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Research Note

## Sobol method application in sensitivity analysis of LuGre friction model during 2D manipulation

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### KEYWORDS

Sensitivity analysis;  
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**Abstract.** Simulation of manipulation is a basic tool for accurate and controllable displacement of bodies and particles at micro and nano scale. The Atomic Force Microscope (AFM) system has become a useful tool for direct measurement of micro and nano structural parameters and for unraveling the intermolecular forces at nanoscale level with atomic-resolution characterization. Friction forces are a part of the surface properties that play an important role in the manipulation of nanoparticles. In order to gain more precise manipulation, different friction models have been developed, one of which is the LuGre model. In this paper, the sensitivity of the manipulation of nanoparticles has been analyzed using dimensional and environmental parameters, based on the LuGre friction model, using the Sobol method. In previous work, sensitivity analysis has been performed using graphical sensitivity analysis. Hence, the importance percentages of the parameters are not clear, but, the Sobol method, which is a statistical model, solves this problem. Results show that cantilever thickness is the most effective dimensional parameter on the critical force value, while cantilever length and width are less important. Environmental parameters, such as cantilever elasticity modulus, substrate velocity and adhesion, respectively, take the next orders.

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### 1. Introduction

Sensitivity Analysis (SA) is the study of how ambiguity in the output of a model can be due to different causes of ambiguity in the model input [1]. Sensitivity analysis can be used to coarsen models and investigate their soundness [2]. Despite the benefits and acquisitions of Sensitivity Analysis (SA), its application has hardly been studied for friction models in cases of manipulation. The SA outcome will be extremely important for changing the critical force in manipulation and

particle movement and also for selecting a suitable instrument for the precise planning of the manufacture and assembly of nano-objects.

The issue of manipulation of micro/nano-particles has become the center of attention in recent years. The Atomic Force Microscope (AFM) is a useful instrument for direct measurement of intermolecular forces and can be used in a wide spectrum of applications. AFM can also be utilized for imaging, indenting, moving the sample etc. The assembly of nano-particles and their connection to electrical leads, such as the random deposition of clusters between electrodes, binding by wet chemistry, and electrostatic trapping, all serve as other important applications of the AFM technique [3].

The area in nano-scale is too large in comparison with volume. Therefore, surface forces such as friction

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and adhesion become more significant and can no longer be ignored [4]. Consequently, several friction models have been extended to foretell frictional behavior on a nano-scale. Accurate prediction of the friction condition will lead to more precise manipulation modeling, so, using a suitable friction model plays an important role in nano-manipulation. For a case of pure adhesion, continuum mechanics models exist, but, Johnson tried to extend these models in order to include both static and sliding friction [5]. A new 2D dynamic nanoscale friction model was presented by Landolsi et al. [6]. The proposed model is based on the bristle interpretation of asperity contacts introduced by the Bristle and the LuGre friction models. It involves a new jumping mechanism to take into consideration the instantaneous jumps of the FFM cantilever during the scanning process. The model is shown to represent the 2D stick-slip phenomena with lattice periodicity. The different parameters of the model were obtained from FFM experiments. These experiments were performed using muscovite mica samples, having an atomically flat surface. The corresponding parameters were plugged into the model and, then, the simulated and experimental results were compared. Another model for friction has been presented by Canudas de Wit et al. [7]. The model is simple yet captures most friction phenomena that are of interest to feedback control. The low velocity friction characteristics are particularly important for high performance pointing and tracking [8]. The model can describe arbitrary steady state friction properties. It supports hysteretic behavior due to frictional lag and spring-like behavior in friction, and gives a changeable break-away force, depending on the rate of change of the applied force. All these phenomena are unified into a first-order nonlinear differential equation. The model can readily be used in the simulations of systems with friction. Choosing an appropriate friction model has a great effect on manipulation, so, Korayem et al. compared and analyzed different friction models [9].

