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Research Note

Pseudospectral optimal control of active magnetic bearing systems

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Abstract. In this paper, an optimal control framework is formed to control rotor-Active Magnetic Bearing (AMB) systems. The multi-input-multi-output non-affine model of AMBs is well established in the literature and represents a challenging problem for control design, where the design requirement is to keep the rotor at the bearing centre in the presence of external disturbances. To satisfy the constraints on the states and the control inputs of the AMB nonlinear dynamics, a nonlinear optimal controller is formed to minimize tracking error between the current and desired position of the rotor. To solve the resulted nonlinear constrained optimal control problem, the Gauss Pseudospectral Collocation Method (GPCM) is used to transcribe the optimal control problem into a nonlinear programming problem (NLP) by discretization of states and controls. The resulted NLP is then solved by a well-developed algorithm known as SNOPT. The procedure for modeling, compilation and solving of the resulted optimal control problem is done using the Matlab optimal control software known as PROPT. The results illustrate the effectiveness of the proposed approach to deal with the control of AMBs.

1. Introduction

AMB usage has become more widespread because of their benefits in comparison with conventional bearings. They need no lubrication between their rotating parts and, in addition, due to electromagnetical levitation of their rotors in the air, the contactless behavior of AMBs decreases the cost of their maintenance. Also, the variable stiffness and damping of AMBs attenuate their vibrations [1].

Controlling AMBs is a representative control problem and has attracted some attention from the control community [1-6]. There are some challenges in this area. The first is the nolinear control non-affine dynamics of AMBs. The electromagnetic actuator force is one of the main causes of nonlinearity in AMB systems [7]. Other sources of nonlinearities in AMB systems are external disturbances and mass unbalance in the nonlinear model of the rotor-AMB system. In addition, because of the limitation on the values of currents applied to generate electromagnetic forces and dimensional tolerances, it is required to design a control approach to overcome these challenges and consider the mentioned limitations. Tombul et al. (2009) designed a Sliding Mode Controller (SMC) for this purpose. Their SMC design for the nonlinear system is simplified by using the successive LTV approximations of the nonlinear system. Hence, difficulties in the design of the hyperplane for the non-affine nonlinear system are eliminated [1]. However, their approach is very tedious and requires many analyt-

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ical calculations, and is suitable for slowly varying dynamics. In addition, limitation on the states and controls of the AMB systems was not considered. A more important issue in their design is that the error between two successive iterations of the control error is not feasible, so, the usage of their approach is risky.

A better choice to deal with the control of AMB systems, while satisfying the input and state constraints, is nonlinear optimal control. Nonlinear optimal control satisfies any of the desirable constraints and is also suitable for nonlinear systems [8]. There is also no limitation in the optimal control method on how the system varies, either slowly or rapidly. In an optimal control problem, the goal is determination of the states and controls that minimize a cost functional, subject to nonlinear dynamic constraints, the boundary condition and inequality path constraints. There are two methods for resolving optimal control problems: direct and indirect [9]. In an indirect method, first-order necessary conditions for optimality are derived from the optimal control problem via the calculus of variations and Pontryagin's minimum principle [10]. These necessary conditions form a Hamiltonian Boundary-Value Problem (HBVP), which is then solved numerically for extremal trajectories. The optimal solution is then found by choosing the extremal trajectory with the lowest cost. The primary advantages of indirect methods are a high accuracy in the solution and the assurance that the solution satisfies first-order optimality conditions. However, indirect methods have several disadvantages. First, the solution of HBVP must usually be derived analytically, which can often be non-trivial. Second, if one wants to obtain the solution numerically, because numerical techniques used in indirect methods typically have small radii of convergence, an extremely good initial guess of the unknown solution or boundary conditions is generally required. Finally, for problems with path constraints, it is necessary to have a priori knowledge of the constrained and unconstrained arcs or switching structure [11]. Lee and Jeong (1996) used an indirect method for the active control of the linearized model of a cone-shaped AMB system [6].

