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Similarity solutions of axisymmetric stagnation-point flow and heat transfer of a viscous, Boussinesq-related density fluid on a moving flat plate

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Abstract. The problem of unsteady three-dimensional axisymmetric stagnation-point flow and heat transfer of a viscous compressible fluid on a flat plate is solved when the plate can move with any arbitrary time-dependently variable or constant velocity. An external low Mach number potential flow impinges, along z-direction, on the flat plate with strain rate a to produce three-dimensional axisymmetric stagnation-point flow where the plate moves toward or away from impinging flow, concurrently. An exact solution of the governing Navier-Stokes and energy equations is obtained by the use of suitably-introduced similarity transformations. The temperature of the plate wall is kept constant which is different with that of the main stream. A Boussinesq approximation is used to take into account the density variations of the fluid. The results are presented for a wide range of parameters characterizing the problem including volumetric expansion coefficient (β), wall temperature, Prandtl number and plate velocity at both steady and unsteady cases. According to the results obtained, it is revealed that when the plate moves away from the impinging flow, thermal and velocity boundary layer thicknesses get higher values compared to the plate moving upward. Besides, it is captured that the value of β and Pr number do not have any significant effect on shear stress and, also, heat transfer for a plate moving away from the incoming potential flow.

1. Introduction

The study of impinging jet problems is of considerable interest in last decades because of its great technical importance in many industrial branches specially cooling applications of electronic components, gas-turbine combustion chambers and mechanical devices. Obtaining exact solutions of Navier-Stokes and energy equations regarding the impinging problems is one of the most efficient methods to solve such problems. There are some publications available in the literature which studied stagnation flow and heat transfer based on incompressible or compressible fluids. The incompressible-based papers were started by Hiemenz [1] and Homann [2] who discussed steady two-dimensional and axisymmetric three-dimensional, respectively, and stagnation flow towards a circular cylinder. A three-dimensional stagnation-point flow on a plane boundary was considered firstly by Howarth [3]. In the more general context of a three-dimensional stagnation point, the flat plate can be allowed to slide in its own plane with constant velocity [4] and, also, can be assumed to be porous to allow for transpiration across it [5]. In another paper, Wang [6] considered axisymmetric case of stagnation flow against a sliding plate. Axisymmetric and nonaxisymmetric stagnation-point flow and heat transfer of a viscous,

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incompressible fluid on a moving cylinder in different physical situations are the main subjects of papers conducted by Saleh and Rahimi [7-9]. The steady three-dimensional stagnation-point flow of a second grade fluid against a moving flat plate is another research written by Baris [10]. In another study, exact solutions of the Navier-Stokes and energy equations of a viscous obliquely impinging flow on a moving cylinder were studied by Rahimi and Esmaeilpour [11]. Exact solutions of the Navier-Stokes and energy equations were derived by Shokrgozar Abbasi and Rahimi [12,13] to solve the problem of stagnation-point flow and heat transfer of an incompressible fluid on a flat plate with and without transpiration. Also, Abbasi et al. [14] investigated the unsteady case of this problem. Moreover, it was shown by Weidman et al. [15] that self-similar solution of the Navier-Stokes equation exists if the isolated infinite flat plate moves at a constant speed normal to the oncoming stagnation point flow.

Some papers available in the literature studied the compressible flow in the stagnation region of bodies using boundary layer equations. The characteristics of such a flow were scrutinized under different physical considerations in [16-17]. Kumari and Nath [18] studied the theory of the response of the compressible laminar boundary layer flow to the variation of the external stream velocity with time at a three-dimensional stagnation point, numerically. They solved such a problem when the flow is asymmetric with respect to the stagnation point [19]. Subsequently, in another paper, Kumari and Nath [20] gained the self-similar solution of the forgoing problem with mass transfer only when the free stream velocity varies inversely as a linear function of time. Vasantha and Nath [21,22] obtained solutions of the unsteady compressible second-order boundary layer flow at the stagnation point, analytically. Additionally, Zheng et al. [23] obtained similarity solutions to a second order heat equation with convection in an infinite medium. They used suitable similarity transformations in order to reduce the parabolic heat equation to a class of singular nonlinear boundary value problems. The same authors, in another research [24], solved the problem of compressible boundary layer behind a thin expansion wave by using the application of the similarity transformation and shooting technique. The objective of the article presented by Zuccher et al. [25] was to analyze the compressible, non-parallel boundary layer of the flow passing a flat plate and sphere. Furthermore, Turkyilmazoglu [26] was concerned with the case in which exact solution of the steady laminar flow of a compressible viscous fluid over a rotating disk was obtained in the presence of uniformly applied suction or blowing. The steady stagnation-point flow and heat transfer of a viscous, compressible fluid on an infinite stationary cylinder is the subject of the paper recently written by Mohammadiun and Rahimi [27]. The potential flow impinging on the cylinder was assumed to be low Mach number one. They found similarity parameters of their problem for the first time.