In previous work [9], the effect of different dimensional and environmental parameters on critical force and time of manipulation movement, for three friction models, Coulomb, HK and LuGre models, has been investigated and the results have been shown in graphical form. This does not indicate the importance percentage of these parameters. To our knowledge, experiments in such situations have rarely been done, but, Dietzel et al. have presented some valuable and comprehensive work in the field of friction [10–12]. In these experiments, frictional forces for the manipulation of antimony nanoparticles and their dependency on contact area, air exposure, particle size and structure have been studied. The friction duality observed in these experiments has been explained. Examination of a possible morphological influence on

friction yielded negative results. Interface oxidation is shown to have a great effect on frictional properties but there was no dependence of friction on regular angular orientation between particle and substrate. Power dissipation and its relativity to the contact area, and, also, frictional forces, were studied, and results showed that the threshold value of the power dissipation needed for translation depends linearly on the contact area between the antimony particles and the substrate. Assuming a linear relationship between dissipated power and frictional forces implies a direct proportionality between friction and contact area.

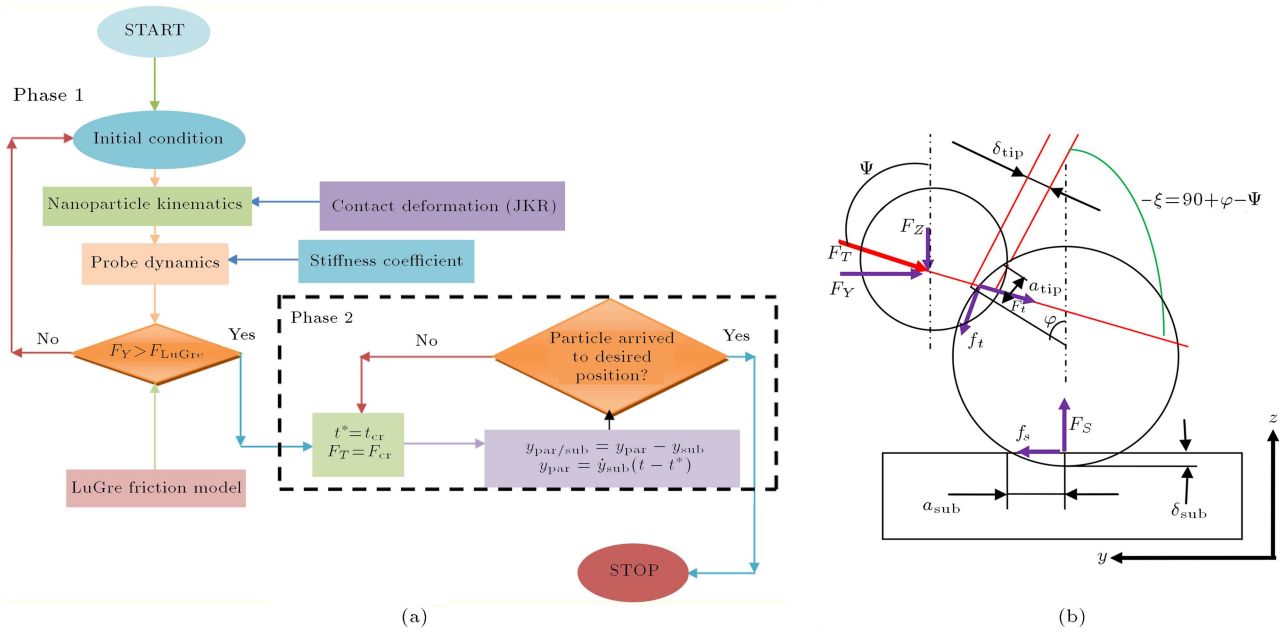
In this article, the LuGre friction model and its application in the manipulation of nano-particles will be studied. According to the importance of critical force in the manipulation of nano-particles, different dimensional and environmental parameters have been chosen to analyze, statistically, their effects on this force. Therefore, the Sobol method has been used to investigate the effect of parameter variations on critical force for the LuGre model and these results are presented with percentage.

## 2. Formulation

In recent years, AFM, as a fundamental tool for moving, manufacturing and assembling nano-particles, has been widely noticed. Manipulation modeling is a basic instrument to conduct a precise and controlled displacement of particles in micro/nano scale. Transition from the macro to the nano world results in an increase in area to volume ratio and, consequently, surface forces such as adhesion and friction become more important. Thus, manipulation modeling and friction are fundamentally co-dependent. In fact, the success of manipulation modeling in the prediction of experimental results is very much related to the accuracy of the friction model.

