems. This algorithm is inspired by the social behavior of bird flocking or fish schooling. The first population of particles is generated randomly and each particle's velocity and position is updated during each iteration, considering the best solution so far reached by the particle (personal best) and the best current solution obtained so far by any particle (global best). A specific number of iterations are considered for the stopping criteria and the last global best will be the final solution of the algorithm.

5. Numerical examples

After tuning the two proposed algorithm's parameters, a small size example is solved by Lingo and the result is compared to GA and PSO algorithms. The proposed example parameters are shown in Tables 2 and 3. In this example, the maximum capacity is considered to be 1000; the first product benefits the all-units discount and other products benefit from incremental discounts. As shown in Table 4, GA and PSO do not reach the optimal solution, but GA's answer was closer to Lingo's answer and the running time of GA was less than the running time of PSO. On the other hand, the single optimal solution can be obtained by Lingo when the problem is small. The scale of the model mainly depends on the number of constraints, which increases with the number of products. Therefore, to solve large problems, GA and PSO algorithms are used. As shown in Table 5, Lingo had a longer running time than the heuristic algorithms when the number of the products became more than 10 (500 iterations are considered the stopping criteria for heuristic algorithms). Moreover, Lingo obtained a local optimum for examples with 35 and 40 products, and it could not gain an answer for examples with 45 and 50 products after 7200 seconds. Comparing heuristic algorithms, the computational time of GA is better than PSO in all examples. Moreover, GA gained a better objective for most of the examples and PSO worked better in examples with 20 and 35 products.

Table 2. Small example purchasing costs and permissible delay times.

Q_i	C_1	C_2	C_3	M_i
$0 < Q_i \leq 200$	99	96	80	0.1
$200 < Q_i \leq 400$	91	89	72	0.2
$400 < Q_i$	73	52	55	0.4

6. Conclusions

In this paper, an inventory model was studied considering defective items and their shortages, which are backordered. Screening rate is always greater than demand rate and all products are sold only after a screening process. There is a delay in payment,

Table 3. Small example parameters.

Product	D_i	p_i	h_i	A_i	b_i	π_i	γ_i	f_i	x_i	ϑ_i	V_i	d_i
1	1000	0.2	0.4	194	18	20	24	5	8200	222	112	10
2	2200	0.3	0.6	165	16	11	37	6	9400	235	122	6
3	1800	0.15	0.3	125	13	11	22	4	9000	242	130	8

Table 4. Results of small example.

Solving method	Q_1	Q_2	Q_3	B_1	B_2	B_3	Objective	Time (second)
Lingo	1.25	1.429	246.29	1	1	1	35878.93	1
GA	41.49	6.63	187.39	5.36	3.37	33.38	31082	27.961231
PSO	67.8342	19.6408	135.57	3.701	7.94	18.20	27740	32.798773

Table 5. Comparison of GA and PSO solutions.

Number of	GA		PSO		Lingo	
products	Objective	Time (second)	Objective	Time (second)	Objective	Time (second)
5	591790	29.693927	524060	34.470654	725045	6
10	1007200	56.297679	726480	58.977584	1383250	28
15	1812500	80.239558	1320800	93.566596	2455254	109
20	4662100	106.608653	5597400	115.505749	5708085	559
25	3438000	132.968956	2349400	143.164634	4236878	993
30	6893800	156.156919	6216800	173.915791	7245396	2214
35	7481900	178.252049	10481000	202.076659	7870036	3911
40	4943300	201.447994	3074200	238.791332	2870065	5380
45	7246000	231.502811	4482600	247.241498	...	7200
50	5972100	254.552539	3881500	281.836228	...	7200

which depends on the order quantity, and the retailer is able to benefit from discount prices only in cases where payment occurs during the grace period. After describing the mathematical model, a small example is solved by Lingo and the optimal solution is gained. As the number of products increases, the number of constraints and the scale of the problem will change. Lingo can only gain local optimum solutions after a longer running time. Therefore, for large scale problems, GA and PSO algorithms are used and compared to each other, considering 10 different problems. Numerical examples indicated that GA has a better performance for the proposed model. GA solved the examples in less time and achieved better solutions for most of them.

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