On the other hand, the direct methods transform the optimal control problem into a nonlinear programming problem (NLP). Direct methods have the advantage that the first-order necessary conditions do not need to be derived. Furthermore, they have much larger radii of convergence than indirect methods and, thus, do not require as good an initial guess. Lastly, the switching structure does not need to be known a priori [11]. In this paper, to solve the stabilization problem of AMB systems, we address a kind of direct method known as the Gauss Pseudospectral Collocation Method (GPCM) to transform the optimal control problem into an NLP by parameterization of the states and the controls. These parameterization techniques have an important role in the convergence and accuracy of the solution and low computation time [11]. The resulted NLP is then solved by a well-developed algorithm called SNOPT. The proposed method is suitable for precision positioning of AMB systems in the presence of extraneous disturbances while satisfying dimensional and current related constraints.

This paper is organized as follows: At first, the mathematical model of a rotor-AMB system adopted in the controller design is presented. Afterwards, the Gauss pseudospectral method is presented in its most current form and provides a complete NLP, which includes both inequality constraints and differential equations in the optimal control problem formulation. The last sections present the simulation results together with some discussions and conclusions.

2. Description of mathematical model of rotor-AMB systems

Figure 1 depicts a schematic view of a rotor-AMB system with coordinates that describes its motion. The rotor is levitated by magnetic bearings providing variable stiffness and damping in each axis separately. When the rotor approaches the bearing stator, AMB produces a net force in the opposite direction to move the rotor to the bearing rotation centre.

Figure 2 shows the differential driving mode for an 8-pole AMB. Ignoring the geometric coupling between horizontal and vertical axes, the net forces acting on the rotor in positive and negative directions are [1]:

$$F_{x+}=\frac{1}{4}\mu_0 N^2 A\cos\alpha\frac{(i_b+i_x)^2}{(s-x)^2}, \tag{1}$$

$$F_{x-}=\frac{1}{4}\mu_0 N^2 A\cos\alpha\frac{(i_b-i_x)^2}{(s+x)^2}, \tag{2}$$

where μ_0 is the permeability of vacuum, A is the electromagnet cross sectional area, N is the number of windings, s is the nominal air gap between the rotor

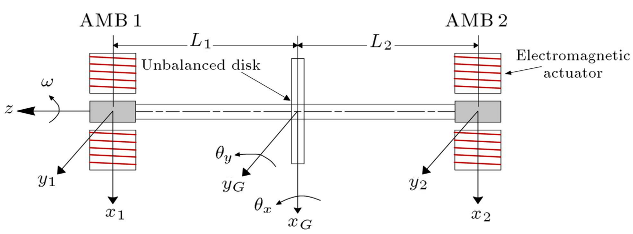

Figure 1. Rotor-AMB system with coordinates that describes its motion [1].

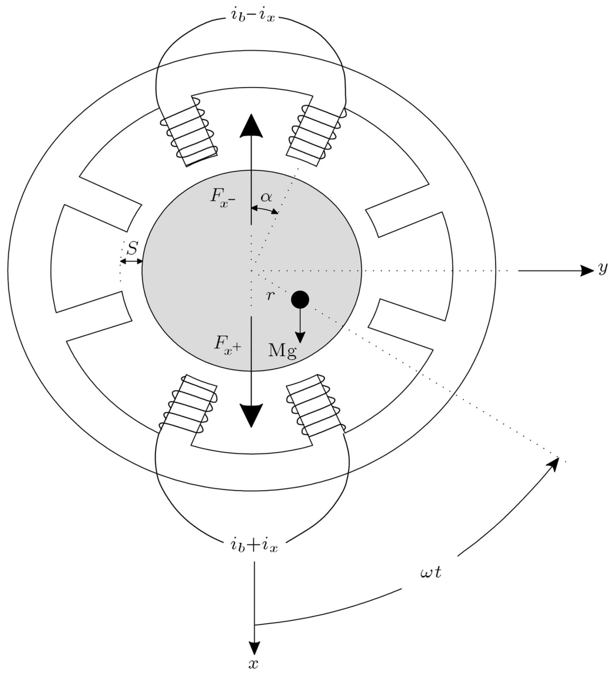

Figure 2. Differential driving mode for an 8-pole AMB [1].