### 2.1. Nano-particle manipulation modeling

Generally, the manipulation process includes the imaging of the substrate and the particles on it, locating the probe tip on the target nano-particle and manipulating the movements of the substrate or tool base with constant velocity. Movement of the probe tip or substrate, with the constant velocity of  $V_{sub}$ , leads to an increase in applied load,  $FT$ , from the nano-manipulator on the nano-particle to a critical value of  $F_{cr}$ , in order to overcome adhesion forces such as contact and friction forces between the particle and substrate, and from then, the movement is started. The AFM manipulation tool consists of a probe, including a cantilever and a tip at the end. The fundamental of AFM reaction and dynamic simulation is the precise modeling of cantilever deformations and adhesion forces of contact surfaces.



**Figure 1.** (a) Algorithm for dynamic modeling and displacement of the particle. (b) Free body diagram of manipulation.

A dynamical modeling algorithm is shown in Figure 1(a). Phase one is separated from phase two with a dashed line. As can be observed, the input of the problem prior to the movement of the particle on the substrate consists of the position of the particle, and the output is the exerted force,  $F_T$ , by the probe tip on the particle. At this point, the normal bending and twisting of the cantilever are directly measured by the light beam or other methods. As the second phase of the simulation algorithm shows, by increasing the applied force to the critical limit,  $F_T$  remains constant and the particle starts moving on the substrate. The output of this segment demonstrates the dynamic performance and the amount of displacement of the particle.

To determine the amount of particle displacement during a certain period in which the substrate moves with a defined speed, one should know the starting moment of the particle's motion. In previous research, the kinematic and dynamic equations regarding the movement of the probe and particle have been obtained from the free body diagram of the problem, and the initial conditions, based on the specific and fixed velocity of the substrate, and the geometry and material of the cantilever, probe, and particle have been determined [9].

Through dynamic modeling of the movement of the probe tip and particle, the onset of movement at the instant when the force of the probe tip overcomes the forces of friction and adhesion, is obtained from Figure 1(b), where  $F_Y$  and  $F_Z$  are the exerted forces on the particle by the probe tip, and  $\Psi$  and  $\varphi$  are angle of probe force, and angle of contact of the probe tip with the particle, respectively:

$$F_Z = \left( \frac{I_P \ddot{\theta} + M_\theta}{H} \right) \sin \theta + K_z Z_p \cos^2 \theta - K_y y_p \sin \theta \cos \theta - m \cos \theta \left( \frac{H}{2} \dot{\theta}^2 + \ddot{\delta}_t \cos(\theta + \phi) + \ddot{\delta}_s \cos \theta \right), \quad (1)$$

$$F_Y = F_y + m \left[ \frac{H}{2} (-\ddot{\theta} \cos \theta + \dot{\theta}^2 \sin \theta) - \ddot{\delta}_t \sin \phi \right], \quad (2)$$

$$F_T = \sqrt{F_Y^2 + F_Z^2}, \quad (3)$$

$$\Psi = \tan^{-1} \left( \frac{F_Y}{F_Z} \right). \quad (4)$$

## 2.2. Friction models

### 2.2.1. Classical models

Friction classical models are composed of several parts, each of which includes one frictional force feature. The main idea shows that friction is opposing movement, and that its magnitude does not depend on velocity and contact area. This means that [13]:

$$F = F_c \text{sgn}(v), \quad (5)$$

in which frictional force,  $F_c$ , is proportional to vertical surface force,  $F = \mu F_N$ . This description shows the Coulomb friction term. The Coulomb friction model has no definition for friction force in zero velocity. Development of the hydrodynamic theory in the 19th century resulted in an explanation of frictional force which was dependant on lubrication. A viscous friction

term was used, presented as follows:

$$F = F_v v. \quad (6)$$

Viscous and Coulomb frictions are usually combined. The best experimental results can be obtained by nonlinear dependency on velocity:

$$F = F_v |v|^{\delta_v} \text{sgn}(v). \quad (7)$$

Obviously, friction cannot be explained as a function of velocity when there is no movement. So, using external force,  $F_e$ , it can be modeled as:

$$F = f(x) = \begin{cases} F_e & \text{if } v=0 \text{ and } |F_e| < |F_S| \\ F_s \text{sgn}(F_e) & \text{if } v=0 \text{ and } |F_e| \geq |F_S| \end{cases} \quad (8)$$

