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A novel Pareto-based multi-objective vibration damping optimization algorithm to solve multi-objective optimization problems

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Abstract. This paper presents a Vibration Damping Optimization (VDO) algorithm to solve multi-objective optimization problems for the first time. To do this, fast non-dominated sorting and crowding distance concepts were used in order to find and manage the Pareto-optimal solution. The proposed VDO is validated using several examples taken from the literature. The results were compared with Multi-Objective Simulated Annealing (MOSA) and Non-dominated Sorting Genetic Algorithms (NSGA-II) presented as state-of-the-art in evolutionary multi-objective optimization algorithms. The results indicate that Multi-Objective VDO (MOVDO) gives better performance with a significant difference in terms of computational time, while NSGA-II is better in finding Pareto solutions. In other standard metrics, MOVDO is able to generate true and well-distributed Pareto optimal solutions and compete with NSGA-II and MOSA.

1. Introduction

Nowadays, most problems encountered by Decision Makers (DMs) in the real world are recognized to be multi-objective optimizations. In fact, the DM likes to pursue more than one target or consider more than one factor or measure. Unlike single optimization problems, which deal with only one global optimum value, there is a set of solutions, called the Pareto solution, in the area of multi-objective optimization [1].

Unlike hard computing schemes, which strive for exactness and full truth, soft computing deals with imprecision, uncertainty, partial truth, and approximation to achieve practicability, robustness and low solution cost. Components of soft computing include terms such as neural networks, fuzzy logic, Evolutionary Computation (EC), and ideas about probability (Bayesian network, Chaos theory, and perceptron). Among these terms, EC is a practicable and common method to solve the problems. Several evolutionary algorithms have been developed which combine rules and randomness mimicking natural phenomena. These phenomena include biological evolutionary processes; for example, evolutionary algorithms [2], Genetic Algorithms (GA) [3,4], animal behavior [5-7], physical annealing processes [8], and the musical process of searching for a perfect state of harmony [9,10]. Many researchers have studied these meta-heuristics to solve various optimization problems recently.

Classical optimization methods suggest converting the multi-objective optimization problem to a single-objective one by emphasizing one particular Pareto-optimal solution at a time, such as classical weighted sum [11], desirability function [12], and Lp-metric [13]. On the other hand, such a method can

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be utilized to find multiple solutions. Over the past decades, a number of Multi-Objective Evolutionary Algorithms (MOEAs) have been suggested [14-18]. It is worth noting that the main reason for the popularity of evolutionary algorithms for solving multi-objective optimization is their population-based nature and their ability to find multiple optima simultaneously. The Non-dominated Sorting Genetic Algorithm (NSGA-II) introduced by Deb et al. [19] is an MOEA that has been applied to find Pareto front solutions in different fields of study, such as facility location problems [20], scheduling problems [21] and inventory control problems [22]. Moreover, in addition to the earlier aggregating approaches of Multi-Objective Simulated Annealing (MOSA), there have been a few methods that incorporate the concept of Pareto dominance. Some of these methods are proposed in [23,24], which use Pareto-domination based acceptance criterion in MOSA. A good review of several MOSA algorithms and their comparative performance analysis can be found in [25]. Several researchers developed multi-objective algorithms and justified them by comparing with NSGA-II and MOSA. Rahmati et al. [10] developed a multi-objective harmony search algorithm and compare it with NSGA-II and a non-dominated ranking genetic algorithm (NRGA). Sadeghi et al. [26] solved their bi-objective model with NSGA-II and NRGA. Bandyopadhyay et al. [27] developed a special type of MOSA and demonstrated its performance in contrast with NSGA-II.

Mehdizadeh and Tavakkoli-Moghaddam [28] proposed a new meta-heuristic optimization algorithm, namely, Vibration Damping Optimization (VDO), which is based on the concept of vibration damping in mechanical vibration. This algorithm simulates the vibration phenomenon. They first utilized the VDO algorithm to solve the parallel machine scheduling problem. To demonstrate the performance of VDO, they applied GA. They showed that when problems are small, the proposed VDO and GA have the same results. But, with increasing problem sizes, the VDO algorithm finds a better solution in terms of computational time and average objective function values. Mehdizadeh et al. [29] proposed a hybrid VDO algorithm to solve the multi-facility stochastic-fuzzy capacitated location allocation problem. Following this, Mousavi et al. [30] developed a special type of VDO algorithm to solve a capacitated multi-facility location–allocation problem with probabilistic customer locations and demands. They formulated their mathematical model in the frameworks of the Expected Value Model (EVM) and the Chance-Constrained Programming (CCP) based on two different distance measures. They demonstrated the performance of their proposed algorithm with GA. Based on their results analysis in 16 test problems, it was shown that there was no statistical significant difference between the performance of the two algorithms; it seems that not only does VDO perform better than GA for the CCP model with Euclidean distance, but also has better performance for both CCP and EVM models with squared Euclidean distance.

