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# A novel practical fair nodal price for DC microgrids and distribution systems

**R. Asad*** **and A. Kazemi**

*Center of Excellence for Power Systems Automation and Operation, Department of Electrical Engineering, Iran University of Science and Technology, Tehran, Iran.*

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Power loss;
Distributed generation;
DC system;
Microgrid;
Distribution system;
Demand response;
Smart grids.

**Abstract.** DC microgrids and distribution systems will have an important role to play in future electrical power systems, i.e. smart grids. One of the most important issues of dc microgrids and distribution systems is the correct and fair determination of the energy prices of the consumers. In this paper, a novel, practical, accurate, fair nodal price, namely, the real nodal price, is proposed to determine the energy payments of consumers in dc microgrids and distribution systems. Applying the real nodal price, each consumer will exactly pay for their real energy costs, i.e. their accurate consumed energy and originated energy loss. The real nodal price is applicable to different dc microgrids and distribution systems, even those with mesh configuration and numerous distributed generators. The real nodal price leads to zero merchandising surplus. Besides, through the presented novel method in this paper, the real nodal price is computed by a few simple and fast calculations, and, also, no slack bus is necessary to be assigned to compute it. In this paper, the real nodal prices of a dc microgrid are derived through the presented method. The analyses and simulation results confirm the mentioned remarkable features of the real nodal price.



## 1. Introduction

Many electrical components that are more compatible with dc microgrids and distribution systems have emerged in recent decades [1-6]. Numerous residential, administrative and commercial electrical loads (office supplies, multimedia appliances, advanced lighting systems, communication appliances, electric vehicles, home appliances equipped with drive systems, high efficiency dc air conditioners, etc.), the majority of growing renewable energy power sources (solar arrays, micro turbines, wind turbines and wave and tidal generators, etc.) and most important storage systems are some of these electrical components. Furthermore, important problems corresponding to the reactive power are completely eliminated in dc microgrids and distribution systems, since there is no reactive power in dc electrical systems [7]. Consequently, as demonstrated in [1-7], dc microgrids and distribution systems will play an important role in future electrical power systems, i.e. smart grids.

One of the most important issues of microgrids and distribution systems is the correct and fair determination of the energy prices of the consumers. The energy price should be as fair as possible. For this purpose, the energy price must consider the energy loss originated by the consumer in addition to their energy consumption. Also, real and accurate data are to be used to compute the energy price. Any approximation or unreal assumption in the computation process of the energy price will make the energy price imprecise and unfair. Besides, the energy price should be derived through a few simple and fast calculations

*. *Corresponding author. Mobile: +98 912 6094806;*
*Fax: +98 21 73225777*
*E-mail addresses: asad@iust.ac.ir (R. Asad);*
*kazemi@iust.ac.ir (A. Kazemi)*

to be applicable to microgrids and distribution systems.

The simplest energy price is the uniform energy price. This price is applied to calculation of the energy payments of all consumers. It is derived through dividing the cost of the total generated energy by the total energy consumption of microgrids or distribution systems. Using the uniform energy price or its modifications [8], each consumer pays for the energy loss proportional to his/her energy consumption and not his/her originated energy loss. On the other hand, regarding the high resistance to the inductance ratio and the low voltage of microgrids and distribution systems, the energy loss is not negligible and has a considerable share in the energy payments of consumers in microgrids and distribution systems [8]. Consequently, the uniform energy price leads to really unfair energy payments.

To remove the mentioned drawback, in recent years, some research has proposed some Locational Marginal Prices (LMP's) for ac distribution systems [9-12]. The LMP's presented in [9,10,12] result in nonzero Merchandising Surplus (MS), which is not desirable [8,11,13]. Although a method is proposed in [11] to redistribute nonzero MS among the consumers, the redistribution is not fair enough [8]. Also, the LMP's presented in [9-12] are generally inaccurate and slack bus dependent. In other words, by assigning different slack buses, the derived amounts of the LMP's change. To remove this disadvantage, Peng et al. [14] and Litvinov et al. [15] presented slack bus independent LMP's for ac distribution systems. All the energy prices presented in [9-12,14,15] are locational marginal prices. Although LMP's are widely used in different power systems, they do not represent the real energy price of nodes [16]. In addition, in [9-12,14,15], the energy loss payment of each consumer is determined proportional to the sensitivity of the total energy loss to the increase of the energy consumption of the consumer and not proportional to the energy loss originated by the consumer. Also, in [9-12,14,15], the LMP of each consumer is determined, regardless of the share of each power source in feeding the consumer. In other words, being fed by cheaper power sources, the energy price of a consumer will not decrease. Therefore, in the mentioned research, the energy payments of the consumers are not still fair. It is noteworthy that if the share of each power source in feeding a consumer is considered to determine the energy price of the consumer, load expansions are motivated to be located as near as possible to the cheapest power sources. This results in more optimal microgrids and distribution systems.

