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Theoretical and experimental investigation of density jump on an inclined surface

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Abstract. The density jump on an inclined surface is analyzed using an integral method by applying mass and momentum conservation equations. The jump occurs in a two-layered fluid flow in which the upper layer is stagnant and very deep. A relation is derived, which gives the conjugate depth ratio as a function of inlet densimetric Froude number, inlet concentration ratio, bed slope and entrainment. A set of experiments are performed to verify the relation. The theory and the measurements are in good agreement. The analysis reveals that increasing the surface inclination results in a decrease in the conjugate depth ratio. This analysis also shows that the densimetric Froude number just after the jump is a function of the inlet densimetric Froude number and surface inclination and not inlet concentration. The model predicts a critical Froude number of 1.12 for horizontal internal hydraulic jumps in salt-water density flows. It also reveals that the critical Froude number for internal hydraulic jumps in salt-water density flows, increases with surface inclination and decreases with inlet concentration of the flow.

1. Introduction

Any current, in either a liquid or a gas that is kept in motion by the force of gravity acting on differences in density, is a density or gravity current. Density difference can be due to temperature differences, and dissolved or suspended material. Some examples of density currents are thunderstorm outflows, sea-breeze fronts, estuarine effluences, discharge of industrial wastewater into rivers, lakes or oceans, the sudden release of a foreign gas into the atmosphere [1], avalanches of airborne snow particles, fiery avalanches, and base surges formed from gases and solids issuing from volcanic eruptions. Gravity currents are important in aircraft safety, atmospheric pollution, entomology and pest control [2], and smoke control systems [3,4]. In reservoirs and lakes, density currents are important to the management of water quality [5]. Density currents may be supercritical and, therefore, need to undergo an internal hydraulic jump to change the regime of the flow to maintain their momentum.

The jump may be a result of an obstruction, discharge through a gate, or a change in bed slope [6,7].

When a fluid is discharged into a stagnant fluid with slightly different density, an internal hydraulic jump is observed if the flow is supercritical. If two fluids are miscible, there is entrainment from the stagnant fluid into the flowing layer, resulting in a change in the flowing layer density. Therefore, the jump is known as a density jump [6]. Density jumps have been observed in the atmosphere and in the ocean at certain locations; for instance, “Loewe’s Phenomenon” on the slopes of the Polar Ice Cap [8,9], the “Morning Glory” of the Gulf of Carpentaria in the north of Australia [10], and in Scott Reef on the edge of the Australian North-West Shelf [11]. Figure 1(a) shows a density jump forming in

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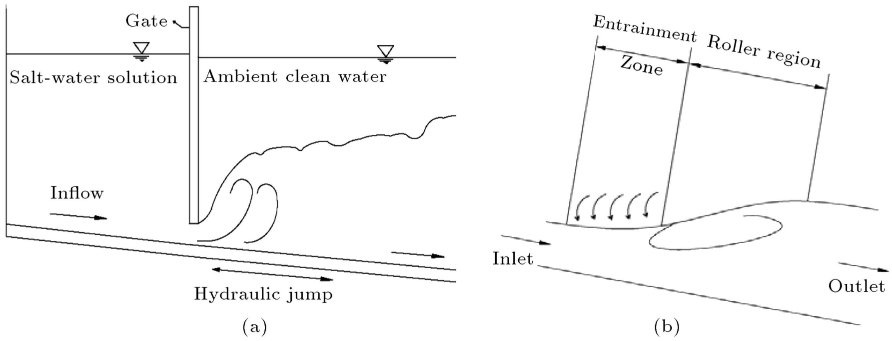


Figure 1. a) Schematic of a density jump, and b) the entrainment zone and the roller region in a hydraulic jump.

the current discharging through a gate, and its regions are depicted in Figure 1(b).

Many investigators have studied horizontal density jumps and have tried to determine their parameters just downstream of the jump, i.e. velocity, depth, and densimetric Froude number, having their relevant parameters just upstream of the jump. Usually, the integral method is used to analyze a density jump. Using mass and momentum conservation relations and considering the miscibility of the two fluid layers results in a system of two equations with three unknowns: velocity, depth, and density after the jump. It should be mentioned that energy equation application is restricted because energy dissipation is not known within a hydraulic jump. Finding the third relation to determine these three unknown parameters has been the objective of many investigations. Applying energy conservation limited to specific assumptions, which has been the subject of discussion among many investigators, provides this relation.

Yih and Guha [12] studied the internal hydraulic jump at the interface of two moving immiscible layers of fluid. As miscibility was ignored, their analysis was complete with mass and momentum conservation equations. Neglecting friction and focusing on a special case in which one layer is moving and the other is stagnant, they developed a relation to calculate the ratio of depth of the moving layer after the jump to the depth of the moving layer before the jump (conjugate depth ratio).

Wilkinson and Wood [6] considered the miscibility of the layers and proposed two regions for a density jump: the entrainment zone and the roller region in which no entrainment occurs (see Figure 1(b)). They discussed the effect of weir height, which is placed downstream of the flow on the density jump. They showed that mass and momentum conservation equations are insufficient to solve the problem completely. With the assumption of critical flow over the weir, they used energy conservation downstream of the jump between the jump and the weir to acquire the third relation to fully solve the problem. They did not give an explicit equation to calculate the jump parameters after the jump, but, with a curve they proposed, one can calculate the Froude number downstream of the jump from the relevant upstream and downstream parameters. They conducted their experiments in a flume 8 ft long, 6.1 ft wide and 3 ft high, and used thermal density current in the laboratory; the warm layer flowing over the cooler ambient layer.