In this paper, the VDO algorithm is extended using Fast Non-Dominated Sorting (FNDS) and ranking procedures to find Pareto-optimal solutions for multi-objective optimization problems with conflicting objectives. Thus, this paper proposes the first proposal of applying multi-objective versions of the VDO algorithm in the literature. In fact, FNDS and Crowding Distance (CD) have been used to find and manage the Pareto-optimal front. The proposed Multi-Objective VDO (MOVDO) algorithm was tested on multi-objective facility location problems with competing objectives presented in [13]. To demonstrate the performance of the MOVDO, two well-developed MOEAs, including NSGA-II and MOSA, were applied.

The remainder of this paper is as follows. Section 2 provides some main concepts and definitions of multi-objective optimization algorithms. In Section 3, the first proposal of the multi-objective version of the VDO algorithm is illustrated in detail. In Section 4, the results are analyzed and discussed by graphical and statistical comparisons. Finally, Section 5 presents the conclusions.

2. Concepts and definitions

A mono-objective optimization algorithm will be terminated upon to obtain an optimal solution, yet it is unlikely to find a single solution for a multi-objective problem. Since objectives are contradictory, we generally find a set of solutions. In order to clarify the point, some basic multi-objective concepts are required to be reviewed [31]. Consider a multi-objective model with a set of conflict objectives as follows:

$$f(\vec{x}) = [f_1(\vec{x}), \cdots, f_m(\vec{x})],$$

S.t.:

$$g_i(\vec{x}) \leq 0, \qquad i = 1, 2, \cdots, I, \quad \vec{x} \in X, \tag{1}$$

where $\vec{x}$ denotes m-dimensional vectors that can get real, integer, or even Boolean values, and X is the feasible region. Then, for a minimization model, we say solution $\vec{a}$ dominates solution $\vec{b}(\vec{a}, \vec{b} \in X)$ if:

(I) $f_i(\vec{a}) \leq f_i\left(\vec{b}\right), \forall i = 1, 2, \cdots, m$

(II) $\exists i \in \{1, 2, \cdots, m\} : f_i(\vec{a}) < f_i\left(\vec{b}\right).$

Moreover, a set of solutions that cannot dominate one other is called the Pareto solutions set or Pareto front. The main goal of multi-objective problems are stated as appropriate convergence and appropriate diversity, which forms a good Pareto front. Accordingly, Pareto-based algorithms aim to achieve the Pareto-optimal front during the evolution process. The Pareto-optimal front is called to the front of the last iteration of the algorithms. This front is expected to have the most convergence and the highest diversity [17]. In fact, two major problems should be addressed when an evolutionary algorithm is applied to multi-objective optimization. The first is how to accomplish fitness assignment and selection, respectively, to guide the search towards the Pareto-optimal front; and the second is how to maintain a diverse population to prevent premature convergence and achieve a well-distributed trade-off front [1].

3. Proposed methodology

In this section, we first introduce the multi-objective version of the Vibration Damping Optimization (VDO) algorithm to solve the multi-objective optimization problem. Then, to demonstrate the performance of MOVDO, two well-developed Pareto-based MOEAs, including NSGA-II and MOSA, are employed.

3.1. Multi-Objective Vibration Damping Optimization (MOVDO)

In the vibration theory, the concept of vibration can be considered to be the oscillation. If the damping is small, it has very little influence on the natural frequencies of the system, and, hence, the calculation for the natural frequencies is nearly made on the basis of no damping [32]. In the VDO algorithm, at high amplitudes, the scope of a solution is bigger and the probability of obtaining a new solution is further away. Therefore, when the amplitude is reduced, the probability of obtaining a new solution decreases, and then the system stops from the amplitude state [28-30].

In the analogy between an optimization problem and the vibration damping process,

- The states of the oscillation system represent feasible solutions of the optimization problem;
- The energies of the states correspond to the objective function value computed at those solutions;
- The minimum energy state corresponds to the optimal solution to the problem, and rapid quenching can be viewed as local optimization.

The VDO algorithm starts working by generating random solutions in search space. Then, the algorithm parameters, including initial amplitude (A_0), max of iteration at each amplitude (L), damping coefficient (γ) and standard deviation (σ), are initialized. Then, the solutions are evaluated by means of the Objective Function Value (OFV). The algorithm contains two main loops. The first loop generates a solution randomly and then, using a neighborhood structure, new solutions are obtained and the best is chosen. However, similar to the SA algorithm, the solution with a worse OFV can be selected, with regards to the Rayleigh distribution function. At given amplitude, it states the probability of an oscillatory system determined by the Rayleigh probability distribution function. In fact, the new solution is accepted if Δ=OFV (new solution)-OFV (current solution) < 0. Besides, if $\Delta > 0$, a random number (r) is selected between (0, 1). When r is less than or equal to this function, the worse solution is chosen. This approach causes escape from local and convergence to global optimum:

