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Analytical investigation into simultaneous effects of friction and heating on a supersonic nucleating Laval nozzle

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Abstract. In supersonic water vapor flow of low pressure turbines, nucleation phenomena and consequent condensation are commonly observed. Internal heat transfer, which is caused by phase change, is strongly irreversible and has unwanted effects on turbine efficiency. Also, the strike of formed droplets on the surfaces results in large amounts of mechanical damage. Condensation heat release to supersonic flow, named condensation shock, leads to considerable pressure rise, which, in turn, reduces outlet velocity and occasionally causes severe oscillations, making the flow supercritical. The authors have presented a novel analytical approach for the reduction of these unwanted results in Laval nozzles by volumetric heating of the convergent section. In this paper, and in continuation of the series of papers by the same authors, one dimensional, supersonic and two-phase flow is modeled analytically, and the simultaneous effects of volumetric heat transfer and friction in the convergent nozzle are investigated. It is concluded that the simultaneous use of friction and volumetric heating can be an appropriate and useful technique for the control of two-phase flow conditions, keeping them within the desired range.

1. Introduction

Generating electricity using steam turbines is a common method of electricity production. Steam at high temperature and pressure is expanded inside the turbines and the mechanical work in the generators is converted to electrical energy.

Due to the decrease in the pressure and temperature of the steam caused by expansion in the low pressure steam turbines, the saturation line is crossed. The steam enters the saturation zone when it is in a single phase and unstable. When the steam flow is supersonic, this phenomenon causes the steam to become even more unstable. Under this condition, when supercooling reaches its maximum magnitude, suddenly and spontaneously, the initial nuclei of the droplets are formed and, due to condensation, the steam flow becomes two-phase.

Condensation, the phase change of vapor in power production cycles and their relevant phenomena lead to several technical and scientific issues, which have attracted great interest during the last century [1-8]. The condensation process starts with nucleation, which can be homogeneous or heterogeneous. In homogeneous nucleation, thermal fluctuations of supersaturated vapor form the stable nucleus, while, in heterogeneous nucleation, vapor molecules incline on the existing surfaces.

It must be noted that nucleation always occurs,

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but it is only in supersaturated flow that the nucleus can become stable and grow into water droplets. Following the formation of a stable nucleus, the growth process begins. The formed nucleus absorbs supersaturated vapor molecules and grows in the form of water droplets. This process reduces the degree of supersaturation and finally causes a two-phase equilibrium in a supersonic convergent-divergent nozzle, as given in reference [9].

Also, the latent heat of condensation increases the temperature of the droplets and surrounding vapor, which can reduce the degree of supersaturation, as well as nucleation rate. On the other hand, heat release to supersonic flow causes deceleration. This, consequently, raises the pressure, which is called condensation shock. These processes create much mechanical damage and strongly increase thermodynamic and aerodynamic losses [7].

Building on previous studies [1,2,3], and in continuation of the authors' recent publications [1,10], in this paper, using an analytical Lagrangian-Eulerian model, the condensation phenomenon in supersonic Laval nozzles is investigated one dimensionally, and, for the first time, the simultaneous effects of volumetric heating and friction in the convergent section on the flow specifications is investigated analytically.

The main goal of this theoretic research is to qualitatively inspect and study the effects of heat transfer and friction on the important parameters of two-phase water vapor, such as their effects on nucleation rate, droplet diameter, wetness fraction and condensation shock. It will be shown that thermodynamic losses could be controlled by this technique, due to reduction of condensation shock losses.

Friction and heating are only imposed on the converging section as the flow becomes two-phase in this section of the nozzle. In other words, in the converging section, the steam flow is dry and single phase, but friction and heating are present. However, in the diverging section, due to the presence of nucleation, the flow is two-phase and no internal or external element, such as heating or friction, is present. Therefore, it is under these conditions that we can analyze and study the effects of direct friction and heating on nucleating flow. Obviously, imposing friction and heating on the diverging section (in addition to the converging section) increases the effects of these two elements on the two-phase flow.

The findings of this research can be used in the process of designing converging diverging channels for different purposes, such as generating liquid droplets with specific radius or eliminating condensation shock. Obviously in real-world cases, friction and heating can be imposed along the channel, but, this research gives an idea how to control and engineer the magnitude of friction and heat transfer at different locations.