Reveg et al. [13] developed an equation giving conjugate depth ratio as a function of upstream Froude number and entrainment. Their method to go further through the problem was like that used by Wilkinson and Wood [6]. They analyzed two cases: a jump controlled by an obstruction downstream of the jump, and a jump controlled by a contraction downstream of the jump. In both cases, assuming a critical flow over the obstruction and through the contraction, and applying conservation of energy between the jump downstream and the obstruction (contraction), they obtained a second equation relating conjugate depth ratio to the entrainment and downstream conditions. The curves they proposed give entrainment and conjugate depth ratios for relevant upstream parameters and downstream conditions.

Chu and Baddour [14] used a different suggestion to utilize an energy relation. In a density jump, there is an expanding layer and a contracting layer. By placing dye in the contracting layer near the contact surface, they observed no mixing, while the dye placed in the expanding layer mixed considerably. Using this observation, they assumed that the energy loss in the contracting layer is insignificant with respect to that of the expanding layer. They utilized energy conservation for the upper contracting layer within the jump, which enabled them to propose a closure for the problem.

Wood and Simpson [15] investigated several cases experimentally and analytically: a jump at the interface of two moving immiscible layers, a jump at the lee of a towed obstacle with no entrainment, a jump advancing into stationary layers with no entrainment, and a stationary entraining internal jump. In their analysis, they utilized the assumption of Chu and Baddour [14] and used energy conservation for the contracting layer. For the last case, in which we are interested, for the extremely deep upper fluid, their analysis yielded a relation giving downstream Froude number as a function of upstream flow parameters and ratio of conjugate discharges.

Klemp et al. [16], unlike Chu and Baddoure [14] and Wood and Simpson [15], who assumed that all the energy loss occurs within the lower expanding layer, assumed that the energy loss occurs in the upper contracting layer and neglected entrainment. With the aid of mass conservation for both layers and conservation of energy for the expanding layer, they proposed a closure, and claimed that the predicted jump behavior is more accurate than the theories of Chu and Baddour [14] and Wood and Simpson [15] for larger conjugate depth ratios. They used experimental data from Rottman and Simpson [17] and Bains [18] to prove their claim.

Li and Cummins [19], in a theoretical study, showed that the results given by Wood and Simpson [15] and Klemp et al. [16] for internal jump speeds are the upper and lower bounds for the actual data received from experiments. Holland et al. [20] assumed that the most of energy dissipated at the jump goes to turbulence and this energy is limited by a measure of stratification. This assumption enabled them to employ an energy equation for the jump. They proposed bounds for conjugate depth, velocity, and density ratios for two and three dimensional density jumps. Hassid et al. [21] showed that these bounds are not in agreement with experimental data, proposed a different model and developed new bounds. Garcia [7] studied density jumps near a slope discontinuity in both turbidity and saline currents. His observations revealed that when the currents go through the jump, the flow regime changes from supercritical to subcritical, and entrainment in the subcritical region is negligible with respect to that of the supercritical region. By analyzing the vertical structure of saline and turbidity currents, he showed that both the currents with similar initial conditions behave approximately alike before and after the jump. In addition, the conjugate depth ratio of the jump was found to be similar to the relation of Yih and Guha [12] for density jumps.

Thrope [22] treated a horizontal two dimensional density jump occurring in a stratified flow with a dense layer of finite depth moving in an infinitely deep fluid. Utilizing continuous velocity and density profiles upstream and downstream of the jump, he investigated the conditions under which a jump can occur and estimated the loss of energy flux owing to the internal hydraulic jump having its conjugate depth ration. Thrope [23] used the same procedure used in Ref. [22] to investigate an internal hydraulic jump focusing on critical Froude number. Introducing a special kind of function for upstream and downstream velocity and density profiles, and by applying mass and momentum conservations to the internal hydraulic jump, he investigated the jump parameters for some extreme cases.

Barahmand and Shamsai [24] considered bed roughness and analyzed a horizontal density jump in a density current discharging into a deep stagnant fluid, analytically and experimentally. They developed an equation similar to that of Reveg et al. [13], giving conjugate depth ratio as a function of upstream Froude number, entrainment, and bed roughness. Using their experimental results, they calibrated and validated their equation.

Rayson et al. [11] simulated the flow inside a channel separating the north and south of Scott Reef island using a non-hydrostatic 3D code. They concluded that an internal hydraulic jump occurs when the flow passes through converging to a diverging part of the channel. They claimed that this was the first research conducted regarding the hydraulic jump of flow in the gap of two islands due to tide.

Ellayn and Sun [25] experimentally investigated the hydraulic jump of dense flow on a rough bed with wedged-shape baffle blocks. They proposed a new formula relating the height and length of the hydraulic jump based on Froude number and bed roughness. Comparison between smooth and rough bed results illustrated that height ratio and jump length decreases in rough beds. Moreover, the height ratio and length in wedge shaped barriers are greater than those in rectangular barriers.

Nasrabadi et al. [26] investigated the effect of suspended particles on the submerged hydraulic jump, experimentally. Results showed that energy dissipation and the height of density current in the entrance are not dependent on the concentration of particles. But, the length of jump decreases when particles are in the flow. Also, increasing particle concentration decreases maximum velocity through the jump. This showed that bed shear stress and current resistance increase in the presence of particles in the current.

Sumner et al. [27] investigated the hydraulic jump in a submarine density current for the first time. Previously, the effect of hydraulic jump on current was investigated using small scaled experimental research or numerical studies. Results indicated that experimental investigation overestimates the loss in velocity and the increase in height ratio after the jump. Moreover, they suggested using maximum velocity instead of averaged lateral velocity for the calculation of critical Froude number.