$$r < 1 - \exp\left(-\frac{A^2}{2\sigma^2}\right). \tag{2}$$

The second loop adjusts the amplitude, which is used for reducing amplitude at each iteration. The algorithm is stopped when the stopping criterion is met:

$$A_t = A_0 \exp\left(-\frac{\gamma t}{2}\right). \tag{3}$$

After brief illustration of the VDO algorithm, we introduce the first proposal of applying a multi-objective version of the VDO algorithm, called MOVDO, to solve and manage Pareto-optimal solutions. To do so, we apply two main concepts of multi-objective meta-heuristics, namely, Fast Non-Dominated Sorting (FNDS) and Crowding Distance (CD), to compare the solutions. In FNDS, R initial populations are compared and sorted. To do this, all chromosomes in the first non-dominated front are first found. Assuming that all objective functions are of a minimization type, the chromosomes are chosen using the concept of domination, in which solution x_i is said to dominate solution x_j. If $\forall o \in \{1,2\}$, we have $Z_o(x_i) \leq Z_o(x_j)$ and $\exists o \in \{1,2\}$, such that $Z_o(x_i) < Z_o(x_j)$. In this case, we say x_i is the non-dominated solution within the solution set, $\{x_i, x_j\}$. Otherwise, it is not. Then, in order to find the chromosomes in the next non-dominated front, the solutions of the previous fronts are disregarded temporarily. This procedure is repeated until all solutions are set into fronts.

After sorting the populations, a CD measure is defined to evaluate solution fronts of populations in terms of the relative density of individual solutions [19]. To do this, consider Z and f_k; $k = 1, 2, \cdots, M$ as the number of non-dominated solutions in a particular front (F) and the objective functions, respectively. Besides, let d_i and d_j be the value of the CD on the solutions, i and j, respectively. Then, the CD is obtained using the following steps:

(I) Set $d_i = 0$ for $i = 1, 2, \cdots, Z$;

(II) Sort all objective functions, f_k; $k = 1, 2, \cdots, M$, in ascending order;

(III) The CD for end solutions in each front (d_1 and d_Z) are $d_1 = d_Z \rightarrow \infty$;

(IV) The crowding distance for d_j; $j = 2, 3, \cdots, (Z-1)$ are $d_j = d_j + (f_{k_{j+1}} - f_{k_{j-1}})$.

To select individuals of the next generation, the crowded tournament selection operator, "$\succ$", is applied [18]. In order to do that, the following steps are required to be carried out:

Step 1: Choose n individuals in the population randomly.

Step 2: The non-dominated rank of each individual should be obtained and the CD of the solutions having equal non-dominated rank is calculated.

Step 3: The solutions with the least rank are those selected. Moreover, if more than one individual share the least rank, the individual with the highest CD should be selected.

In other words, the comparison criterion of MOVDO algorithm's solutions can be written as: If $r_x < r_y$ or ($r_x = r_y$ and $d_x > d_y$), then $x \succ y$, where r_x and r_y are the ranks, and d_x and d_y are the CDs. In this paper, a polynomial neighborhood structure for the selected chromosome is performed.

After operating the aforementioned concepts and operators, the parents and offspring population should be combined to ensure the elitism. Since the combined population size is naturally greater than the original population size, N, once more, non-domination sorting is performed. In fact, chromosomes with higher ranks are selected and added to the population until the population size becomes N. The last front also consists of the population based on the crowding distance. The algorithm stops when a predetermined number of iterations (or any stopping criteria) is reached.

Figure 1 illustrates the evolution process of the proposed MOVDO schematically. The process is started by initializing the initial population of the solution vectors, P_j. Then, the new operators are implemented on P_j to create a new population, Q_j. The combination of P_j and Q_j creates R_j for keeping elitism in the algorithm. In this step, vectors of R_j are sorted in several fronts based on FNDS and CD. Using the proposed selection method, a population of the next iteration, P_{j+1}, is chosen to have a predetermined size.

It is worth mentioning that by using Pareto dominance solutions, it becomes a computationally efficient algorithm, implementing the idea of a selection method based on classes of dominance of all the solutions. In order to clarify the trend of the proposed algorithm, we represent a pseudo code of MOVDO, as shown in Figure 2.

To demonstrate the performance of the proposed MOVDO, two Pareto-based MOEAs, including NSGA-II and MOSA, are applied. The main difference between NSGA-II and MOSA in comparison with MOVDO is the evolution process of the algorithms from P_t to Q_t. Furthermore, NSGA-II and MOSA are different in their selection strategies. NSGA-II uses a binary tournament selection and MOSA uses the roulette wheel selection strategy. Accordingly, after generating or modifying populations by means of single-objective operators of the algorithms (e.g., GA, SA or VDO), the population is dealt in a multi-objective way in a similar fashion in all algorithms. Besides, to minimize the impact of using different operators on the performance comparison process of the algorithms, operators are designed identically. To do so, the neighborhood structure of the proposed MOVDO is designed similar to the mutation operator of NSGA-II and the neighborhood structure of MOSA. Moreover, in NSGA-II, the crossover operator is also designed similarly using a uniform crossover operator [33]. A flow chart of the proposed NSGA-II is depicted in Figure 3. Moreover, to clarify the trend of the proposed MOSA, Figure 4 represents a pseudo code of this algorithm.