In this paper, a novel, practical, accurate, fair nodal price, namely, the real nodal price, is proposed to determine the energy payments of consumers in dc microgrids and distribution systems. Applying the real nodal price, each consumer will fairly pay for their exact real energy costs, i.e. their consumed energy and originated energy loss. The remarkable features of the real nodal price are as follows:

- All components affecting the energy price of consumers are accurately considered to determine the real nodal price. In other words, to compute the energy price of each consumer:
  a) The share of each power source in feeding the consumer is precisely considered;
  b) The exact energy loss originated by the consumer is applied.
- No assumption or approximation is applied to determine the real nodal price. Therefore, the calculation process is completely accurate.
- No slack bus is necessary to be assigned to determine the real nodal price. Consequently, the real nodal price is slack bus independent.
- The real and accurate data of consumers and power sources, simply derived through ordinary electricity meters, are applied to determine the real nodal price.
- Regarding the preceding four features, the real nodal price is a completely accurate, real, and, as a result, fair nodal price.
- The real nodal price leads to zero merchandising surplus.
- Through the presented method in this paper, the real nodal price is computed by a few simple and fast calculations in all dc microgrids and distribution systems, including those with mesh configuration and numerous distributed generators.
- Applying the real nodal price in dc microgrids and distribution systems, load and generation expansions are automatically motivated to be situated as near as possible to optimal points. Obviously, this leads to more optimal dc microgrids and distribution systems.
- If proper telecommunication infrastructures are available, the real nodal price can be used as a real-time nodal price. This will result in a demand response and outstanding advantages in microgrids and distribution systems.

The simulation results and analyses in the next sections will confirm the mentioned remarkable features of the real nodal price.

In this paper, at first, the necessary equations to calculate the real nodal price of a node through the energy prices of its neighbouring nodes are derived. Then, a novel method to compute the real nodal prices

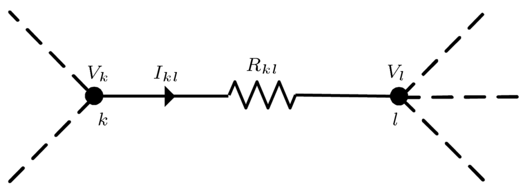


**Figure 1.** Two typical neighbouring nodes.

of all nodes of dc microgrids and distribution systems is presented. Also, the features and characteristics of the real nodal price are discussed in detail. Finally, the real nodal prices of a typical dc microgrid are derived through the presented method in this paper.

## 2. A novel method to calculate the real nodal price

Here, a novel method to calculate the real nodal price of each node of different dc microgrids and distribution systems is presented. For this purpose, the necessary equations to calculate the real nodal price of a node through the energy prices of its neighbouring nodes are initially derived.

### *2.1. Determination of the real nodal price of a node*

Suppose two neighbouring nodes in Figure 1. Node $l$ receives energy from node $k$ with constant power in a period of time, e.g. $T$. To determine the real cost of the received energy to node $l$, the energy loss of the line between node $k$ and node $l$ should be also considered. Thus, the real cost of the received energy to node $l$ is equal to the cost of the sent energy from node $k$, i.e:

$$\mathrm{Cost}_{E_{kl,r}} = \mathrm{Cost}_{E_{kl,s}}. \tag{1}$$

All the variables and parameters are identified in nomenclature. On the other hand:

$$\mathrm{Cost}_{E_{kl,r}} = E_{kl,r}\mathrm{Price}_{E_{kl,r}}, \tag{2}$$

and:

$$\mathrm{Cost}_{E_{kl,s}} = E_{kl,s}\mathrm{Price}_{k}. \tag{3}$$

Also:

$$E_{kl,r} = P_{kl,r}T, \tag{4}$$

and:

$$E_{kl,s} = P_{kl,s}T. \tag{5}$$

Regarding Eqs. (1) to (5), the real price of the received energy to node $l$ can be calculated as follows:

$$\mathrm{Price}_{E_{kl,r}} = \left(P_{kl,s}\mathrm{Price}_{k}\right)/P_{kl,r}. \tag{6}$$

Besides, according to Figure 1:

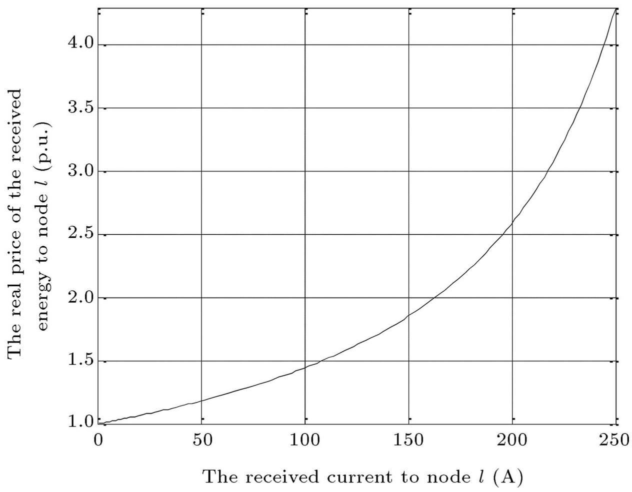


**Figure 2.** The relationship between the real price of the received energy to node $l$ ($\mathrm{Price}_{E_{kl,r}}$) and the received current to node $l$ ($I_{kl}$). $R_{kl} = 1\ \Omega$, $\mathrm{Price}_k = 1$ p.u., and $V_k = 326$ V.