and the stator, and α is the angle between the stator pole and its effecting axis. Here the current, i_b, is called bias current, which is supplied constantly to each 2-pole element. In the differential driving mode, the control current, i_x, is added to the bias current, while, in the opposite direction, it is subtracted from it. The net resulting force is calculated by subtracting Eq. (2) from Eq. (1):

$$F_x = \frac{1}{4}\mu_0 N^2 A \cos\alpha \left[\frac{(i_b+i_x)^2}{(s-x)^2} - \frac{(i_b-i_x)^2}{(s+x)^2}\right]. \quad (3)$$

Assuming that the acceleration sensors are located in the middle of the magnetic bearings, the acceleration components for each end of the rotor are obtained as follows [1]:

$$\begin{aligned}
\ddot{x}_1 =& \frac{1}{M}[F_{x1}+F_{x2}] \\
&+ L_1\left[\frac{\omega I_z}{I_x}\left[\frac{\dot{y}_1-\dot{y}_2}{L_1+L_2}\right] + \frac{F_{x1}L_1}{I_x} - \frac{F_{x2}L_2}{I_x}\right] \\
&+ \omega^2 r\cos\omega t + g, \\
\ddot{y}_1 =& \frac{1}{M}[F_{y1}+F_{y2}] \\
&+ L_1\left[-\frac{\omega I_z}{I_y}\left[\frac{\dot{x}_1-\dot{x}_2}{L_1+L_2}\right] - \frac{F_{y1}L_1}{I_y} + \frac{F_{y2}L_2}{I_y}\right] \\
&+ \omega^2 r\cos\omega t, \\
\ddot{x}_2 =& \frac{1}{M}[F_{x1}+F_{x2}] \\
&- L_2\left[\frac{\omega I_z}{I_x}\left[\frac{\dot{y}_1-\dot{y}_2}{L_1+L_2}\right] + \frac{F_{x1}L_1}{I_x} - \frac{F_{x2}L_2}{I_x}\right] \\
&+ \omega^2 r\cos\omega t + g, \\
\ddot{y}_2 =& \frac{1}{M}[F_{y1}+F_{y2}] \\
&- L_2\left[-\frac{\omega I_z}{I_y}\left[\frac{\dot{x}_1-\dot{x}_2}{L_1+L_2}\right] - \frac{F_{y1}L_1}{I_y} + \frac{F_{y2}L_2}{I_y}\right] \\
&+ \omega^2 r\cos\omega t, \quad (4)
\end{aligned}$$

where I_x and I_y are the transverse moment of inertia about x and y axes, respectively, I_z is the polar moment of inertia about the z-axis, M is the mass of the rotor, L_1 and L_2 are the distances between the centre of mass and magnetic bearings on the left and right-hand sides, respectively. The term, $\omega^2 r\cos\omega t$, is due to mass unbalance in the rotor, while ω is the rotor angular velocity.

The objective of the controller is to keep the rotor at the bearing centre in the presence of external disturbances. This can be represented as a quadratic cost function of error between the desired and current position of the AMB dynamical system. To define the position vector of the AMB system as $\mathbf{p} = [x_1, x_2, y_1, y_2]$, to keep the rotor at the bearing centre, the desired AMB position matrix should be chosen as zero. So, the cost function to provide this objective can be represented as follows:

$$J = \frac{1}{2}\int \mathbf{p}^T\mathbf{Q}\mathbf{p}, \quad (5)$$

where $\mathbf{Q}$ is a tunable symmetric and positive semi definite matrix.

The constraints that should be considered during the control design process are control currents and dimensional limits for the motion of the rotor. i.e.:

$$\begin{aligned}
&|i_{x_1}| < i_{x_{1\max}}, \qquad |i_{x_2}| < i_{x_{2\max}}, \\
&|i_{y_1}| < i_{y_{1\max}}, \qquad |i_{y_2}| < i_{y_{2\max}}, \\
&|x_1|, \quad |x_2|, \quad |y_1|, \quad |y_2| < s. \quad (6)
\end{aligned}$$

The dynamic equations of AMB given in Eq. (4), together with the cost function of Eq. (5) and the inequality constraints of Eq. (6), are used in the following section for their optimal control system design.