Classical friction parts can be combined in several ways and any of these combinations presents a classical model. A more general explanation of friction in classical models can be described as follows:

$$F = f(x) = \begin{cases} F(v) & \text{if } v \neq 0 \\ F_e & \text{if } v=0 \text{ and } |F_e| < |F_S| \\ F_s \text{sgn}(F_e) & \text{if } v=0 \text{ and } |F_e| \geq |F_S| \end{cases} \quad (9)$$

in which,  $F_s$  is the vertical component of the sliding contact force and  $F(v)$  is an optional function mostly used in this form:

$$f(v) = F_C + (F_S - F_C)e^{-|v/v_S|^{\delta_S}} + F_v v. \quad (10)$$

### 2.2.2. Armstrong's model

Armstrong et al. has presented a modified classical model for some dynamic frictional phenomenon. Friction is defined as follows [14]:

$$F(x) = \sigma_0 x, \quad (11)$$

and adhesion will be as follows:

$$F(v, t) = \left( F_C + F_S(\gamma, t_d) \frac{1}{1 + (v(t - \tau_l/v_S))} \right)^2 \text{sgn}(v) + F_v v, \quad (12)$$

and sliding will be as follows:

$$F_S(\gamma, t_d) = F_{s,a} + (F_{S,\infty} - F_{s,a} \frac{t_d}{t_d + \gamma}). \quad (13)$$

### 2.2.3. Dahl model

The Dahl model was established in order to simulate control systems with expanded friction. The starting point of the Dahl model was some experiments on server systems with ball bearings. Dahl extended an approximately simple model which was used for simulation systems with ball bearing friction [15].

The starting point for the Dahl model is the stress-strain curve of classical solid mechanics. Dahl modeled the strain-stress curve with different equations. Assume that  $x$  is displacement,  $F$  is friction force, and  $F_C$  is the Coulomb friction force. Then, the Dahl model will be as follows:

$$\frac{dF}{dx} = \sigma \left( 1 - \frac{F}{F_C} \text{sgn}(v) \right)^\alpha, \quad (14)$$

in which  $\sigma$  is the stiffness coefficient and  $\alpha$  is the parameter which determines the strain-stress curve form. To have a time-dependent model, Dahl suggested that:

$$\frac{dF}{dt} = \frac{dF}{dx} \frac{dx}{dt} = \frac{dF}{dx} v = \sigma \left( 1 - \frac{F}{F_C} \text{sgn}(v) \right)^\alpha v. \quad (15)$$

This model is the general form of the Coulomb model. For  $\alpha = 1$ , the Dahl model will be:

$$\frac{dF}{dt} = \sigma v - \frac{F}{F_C} |v|. \quad (16)$$

### 2.2.4. LuGre friction model

Canudas de Wit et al. presented the LuGre model, in which the Dahl model has been combined with frictional features of arbitrary steady state. The Stribeck effect has been considered in this model, which produces a non-constant effect at low velocities. The LuGre model consists of a nonlinear state and a frictional force [7]:

$$\frac{dz}{dt} = v - \frac{|v|}{g(v)} z, \quad (17)$$

$$F_f = \sigma_0 + \sigma_1 \frac{dz}{dt} + \sigma_2 v, \quad (18)$$

in which  $\sigma_0$  is equivalent stiffness for the related force position at reverse velocity;  $\sigma_1$  is the micro viscous frictional coefficient;  $\sigma_2$  is the viscous frictional coefficient;  $v$  is the relative velocity between sliding surfaces;  $z$  is the state variable, which shows the average deformation of the bristles;  $g(v)$  is a positive function which models constant velocity behavior and depends on some factors, such as material properties, lubrication and temperature. The  $g(v)$  function, which explains the Stribeck effect, is:

$$\sigma_0 g(v) = (F_S - F_C) e^{-(v/v_S)^2}, \quad (19)$$

where  $v_s$  is the Stribeck velocity.

A proper description of the constant velocity behavior and a smooth transition at reverse velocity is given by the LuGre model. Both LuGre and Dahl models show semi hysteresis behavior only when frictional force is less than maximum static frictional force.