$$P_{kl,r} = P_{kl,s} - P_{\mathrm{loss}_{kl}}, \tag{7}$$

$$P_{kl,s} = V_k I_{kl}, \tag{8}$$

$$P_{\mathrm{loss}_{kl}} = R_{kl} I_{kl}^2. \tag{9}$$

Combining Eqs. (6) to (9) gives:

$$\mathrm{Price}_{E_{kl,r}} = \left(1 + \frac{R_{kl}I_{kl}}{V_k - R_{kl}I_{kl}}\right)\mathrm{Price}_{k}. \tag{10}$$

Eq. (10) demonstrates the relationship between the real price of the received energy to node $l$ ($\mathrm{Price}_{E_{kl,r}}$) and the received current to node $l$ ($I_{kl}$). The relationship is depicted in Figure 2. Figure 2 implies that the real price of the received energy to node $l$ grows as the received current or, in other words, the received power to node $l$ ($P_{kl,r}$) increases. Since:

$$R_{kl}I_{kl} = V_k - V_l. \tag{11}$$

Eq. (10) can be rewritten as follows:

$$\mathrm{Price}_{E_{kl,r}} = \left(V_k/V_l\right)\mathrm{Price}_{k}. \tag{12}$$

Since there is energy loss in the line between node $k$ and node $l$, the real price of the received energy to node $l$ is greater than $\mathrm{Price}_k$, i.e. the price of energy in node $k$.

Suppose node $j$ with $n$ neighbouring nodes in Figure 3. Node $j$ receives energy from $m$ number of its neighbouring nodes and sends it to the rest of the neighbouring nodes with constant power over a period of time, e.g. $T$. Obviously, the real nodal price of node $j$, i.e. the real price of energy in node $j$, is derived by

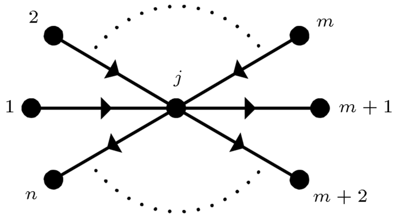


**Figure 3.** A typical node connected to $n$ other nodes.

dividing the sum of the real cost of the received energies by the sum of the received energies:

$$\text{Price}_j = \frac{\sum_{i=1}^{m} \text{Cost}_{E_{ij,r}}}{\sum_{i=1}^{m} E_{ij,r}}. \tag{13}$$

Combining Eqs. (2), (4) and (13) equals:

$$\text{Price}_j = \frac{\sum_{i=1}^{m} \left(P_{ij,r} \text{Price}_{E_{ij,r}}\right)}{\sum_{i=1}^{m} P_{ij,r}}, \tag{14}$$

where $i$ is the number of nodes which send energy to node $j$. Regarding Eqs. (6) and (14), the real nodal price of node $j$ is desirably a function of the share of each neighbouring node in feeding node $j$, the price of each neighbouring node feeding node $j$, and the energy loss originated by each received energy.

According to Eq. (12), the real price of the received energy from node $i$ to node $j$ is equal to:

$$\text{Price}_{E_{ij,r}} = (V_i/V_j)\,\text{Price}_i. \tag{15}$$

Considering Eq. (15), Eq. (14) is rewritten as follows:

$$\text{Price}_j = \frac{1}{V_j} \frac{\sum_{i=1}^{m} \left(P_{ij,r} V_i \text{Price}_i\right)}{\sum_{i=1}^{m} P_{ij,r}}. \tag{16}$$

If only one of the neighbouring nodes feeds node $j$, Eq. (16) can be further simplified:

$$\text{Price}_j = (V_i/V_j)\,\text{Price}_i. \tag{17}$$

According to Eqs. (16) and (17), the real nodal price of a node can be simply calculated without knowing the conductor type and the length of the lines between the node and its neighbouring nodes. The necessary data to determine the real nodal price of a node are: the voltage of the node, the voltages and real nodal prices of the neighbouring nodes feeding the node, and the share of each neighbouring node in feeding the node. If a node is fed by only one of its neighbouring nodes, only the voltage of the node, and the voltage and real nodal price of the neighbouring node feeding the node, are required to determine the real nodal price of the node. It is noteworthy that both Eqs. (16) and (17) are very simple equations. Also, no approximation or assumption is applied to derive Eqs. (16) and (17). Therefore, the real nodal price of a node can be simply and accurately calculated by the two simple equations through the real nodal prices of their neighbouring nodes.

### *2.2. Determination of the real nodal prices of dc microgrids and distribution systems*

To determine the real nodal price of all the nodes of a dc microgrid or distribution system for a period of time, the voltage of all the nodes and the power flow of all the lines are initially calculated. For this purpose, firstly, the power consumption of consumers, the terminal voltage and the generated power of power sources are simply and precisely extracted through the data gathered from the electricity meters of the consumers and the power sources. Then, knowing the branch data of the dc network, the voltage of all the nodes and the power flow of all the lines are precisely calculated by a simple, fast and accurate distribution load flow method, e.g. [17] or [18], in a few milliseconds. Since the accurate generated powers of the power sources are accessible through their electricity meters, the total power loss is initially and precisely allocated to the power sources. Therefore, no slack bus is necessary to be assigned in the load flow process. Furthermore, the accurate and real data of consumers and power sources, simply derived through electricity meters, are applied to the load flow. Consequently, the results of the load flow are completely precise and real. Also, since there are some uncomplicated and fast distribution load flow methods, e.g. [18], which are applicable to even meshed networks with numerous distributed generators, the voltage of the nodes and the power flow of the lines of all dc microgrids and distribution systems can be simply and quickly determined.