3. Gauss pseudospectral collocation method

Consider the following general optimal control problem. Determine the state, $x(t)$, and control, $\mathbf{u}(t)$, that minimize the cost functional:

Minimize:

$$J = \Phi(\mathbf{x}(t_0), t_0, \mathbf{x}(t_f), t_f) + \int_{t_0}^{t_f} g(\mathbf{x}(t), \mathbf{u}(t), t)dt,$$

such that:

$$\begin{cases} \text{the dynamic constraints:} \\ \quad \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t), \quad t \in [t_0, t_f] \\ \text{the boundary conditions:} \\ \quad \mathbf{h}(\mathbf{x}(\tau_0), t_0, \mathbf{x}(\tau_f), t_f) = \mathbf{0} \\ \text{the inequality path constraints :} \\ \quad \mathbf{C}(\mathbf{x}(t), \mathbf{u}(t), t) \leq \mathbf{0}. \end{cases} \tag{7}$$

where t_0 is the fixed or free initial time, t_f is the fixed or free final time, and $t \in [t_0, t_f]$. Eq. (1) is referred to as the continuous Bolza problem [11]. For the problem studied in this paper, the cost function is according to Eq. (5), dynamic constraints are according to Eq. (4) and the boundary conditions will be determined in the next section. The inequality path constraints are according to Eq. (6).

The GPCM method requires a fixed time interval, such as $[-1, 1]$. So, the time variable is mapped to the general interval, $\tau \in [-1, 1]$, via the affine transformation:

$$\tau = \frac{2t}{t_f - t_0} - \frac{t_f + t_0}{t_f - t_0}.$$

Now, the optimal control problem is rewritten as:

$$J = \Phi(\mathbf{x}(\tau_0), \tau_0, \mathbf{x}(\tau_f), t_f) + \frac{t_f - t_0}{2} \int_{\tau_0}^{\tau_f} g(\mathbf{x}(\tau), \mathbf{u}(\tau); t_0, t_f)d\tau,$$

such that:

$$\begin{cases} \text{the dynamic constraints:} \\ \quad \frac{d\mathbf{x}}{d\tau} = \frac{t_f - t_0}{2}\mathbf{f}(\mathbf{x}(\tau), \mathbf{u}(\tau), \tau; t_0, t_f) \\ \text{the boundary conditions:} \\ \quad \mathbf{h}(\mathbf{x}(\tau_0), t_0, \mathbf{x}(\tau_f), t_f) = \mathbf{0} \\ \text{the inequality path constraints:} \\ \quad \mathbf{C}(\mathbf{x}(\tau), \mathbf{u}(\tau), \tau; t_0, t_f) \leq \mathbf{0}. \end{cases} \tag{8}$$

In the GPCM, this optimal control problem is discretized at some specific discretization points called the Legendre-Gauss (LG) points, and then transcribed into a nonlinear program (NLP) by approximating the states and controls using Lagrange interpolating polynomials [8]. The set of N discretization points includes $K = N - 2$ interior LG collocation points, defined as the roots of the Kth-degree Legendre polynomial, the initial point, $\tau_0 = -1$, and the final point, $\tau_f = 1$. An approximation to the state, $\mathbf{X}(\tau)$, is formed with a basis of $K + 1$ Lagrange interpolating polynomials. The control is approximated using a basis of K Lagrange interpolating polynomials, namely $\mathbf{U}(\tau)$. The continuous dynamics are then transcribed into a set of K algebraic constraints via orthogonal collocation. In addition, the integral term in the cost functional can be approximated with a Gauss quadrature.