2. Methodology

Condensing steam flow is considered a continuum flow of vapor containing small droplets of liquids with the same velocities. Assuming steady and one dimensional flow, the gas dynamics of the flow are derived. Also, using suitable equations for nucleation rate and droplet growth, the condensation rate is determined.

Gas dynamic equations are solved in an Eulerian coordinate, while the droplet growth equations are performed in a Lagrangian frame. Therefore, the model is a combined Eulerian-Lagrangian model. Volumetric heat transfer is applied to gas dynamics equations as a heat source [10]. Since the solution is one dimensional, the surface integral of the heat source intensity ($Qc = \dot{q}A$) with the unit (J/mm.s) is used for the evaluation of heating rate.

To combine the two frameworks (Eulerian-Lagrangian), the nozzle length is divided into a large number of small elements, and, at each step, all flow variables, such as density, ρ, velocity, u, pressure, P and temperature, T, are determined by gas dynamics equations. By calculating pressure and temperature, the Lagrangian equations are solved and the droplet size distribution and wetness fraction, w, are obtained. Considering a one dimensional control volume with length dx (Figure 1), and assuming that the occupied volume by the liquid phase is negligible compared to the volume of the gas phase [8,9], the basic governing equations of gas dynamics are written as follows.

2.1. Mass balance

The mass balance along the axis of a one dimensional control volume is presented as:

$$M_L = \rho_G U_G A = \text{Const}, \tag{1}$$

where M is the mass flow rate, and subscripts L and G refer to the liquid and gas phases, respectively.

Differentiating the above equation and performing a few mathematical operations, the differentiation form

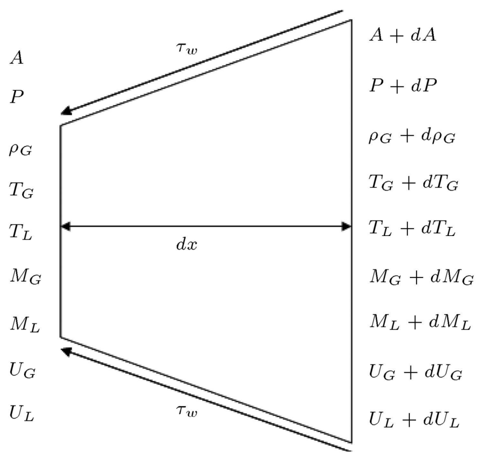

Figure 1. Control volume of flow in Eulerian framework.

of the mass conservation law for one dimensional condensing flow is obtained as:

$$\frac{d\rho_G}{\rho_G}+\frac{dA}{A}+\frac{dU_G}{U_G}+\frac{dM_L}{M_G}=0. \tag{2}$$

2.2. Momentum balance

The momentum balance for a one dimensional control volume is given as:

$$\frac{dP}{P}+\frac{f\rho_G U_G^2}{2PD_e}dx+\frac{(M_G)U_G}{AP}\frac{dU_G}{U_G}+\frac{(M_L)U_L}{AP}\frac{dU_L}{U_L}=0. \tag{3}$$

Neglecting the slip velocity between the phases due to the microscopic size of the liquid droplets:

$$\frac{dP}{P}+\frac{f\rho_G U_G^2}{2PD_e}dx+\frac{(M_G+M_L)}{AP}dU_G=0, \tag{4}$$

where f and D_e are the wall friction factor and channel's hydraulic diameter, respectively.

2.3. Energy balance

Imposing the energy balance on one dimensional, compressible, non-adiabatic and steady state nucleating flow, the energy equation is written as:

$$d\left[(M-M_L)\left(h_G+\frac{U_G^2}{2}\right)+M_L\left(h_L+\frac{U_L^2}{2}\right)\right]+d\dot{Q}=0. \tag{5}$$

Neglecting the slip velocity and doing some mathematical operations:

$$\frac{dT_G}{T_G}+\frac{P}{\rho_G C_P T_G}\left(1-\frac{Y}{X}\right)\frac{dP}{P}+\frac{U_G^2}{C_P T_G}\frac{dU_G}{U_G}+\frac{d\dot{Q}}{(M_L+M_G)C_P T_G}=\frac{L_h}{C_P T_G}\frac{dM_L}{(M_L+M_G)}, \tag{6}$$

where L_h is the latent heat, C_P is the specific heat and $d\dot{Q}$ is the rate of heat transfer in differential form as follows:

$$d\dot{Q}=\dot{q}A\,dx, \tag{7}$$

where $\dot{q}$ $(\frac{\mathrm{J}}{\mathrm{m}^3.s})$ is the volumetric heat transfer rate.