The real nodal prices of all the nodes of dc microgrids or distribution systems are calculated through a simple, fast and one-step sweep in the dc network. The sweep starts from the power sources. It is noteworthy that the real nodal prices of the power sources are definite and equal to their energy prices. Depending on the type (regulated or deregulated) of the dc microgrid or distribution system, the energy price of each power source is determined through either its cost function or bid. During the sweep, the real nodal prices of nodes with unknown real nodal prices are determined through neighbouring nodes with known real nodal prices by Eqs. (16) and (17). Upon deriving the real nodal prices of all the nodes of the dc microgrid or distribution system, the sweep finishes. When the real nodal prices of all the nodes are determined, the real energy cost of each consumer at each node is simply and precisely calculated through the product of the energy consumption of the consumer and the real nodal price of the node.

### *2.3. Numerical example*

To further clarify the presented method, a numerical example is mentioned here. Suppose the small dc microgrid in Figure 4. The real nodal prices of the dc microgrid will be calculated in detail through the presented method. The branch data of the dc microgrid is presented in Table 1. Also, the accurate and real data of the power sources and the consumers, derived through their electricity meters, are listed in Tables 2 and 3. Moreover, the energy prices of the power sources are demonstrated in Table 2. It is noteworthy that all the values mentioned in this numerical example are single-phase.

As mentioned before, to determine the real nodal price of all the nodes of a dc microgrid or distribution system through Eqs. (16) and (17), firstly, the voltage of all the nodes and the power flow of all the lines should be calculated through the load flow. Knowing the dc microgrid branch data, the terminal voltage and the generated power of the power sources, and the power consumption at each node, the current flow of each branch and the voltage of each node can be easily derived through different, existent, simple, fast and precise distribution load flow methods, such as forward-backward sweep load flow methods [17-18]. Table 1 demonstrates the current flow of different branches, derived by the load flow. Also, the direction of the current flow at each branch is shown in Figure 4. Moreover, the voltages of different nodes are demonstrated in Table 3.

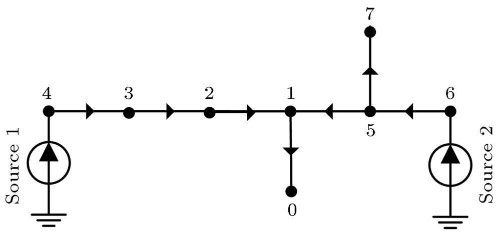


**Figure 4.** A small dc microgrid.

**Table 1.** The branch data of the dc microgrid.

| Branch number | From node | To node | Resistance (mΩ) | Current flow (A) |
|---|---|---|---|---|
| 1 | 0 | 1 | 0.9108 | -77.7367 |
| 2 | 1 | 2 | 5.0204 | -28.6027 |
| 3 | 2 | 3 | 141.4116 | -28.6027 |
| 4 | 3 | 4 | 40.3524 | -268.8586 |
| 5 | 1 | 5 | 29.986 | -49.1340 |
| 6 | 5 | 6 | 8.8968 | -298.8291 |
| 7 | 5 | 7 | 2.4684 | 245.3958 |

Now, to calculate the real nodal price of all the nodes, the sweep should start. As mentioned before, the sweep starts from a position with known real nodal prices, i.e. a power source. Sometimes, there is more than one power source. In this situation, it makes no difference which power source is selected for the start of the sweep.

Here, the sweep starts from source 1. Obviously, the real nodal price of node 4, which is connected directly to source 1, is equal to the energy price of source 1, i.e. 100 $/MWh. Since node 3 is fed by only one of its neighbouring nodes, i.e. node 4, its real nodal price can be calculated by Eq. (17). Therefore:

$$\text{Price}_3 = (V_4/V_3)\,\text{Price}_4 = (342.1859/331.3368) \times 100 = 103.2743.$$

**Table 2.** The power source data of the dc microgrid.

| Power source number | Node | Energy price ($/MWh) | $V_S$ (V) | $P_S$ (kW) | Income ($) |
|---|---|---|---|---|---|
| 1 | 4 | 100 | 342.1859 | 130.1574 | 13.0158 |
| 2 | 6 | 100 | 331.2809 | 110.3280 | 11.0328 |
| Total | | — | — | 240.4854 | 24.05 |

**Table 3.** The node and consumer data of the dc microgrid.

| Node | Power consumption (kW) | Node voltage (V) | Real nodal price ($/MWh) | Energy payment($) |
|---|---|---|---|---|
| 0 | 25.4259 | 327.0777 | 102.5119 | 2.6065 |
| 1 | 0 | 327.1484 | 102.4897 | 0 |
| 2 | 0 | 327.2921 | 104.5506 | 0 |
| 3 | 79.6101 | 331.3368 | 103.2743 | 8.2217 |
| 4 | 38.1578 | 342.1859 | 100.0000 | 3.8158 |
| 5 | 1.4133 | 328.6217 | 100.8092 | 0.1424 |
| 6 | 11.306 | 331.2809 | 100.0000 | 1.1306 |
| 7 | 80.4938 | 328.0164 | 100.9952 | 8.1295 |
| Total | 236.4069 | — | — | 24.05 |