Borden et al. [28,29] simulated lock-exchange density current using a 2D and 3D direct numerical simulation method. They showed that global energy loss occurs during a hydraulic jump. In addition, for density ratios more than 0.5, energy is transferred from the contracted to the expanded layer. Based on the results, they improved the two-layer internal bore model. Their improved model represents a more accurate propagating velocity in terms of geometrical parameters, Reynolds number and Schmidt number.

Qu and Chow [30] investigated, numerically, the characteristic of a density jump at a smoke barrier. They compared their results with analytical investigations [3,13]. Due to high temperature conditions, entrainment was higher in comparison with analytical results. To conclude, 2D simulations are consistent with analytical results, while 3D simulation results differ noticeably from analytical results.

Despite the fact that density jumps in the atmosphere have usually been observed on slopes [6-8], little work has been devoted to density jumps on slopes, and to study of the effect of sloping beds on density jumps, which is the main objective of this study.

2. Analysis

The method used to analyze the internal jump on an inclined surface here is the integral method, which is used to analyze a hydraulic jump in one layer flows in an open channel as well. Entrainment is considered and, therefore, the lower layer density changes through the jump. In a hydraulic jump, an energy conservation equation cannot be used due to unknown energy loss, while momentum balance can be applied to the control volume.

Consider a one dimensional internal hydraulic jump on an inclined surface, shown in Figure 2. The upper ambient layer is infinitely deep, thus, no lateral velocity is considered in this layer. Mass flux conservation per unit width for the control volume gives:

$$\rho_1 v_1 h_1 + \rho_a q_e = \rho_2 v_2 h_2, \tag{1}$$

where ρ_1, ρ_2, v_1, v_2, h_1 and h_2 are density, velocity, and depth before and after the jump, respectively, and

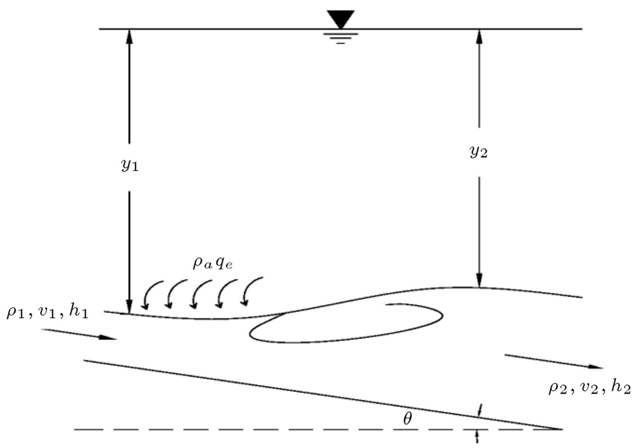


Figure 2. The schematic diagram of 1D internal hydraulic jump on slope.

ρ_a and q_e refer to ambient water density and entraining ambient water volume flux per width, respectively.

Assuming a dilute upstream flow, i.e. $\rho_a \simeq \rho_1$, the mass conservation relation changes into a volume conservation relation:

$$v_1 h_1 + q_e = v_2 h_2. \tag{2}$$

Let ε be the entrainment coefficient, which is defined as the ratio of entraining mass flux to the upstream mass flux, $\varepsilon = \frac{\rho_a q_e}{\rho_1 v_1 h_1}$ [13]. Then, Eq. (1) may be written as:

$$1 + \varepsilon = \frac{\rho_2 v_2 h_2}{\rho_1 v_1 h_1}. \tag{3}$$

Substituting q_e from Eq. (2) into Eq. (1), one has:

$$\Delta\rho_1 v_1 h_1 = \Delta\rho_2 v_2 h_2, \tag{4}$$

in which $\Delta\rho_i = \rho_i - \rho_a \quad (i = 1, 2)$.

Densimetric Froude numbers for upstream and downstream of the flow are defined, respectively, as:

$$\mathrm{Fr}_1 = \frac{v_1}{\sqrt{\frac{\Delta\rho_1}{\rho_1} g h_1}}, \tag{5}$$

$$\mathrm{Fr}_2 = \frac{v_2}{\sqrt{\frac{\Delta\rho_2}{\rho_2} g h_2}}. \tag{6}$$

With the assumption of dilute current, Eq. (6) can be rewritten as:

$$\mathrm{Fr}_2 = \sqrt{\frac{v_2^2 \rho_2^2}{\frac{\Delta\rho_2}{\rho_2} \rho_1^2 g h_2}}. \tag{7a}$$

By substituting $\Delta\rho_2$ from Eq. (4) into Eq. (7a), multiplying the numerator and denominator by h_2^3, and substituting $\rho_2 v_2 h_2$ from Eq. (3), one obtains:

$$\mathrm{Fr}_2 = \sqrt{\frac{(1+\varepsilon)^3 v_1^3 \rho_1^3 h_1^3}{\Delta\rho_1 v_1 h_1 \rho_1^2 g h_2^3}}, \tag{7b}$$

or:

$$\mathrm{Fr}_2 = \frac{v_1}{\sqrt{\frac{\Delta\rho_1}{\rho_1} g h_1}} \left[(1+\varepsilon)\frac{h_1}{h_2}\right]^{\frac{3}{2}}, \tag{7c}$$

and, bearing in mind the upstream Froude number definition, we have:

$$\mathrm{Fr}_2 = \mathrm{Fr}_1 \left[(1+\varepsilon)\frac{h_1}{h_2}\right]^{\frac{3}{2}}. \tag{7d}$$