In the next section, we analyze the results and

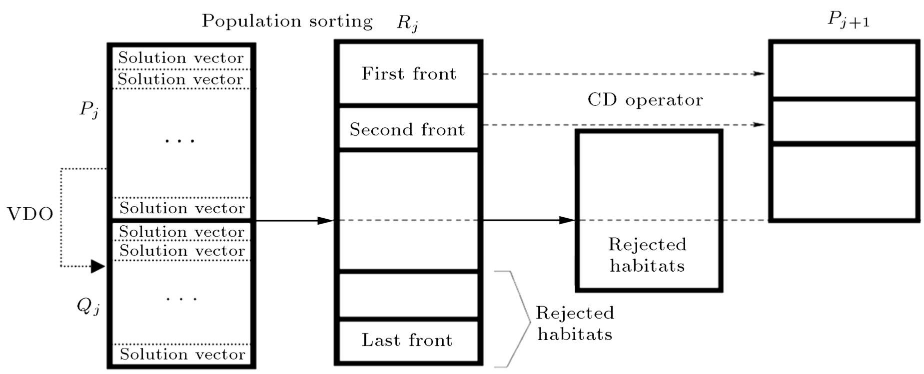

Figure 1. Evolution process of the proposed MOVDO.

Begin
Input $nPop$ (Population number, γ (damping coefficient); and σ (*Rayleigh* distribution constant)
Initialize (X; A; L and t, $t = 1$);
Evaluate solutions
Perform non-dominate sorting and calculate ranks
Calculate Crowding Distance (CD)
Sort population according to ranks and CDs
For $j = 1 : nPop$
P_j=population
For $i = 1 : L$
Y=PERTURB(X); {Generate new neighborhood solution}
$\Delta = E(Y) - E(X)$;
If $\Delta \leq 0$ or $(1 - e^{-A^2/2\sigma^2} > Random(0, 1))$
Then $X = Y$; {Accept the movement if dominates final Pareto solution}
End if
Update (A *and* t, $A = A_0 e^{-\gamma t/2}$, $t = t + 1$)
Until (Stop-Criterion)
End for
Q_j=new population
$R_j = P_j \cup Q_j$
Perform non-dominate sorting on R_j and calculate ranks
Calculate Crowding Distance (CD) of R_j
Sort population according to ranks and CDs on R_j
Create P_{j+1} as size as population size (population=P_{j+1})
End for
End

Figure 2. Pseudo code of MOVDO.

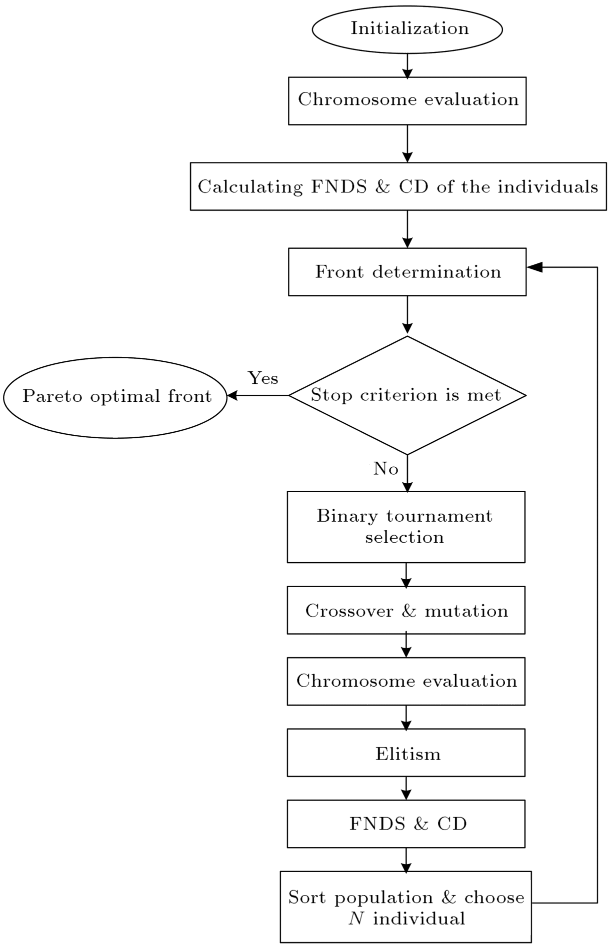

Figure 3. Flowchart of NSGA-II.

demonstrate the performance of the proposed MOVDO in the area of multi-objective optimization problems.