Similarly, the real nodal price of node 2 is computed as follows:

$$\text{Price}_2 = (V_3/V_2)\,\text{Price}_3 = (331.3368/327.2921)$$

$$\times\, 103.2743 = 104.5506.$$

Since node 1 is fed by more than one of its neighbouring nodes, i.e. nodes 2 and 5, its real nodal price should be calculated by Eq. (16). Thus:

$$\text{Price}_1 = \frac{1}{V_1}\left(\frac{P_{21,r}V_2\text{Price}_2 + P_{51,r}V_5\text{Price}_5}{P_{21,r} + P_{51,r}}\right), \quad (18)$$

where:

$$P_{21,r} = V_1 I_{21} = 327.1484 \times 28.6027 = 9357.3 \quad \text{watts},$$

$$P_{51,r} = V_1 I_{51} = 327.1484 \times 49.1340 = 16074 \quad \text{watts}.$$

Therefore the real nodal price of node 1 is calculated by the equation shown in Box I.

According to Eq. (18), to calculate the real nodal price of node 1, the real nodal price of the feeding neighbouring nodes, i.e. nodes 2 and 5, should be known. Hence, since the real nodal price of node 5 is unknown, the real nodal price of node 1 cannot be calculated now. As a result, the sweep jumps to another position with known real nodal price, i.e. source 2.

Obviously, the real nodal price of node 6, which is connected directly to source 2, is equal to the energy price of source 2, i.e. 100 \$/MWh. The real nodal price of node 5, which is fed by only one of its neighbouring nodes, i.e. node 6, can be computed by Eq. (17):

$$\text{Price}_5 = (V_6/V_5)\,\text{Price}_6 = (331.2809/328.6217)$$

$$\times\, 100 = 100.8092.$$

Now, since the real nodal price of node 5 is determined, the real nodal price of node 1 can be calculated through Eq. (18) as shown in Box II.

The real nodal price of node 0, which is fed by only node 1, can be computed by Eq. (17):

$$\text{Price}_0 = (V_1/V_0)\,\text{Price}_1 = (327.1484/327.0777)$$

$$\times\, 102.4897 = 102.5119.$$

Finally, since node 7 is fed by only node 5, the real nodal price of node 7 can be determined as follows:

$$\text{Price}_7 = (V_5/V_7)\,\text{Price}_5 = (328.6217/328.0164)$$

$$\times\, 100.8092 = 100.9952.$$

In this manner, the real nodal prices of all the nodes are determined and the sweep finishes. The real nodal prices of the nodes are listed in Table 3. Also, the sum of incomes of the power sources and the sum of energy payments of the consumers are mentioned in Tables 2 and 3, respectively. According to Tables 2 and 3, the sum of the incomes of the power sources and the sum of the energy payments of the consumers are exactly equal. Therefore, as expected, the real nodal prices derived though the presented method result in zero Merchandising Surplus (MS).

According to above discussions, the presented method to determine the real nodal prices of nodes consists of only two parts, i.e. the load flow and the sweep. As mentioned before, there are some simple, fast and accurate distribution load flow methods, e.g. [18], which are applicable to even meshed dc networks with numerous distributed generations. On the other hand, by applying the two uncomplicated equations, i.e. Eqs. (16) and (17), the sweep contains very simple and fast calculations and finishes rapidly, even in meshed dc microgrids or distribution systems. Consequently, the presented method in this paper to determine the real nodal prices of nodes is applicable to different dc microgrids and distribution systems, even those with mesh configuration and numerous distributed generations. Also, the presented method is an uncomplicated, fast, practical method, which can be implemented simply and inexpensively. Furthermore,

$$\text{Price}_1 = \frac{1}{327.1484} \times \left(\frac{9357.3 \times 10^{-6} \times 327.2921 \times 104.5506 + 16074 \times 10^{-6} \times 328.6217 \times \text{Price}_5}{9357.3 \times 10^{-6} + 16074 \times 10^{-6}}\right).$$

Box I

$$\text{Price}_1 = \frac{1}{327.1484} \times \left(\frac{9357.3 \times 10^{-6} \times 327.2921 \times 104.5506 + 16074 \times 10^{-6} \times 328.6217 \times 100.8092}{9357.3 \times 10^{-6} + 16074 \times 10^{-6}}\right) = 102.4897.$$

Box II

no approximation or unreal assumption is applied to the derivation process of Eqs. (16) and (17), the utilized load flow method and the sweep, i.e. the presented method. Thus, using the real and precise data of the consumers and the power sources, the real nodal prices of nodes derived through the presented method in this paper are completely accurate, real and, as a result, fair.

With respect to the preceding discussions, applying the real nodal price in dc microgrids and distribution systems, the energy payments of the consumers are completely real, precise and fair. In this way, the energy payment of each consumer includes the exact energy loss originated by the consumer too. In addition, the sum of energy payments of the consumers exactly equals the total energy cost of the electrical power system, i.e. the total income of the power sources. Consequently, the real nodal price results in zero Merchandising Surplus (MS). It is noteworthy that nonzero MS is not desirable [8,11,13] and it should be redistributed among the consumers. On the other hand, the fair redistribution of MS among the consumers is very difficult to satisfy. The determination of the real nodal prices of different dc microgrids and distribution systems is simple, fast and inexpensive through the presented method in this paper. Therefore, the real nodal price is a practical nodal price too. Moreover, if proper telecommunication infrastructures are available and, as a result, the data of power sources and consumers can be transmitted in real time, the real nodal price of each consumer can be computed and sent back to the consumer in real time. In other words, the real nodal price can be used as a real-time nodal price. This leads to the demand response and its outstanding advantages, e.g. peak-shaving in dc microgrids and distribution systems.