To apply the momentum balance to the control volume, one has to find the forces acting on the control volume, which are pressure and gravity (shear forces are neglected). f_1, f_2 and f_3 are pressure forces acting on

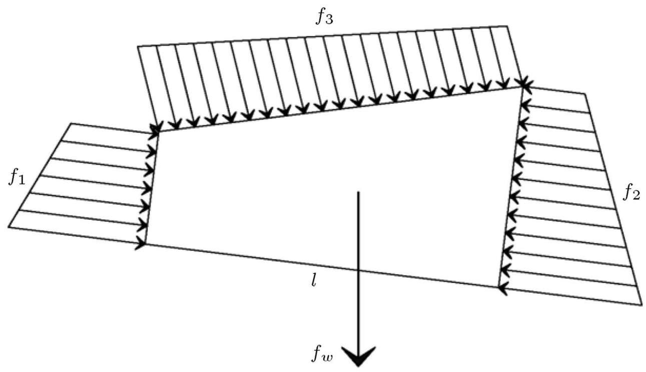


Figure 3. Forces acting on the control volume.

the control volume, as shown in Figure 3. Assuming hydrostatic pressure distribution, the pressure forces are:

$$f_1 = \frac{\rho_1 g h_1^2}{2} + \rho_a g y_1 h_1, \tag{8}$$

$$f_2 = \frac{\rho_2 g h_2^2}{2} + \rho_a g y_2 h_2. \tag{9}$$

Let f_{3x} be the flow direction component of pressure force f_3:

$$f_{3x} = \rho_a g \frac{y_1 + y_2}{2}(h_2 - h_1). \tag{10}$$

Assuming a dilute current, the flow direction gravity force is:

$$f_{wx} = \rho_a g \frac{h_1 + h_2}{2} l \sin\theta. \tag{11}$$

From the geometry of the jump, one obtains:

$$y_1 - y_2 + l\sin\theta = (h_2 - h_1)\cos\theta. \tag{12}$$

Assuming a uniform velocity profile, the momentum balance for the control volume gives:

$$f_{3x} + f_1 + f_{wx} - f_2 = \rho_2 v_2^2 h_2 - \rho_1 v_1^2 h_1. \tag{13}$$

Substituting Eqs. (8)-(11) into Eq. (13) yields:

$$\begin{aligned}&\rho_a g \frac{y_1 + y_2}{2}(h_2 - h_1) + \rho_a g y_1 h_1 + \frac{\rho_1 g h_1^2}{2}\\ &\quad + \rho_a g \frac{h_1 + h_2}{2} l \sin\theta - \rho_a g y_2 h_2 - \frac{\rho_2 g h_2^2}{2}\\ &\quad = \rho_2 v_2^2 h_2 - \rho_1 v_1^2 h_1.\end{aligned} \tag{14a}$$

Assuming a dilute current, therefore, taking ρ_1 and ρ_2 equal to ρ_a on the right hand side and with the aid of Eqs. (5), (6), (7d) and (12) after some manipulations, one obtains:

$$\begin{aligned}&\left(\frac{h_2}{h_1}\right)^3 - \frac{(1+\varepsilon)\left[1 + 2\mathrm{Fr}_1^2 - \frac{\rho_a}{\Delta\rho_1}(\cos\theta - 1)\right]}{\left[1 - \frac{\rho_a}{\Delta\rho_1}(\cos\theta - 1)(1+\varepsilon)\right]}\left(\frac{h_2}{h_1}\right)\\ &\quad + \frac{2\mathrm{Fr}_1^2(1+\varepsilon)^3}{\left[1 - \frac{\rho_a}{\Delta\rho_1}(\cos\theta - 1)(1+\varepsilon)\right]}.\end{aligned} \tag{15}$$

Eq. (15) relates conjugate depth ratio to upstream parameters, surface inclination, and entrainment as well. When $\theta = 0$, this equation reduces to one, proposed in Ref. [13].

One can solve Eq. (15) to acquire $\frac{h_2}{h_1}$ through the following procedure:

$$\frac{h_2}{h_1} = 2\sqrt{n}\cos(u/3), \tag{16}$$

where:

$$\cos(u) = \frac{m}{n\sqrt{n}}, \tag{17}$$

$$n = \frac{1}{3}\frac{(1+\varepsilon)\left[1 + 2\mathrm{Fr}_1^2 - \frac{\rho_a}{\Delta\rho_1}(\cos\theta - 1)\right]}{\left[1 - \frac{\rho_a}{\Delta\rho_1}(\cos\theta - 1)(1+\varepsilon)\right]}, \tag{18}$$

$$m = -\frac{\mathrm{Fr}_1^2(1+\varepsilon)^3}{\left[1 - \frac{\rho_a}{\Delta\rho_1}(\cos\theta - 1)(1+\varepsilon)\right]}. \tag{19}$$

To examine the accuracy of this model, a set of experiments are performed, which will be explained in detail in the following parts. The conjugate depth ratio obtained from the experiments will be compared with that of the model prediction and, finally, the model will be investigated in detail.