2.4. Equation of state

A density based virial state equation can be used for supercooled steam [9]:

$$\frac{P}{\rho_G R T_G}=1+B_1\rho_G+B_2\rho_G^2, \tag{8}$$

where B_1 and B_2 are virial density coefficients and are functions of the vapor temperature. The state equation in a differential form is given by:

$$\frac{dP}{P}-X\frac{d\rho_G}{\rho_G}-Y\frac{dT_G}{T_G}=0, \tag{9}$$

where:

$$X=\frac{\rho_G}{P}\left(\frac{\partial P}{\partial\rho_G}\right)_{T_C}=\frac{1+2B_1\rho_G^2+3B_2\rho_G^2}{1+B_1\rho_G^1+B_2\rho_G^2}, \tag{10}$$

$$Y=\frac{T_G}{P}\left(\frac{\partial P}{\partial\rho_G}\right)_{\rho_G}=1+\frac{\rho_G T_G}{1+B_1\rho_G^1+B_2\rho_G^2}\times\left[\frac{dB_1}{dT_G}+\rho_G\frac{dB_2}{dT_G}\right]. \tag{11}$$

2.5. Vapor much number

The speed of sound in the vapor phase, assuming that the impact of the liquid phase is negligible, is known as the frozen speed of sound, and is defined as follows:

$$Z=Ma^2=\left(\frac{U_G}{C}\right)^2, \tag{12}$$

where C is the frozen speed of the sound and which can be expressed as:

$$C=\left(\frac{\partial P}{\partial\rho_G}\right)^{0.5}=RT_G(1+2B_1\rho_G+2B_2\rho_G^2). \tag{13}$$

Finally, the differential form of the Mach number equation is derived as:

$$\frac{dZ}{Z}=2\frac{dU_G}{U_G}+\frac{d\rho_G}{\rho_G}-\frac{dP}{P}+\frac{d(1+B_1\rho_G+B_2\rho_G^2)}{1+B_1\rho_G+B_2\rho_G^2}-\frac{d(1+2B_1\rho_G+2B_2\rho_G^2)}{1+2B_1\rho_G+3B_2\rho_G^2}. \tag{14}$$

Based on the obtained equations (Eqs. (2), (4), (6), (9) and (14)), the velocity, pressure, temperature and density fields are calculated in an Eulerian framework. The mass flow rate of the liquid phase is unknown in the equations and is obtained in a Lagrangian framework, as will be explained in the following sections.

2.6. Nucleation model

Nucleation is a mechanism to form a new phase within a metastable main phase. Despite significant advances in experimental equipment and computational methods, the classical nucleation theory presented by several researchers, for example Zeldovich [11], is still reliable. Furthermore, many researchers work on the nucleation theory but their studies are based on the classical theory of nucleation.

The classical nucleation theory has two thermodynamic and kinetic aspects, and the changes of free energy of the system, due to the appearance of a droplet of radius r, as shown in Figure 2, are as follows:

$$\Delta G=\Delta G_v+\Delta G_s=-m_r R T_G Ln\left(\frac{P}{P_s(T_G)}\right)+4\pi r^2\sigma_r, \tag{15}$$

where m_r is the mass of a droplet, and σ_r is the surface tension of the liquid droplet, which is equivalent to the surface tension of the bulk of water.

The classical (standard) nucleation rate can be expressed in terms of the required free energy for critical cluster formation (ΔG^*) by means of Boltzmann's expression as:

$$J_{\mathrm{st}} = K\exp\left(-\frac{\Delta G^*}{kT_G}\right), \tag{16}$$

where the kinetic factor, K, depends mainly on the impingement rate of vapor molecules and the effective surface area of the critical cluster, and k is Boltzmann's constant [11]:

$$K = \sqrt{\frac{2\gamma_\infty}{\pi m}}v_m\left(\frac{P_G}{kT_G}\right)^2, \tag{17}$$

where v_M and m are the molecular volume and molecular mass, respectively. Also, ΔG^* can be calculated as:

$$\Delta G^* = \frac{16\pi\sigma_r^3}{\rho_L kT\ln S}. \tag{18}$$

The number of molecules in the critical cluster is expressed by the Gibbs-Thomson equation as:

$$n^* = \frac{32\pi v_L^2\sigma_r^3}{3\Delta\mu^3}, \tag{19}$$

where:

$$\Delta\mu = kT_G\ln\left(\frac{P_G}{P_s(T_G)}\right). \tag{20}$$

Many researchers have worked to modify the classical theory of nucleation, such as Kantrowitz [12], Courtney [13], Girshick and Chiu [14,15] and Wölk et al. [16].