Regarding Eqs. (16) and (17) and the sweep, to determine the real nodal price of each consumer, in addition to the energy loss originated by the consumer, the share of each power source in feeding the consumer is precisely considered too. Therefore, the more expensive the feeding power sources of a node and the more the energy loss originated by the node, the more the real nodal price of the node will be. Consequently, applying the real nodal price, load expansions are desirably motivated to be located as near as possible to the cheapest power sources. On the other hand, generation expansions are desirably motivated to be situated as near as possible to the consumers at the most expensive nodes. Therefore, utilizing the real nodal price in dc microgrids and distribution systems, load and generation expansions are automatically motivated to be situated as near as possible to the optimal points. Obviously, this leads to more optimal dc microgrids and distribution systems and, as a result, a further increase in social welfare.

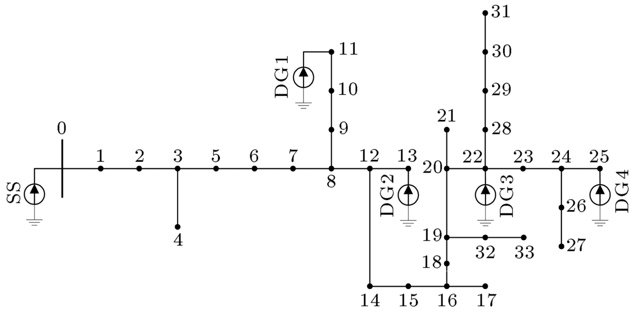


**Figure 5.** A dc microgrid diagram.

## 3. Simulation results

Here, the real nodal prices of the dc microgrid in Figure 5 are calculated through the presented method in this paper. The dc microgrid is a version of the IEEE 34 node test feeder [19,20]. The dc microgrid is a two-phase and three-wire (+326 V, −326 V and neutral) dc system. The lengths of different branches of the microgrid are listed in Table 4. The resistance of the electrical lines is considered to be 0.44 Ω/km. The dc microgrid contains four distributed generators and one substation. The substation is connected to a transmission system through a bidirectional ac/dc converter. The specifications of each power source of the dc microgrid are demonstrated in Table 5. Besides, the terminal voltage and the generated power of different power sources, for an hour, are mentioned in Table 5. The loads of the consumers are two-phase constant-power. Table 6 demonstrates the power consumption at each node for an hour. It is remarkable that all the values mentioned in this section are single-phase.

Knowing the dc microgrid data, the terminal voltage, the generated power of the power sources and the power consumption at each node, the current flow of each branch and the voltage of each node can be easily derived through different existent simple, fast and precise distribution load flow methods, such as forward-backward sweep load flow methods [17,18]. Table 4 and Figure 6 show the current flow of different branches. Also, the voltages of different nodes are demonstrated in Table 6 and Figure 7. Regarding Tables 5 and 6, the total generated power, the total consumed power and the total power loss are, respectively, 1921.6 kW, 1840 kW and 81.6 kW, i.e. 4.25%. Using the results of the load flow, the real nodal price of all the nodes are calculated through a simple, fast and one-step sweep. The sweep is executed by a MatLab mFile on Windows Vista-based, Intel Core 2Duo, 2.50 GHz PC in less than 0.02 seconds. The real nodal prices of the nodes of the dc microgrid are demonstrated in Table 6 and Figure 8.

It is noteworthy that although the total consumed power is not equal to the total generated power,

**Table 4.** The branch data of the dc microgrid.

| Branch number | From node | To node | Length (m) | Current flow (A) |
|---|---|---|---|---|
| 1 | 0 | 1 | 17.22 | 457.7570 |
| 2 | 1 | 2 | 11.55 | 263.8623 |
| 3 | 2 | 3 | 215.12 | 263.8623 |
| 4 | 3 | 4 | 38.74 | 341.5989 |
| 5 | 3 | 5 | 250.3 | -77.7367 |
| 6 | 5 | 6 | 198.44 | -77.7367 |
| 7 | 6 | 7 | 0.07 | -77.7367 |
| 8 | 7 | 8 | 2.07 | -77.7367 |
| 9 | 8 | 9 | 11.41 | -28.6027 |
| 10 | 9 | 10 | 321.39 | -28.6027 |
| 11 | 10 | 11 | 91.71 | -268.8586 |
| 12 | 8 | 12 | 68.15 | -49.1340 |
| 13 | 12 | 13 | 20.22 | -298.8291 |
| 14 | 12 | 14 | 5.61 | 245.3958 |
| 15 | 14 | 15 | 136.43 | 241.9499 |
| 16 | 15 | 16 | 3.47 | 118.5507 |
| 17 | 16 | 17 | 155.72 | 155.6586 |
| 18 | 16 | 18 | 245.83 | -37.1079 |
| 19 | 18 | 19 | 0.07 | -37.1079 |
| 20 | 19 | 20 | 32.71 | -376.7845 |
| 21 | 20 | 21 | 10.81 | 1.7513 |
| 22 | 20 | 22 | 38.91 | -772.5612 |
| 23 | 22 | 23 | 13.48 | -302.9063 |
| 24 | 23 | 24 | 17.89 | -353.1117 |
| 25 | 24 | 25 | 5.74 | -444.4316 |
| 26 | 24 | 26 | 1.87 | 23.2629 |
| 27 | 26 | 27 | 32.44 | 23.2629 |
| 28 | 22 | 28 | 1.87 | 550.8016 |
| 29 | 28 | 29 | 9.01 | 543.2205 |
| 30 | 29 | 30 | 24.3 | 199.8710 |
| 31 | 30 | 31 | 3.54 | 141.8506 |
| 32 | 19 | 32 | 0 | 339.6766 |
| 33 | 32 | 33 | 70.48 | 339.6766 |