### 2.2.5. Lemaître and Carlson model

This model is also a pulling/pushing nano friction model in which we have [16]:

$$\ddot{x} = -E_1 e^{-k/\dot{x}} + \alpha \sigma \dot{\gamma}, \quad (20)$$

$$\dot{\sigma} = \mu(\dot{\epsilon} - \dot{\gamma}), \quad (21)$$

$$\dot{\epsilon} = V/2h, \quad (22)$$

$$\dot{\gamma} = E_0 e^{-1/\dot{x}} \sinh(\sigma/\bar{\mu}), \quad (23)$$

where  $V$  is an optimum pulling/pushing velocity and  $t$  is time. The velocity and the position of the pulled/pushed object are defined, respectively, by  $x$  and  $\dot{x}$ . The state variable of  $\theta$  in this model also shows the lubrication state in the intervening contact area. When  $\theta = 1$ , the lubrication is in pure solid state (rolling phase). In other words, when  $\theta = 0$ , the lubrication is in a pure fluid state (sliding phase).

## 3. Simulation of nano-particle manipulation

In this section, the necessary initial values and initial conditions for solving the problem will be presented, respectively.

### 3.1. Initial values of the problem

In the present study, the simulation is verified using the available results [17]. Then, the mathematical model development is done considering mechanical properties; namely,  $E = 102.87$  (GPa),  $\nu = 0.27$ ,  $G = 40.5$  (GPa) and  $\rho = 2330$  (kg/m<sup>3</sup>). In this simulation, a  $R_p = 50$  nm gold particle has been pushed on the silicon oxide substrate that moves with constant velocity. The ranges of the geometrical properties of the AFM are: length = 200–700  $\mu\text{m}$ , width = 5–80  $\mu\text{m}$ , thickness = 0.25–2.5  $\mu\text{m}$  and height = 5–20  $\mu\text{m}$ . The environmental parameter ranges are: velocity = 0–800 nm/s,  $E = 120 - 200$  GPa,  $K = 10 - 100$  GPa and adhesion = 0–3 J/m<sup>2</sup>.

Contact mechanics and tribological parameters can be obtained experimentally for the different materials that are in contact. The surface energy between the nanoparticle and the tip/substrate is  $\omega = 0.2$  J/m<sup>2</sup>. The constant friction coefficients for the static and dynamic movements of the nanoparticle on the substrate are  $\mu_s = 0.8$ , and  $\mu_d = 0.7$ , respectively. Shear strength is assumed to be constant on both contact surfaces between the particle/substrate and the tip/substrate. Tip radius is  $R_t = 20$  nm and contact angle is  $\phi = 60$  [17].

### 3.2. Initial conditions

Assume that the substrate velocity is 100 nm/s,  $\ddot{\delta}_{\text{tip}}$  and  $\ddot{\delta}_{\text{sub}}$  are negligible, and initial conditions have been obtained by simplifying the equations at  $t = 0$ . These

initial conditions, given in Eq. (12), have been used throughout the analysis [17]. As mentioned before, to be certain of the desired contact, a small normal preload,  $F_{z0}$ , is exerted by providing normal deflection offset,  $z_{P0}$ , on the AFM probe. By measuring  $\phi_0$  in the AFM system,  $z_{P0}$  is obtained:

$$\left\{ \begin{array}{l} \phi_0 = 0.7^\circ \\ z_{P0} = L \sin(\phi_0) \\ \ddot{z}_{P0} = -\frac{V_{\text{sub}}^2}{H} \\ z_{T0} = z_{P0} - H \end{array} \right\}, \quad \left\{ \begin{array}{l} \dot{\delta}_{\text{tip}} = \dot{\delta}_{\text{sub}} = \text{cte}, \\ \ddot{\delta}_{\text{tip}} = \ddot{\delta}_{\text{sub}} = 0 \end{array} \right\},$$

$$\left\{ \begin{array}{l} \theta_0 = 0, \\ \dot{\theta} = \frac{V_{\text{sub}}}{H}, \\ \ddot{\theta} = 0 \end{array} \right\}, \quad \left\{ \begin{array}{l} y_{P0} = y_{T0} = 0 \\ \ddot{y}_P = \ddot{y}_T = 0 \end{array} \right\}. \quad (24)$$