The resulted NLP are finally found as:

Minimize:

$$\underset{\mathbf{X}(\tau_k),\mathbf{U}(\tau_k)}{J} = \Phi(\mathbf{X}(\tau_0), t_0, \mathbf{X}(\tau_f), t_f) + \frac{t_f - t_0}{2}\sum_{i=1}^{K}\omega_i g(\mathbf{X}(\tau_i), \mathbf{U}(\tau_i), \tau_i),$$

s.t.:

$$\begin{cases} \mathbf{X}(\tau_f) - \mathbf{X}(\tau_0) - \frac{t_f - t_0}{2} \\ \qquad \sum_{i=1}^{K} \omega_i \mathbf{f}(\mathbf{X}(\tau_i), \mathbf{U}(\tau_i), \tau_i; t_0, t_f) = \mathbf{0} \\ \sum_{i=0}^{K}\sum_{\ell=0}^{K} \frac{\prod_{j=0, j\neq i,\ell}^{K}(\tau_k - \tau_j)}{\prod_{j=0, j\neq i}^{K}(\tau_i - \tau_j)}\mathbf{X}(\tau_i) \\ \qquad - \frac{t_0 - t_f}{2}\mathbf{f}(\mathbf{X}(\tau_k), \mathbf{U}(\tau_k), \tau_k; t_0, t_f) = \mathbf{0} \\ \mathbf{h}(\mathbf{X}(\tau_0), t_0, \mathbf{X}(\tau_f), t_f) = \mathbf{0} \\ \mathbf{C}(\mathbf{X}(\tau_k), \mathbf{U}(\tau_k), \tau_k; t_0, t_f) \leq \mathbf{0}, \quad (k = 1, \cdots, K) \end{cases} \tag{9}$$

In Eq. (9), ω_i are the Gauss weights.

The solution of Eq. (9) is an approximate solution to the continuous Bolza problem. In this paper, to solve this NLP, we use SNOPT solvers. SNOPT is a software package for solving large-scale optimization problems. It has been designed for problems with many thousands of constraints and variables, but is best suited for problems with a moderate number of degrees of freedom (up to 2000) [12]. It helps us to solve resulted non-convex optimization problems.

The GPCM method for solving optimal control problems has been implemented in a Matlab based pseudospectral optimal control software known as PROPT, together with the NLP solver SNOPT.

PROPT is a combined modeling, compilation and solver engine for generation of highly complex optimal control problems. Further information about PROPT can be found in [13]. PROPT software transcribes the optimal control structure of this study automatically to corresponding NLP. The resulted NLP is solved by the SNOPT solver without having to worry about the mathematics of the solver. In other words, once a problem has been properly defined, PROPT will take care of all the necessary steps in order to return a solution.

In order to obtain a solution to the optimal control problem of Eq. (7), as efficiently as possible, while obtaining an accurate solution, 90 Legendre-Gauss collocation points are chosen. While it is beyond the scope of this paper to provide a detailed explanation of various pseudospectral methods and their accuracy, more detailed information can be found in [10,11,14].

4. Simulations and results

In the current section, the results of simulation with PROPT software are presented. Using a rotational speed of 250 rad/s, initial positions of:

$$[x_1, x_2, y_1, y_2] = [0.35, -0.35, -0.35, 0.35] \text{ mm}$$

and model parameters of the rotor and electromagnetic actuators given in [1], the response of the system is depicted in Figures 3 to 6 for x_1, y_1, x_2, and y_2. It can be seen from the figures that the position states are converging to zero after little time. In Figures 7 to 10, the control currents for magnetic bearings are given. Control currents oscillate in order to compensate for the unbalance force and keep the rotor at the bearing centre without exceeding their specified limits of 2 A. The advantage of the used approach over that of [1] is that the control current of [1] oscillates between the positive and negative current values in order to maintain the rotor at the centre of the bearings, which may be damaging to the switching circuits.