according to Tables 5 and 6, both the sum of the energy payments of the consumers and the total income of the power sources are equal to 219.5$ in the dc microgrid. That is, the real nodal price expectedly leads to zero merchandising surplus in the dc microgrid. The reason is that the real nodal prices derived through the presented method in this paper are completely real and accurate. Besides, they precisely and fairly

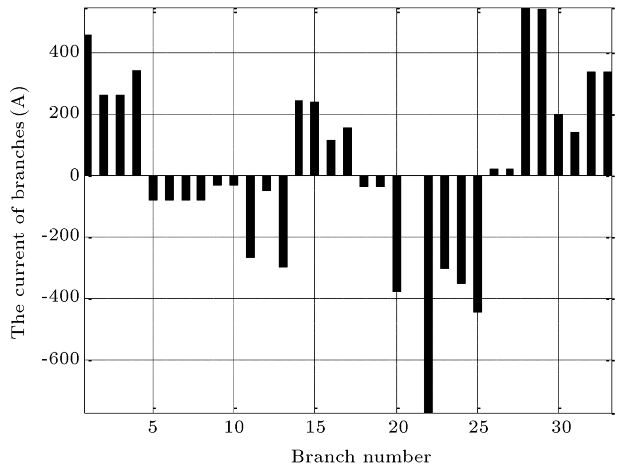


**Figure 6.** The current of the branches of the dc microgrid.

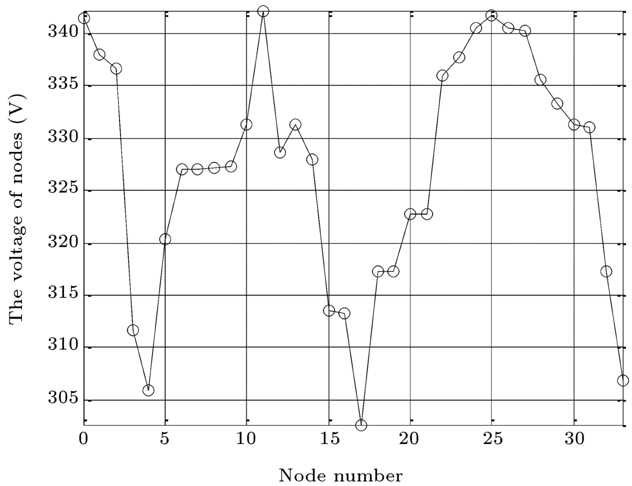


**Figure 7.** The voltage profile of the dc microgrid.

**Table 5.** The power source data of the dc microgrid.

| Power source number | Generation type | Node | Energy price ($/MWh) | $V_S$ (V) | $P_S$ (kW) | Income (two-phase) ($) |
|---|---|---|---|---|---|---|
| 1 | DG | 13 | 100 | 331.2809 | 110.3280 | 22.0656 |
| 2 | DG | 11 | 100 | 342.1859 | 130.1574 | 26.0315 |
| 3 | DG | 22 | 110 | 335.9759 | 393.1917 | 86.5022 |
| 4 | DG | 25 | 125 | 341.6747 | 170.7888 | 42.6972 |
| 5 | Substation | 0 | 135 | 341.5116 | 156.3294 | 42.2089 |
| Total (two-phase) | | — | — | 1921.6 | 219.5 | |