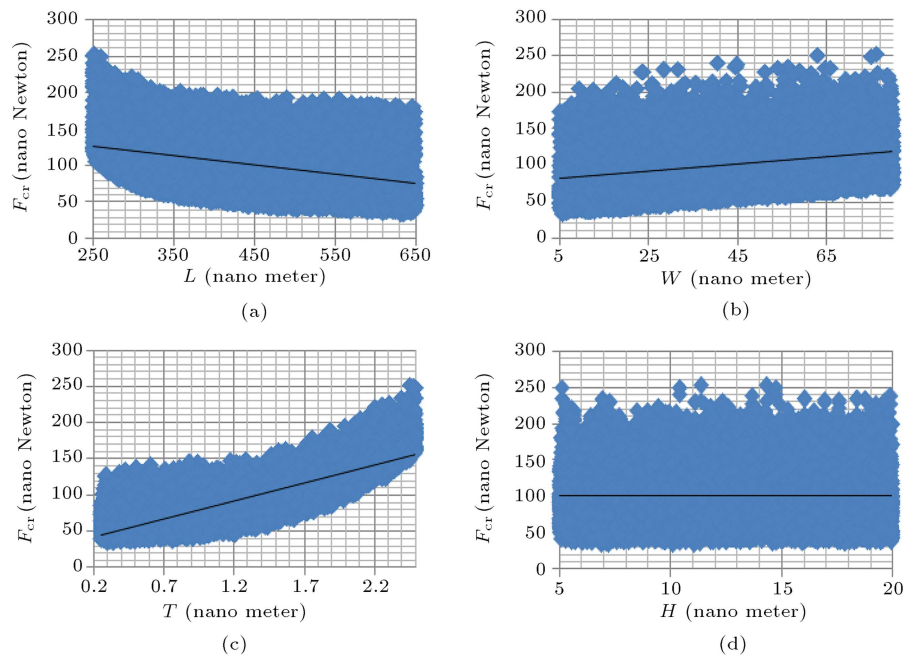
Note that the second derivative of cantilever deformation and contact elastic deformation is negligible.

## 4. Results

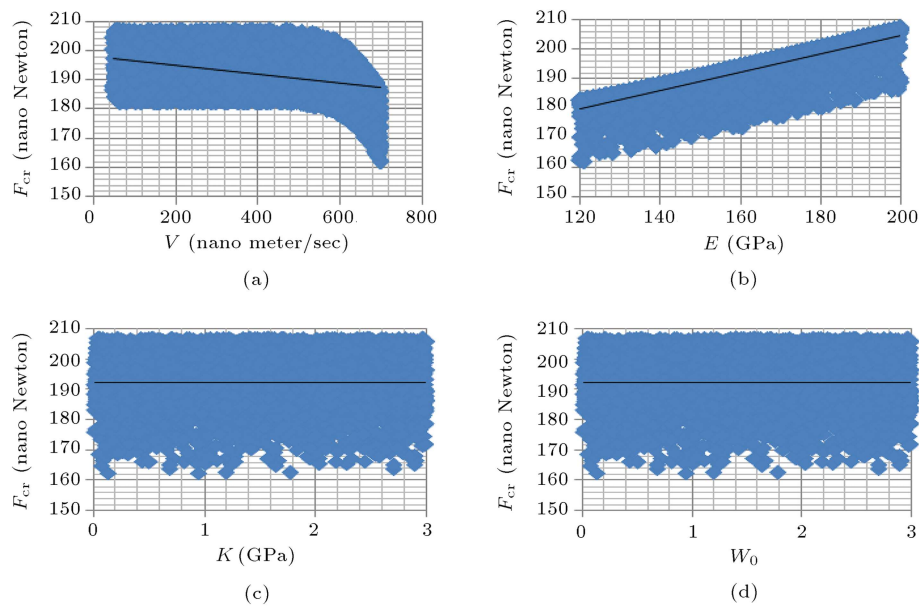
### 4.1. Dimensional sensitivity analysis results

Calculation of movement critical force in nano-particle manipulation is very important, because precise determination of this force causes accurate and controlled movement and manipulation of the particle in order to manufacture nano/micro instruments. Since the most effective factors on movement critical force value are cantilever dimensions, including length, width, thickness and height, investigating the dimensional effect of the cantilever on movement critical force precisely is crucial and affects nano-particle manipulation analysis. Therefore, in this article, a cantilever dimensional sensitivity analysis, based on the Sobol sensitivity analysis method, has been undertaken, and its effect on the manipulation of critical force using a friction model, such as LuGre, has been investigated.