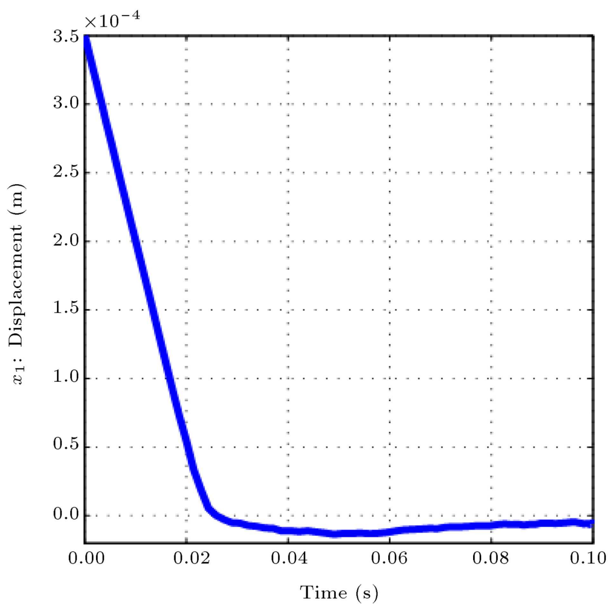

Figure 3. x_1 displacement graph.

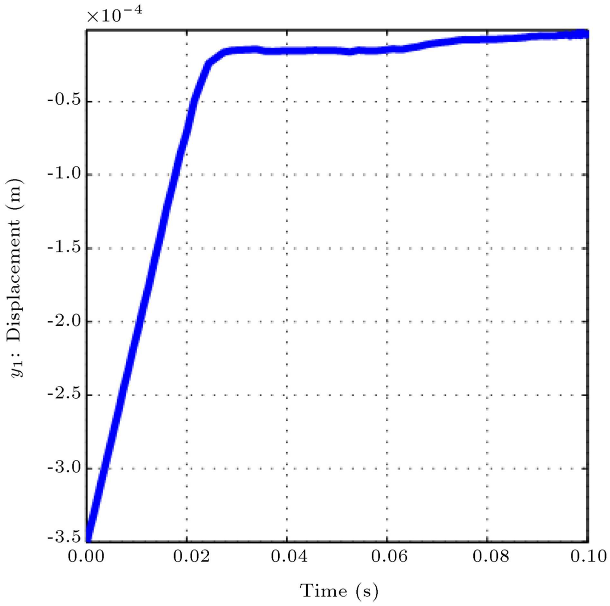

Figure 4. y_1 displacement graph.

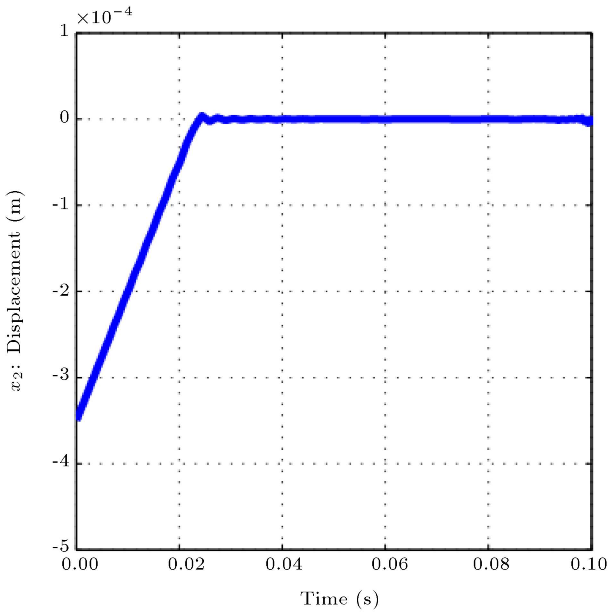

Figure 5. x_2 displacement graph.

This simulation was run on a laptop with a Windows® 7 Operating System (OS), an Intel Core i5 2.27GHz processor, and 4GB of Random Access Memory (RAM). The mean computation time for the 90-node solution is approximately 45 seconds, meaning that the computational time is low. The value of the cost function for strictly penalizing the AMB system