**Table 6.** The node and consumer data of the dc microgrid.

| Node | Power consumption (kW) | Node voltage (V) | Real nodal price ($/MWh) | Energy payment (two-phase) ($) |
|---|---|---|---|---|
| 0 | 0 | 341.5116 | 135.0000 | 0 |
| 1 | 65.5458 | 338.0432 | 136.3852 | 17.8789 |
| 2 | 0 | 336.702 | 5136.9282 | 0 |
| 3 | 0 | 311.7267 | 138.7192 | 0 |
| 4 | 104.5224 | 305.9039 | 141.3597 | 29.5505 |
| 5 | 0 | 320.2880 | 104.6850 | 0 |
| 6 | 0 | 327.0754 | 102.5126 | 0 |
| 7 | 0 | 327.0777 | 102.5118 | 0 |
| 8 | 0 | 327.1484 | 102.4897 | 0 |
| 9 | 0 | 327.2921 | 104.5506 | 0 |
| 10 | 79.6101 | 331.3368 | 103.2743 | 16.4434 |
| 11 | 38.1578 | 342.1859 | 100.0000 | 7.6315 |
| 12 | 1.4133 | 328.6217 | 100.8092 | 0.2849 |
| 13 | 11.306 | 331.2809 | 100.0000 | 2.2612 |
| 14 | 1.1306 | 328.0164 | 100.9952 | 0.2284 |
| 15 | 38.6978 | 313.4922 | 105.6744 | 8.1787 |
| 16 | 0 | 313.3112 | 109.5489 | 0 |
| 17 | 47.1306 | 302.6459 | 113.4094 | 10.6901 |
| 18 | 0 | 317.3250 | 120.1923 | 0 |
| 19 | 0 | 317.3260 | 120.1919 | 0 |
| 20 | 127.1925 | 322.7482 | 118.1727 | 30.0614 |
| 21 | 0.5653 | 322.7399 | 118.1757 | 0.1336 |
| 22 | 50.3117 | 335.9759 | 113.5201 | 11.4228 |
| 23 | 16.959 | 337.7729 | 126.4439 | 4.2887 |
| 24 | 23.1773 | 340.5522 | 125.4120 | 5.8134 |
| 25 | 18.9376 | 341.6747 | 125.0000 | 4.7344 |
| 26 | 0 | 340.5330 | 125.4191 | 0 |
| 27 | 7.9142 | 340.2010 | 125.5415 | 1.9871 |
| 28 | 2.5438 | 335.5230 | 113.6733 | 0.5783 |
| 29 | 114.4733 | 333.3692 | 114.4077 | 26.1933 |
| 30 | 19.2202 | 331.2326 | 115.1457 | 4.4262 |
| 31 | 46.959 | 331.0118 | 115.2225 | 10.8215 |
| 32 | 0 | 317.3260 | 120.1919 | 0 |
| 33 | 104.2398 | 306.7916 | 124.3190 | 25.9180 |
| Total (Two-phase) | 1840 | — | — | 219.5 |

compensate for the cost of the energy loss originated by each consumer.

## 4. Conclusion

In this paper, a novel, practical, accurate, fair nodal price, namely, the real nodal price, was proposed to determine the energy payments of consumers in dc microgrids and distribution systems. The real nodal price is applicable to different dc microgrids and distribution systems, even those with mesh configuration and numerous distributed generators. Applying the real nodal price, each consumer will fairly pay for their exact real energy costs. To determine the real nodal

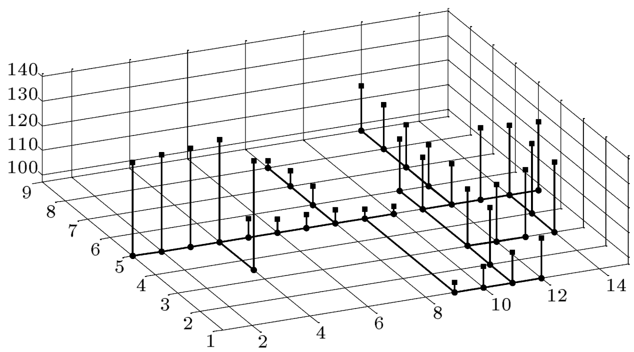


**Figure 8.** The real nodal prices of the dc microgrid (\$/MWh) (square marker), and the dc microgrid diagram (circle marker).

price of each consumer, the energy loss originated by the consumer and the share of each power source in feeding the consumer are precisely considered. The real nodal price is slack bus independent. Also, it desirably leads to zero merchandising surplus. No assumption or approximation is applied to compute the real nodal price. Through the presented novel method in this paper, the real nodal price is computed by a few simple and fast calculations. Consequently, the implementation of the real nodal price is simple and inexpensive. It is demonstrated that the real nodal price leads to more optimal dc microgrids and distribution systems. Also, if proper telecommunication infrastructures are available, the real nodal price can be used as a real-time nodal price, resulting in a demand response.

## Nomenclature

| | |
|---|---|
| $I_{kl}$ | Current flowing through the branch (line) between node $l$ and node $k$. |
| $E_{kl,r}$ | Received energy to node $l$ from node $k$. |
| $E_{kl,s}$ | Sent energy from node $k$ to node $l$. |
| $P_{kl,r}$ | Received power to node $l$ from node $k$. |
| $P_{kl,s}$ | Sent power from node $k$ to node $l$. |
| $P_{\text{loss}_{kl}}$ | Power loss of the branch (line) between node $l$ and node $k$. |
| $\text{Price}_k$ | Real nodal price of node $k$. |
| $R_{kl}$ | Resistance of the branch (line) between node $l$ and node $k$. |
| $V_k$ | Voltage of node $k$. |

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## Biographies

**Roozbeh Asad** was born in Shiraz, Iran. He received an MS degree in Electrical Engineering, in 2007, from Iran University of Science and Technology, Tehran, Iran, where he is currently a PhD degree student in the same subject. His research interests are distribution systems and microgrids, smart grids, stability and control, dc power systems, and power electronics.

**Ahad Kazemi** was born in Tehran, Iran. He received an MS degree in Electrical Engineering from Oklahoma State University, Stillwater, USA, in 1979, and is currently Associate Professor and Power Systems Research Group Coordinator in the Department of Electrical Engineering at Iran University of Science and Technology, Tehran, Iran. His research interests are reactive power control, power system dynamics, stability and control, and FACTS devices.