3. Experiments

3.1. Experimental setup

A laboratory apparatus was built to study two-dimensional (2D) flows resulting from the release of the salt-water solution density currents on a sloping surface in a freshwater channel. The channel was 12 m long, 0.20 m wide and 0.60 m deep. One channel side was made in glass for visual observation. The channel was divided into two sections in the longitudinal direction using a separating Plexiglas sheet (Figure 4). The shorter upstream section accumulated dense water with a sluice gate in its rectangular bottom. The adjustable opening allowed changing the inlet velocity of the dense water. In all tests, the opening was 4 or 7 cm high. The channel was previously filled with fresh water and its temperature

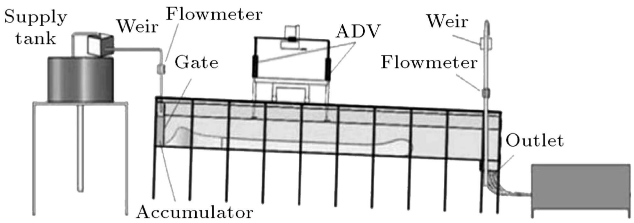


Figure 4. Schematic sketch of the experimental setup.

was as in the laboratory. At test start, the dense water continuously left the accumulator through the gate flowing down the sloping bottom. The slope of the channel bed was adjusted to 1%. The salt-water solution density current gradually spread under the fresh water. At the channel end, the dense water impacted a back step and was withdrawn through the bottom drains.

Another tank, called the reservoir tank, with a maximum capacity of 2 m^3, was used to prepare the dense water mixture. It was made of stainless steel and installed 2.5 m from the ground. A supplying pipe fed the dense water from the reservoir into the accumulator. A gate valve controlled the feed rate measured by a discharge meter and fixed at a desired rate (35.6 lit/min).

To avoid a return flow, a 25 cm step was located at the channel end. Sixty-four valves with a discharge of about 0.6 lit/min were installed under the step surface. The number of opening valves was dictated by the inlet flow. The outflow discharge was set equal to the inflow; then, the inflow was kept constant during a test. In all tests, salt was used as the soluble material. The density of the salt mixture was measured with a hydrometer prior to entering the channel. This test was conducted to verify the analytical modeling developed in the previous section. Several experimental sets were conducted with different inlet conditions, and conjugate depth ratio was measured in each test. For measuring the density current depth after the jump, three rulers were attached to the glass wall of the channel. The rulers were set 10, 25, and 50 cm from the inlet gate. A high resolution camera was utilized to record the jump, and the jump height was measured carefully from the recorded movies.

3.2. Experimental results

Initial conditions and results of experiments are introduced in Table 1. h_0 and v_0 represent the opening inlet height and mean inlet velocity of dense layer, respectively. Mean inlet density current velocity is calculated having flow rate, Q, which was constant and equal to 35.6 lit/min, gate width b, and opening inlet height h_0 ($v_0 = Q/bh_0$). C_0 is the inlet concentration of salt-water solution, which varies from 0.5 to 6 gr/lit. The inlet densimetric Froude number Fr_1 is defined as:

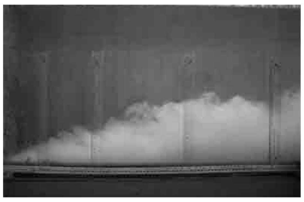
Table 1. Inlet conditions and observed conjugate height ratio for experiments.

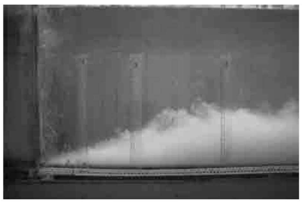
Run	h_0 (cm)	C_0 (gr/lit)	v_0 (cm/s)	Fr_1	$\frac{h_2}{h_1}$ (measured)
1	2	2.5	14.84	6.71	9.25
2	2	3.5	14.84	5.67	7.5
3	2	6	14.84	4.34	5.5
4	3	0.5	9.89	8.16	11
5	3	1.5	9.89	4.71	6
6	3	2.5	9.89	3.65	4.83
7	3	3.5	9.89	3.09	3.67
8	3	6	9.89	2.36	3.17
9	4	0.5	7.42	5.3	6
10	4	1.5	7.42	3.06	4.625
11	4	2.5	7.42	2.37	3.125
12	4	3.5	7.42	2	2.25
13	4	6	7.42	1.53	2
14	5	0.5	5.94	3.79	5.3
15	5	1.5	5.94	2.19	2.9
16	5	2.5	5.94	1.7	2.2
17	5	3.5	5.94	1.44	2
18	5	6	5.94	1.1	1.7
19	7	0.5	4.24	2.29	3.28
20	7	1.5	4.24	1.32	2.07
21	7	2.5	4.24	1.02	1.36
22	7	3.5	4.24	0.87	No jump
23	7	6	4.24	0.66	No jump

$$\mathrm{Fr}_1 = \frac{v_0}{\sqrt{\frac{\Delta\rho_0}{\rho_0} g h_0}}. \tag{20}$$

Figure 5 shows sample photos of internal hydraulic jumps observed in the laboratory. After opening the inlet gate, a sudden change in the density current depth is observed, which is due to a change in the current regime as a result of the density jump. In fact, before the jump, due to the momentum of the density current, the inlet Froude number is high (e.g. $\mathrm{Fr}_1 = 6.71$ for Run 1). In this condition, the inertial force of the current is so high that none of the opposite forces can neutralize it, and the current tends to experience a hydraulic jump, during which its depth drastically increases. After the jump, the Froude number reduces and the current is stable afterwards.

It was observed that the density jump initiated immediately after the channel opening in all sets of experiments having an inlet Froude number more than unity. As a result, the current depth before jump, h_1, was considered the opening inlet height. The current depth after jump, h_2, was measured visually.