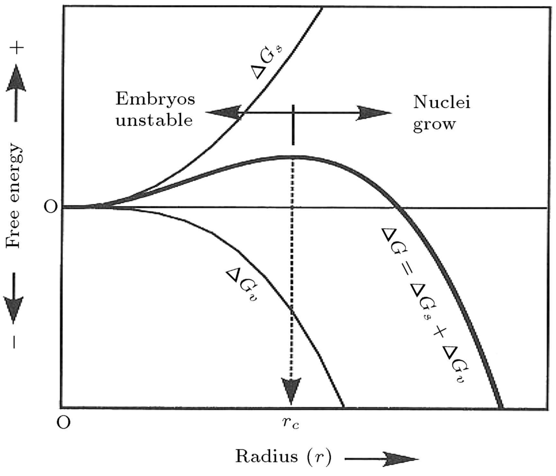

Figure 2. Gibbs free energy diagram of droplet formation.

Kantrowitz proposed a non-isothermal correction factor and Courtney considered cluster compressibility. The combination of Kantrowitz and Courtney's corrections led to [12,13]:

$$J_{Ka-Co} = \frac{\rho_s(T_G)}{\rho_G\times(1+\phi)}J_{\mathrm{st}}, \tag{21}$$

where ϕ is defined as follows:

$$\phi = \frac{q_c\rho_G}{\alpha_r}\left(\frac{RT_G}{2\pi}\right)^{0.5}\left(\frac{L_h^2}{RT_G^2}-\frac{L_h}{2T_G}\right), \tag{22}$$

where α_r is the heat transfer coefficient, q_c is the condensation coefficient, which, in this research, is considered as unity ($q_c = 1$) [2].

Girshick and Chiu have obtained an additional factor of $1/S$ in the classical expression for the nucleation rate, where S is the vapor supersaturation. In addition, they attempted to achieve the self-consistency of the classical theory and suggested the following expression [14,15]:

$$J_{\mathrm{GC}} = \frac{e^\theta}{S}J_{\mathrm{st}}, \tag{23}$$

where:

$$\theta = \frac{(36\pi)^{\frac{1}{3}}v_L^{\frac{2}{3}}\sigma_r}{kT_G}. \tag{24}$$

Wölk and Strey analyzed the data's temperature dependence and presented an empirical fitting function for H_2O, which brings the classical nucleation expression into a close agreement with experiment:

$$J_{H_2O} = J_{\mathrm{st}}\exp\left(-27.56+\frac{6.5\times10^3}{T_G}\right). \tag{25}$$

In this research, for the given condition, three nucleation models are used in order to find the best, as shown in Figure 3.

In Figure 4, the pressure ratios related to each model are presented and the results are also compared with experimental data as used in reference [3].

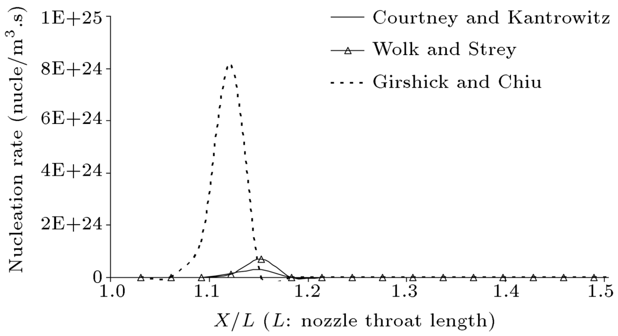

Figure 3. Nucleation rate along the nozzle for different nucleation models.