4. Result analysis and comparisons

To evaluate the performances of the proposed MOVDO, five standard metrics of multi-objective algorithms are applied as follows:

- Diversity: Measures the extension of the Pareto front, in which bigger value is better [34].
- Spacing: measures the standard deviation of the distances among solutions of the Pareto front, in which smaller value is better [35].
- Mean Ideal Distance (MID): Measures the convergence rate of Pareto fronts to a certain point (0, 0), in which smaller value is better [34].
- Number Of found Solutions (NOS): Counts the number of Pareto solutions in the Pareto optimal front in which bigger value is better [35].
- The computational (CPU) time of running the algorithms to reach near optimum solutions.

As mentioned before, the proposed multi-objective algorithm is applied to solve multi objective facility location problems in the literature [13]. The experiments are implemented on 20 test problems. Furthermore, to eliminate uncertainties of the solutions obtained, each problem is used three times under different random environments. Then, the averages of

```
Parameter setting: T_f, T_0, nGen, β, frontmax
Initialization: Generate initial solutions
Evaluation: Evaluate initial solutions

Perform non-dominate sorting and calculate ranks
Calculate Crowding Distance (CD)
Sort population according to ranks and CDs

P_t=population
For it=1: num.it

    if T < T_f
    break
    end

    for i = 1: popsize
        S_t(i)=perform neighborhood structure on the
        solution i of the population
    end

    Perform non-dominate sorting and calculate ranks (S_t)
    Calculate Crowding Distance (CD) (S_t)
    Sort population according to ranks and CDs (S_t)

    for i = 1: popsize
      if~Dominates (P_t(i), S_t(i))
         Q_t(i) = S_t(i)
      else
        Δ=Cost P_t(i)-Cost S_t(i)
        P = exp(−Δ/T(it))
        if rand < p
              Q_t(i) = S_t(i)
        end
      end
    end

    R_t = P_t ∪ Q_t
    Perform non-dominate sorting and calculate ranks (R_k)
    Calculate Crowding Distance (CD) (R_t)
    Sort population according to ranks and CDs (R_t)

    if size(Rt)>frontmax
      Pt=Select frontmax number of the solution
      non-dominate sorting and calculate ranks (Pt)
      Calculate crowding distance (Pt)
    else
      Pt = Rt
    end
Update T
End
```

Figure 4. Pseudo code of MOSA.

these three runs are treated as the ultimate responses. Then, we compare the proposed MOVDO algorithm with MOSA and NSGA-II, as the most applicable Pareto-based MOEAs in the literature, to demonstrate the performance of the proposed algorithm in solving multi-objective optimization problems. It should be mentioned that the input parameters of the algorithms are reported in Table 1.

To evaluate the performance of the proposed MOVDO, Table 2 reports the multi-objective metric amounts on the 20 test problems, in which "NAS" shows that the algorithm cannot find the Pareto front in the reported time. It is noted that MATLAB software [36] has been used to code the proposed meta-heuristic algorithms, and the programs have been executed on a 2 GHz laptop with eight GB RAM.

Table 1. Values of algorithms parameters along with their tuning procedure.

Multi-objective algorithms	Algorithm parameters	Optimum amount	Selected policy
NSGA-II	$nPop$	25	RSM [13]
	P_c	0.6	
	P_m	0.4	
	$Ngen$	100	
MOSA	T_0	500	RSM [13]
	$Popsize$	5	
	$Ngen$	500	
	β	0.99	
MOVDO	$nPop$	5	Taguchi [30]
	A_0	6	
	L	40	
	σ	1.5	
	γ	0.05	

The algorithms are statistically compared based on the properties of their obtained solutions via the analysis of variance (ANOVA) test. These outputs are reported in Tables 3 to 7 in terms of defined metrics. In these tables, the abbreviation of first row show Degrees of Freedom (DF), Sum of Squares (SS), Mean Square (MS), F-value test (F), P-value (P). In order to clarify our statistical results, box-plots are represented in Figures 5 and 6. Moreover, graphical comparisons of all metrics on 20 test problems are shown in Figure 7.

Based on the statistical outputs in Tables 3 and 4, along with Figure 5, NSGA-II shows better performances, in terms of NOS, while MOVDO has better performance in terms of CPU time. Moreover, Tables 4 to 7, along with Figure 6, show the comparability of MOVDO, in comparison with NSGA-II and MOSA, on MID, spacing, and diversity metrics in which the algorithms have no significant differences and, statistically, work the same. It should be mentioned that this conclusion is confirmed at 95% confidence level. Based on the outputs in Table 2, with increasing the size of problems, in test problems 19 and 20 and in test problem 20, NSGA-II and MOSA cannot find the Pareto front, respectively. However, in these large sizes, MOVDO can find the Pareto front. The MOVDO algorithm performs better in terms of CPUT metric. These features conclude the robustness of the proposed MOVDO in large-sized problems in the area of multi-objective optimization problems.

To increase the readability of the proposed MOVDO, Figure 8 represents the non-dominated solutions of a single run of the proposed MOVDO algorithm in the initial and final iterations in the left and right sides, respectively. These two figures indicate the

Table 2. Multi-objective metrics computed for three proposed MOEAs.