(a) Run 1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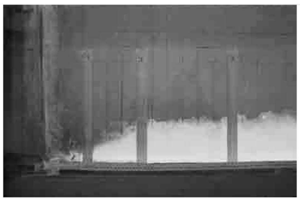
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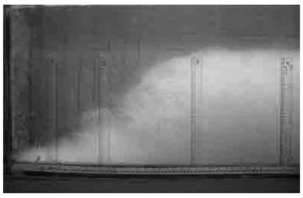
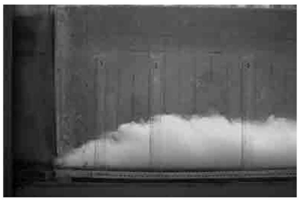
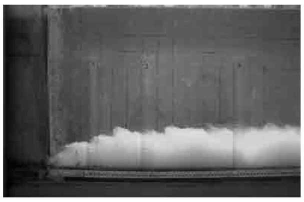
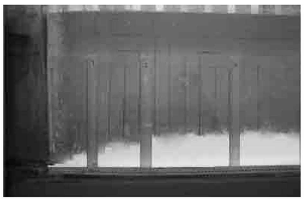
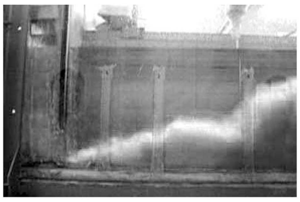

(d) Run 4

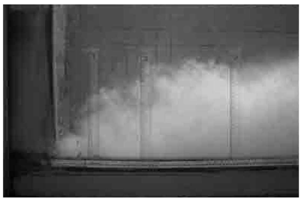

(e) Run 5

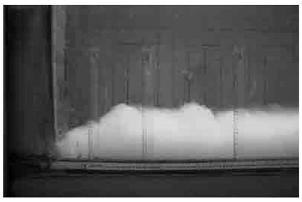

(f) Run 6

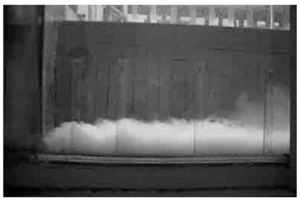

(g) Run 7

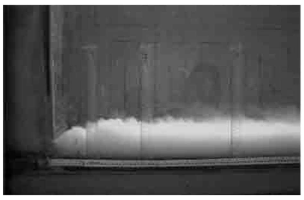

(h) Run 8


(i) Run 9


(j) Run 14


(k) Run 15


(l) Run 16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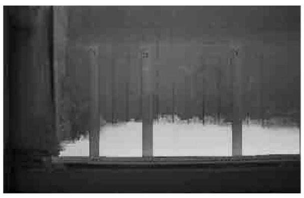
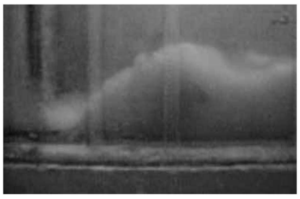

(n) Run 18


(o) Run 19

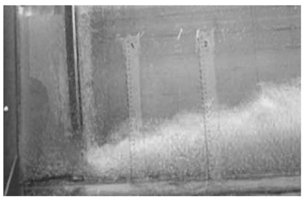

(p) Run 21

Figure 5. Internal hydraulic jumps observed in the laboratory channel in different experiments. Potassium permanganate was used to visualize the salt-water density current.

4. Results and discussion

4.1. Validation of the model

In this part, the accuracy of the analytical model developed in Section 2 is examined. The conjugate depth ratio obtained from experiments is used to compare with the conjugate depth ratio predicted by Eqs. (15)-(19). In order to obtain $\frac{h_2}{h_1}$ from Eqs. (15)-(19), one needs the entrainment coefficient in each initial condition, as Eq. (15) gives $\frac{h_2}{h_1}$ as a function of entrainment. Dallimore et al. [31] proposed a relation for entrainment coefficient in salt-water currents, which is a function of the inlet bulk Richardson number, as follows:

$$\varepsilon = \frac{C_K C_D^{3/2} + C_S}{Ri + 10(C_K C_D^{3/2} + C_S)}, \tag{21}$$

where Ri denotes upstream bulk Richardson number,

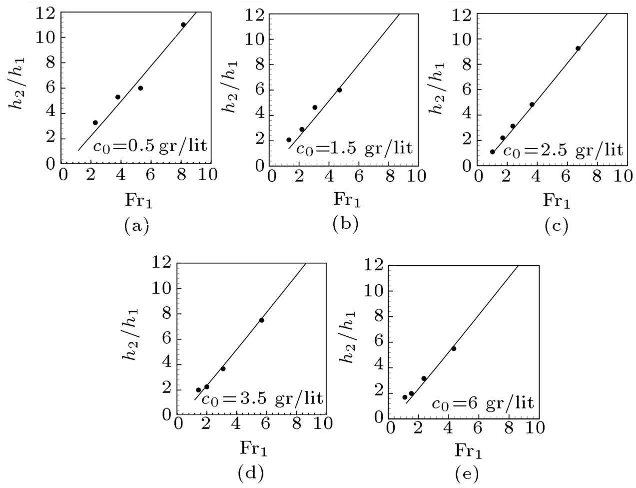


Figure 6. Conjugate depth ratio vs. Fr_1 for five different inlet concentrations of salt-water solution: — theory; • experimental points.

which is *the inverse square* root of the Froude number, and C_K, C_D and C_S are constants that are obtained from experiments with values of 2.2, 0.015 and 10^{-4}, respectively.