In this paper, the general unsteady three-dimensional axisymmetric stagnation-point flow and heat transfer of a viscous, compressible fluid of a low Mach number flow impinging on a flat plate are intended to be solved for the first time. The flat plate is moving toward or away from the impinging flow at both constant and time-dependently variable velocity. The density of the fluid, also, changes due to the temperature difference existing between the plate and incoming infinite fluid. New similarity transformations are introduced in order to reduce the governing Navier-Stokes and energy equations to ordinary differential equations which are much easier to solve. The results are presented over a wide range of parameters characterizing the problems such as coefficient of volumetric expansion, wall temperature and Prandtl number at both steady and unsteady cases.

2. Problem formulation

The problem of steady and unsteady three-dimensional axisymmetric stagnation-point flow and heat transfer of a viscous compressible fluid on a flat plate is aimed to be solved when the plate is moving toward or away from the oncoming low Mach number flow at both time-dependently variable and constant velocity. In order to solve this problem, the axisymmetric cylindrical coordinate system (r, z) with corresponding velocity components (u, w) is selected, as illustrated in Figure 1. An external potential flow impinges along z-direction on the moving plate, firstly centered at $z = 0$, with strain rate a. Moreover, the temperature of the plate wall is maintained constant which is different with that of the main stream fixed at 25°C. The Navier-Stokes

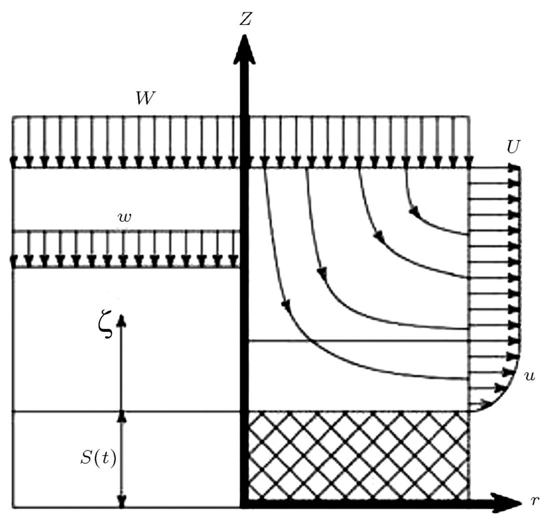

Figure 1. Schematic of the problem.

and energy equations governing this problem are:

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r u)+\frac{\partial}{\partial z}(\rho w)=0, \tag{1}$$

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial r}+\rho w\frac{\partial u}{\partial z}=-\frac{\partial P}{\partial r}$$
$$+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r})-\frac{u}{r_2}+\frac{\partial^2 u}{\partial z^2}\right), \tag{2}$$

$$\rho\frac{\partial w}{\partial t}+\rho u\frac{\partial w}{\partial r}+\rho w\frac{\partial w}{\partial z}=-\frac{\partial P}{\partial z}$$
$$+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial w}{\partial r})+\frac{\partial^2 w}{\partial z^2}\right), \tag{3}$$

$$\rho c_p\left[\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right]=k\left[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})+\frac{\partial^2 T}{\partial z^2}\right], \tag{4}$$

where, p, ρ, μ, k and T are pressure, density, dynamic viscosity, thermal conductivity and temperature, respectively. It is worth noting that dynamic viscosity and thermal conductivity of the fluid are assumed to be constant. Furthermore, the dissipation terms of the energy equation are negligible at the stagnation region.

3. Similarity solutions

3.1. Fluid flow solution

By solving the momentum equations in potential region, the velocity components can be gained as:

$$U=a(t)r, \tag{5}$$

$$W=-2a(t)\zeta, \tag{6}$$

where $\zeta=z-S(t)$ and $a(t)=\frac{\partial w}{\partial \zeta}$. Here, $S(t)$ is the amount of vertical displacement of the plate, positively defined when the plate moves toward the incoming far field flow, and is a function of time. Hence, ζ and then flow strain rate $a(t)$ can be expressed as time-dependent functions.