Problem no.	Time			NOS			MID			Spacing			Diversity		
	NSGA-II	MOSA	MOVDO	NSGA-II	MOSA	MOVDO	NSGA-II	MOSA	MOVDO	NSGA-II	MOSA	MOVDO	NSGA-II	MOSA	MOVDO
1	17.76	16.43	11.33	24	6	5	2.40E+07	2.46E+07	2.14E+07	2.55E+08	3.46E+07	1.78E+07	2.15E+08	2.20E+08	1.34E+07
2	32.67	25.23	13.11	20	4	8	4.57E+08	2.89E+08	3.21E+08	3.89E+08	4.52E+08	4.17E+05	1.89E+09	2.74E+09	2.38E+07
3	35.98	22.13	18.73	24	5	6	5.69E+08	5.91E+09	4.32E+09	3.94E+08	8.98E+09	8.32E+06	1.98E+09	1.78E+09	5.62E+07
4	33.57	29.12	18.31	21	6	5	2.47E+08	2.20E+08	1.43E+09	2.76E+09	2.40E+09	3.17E+06	4.52E+07	1.57E+10	2.62E+07
5	33.94	29.91	17.13	23	4	6	3.99E+08	7.55E+09	8.91E+09	2.47E+08	2.98E+10	2.36E+06	3.50E+09	5.85E+09	8.18E+08
6	31.83	28.12	19.84	22	5	6	2.89E+08	4.87E+09	5.32E+09	2.47E+08	2.19E+11	4.69E+07	1.57E+09	8.92E+10	2.35E+09
7	44.87	31.13	22.90	23	6	7	3.98E+08	8.77E+09	7.40E+09	2.41E+11	3.18E+11	4.72E+06	5.19E+10	8.92E+09	2.19E+09
8	40.90	32.24	22.73	25	NAS	6	6.69E+09	NAS	5.49E+09	2.93E+10	NAS	5.84E+06	1.87E+11	NAS	9.27E+08
9	44.23	29.44	29.82	25	7	7	1.29E+08	5.70E+08	4.32E+08	6.78E+08	8.80E+07	6.22E+06	2.35E+08	2.90E+09	1.23E+09
10	48.98	33.92	27.61	24	5	8	3.71E+08	3.85E+08	2.25E+08	3.43E+09	9.46E+06	2.68E+07	2.32E+08	2.40E+08	2.16E+09
11	59.47	39.23	29.28	24	4	8	3.28E+08	1.34E+10	2.49E+10	3.39E+08	7.96E+09	3.62E+07	1.84E+08	2.18E+10	2.80E+08
12	61.11	44.23	25.88	22	5	6	3.29E+08	2.85E+09	2.82E+10	5.48E+08	1.49E+10	2.34E+07	1.24E+09	5.72E+10	2.71E+08
13	75.12	55.32	30.91	21	4	8	7.80E+08	3.49E+10	2.39E+10	2.39E+08	2.98E+11	1.32E+07	1.94E+08	4.18E+10	4.18E+09
14	77.10	51.13	31.99	22	6	6	1.99E+09	2.26E+09	2.46E+09	6.88E+08	1.87E+08	1.48E+07	2.20E+10	1.87E+09	5.20E+10
15	98.13	77.22	39.51	25	6	7	3.66E+09	6.46E+09	5.88E+09	6.49E+10	3.42E+09	1.44E+07	2.99E+10	2.72E+10	4.62E+10
16	89.49	73.21	37.91	25	7	8	2.87E+09	2.86E+09	2.45E+09	1.89E+09	7.69E+07	5.88E+07	2.87E+09	1.48E+09	8.92E+09
17	92.13	89.12	45.60	25	5	7	5.06E+09	4.88E+09	4.38E+10	2.01E+10	4.67E+08	2.61E+09	2.43E+10	2.72E+10	1.63E+10
18	99.71	99.74	62.12	19	4	8	5.08E+10	5.68E+09	5.37E+09	1.90E+10	6.54E+09	2.70E+10	1.86E+11	1.90E+10	2.36E+11
19	165.89	101.88	77.19	NAS	5	8	NAS	7.64E+09	8.54E+09	NAS	5.87E+09	1.57E+10	NAS	1.43E+10	1.96E+10
20	264.42	232.83	123.88	NAS	NAS	7	NAS	NAS	3.42E+10	NAS	NAS	2.92E+10	NAS	NAS	1.26E+11

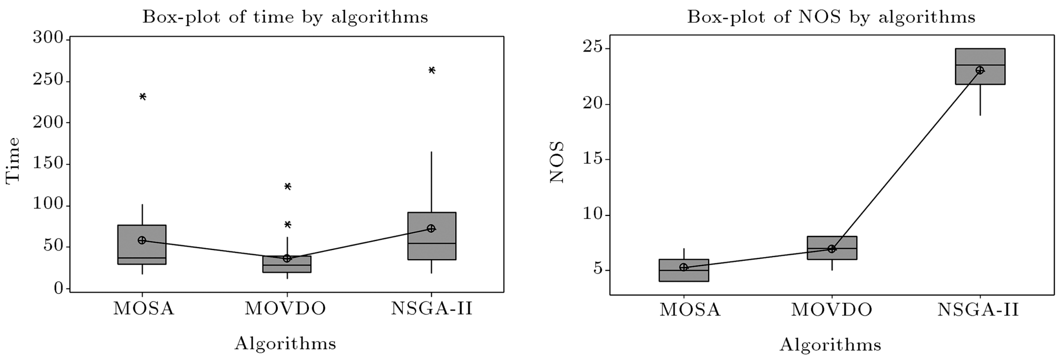

Figure 5. Box-plots of metrics with significant difference.