We utilize Eq. (21) to calculate the conjugate depth ratio from Eq. (15) and it is plotted vs. the inlet Froude number for five inlet concentrations and bed slope of 1% in Figure 6. A good agreement is observed between the experimental and analytical results. It can be seen that the conjugate depth ratio increases with the inlet Froude number, Fr_1. For Fr_1 of less than unity, Eq. (15) predicts no jump, which is consistent with our observations.

4.2. Discussion

Eq. (15) shows that the conjugate depth ratio not only depends on the inlet Froude number but on the surface inclination and inlet concentration. These dependencies are examined in this part. Figure 7 shows the conjugate depth ratio against the inlet concentration ratio, $\frac{c_0}{\rho_a}$, for four inlet Froude numbers, Fr_1, and bed slope of 1%. One can observe that the conjugate depth ratio drastically increases with concentration for small concentrations and is nearly constant for concentration ratios larger than 0.005.

Figure 8 shows the conjugate depth ratio against surface inclination for four different inlet Froude numbers and an inlet concentration ratio of 0.001. The surface inclination is usually small and, therefore, a maximum of 6° is chosen. It can be seen that for all inlet Froude numbers, the conjugate depth ratio decreases with surface inclination. It can also be seen that the effect of surface inclination is more severe for currents with higher inlet Froude numbers. Thus, the conjugate depth ratio for a current with an inlet Froude number of 10 falls from 14 for a horizontal bed to 5 for a 6 degrees inclined bed, while this change

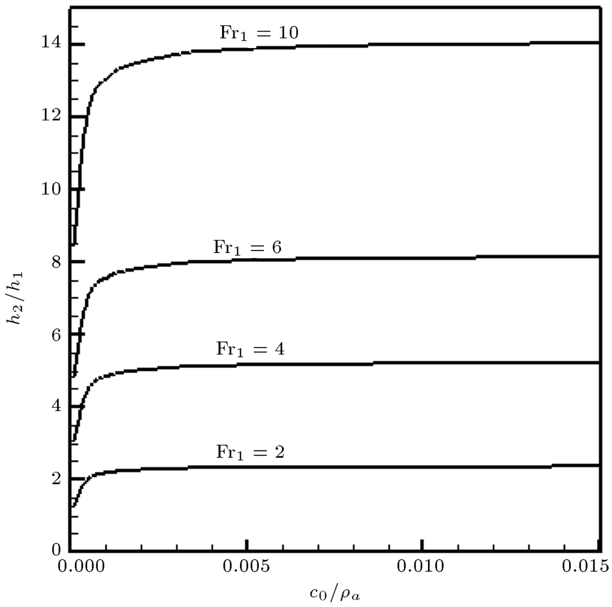


Figure 7. $\frac{h_2}{h_1}$ vs. inlet concentration ratio $\frac{c_0}{\rho_a}$ for different inlet Froude numbers ($\theta = 1\%$).

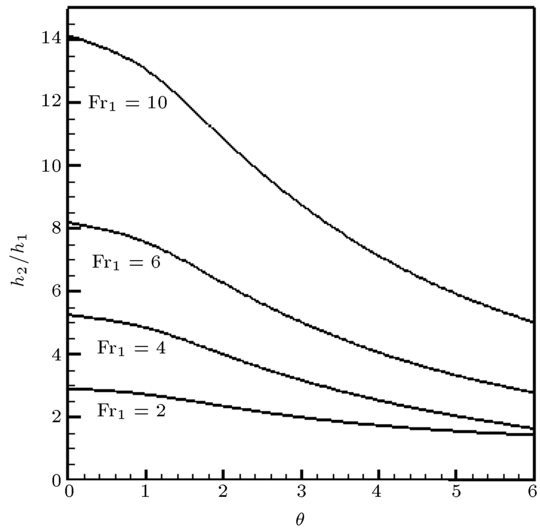


Figure 8. $\frac{h_2}{h_1}$ vs. surface inclination for different inlet Froude numbers ($\frac{c_0}{\rho_a} = 0.001$).

for a current with a Froude number of 2 is 2.9 to 1.4.

Another interesting parameter that may be noticed is the Froude number right after the internal jump, Fr_2. Eq. (7d) relates Fr_2 to upstream jump parameters. Figure 9 shows $\frac{\mathrm{Fr}_2}{\mathrm{Fr}_1}$ as a function of Fr_1 for different inlet concentration ratios. As can be seen, the graphs are the same for all concentration ratios. Hence, the Froude number after the jump is not a function of the inlet concentration. Figure 10 shows $\frac{\mathrm{Fr}_2}{\mathrm{Fr}_1}$ as a function of surface inclination for a current concentration ratio of 0.006. It is observed that the conjugate Froude number ratio increases slightly with the surface inclination.

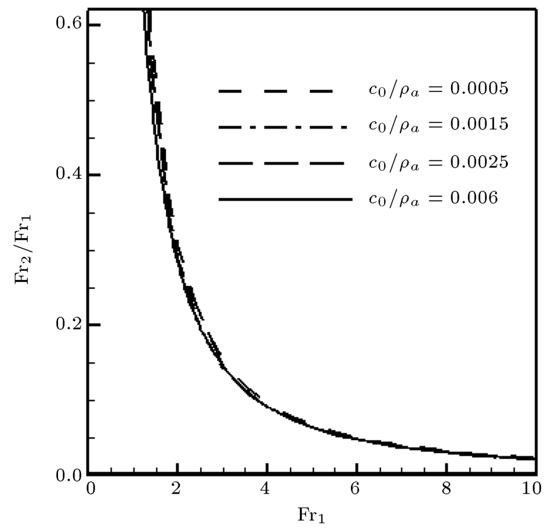


Figure 9. $\frac{\mathrm{Fr}_2}{\mathrm{Fr}_1}$ vs. Fr_1 for four inlet concentration ratios.