As shown in Figure 2(a), in the LuGre model, an increase in cantilever length results in decreasing critical force. But, with further increase of cantilever length, the slope becomes slower, which shows that in cases of increasing length, using different friction models cannot affect simulation results. As observed from Figure 2(b), an increase in cantilever width leads to a linear increase in the critical force LuGre model. So, using wider cantilevers requires further accuracy in choosing a proper friction model for the manipulation process. Figure 2(c) shows that in the LuGre model, increasing cantilever thickness causes an increase in critical force magnitude. Critical force growth with an increase in cantilever thickness is significant. In particular, when the nano-particle is too small, it is not desirable and should be prevented. According to Figure 2(d), increasing cantilever height has no especial effect on critical force, and the movement critical force remains constant with cantilever height variations.



**Figure 2.** (a) Cantilever length effect on critical force of manipulation. (b) Cantilever width effect on critical force of manipulation. (c) Cantilever thickness effect on critical force of manipulation. (d) Cantilever height effect on critical force of manipulation.



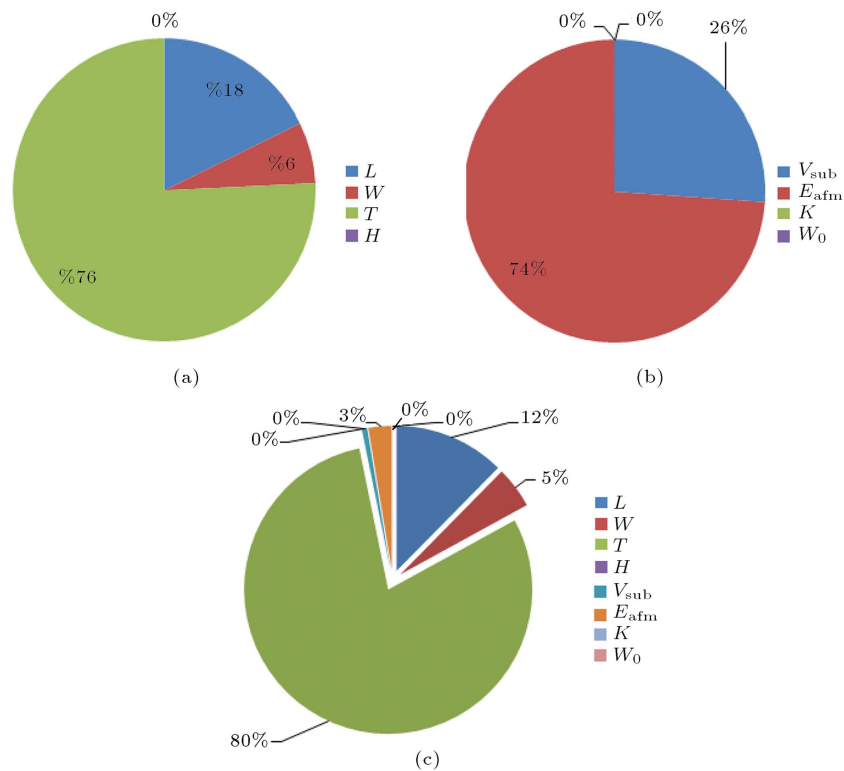
**Figure 3.** (a) Substrate velocity effect on critical force of manipulation. (b) Cantilever elasticity modulus effect on critical force of manipulation. (c) Equivalent elasticity modulus effect on critical force of manipulation. (d) Adhesion coefficient effect on critical force of manipulation.

#### 4.2. Environmental sensitivity analysis results

In this section, contact surface conditions and the effect of variations on the critical force needed to move nano-particles are investigated. Results are shown in Figures 3.

As shown in Figure 3(a), increasing substrate velocity to 600 nm/s has no especial effect on critical force magnitude. Since the applied velocity in the manipulation of nano-particles is smaller than this

value, it can be said that substrate velocity has no significant effect on critical force. Figure 3(b) shows that an increase in cantilever elasticity modulus leads to a linear increase of critical force in the LuGre model, and the more the elasticity modulus increases, the more the critical forces in this model diverge from one another. So, for cantilevers with higher elasticity modulus, choosing a proper friction model can affect simulation results. Since  $K$  variations lead to



**Figure 4.** (a) Cantilever dimensional sensitivity analysis for LuGre friction model. (b) Environmental sensitivity analysis for LuGre friction model. (c) General sensitivity analysis of dimensional and environmental parameters for LuGre friction model.

a change in LuGre frictional model parameters, effects of these variations cannot be investigated (Figure 3(c)). Changes in adhesion parameter will lead to changes in LuGre model parameters. But, since LuGre parameters are considered constant, these changes are invisible and cannot be shown (Figure 3(d)).