Table 3. Analysis of variance for the time metric.

Source	DF	SS	MS	*F*	*P*
Algorithms	2	13887	6944	3.28	0.045
Error	57	120615	2116		
Total	59	134502			

Table 4. Analysis of variance for the NOS metric.

Source	DF	SS	MS	*F*	*P*
Algorithms	2	3522.32	1761.16	955.77	0.000
Error	53	97.66	1.84		
Total	55	3619.98			

Table 5. Analysis of variance for the MID metric.

Source	DF	SS	MS	*F*	*P*
Algorithms	2	4.27263E+20	2.13631E+20	1.71	0.191
Error	53	6.62076E+21	1.24920E+20		
Total	55	7.04802E+21			

Table 6. Analysis of variance for the spacing metric.

Source	DF	SS	MS	*F*	*P*
Algorithms	2	2.13294E+22	1.06647E+22	2.26	0.114
Error	53	2.49807E+23	4.71334E+21		
Total	55	2.71136E+23			

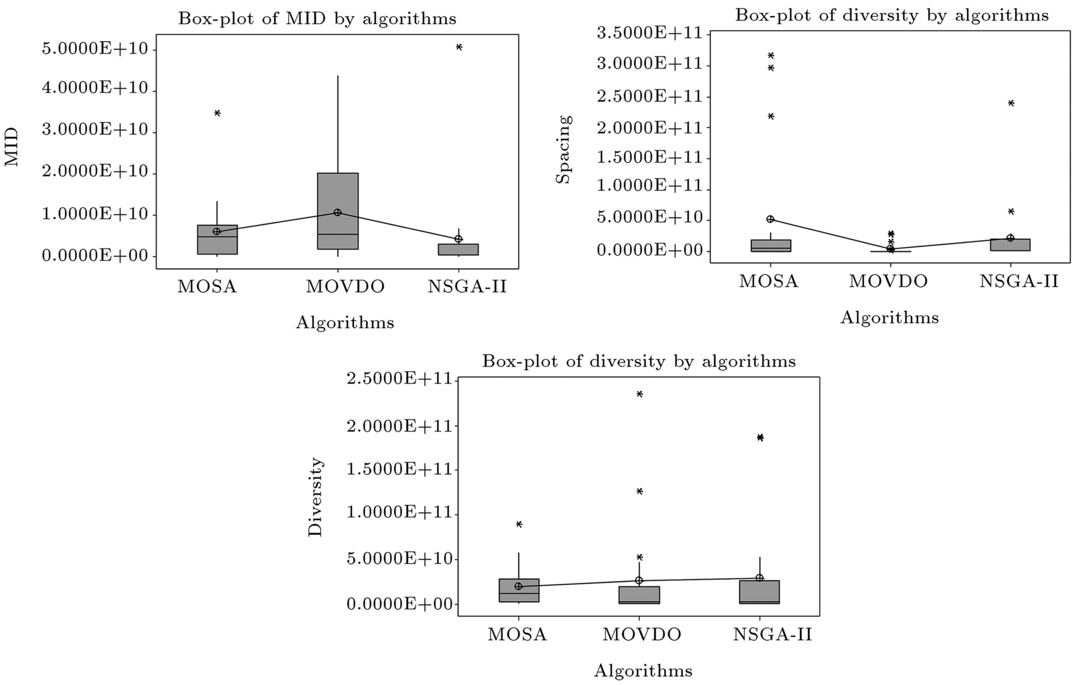

Figure 6. Comparability of MOVDO in comparison with NSGA-II and MOSA on MID, spacing, and diversity metrics.