A reduction of Navier-Stokes and energy equations to ordinary differential equations is sought by using suitably introduced new similarity transformations as below:

$$\eta=\left(\rho_\infty\frac{a_o}{\mu}\right)^{\frac{1}{2}}\left(\int_0^z(\frac{\rho}{\rho_\infty})dz-S(t)\right), \tag{7}$$

$$u=a(t)rf'(\eta), \tag{8}$$

$$w=-\frac{2}{c}a(t)\left(\frac{\mu}{\rho_\infty a_0}\right)^{\frac{1}{2}}f(\eta)+\dot{S}\frac{\ln c}{c}, \tag{9}$$

where η is the similarity variable, the terms involving $f(\eta)$ comprise the cylindrical similarity form for stagnation-point flow, prime denotes differentiation with respect to η, a_o is the reference potential flow strain rate at the time = 0, the subscript w and ∞ refer to the conditions at the wall and in the free stream, respectively, and $\dot{S}$ is the plate velocity. It is interesting to note how the effect of the plate velocity shows itself in w-component of velocity as in Eq. (9). In case of incompressible fluid, $c(\eta)=1$, this part has no role in the results.

In order to capture the effects of variations of temperature on the density of the fluid, a parameter, $c(\eta)$, named density ratio, is introduced as:

$$c(\eta)=\frac{\rho(\eta)}{\rho_\infty}. \tag{10}$$

From Boussinesq approximation for low Mach number flow:

$$\rho\approx\rho_\infty\left[1-\beta(T-T_\infty)-\frac{\beta^2(T-T_\infty)^2}{2}\right.$$
$$\left.-\frac{\beta^3(T-T_\infty)^3}{3!}-...\right]$$
$$\Rightarrow\quad\frac{\rho}{\rho_\infty}=c(\eta)\approx 1-\beta(T-T_\infty)$$
$$-\frac{\beta^2(T-T_\infty)^2}{2}-\frac{\beta^3(T-T_\infty)^3}{3!}-..., \tag{11}$$

in which, β is the volumetric expansion coefficient. It is clear that for the case of incompressible fluid, $\beta=0$. Hence $\rho=\rho_\infty$ and $c(\eta)=1$. Inserting the Transformations (7)-(10) into Eqs. (1)-(3) causes the continuity equation to be satisfied automatically, yeilding an ordinary differential equations reduced from r-momentum, and also an expression for the pressure, obtained by integrating Eq. (3) in z-direction as:

$$cf'''+\left(2\tilde{a}f+\tilde{\dot{S}}(1-\ln c)+c'\right)f''$$
$$+\left(-\frac{1}{\tilde{a}}\frac{\partial\tilde{a}}{\partial\tau}-\tilde{a}f'\right)f'-\frac{1}{c\tilde{a}\xi}\frac{\partial\tilde{P}}{\partial\xi}=0, \tag{12}$$

where:

$$\frac{1}{c\tilde{a}\xi}\frac{\partial\tilde{P}}{\partial\xi}=-\frac{1}{c}(\tilde{a})-\frac{1}{c}\left(\frac{1}{\tilde{a}}\frac{\partial\tilde{a}}{\partial\tau}\right),$$

$$\tilde{P}-\tilde{P}_o=-\frac{\xi^2}{2}\left(\frac{\partial\tilde{a}}{\partial\tau}+(\tilde{a})^2\right)-2\tilde{a}\tilde{\dot{S}}(\frac{f}{c})+(\tilde{\dot{S}})^2\frac{\ln c}{c}$$
$$+\int\left(2\frac{\partial\tilde{a}}{\partial\tau}\frac{f}{c}-\tilde{\ddot{S}}\frac{\ln c}{c}+\frac{\tilde{a}}{c^2}(f'c-c'f)\right.$$
$$.(2\tilde{\dot{S}}\ln c-4\tilde{a}f)+\frac{\tilde{\dot{S}}}{c^2}c'(1-\ln c).(2\tilde{a}f-\tilde{\dot{S}}\ln c)$$

$$- 2\frac{\tilde{a}}{c}\left(f''c - c''f - c'f' + (c')^2\frac{f}{c}\right)$$

$$+ \frac{\dot{\tilde{S}}}{c}\left(c''(1-\ln c) - \frac{(c')^2}{c}(2-\ln c)\right)\Bigg)d\eta. \qquad (13)$$