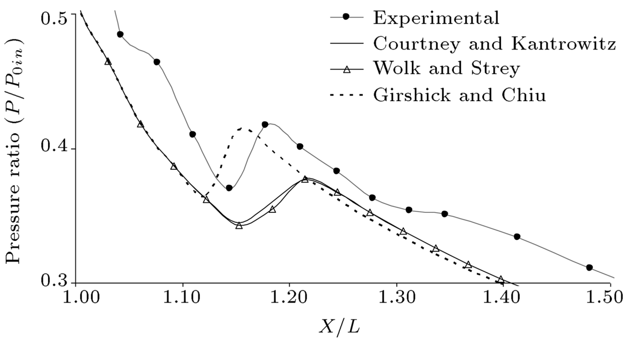

Figure 4. Pressure ratio along the nozzle for different nucleation models.

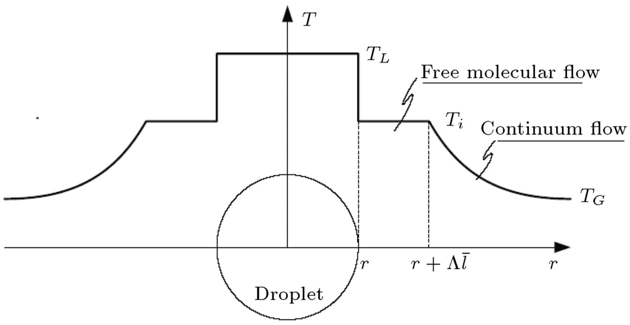

Figure 5. Temperature distribution around the droplet.

Regarding Figure 4, the model of Girshick and Chiu works better for prediction of the pressure ratio along the nozzle and, especially, predicts the place of condensation shock more precisely. Therefore, in this research, the model of Girshick and Chiu is used.

2.7. Droplet growth

Droplet growth is often as important as the nucleation phenomena, and the development of this process is less complicated than the nucleation. The growth of the formed droplets is governed by heat and mass transfers between droplets and the bulk vapor, which is dependent on its Knudsen number, which is the vapor mean free path divided by the droplet's diameter.

When vapor molecules condensate on a droplet, the latent heat of condensation transfers to the droplet, which raises its temperature. Consequently, there is heat transfer from the droplet to the surrounding vapor that governs the growth rate of the droplet. Figure 5 represents a schematic view of the temperature variation around the droplet.

For very small Knudsen numbers, there is a continuum flow regime, and diffusion is the determinant factor of mass transfer based on Fick's law. For very large Knudsen numbers (free molecular regime), the droplet growth is governed by kinetic processes of vapor molecule collision on the droplet surface.

In this regard, Bakhtar and Zidi [9] proposed a reliable droplet growth model for a wide range of Knudsen numbers. They used the continuum and free molecular equations and a correction factor, $f(\mathrm{Kn})$, to introduce a general equation for the whole range of Knudsen numbers (Kn no.) as the following:

$$\frac{dr}{dt} = \frac{\mathrm{Kn}}{\mathrm{Kn} + 0.375 q_c \mathrm{Sc}} \frac{q_c}{\rho_L} \left(\frac{R}{2\pi}\right)^{0.5} \left[\rho_G \sqrt{T_G} - \rho_s(T_L, r)\sqrt{T_L}\right], \tag{26}$$

where T_L is the temperature of the droplet and Sc is the Schmidt number.

Furthermore, for predicting the temperature of a droplet, T_L, surrounded by vapor having a pressure equal to P and temperature equal to T_G, several researchers (e.g. [3,7,8]) have used the following simplified semi-empirical equation:

$$T_L = T_s(P) - \{T_s(P) - T_G\} \frac{r^*}{r}. \tag{27}$$

The droplet radius and temperature can be calculated by simultaneous solving of Eqs. (26) and (27). In addition, the total specific entropy of the two-phase flow is calculated by the following equation, which is described in reference [3].

$$s = (1 - w) s_G + w s_L + \frac{\dot{S}_s}{M}, \tag{28}$$

where $\dot{S}_s$ denotes the rate of entropy generation caused by surface formation, s_L is the specific entropy of liquid, s_G is the specific entropy of the vapor phase and M is the total mass flow rate.