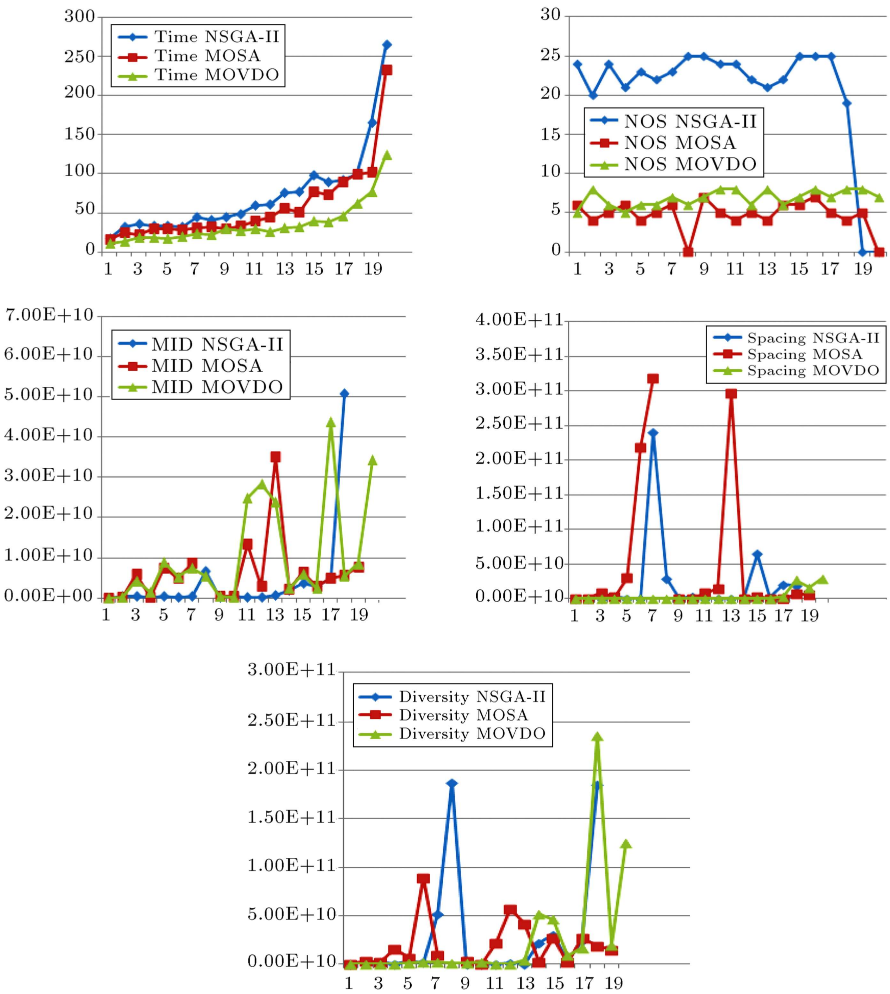

Figure 7. Graphical comparisons of metrics on 20 test problems.

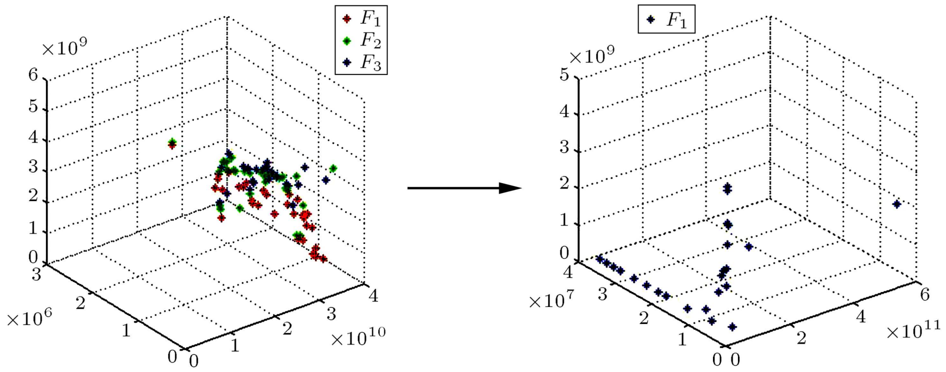

Figure 8. Pareto solutions obtained by the proposed MOVDO.

Table 7. Analysis of variance for the diversity metric.

Source	DF	SS	MS	*F*	*P*
Algorithms	2	9.23360E+20	4.61680E+20	0.18	0.832
Error	53	1.32671E+23	2.50322E+21		
Total	55	1.33594E+23			

intelligence of the proposed algorithm for solving multi-objective Pareto-based meta-heuristic algorithms.

5. Conclusions

In this paper, a Vibration Damping Optimization (VDO)-based multi-objective meta-heuristic algorithm has been presented. In this respect, the concept of domination has been used in solving the multi-objective optimization problems. In the Multi-Objective Vibration Damping Optimization (MOVDO), two main concepts (i.e., Fast Non-Dominated Sorting (FNDS) and Crowding Distance (CD)) have been considered in order to introduce the proposed MOVDO. In order to demonstrate the applicability of the proposed MOVDO, multi-objective facility location problems are applied. The results show the capability of the MOVDO algorithm to solve the multi-objective problems. To justify this, NSGA-II and MOSA algorithms have been implemented to evaluate the performance of the proposed MOVDO. The efficiency of MOVDO in large-size problems has been demonstrated, although, only the CPU time metric has been statistically different in comparison with NSGA-II and MOSA. As future research, one can compare the proposed MOVDO with other multi-objective algorithms (e.g., MOPSO or MOTS) in various optimization problems.

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