A general comparison of dimensional sensitivity analyses in the manipulation of nano-particles for the LuGre friction model has been shown in Figure 4(a). As can be observed, while in this model the height of the cantilever has no effect on movement critical force and can be ignored, cantilever width is the most effective parameter on movement critical force, the second being the cantilever length parameter.

According to Figure 4(a), a cantilever thickness with 76% contribution is the most effective dimensional parameter in the LuGre model. Figure 4(b) shows a general comparison between environmental parameter sensitivity analyses in nano-particle manipulation, based on the LuGre friction model. As can be seen, the cantilever elasticity modulus is the most effective parameter on the critical force of movement and, after that, substrate velocity and surface adhesion, respectively, has the most important effect. Figure 4(c) shows a general comparison of sensitivity analysis between dimensional and environmental parameters in the manipulation of nano-particles using LuGre friction models. As shown, in this friction model, cantilever

thickness, as a dimensional parameter, has the most significant effect on the critical force of manipulation. After that, cantilever length and width, respectively, play the most important role in critical force magnitude variation. Environmental parameters are less effective, but, among them, as mentioned before, the elasticity modulus of the cantilever has the most effect on critical force, while substrate velocity and surface adhesion are less important. As expected, dimensional parameters are more effective than environmental parameters.

## 5. Conclusion

Manipulation modeling is a basic instrument to obtain precise and controlled displacement of particles at micro/nano scale. In general, the manipulation process includes imaging of the substrate and the particles on it, locating the probe tip on the target nano-particle and starting the manipulation substrate or tool base moving with constant velocity. Movement of the probe tip or substrate with a constant velocity of  $V_{sub}$  leads to an increase in applied load,  $FT$ , from the nano-manipulator onto the nano-particle to a critical value of  $F_{cr}$  in order to overcome adhesion forces, such as contact and friction forces, between the particle and the substrate. From then, the movement is started. In this article, the LuGre friction model sensitivity to dimensional and environmental param-

ters has been analyzed using the Sobol method to show the importance of these parameters in variations of the manipulation critical force.

According to Figure 4(a), cantilever thickness with 76% contribution is the most effective dimensional parameter in the LuGre model. After that, length, width and the height of the cantilever are of less importance, respectively. Increasing cantilever width leads to a linear increase in critical force in the LuGre model. So, using wider cantilevers needs more accuracy in choosing a proper friction model for the manipulation process. Increasing cantilever thickness leads to an increase in critical force magnitude. Critical force growth with increasing cantilever thickness is significant. So, especially when the nano-particle is too small, it is not desirable and should be prevented. Cantilever height increase has no especial effect on critical force, and the movement critical force remains constant with cantilever height variations. A general comparison of dimensional sensitivity analyses in manipulation of the nano-particle for the LuGre friction model has been shown in Figure 4(a).

Increasing of substrate velocity up to 600 nm/s has no especial effect on critical force magnitude. Since the applied velocity in manipulation of nano-particles is smaller than this value, it can be said that substrate velocity has no significant effect on critical force. An increase in the cantilever elasticity modulus leads to a linear increase of critical force in the LuGre model, and the more the elasticity modulus increases, the more the critical forces in this model diverge from one another. So, for cantilevers with higher elasticity modulus, choosing the proper friction model can affect simulation results. Since  $K$  variations lead to changing LuGre frictional model parameters, the effects of these variations cannot be investigated. Change in adhesion parameter will lead to changes in LuGre model parameters, but since LuGre parameters are considered constant, these changes are invisible and cannot be shown.

A general comparison shows that the cantilever elasticity modulus is the most effective parameter (74%) on the critical force of movement and, subsequently, substrate velocity (about 26%) and surface adhesion, respectively, have the most important effects.

As shown, in this friction model, cantilever thickness as a dimensional parameter has the most significant effect (about 80%) on the critical force of manipulation. Subsequently, cantilever length and width (12% and 5%), respectively, play the most important role in critical force magnitude variation. Environmental parameters are less effective. But, among them, as mentioned before, the elasticity modulus of the cantilever has the most effect (about 3%) on critical force, while substrate velocity and surface adhesion are less important. As expected, dimensional

parameters are more effective than environmental parameters.

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