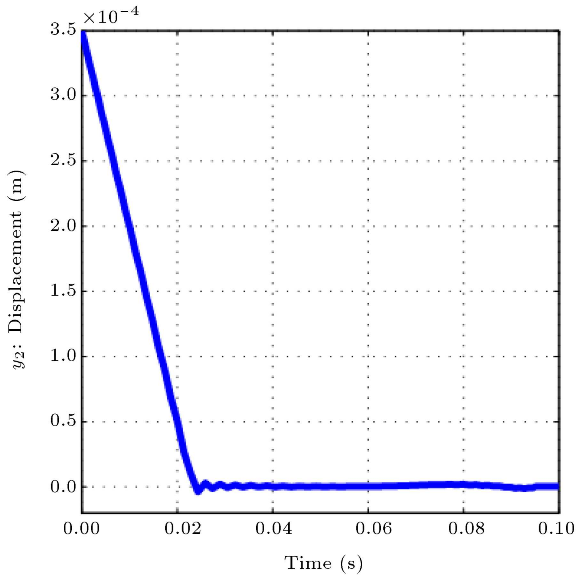

Figure 6. y_2 displacement graph.

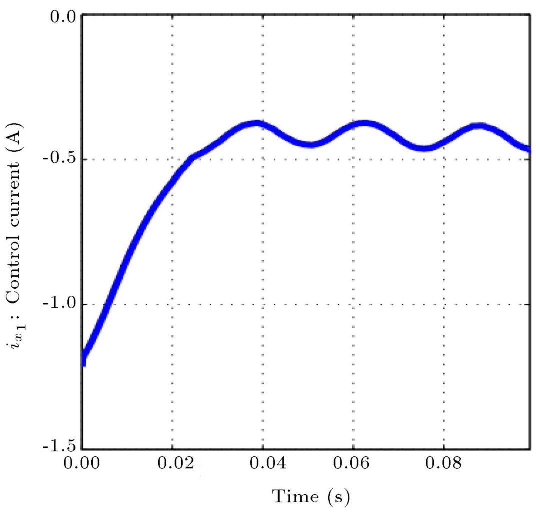

Figure 7. i_{x_1} control current.

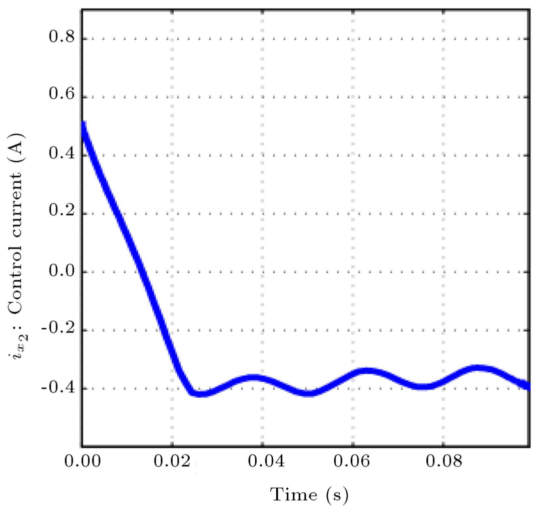

Figure 8. i_{x_2} control current.

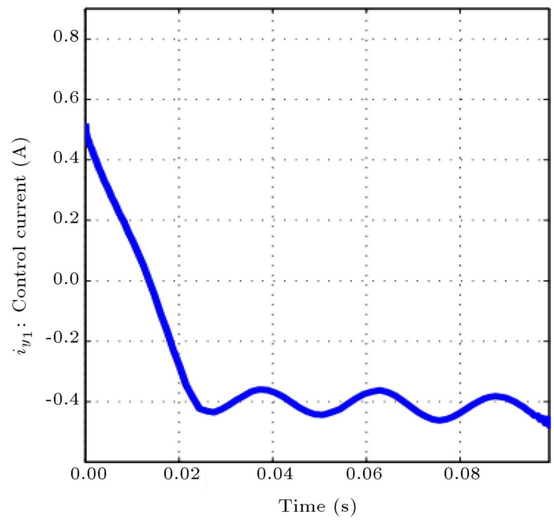

Figure 9. i_{y_1} control current.