Using the state equation and thermodynamic relations, the specific entropy of the vapor phase is calculated from:

$$s_G = R\left[(-B_1 \rho) + \left(-\rho \frac{dB_1}{dT}\right)\right] + s_0, \tag{29}$$

where s_0 can be obtained from the following equation:

$$s_0 = 0.30773 - 0.46153 Ln(\rho) + 1.1095 Ln(T_G) + 7.11756^* 10^{-4} T_G - \frac{3495}{T_G^2}. \tag{30}$$

On the basis of classical thermodynamics, the specific entropy of a water droplet at temperature T_L can be approximated by:

$$s_L = C_L Ln\left(\frac{T_L}{T_D}\right). \tag{31}$$

In the above equation, C_L is the water's specific temperature and T_D is the datum temperature at 273.15 Kelvin.

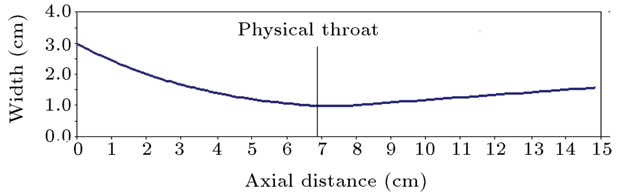

Figure 6. The geometry of convergent divergent nozzle.

Table 1. Different investigated cases.

	Friction factor (f)	**Q_s (J/mm.s)**	**Mass flow rate (gr/s)**
Frictionless-adiabatic	0.00	0	34.2
Case 1	0.00	240	33.0
Case 2	0.05	0	33.0
Case 3	0.02	140	33.0
Case 4	0.05	240	31.5

Based on surface thermodynamics, the specific entropy of droplet surfaces is obtained from [17]:

$$S_s = -\dot{A}_D \left(\frac{\partial \sigma}{\partial T} \right)_P , \tag{32}$$

where $\dot{A}_D$ and σ denote droplet surface and surface tension, respectively.

3. Results and discussion

For investigation into the effects of volumetric heating on flow, $Q_s = \dot{q}A \left(\frac{J}{mms} \right)$ is introduced for evaluation of the heating rate. In this research, different friction factors and heating rates are imposed onto the convergent section of a supersonic, two-phase Laval nozzle (Figure 6) with the inlet stagnation properties of $P_{0in} = 70.67$ kPa and $T_{0in} = 401.5$ K.

Table 1 presents the mass flow rate of the nozzle in each test case. In test case 1, the frictionless flow is heated. In test case 2, the effect of friction on the adiabatic flow is investigated to obtain the same mass flow rate as in test case 1. In test case 3, the friction factor and heating rate have been chosen to obtain the same mass flow rate as in test cases 1 and 2. Finally, in test case 4, the effects of the heating rate of test case 1, and the friction factor of test case 2 are imposed simultaneously. It is obvious that both friction and heating reduce the mass flow rate.

Figure 7 shows the variations of the nucleation rate along the nozzle. Here, the maximum nucleation rate and its location along the nozzle are important.

In test case 1, heating the frictionless flow reduces the supersaturation degree and nucleation rate significantly and postpones its position.

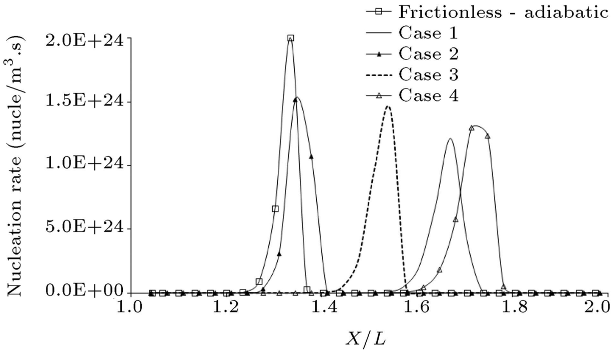

Figure 7. Nucleation rate along the nozzle for different investigated cases.

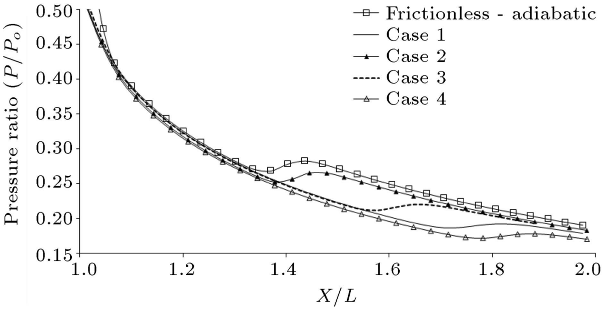

Figure 8. Pressure ratio along the nozzle for different investigated cases.