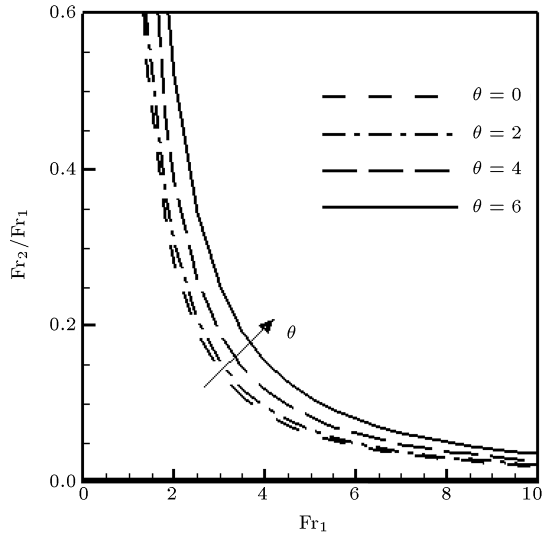


Figure 10. $\frac{\mathrm{Fr}_2}{\mathrm{Fr}_1}$ vs. Fr_1 for four surface inclinations.

In open channel flow, the critical Froude number is $\mathrm{Fr}_c = 1$. However, in internal hydraulic jumps, its value may be different and is determined from experiments [32]. In their numerical study, Huang et al. [32] reported a critical Froude number of 1.21 for a salt-water solution internal jump on a 3° slope with an excess fractional density of 0.02. Also, Nourmohamadi et al. [33] observed a critical Froude number of less than unity (0.6) for turbidity currents in their experiments. Using Eq. (15) with the aid of Eq. (21), one can attain Fr_c for a salt-water current with specified inlet parameters. Fr_c is considered the smallest Fr_1, for which $\frac{h_2}{h_1} > 1$ is calculated by Eq. (15). Figure 11 shows Fr_c for salt-water density currents as a function of surface inclination, θ, for different inlet current concentrations. Figure 11 reveals that Fr_c is 1.12 on the horizontal surfaces for salt-water density currents and increases with inclination for all inlet concentrations. One can also realize, from Figure 11, that Fr_c decreases with inlet concentration in an arbitrary surface inclination.

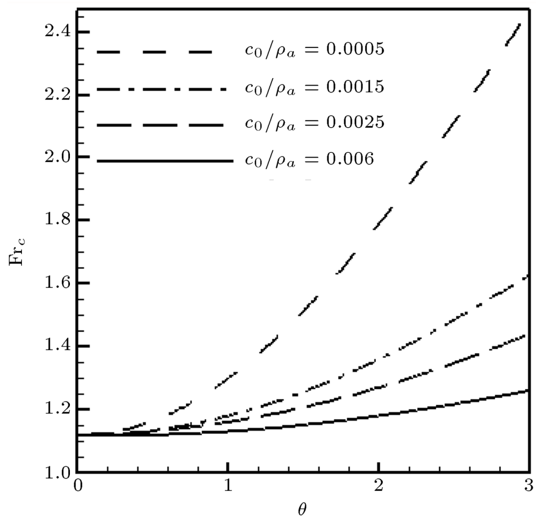


Figure 11. Fr_c as a function of surface inclination θ for different inlet current concentration ratios.

5. Conclusion

Using an integral method analysis, a relation for an internal hydraulic jump on an inclined surface, giving conjugate depth ratio as a function of inlet flow parameters, surface inclination and entrainment, is derived. A set of experiments are conducted to verify the model. A good agreement is observed between the model prediction of the conjugate depth ratio and the measurements. The analysis reveals that the conjugate depth ratio drastically increases with the inlet concentration for small concentrations and is nearly constant for concentration ratios larger than 0.005. It is also shown that increasing the surface inclination results in a decrease in the conjugate depth ratio. This analysis also shows that the densimetric Froude number just after the jump is a function of the inlet densimetric Froude number and surface inclination and not inlet concentration. The model predicts a critical Froude number of 1.12 for horizontal internal hydraulic jumps in salt-water density flows. It also reveals that the critical Froude number for internal hydraulic jumps increases with surface inclination and decreases with inlet concentration of the flow.

Nomenclature

b	Gate width
C_0	Inlet concentration of salt-water solution
f_1	Upstream pressure force of the jump
f_2	Downstream pressure force of the jump
f_3	Upper surface pressure force

f_{3x}	Flow direction component of pressure force f_3
f_w	Gravity force
f_{wx}	Flow direction gravity force f_w
Fr	Froude number
Fr_1	Upstream Froude number of the jump
Fr_2	Downstream Froude number of the jump
g	Gravity acceleration
h_0	Opening inlet height
h_1	Upstream depth of the jump
h_2	Downstream depth of the jump
l	Jump length
m	A mathematical coefficient introduced in the text
n	A mathematical coefficient introduced in the text
q_e	Entrainment volume flux
Q	Inlet water volume flux
u	A mathematical variable introduced in the text
Ri	Bulk Richardson number
v_0	Inlet density current mean velocity
v_1	Upstream water mean velocity of the jump
v_2	Downstream water mean velocity of the jump
y_1	Distance between the upper edge of jump upstream height and ambient water surface
y_2	Distance between the upper edge of jump downstream height and ambient water surface
ρ_1	Upstream water density of the jump
ρ_2	Downstream water density of the jump
ρ_a	Ambient water density
ε	Entrainment coefficient
θ	Slope angle

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