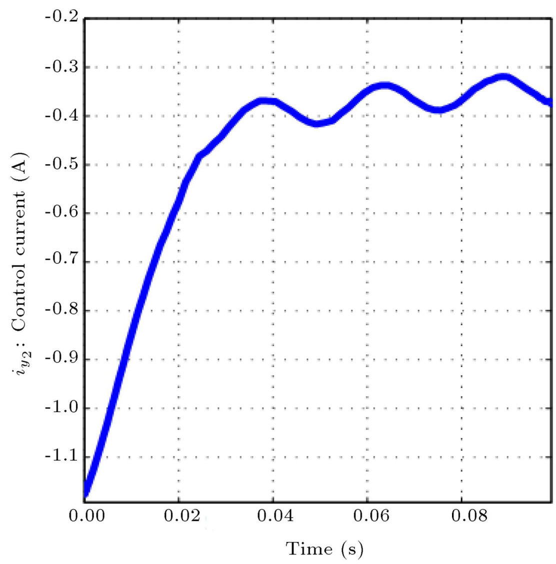

Figure 10. i_{y_2} control current.

with $\mathbf{Q} = \text{diag}\{1, 1, 1, 1\} \times 10^9$, is 12.36, which means that the precision positioning for the rotor is promising.

5. Conclusion

In this study, we have modeled the dynamics of AMBs in PROPT software and formed an optimal control approach in order to stabilize the AMB systems. The constraints considered in this paper are the limitations on the values of control currents and displacements for each end of the rotor. The simulation results show the success of the method, even for disturbances.

References

1. Tombul, G.S., Banks, S.P. and Akturk, N. "Sliding mode control for a class of non-affine nonlinear

systems", *Nonlinear Analysis: Theory, Methods & Applications*, **71**, pp. e1589-e1597 (2009).

2. Nagi, F.H., Inayat-Hussain, J.I. and Ahmed, S.K. "Fuzzy bang-bang relay control of a single-axis active magnetic bearing system", *Simulation Modelling Practice and Theory*, **17**, pp. 1734-1747 (2009).
3. Hsu, C.-T. and Chen, S.-L. "Nonlinear control of a 3-pole active magnetic bearing system", *Automatica*, **39**, pp. 291-298 (2003).
4. Jugo, J., Lizarraga, I. and Arredondo, I. "Nonlinear modelling and analysis of active magnetic bearing systems in the harmonic domain: A case study", *IET Control Theory & Applications*, **2**, pp. 61-71 (2008).
5. Lin, F.J., Chen, S.Y. and Huang, M.S. "Intelligent double integral sliding-mode control for five-degree-of-freedom active magnetic bearing system", *IET Control Theory & Applications*, **5**, pp. 1287-1303 (2011).
6. Lee, C.W. and Jeong, H.S. "Dynamic modeling and optimal control of cone-shaped active magnetic bearing systems", *Control Engineering Practice*, **4**, pp. 1393-1403 (1996).
7. Inayat-Hussain, J.I. "Chaos via torus breakdown in the vibration response of a rigid rotor supported by active magnetic bearings", *Chaos, Solitons & Fractals*, **31**, pp. 912-927 (2007).
8. Cimen, T. and Banks, S.P. "Nonlinear optimal tracking control with application to super-tankers for autopilot design", *Automatica*, **40**, pp. 1845-1863 (2004).
9. Betts, J.T. "Survey of numerical methods for trajectory optimization", *Journal of Guidance, Control, and Dynamics*, **21**, pp. 193-207 (1998).
10. Benson, D. "A Gauss pseudospectral transcription for optimal control", Ph.D. Thesis, Massachusetts Institute of Technology, United States, Massachusetts (2005).
11. Huntington, G. "Advancement and analysis of a Gauss pseudospectral transcription for optimal control problems", Ph.D. Thesis, Massachusetts Institute of Technology, United States, Massachusetts (2007).
12. Gill, P.E., Murray, W. and Saunders, M.A. "SNOPT: An SQP algorithm for large-scale constrained optimization", *SIAM Review*, **47**, pp. 99-131 (2005).
13. Rutquist, P.E. and Edvall, M.M. "PROPT - Matlab optimal control software", *Tomlab Optimization* (2010).
14. Garg, D., Patterson, M., Hager, W.W., Rao, A.V., Benson, D.A. and Huntington, G.T. "A unified framework for the numerical solution of optimal control problems using pseudospectral methods", *Automatica*, **46**, pp. 1843-1851 (2010).

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