In test case 2, friction in the convergent section of adiabatic flow reduces a small amount of supersaturation and, consequently, suppresses nucleation rate and postpones the nucleation zone marginally.

In test case 3, a combination of heating rate and friction rate produce the same mass flow rate as in the previous test cases. The nucleation rate and nucleation zone of this case are between the previous cases.

Test case 4 is a combination of test cases 1 and 2, and its effects on nucleation are stronger than both cases, such that the nucleation zone is postponed by approximately the sum of cases 1 and 2.

Regarding the relationship of the condensation shock location and nucleation zone, it is expected that the condensation shock will behave similar to the nucleation. Figure 8 shows the pressure ratio along the nozzle and represents the same behavior as the nucleation diagram.

Droplet radius is another parameter that must be treated similar to nucleation. In fact, when nucleation occurs sooner, there is more time for droplet growth. Figure 9 shows postponing of the nucleation zone, which makes the droplets smaller.

The wetness fraction along the nozzle is shown in Figure 10. This parameter is controlled by the number and size of the droplets. In the frictionless adiabatic case, the nucleation rate is greater and this translates to more droplets. Also, in this case, the nucleation occurs sooner and the droplets have more time to grow.

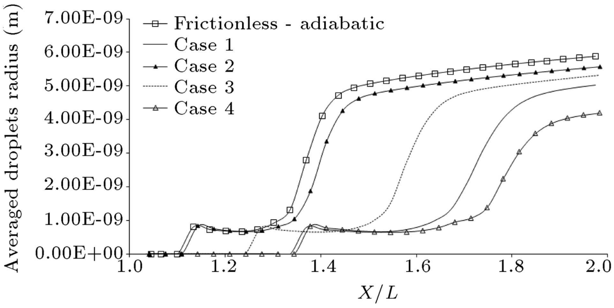

Figure 9. Averaged droplet radius along the nozzle for different investigated cases.

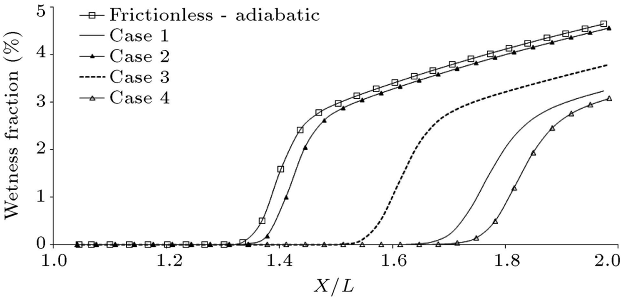

Figure 10. Wetness fraction along the nozzle for different investigated cases.

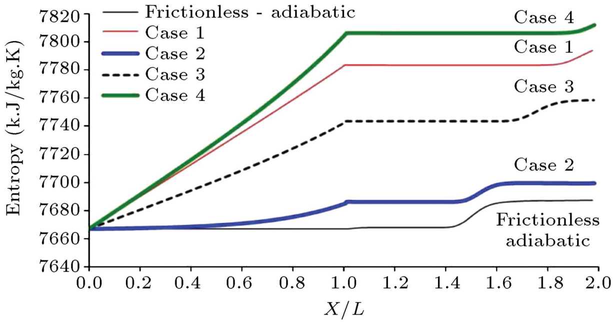

Figure 11. Flow entropy along the nozzle for different investigated cases.

Because of these two factors, the maximum wetness fraction is observed in the frictionless adiabatic case. Regarding the similarity of test cases 1, 2, 3 and 4, the droplet radius governs the wetness fraction in these cases. Therefore, the wetness fraction in these cases behaves like the droplet radius.

Figure 11 represents the entropy variations along the nozzle. Both volumetric heating and friction increase the entropy in the convergent section, which can be seen in Figure 11. In the divergent section, the internal heat transfer between phases causes an increase in entropy in the nucleation and condensation zones. However, the divergent section is adiabatic; therefore, the entropy increase in the divergent section is equal to the entropy generated by the nucleation and droplet growth (condensation) processes.

Entropy generation in the divergent section is a good factor for evaluation of thermodynamic losses. This parameter is shown in Table 2 and the generated entropy depends on nucleation and droplet growth rates. Regarding Table 2, the most thermodynamic loss occurs in the frictionless adiabatic case, where maximum rates of nucleation and droplet growth are observed. The least generated entropy occurs in test case 4, where the minimum wetness fraction and postponed nucleation zone are observed.

Table 2. Entropy generation in divergent nozzle.

	Friction factor (f)	**Q_s (J/mm.s)**	**Sgen (J/kg.K)**
Frictionless-adiabatic	0.00	0	19.884
Case 1	0.00	240	10.422
Case 2	0.05	0	13.229
Case 3	0.02	140	14.904
Case 4	0.05	240	5.778

Generated entropy in test case 4 is less than that of test case 1, which corresponds with the nucleation and droplet growth rates, as shown in Figures 7 and 9. Entropy generation in test cases 2 and 3 are nearly similar.

To better present the quantitative effects of friction and heat on the two-phase phenomenon, particularly their effects on the location of maximum nucleation, droplet radius and wetness fraction percentage, a summary of the results is given in Table 3.

4. Conclusion

According to test cases 1 and 2, heating and friction in the convergent section of a Laval nozzle can postpone the nucleation zone, reduce the nucleation and droplet growth rate and finally suppress the entropy generation of the condensation process. Regarding the constant mass flow rates, the volumetric heating influences are stronger.

In test case 3, the heating rate and friction factor have been chosen in order to deliver the same mass flow rate as test cases 1 and 2. Under these conditions, postponement of the nucleation zone, a decrease in its production and a reduction in droplet growth rate are observed. The significant finding is that the nucleation zone of test case 3 is somewhere between the location of the first and second test cases. That is to say, when we make the mass flow rate constant, the friction coefficient changs from 0.05 to zero and, simultaneously, by increasing the heating rate from zero to 240 J/mm, the nucleation zone will be made to move from the location in test case 2 (pure friction) to the location of test case 1, which is pure heating.

In test case 4, without making the mass flow rate

Table 3. The effects of heating and friction factor on two-phase parameters relative to adiabatic frictionless case.

Case	2-phase parameter			
	Shift of max. nucleation location (%)	Shift of condensation shock (%)	Reduction of wetness fraction at outlet (%)	Reduction of mean droplet radius at outlet (%)
Case 1	25	26	31	17
Case 2	1.3	1.5	4	7
Case 3	15	15.5	22	13.5
Case 4	30	30.5	35	31.0

constant, conditions of 240 J/mm.s heating and friction coefficient 0.05 are used simultaneously. Consequently, the nucleation zone in this state has been postponed almost as much as the sum of the first and second test cases. Furthermore, the rate of entropy in this case is the least among all the test cases, due to the additional effects of volumetric heating and friction.

As a final point, it is concluded that the simultaneous use of friction and volumetric heating can be an appropriate and useful technique for the control of two-phase flow conditions and to keep them within the desired range.

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Nomenclature

A	Area
C_p	Specific heat at constant pressure
D_e	Equivalent diameter
f	Friction factor
ΔG	Change in Gibbs free energy
J	Rate of formation of critical droplets per unit volume and time
Kn	Knudsen number (mean free path/droplet diameter)
L_h	Latent heat
Ma	Mach number
m_r	Mass of droplet
P	Vapour pressure
f	Friction factor
$P_s(T_G)$	Saturation pressure at T_G
q_c	Condensation coefficient
R	Gas constant for water vapour
r	Radius of droplet
T	Temperature
$T_s(P)$	Saturation temperature at P
ΔT	Degree of supercooling $[T_s(P)\text{-}T_G]$
t	Time
U	Velocity
v	Specific volume
Q_s	Volumetric heating (J/mm.s)
$\dot{q}$	Volumetric heat transfer rate (J/m^3.s)
$\dot{Q}$	Total heat transfer rate (J/s)
M	Total mass flow rate
x	Distance along nozzle axis
L	Nozzle throat length
X, Y	Functions of temperature and density in equation of state
α_r	Coefficient of heat transfer
γ	Isentropic component
μ_G	Kinematic viscosity of vapour
ζ	Dryness fraction
ρ	Density of mixture
λ	Coefficient of thermal conductivity
σ_r	Surface tension of droplet
$\rho_s(T_L, r)$	Density corresponding to saturation pressure at temperature T_L over a surface of curvature r
Sc	Schmidt number

Subscripts

G	Vapour phase
L	Liquid phase
0	Stagnation condition
S	Saturation

Superscripts

$*$	Critical droplet

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