There are several dimensionless parameters introduced in Eqs. (12) and (13), which are defined as:

$$\tilde{a}(\tau) = \frac{a(t)}{a_0}, \qquad \tilde{P} = \frac{P}{\rho_\infty a_0 \nu_\infty}, \qquad \tau = a_o t,$$

$$\dot{\tilde{S}} = \dot{S}(t)/\sqrt{(a_0\nu_\infty)}, \qquad \ddot{\tilde{S}} = \frac{\ddot{S}}{a_o(\nu_\infty a_o)^{\frac{1}{2}}},$$

$$\xi = \sqrt{\frac{a_0}{\nu_\infty}}r, \qquad \frac{\partial\tilde{a}}{\partial\tau} = \frac{\ddot{\tilde{S}}}{\eta_0} + \frac{(\dot{\tilde{S}})^2}{\eta_0^2} + \frac{\dot{\tilde{S}}\tilde{W}_o}{\eta_0^2}, \qquad (14)$$

where, $\tilde{a}(\tau)$, $\tilde{P}$, τ, $\dot{\tilde{S}}$, $\ddot{\tilde{S}}$ and ξ are dimensionless forms of the quantities strain rate, pressure, time, plate velocity, plate acceleration and r, respectively. In general, when the plate moves with time-dependently variable velocity, the strain rate, a, can be expressed as a function of time. Hence, $\frac{\partial\tilde{a}}{\partial\tau}$ represents the strain variation with respect to time, and is taken into account when the plate moves with time-dependent velocity and acceleration. The quantity η_o used in this relation expresses the amount of vertical distance from the plate in which the velocity of flow incoming to the plate is affected by the movement of the plate and starts decreasing. The quantity $\tilde{W}_o$ is dimensionless velocity of potential flow at η_o.

The boundary conditions used for Eq. (12) are:

$$\eta = 0: \quad f = \frac{\dot{\tilde{S}}\ln c_w}{2\tilde{a}}, \quad f' = 0, \qquad (15)$$

$$\eta \to \infty: \quad f' = 1, \qquad (16)$$

where:

$$c_w = 1 - \beta(T_w - T_\infty). \qquad (17)$$

Note that for an incompressible fluid impinging on a stationary plate at steady state conditions, $\beta = 0.0$, $\dot{S} = 0$, $\ddot{S} = 0$ and $a(t) = a_o$, Eq. (12) simplifies to the case of Homann flow obtained in [2], and this is one way of validation of the results achieved.

3.2. Heat transfer solution

To transform the energy equation into a non-dimensional form for the case of defined wall temperature, we introduce:

$$\theta = \frac{T(\eta) - T_\infty}{T_w - T_\infty}. \qquad (18)$$

Making use of Transformations (7)-(10) and (18), this equation may be written as:

$$c\theta'' + c'\theta' + \left(2\tilde{a}f + \dot{\tilde{S}}(1-\ln c)\right)\mathrm{Pr}.\theta' = 0, \qquad (19)$$

in which Pr is the Prandtl number.

It is worth noting that the coupled system of (12), (13) and (19) is the most general form for any arbitrary flat plate movement in vertical direction. The boundary conditions needed to solve Eq. (19) are defined as:

$$\eta = 0: \quad \theta = 1$$

$$\eta \to \infty: \quad \theta = 0. \qquad (20)$$

The local heat transfer coefficient on the flat plate is calculated from:

$$h = \frac{q_w}{T_w - T_\infty}. \qquad (21)$$

Using Eq. (21) and dimensionless parameters, the dimensionless form of heat transfer coefficient for this study can be gained as:

$$H = -\theta'(\eta = 0)c_w, \qquad (22)$$

where:

$$H = \frac{h}{k\left(\rho_\infty\frac{a}{\mu}\right)^{\frac{1}{2}}}. \qquad (23)$$

A finite difference procedure consisting of Tri-Diagonal Matrix Algorithm (TDMA) is used to numerically solve the governing equations (12), (13) and (19). The numerical procedure is repeated until the difference between the results of two repeated sequences of each of the equations becomes less than 0.00001.

4. Shear stress

Shear stress at the wall surface is given by:

$$\sigma = \mu\frac{\partial u}{\partial z}_{z=0}. \qquad (24)$$

By introducing the dimensionless parameters defined in Section 3, the dimensionless form of shear stress on the flat plate is obtained as:

$$\tilde{\sigma} = \xi f''(\eta = 0)c_w, \qquad (25)$$

where:

$$\tilde{\sigma} = \frac{\sigma}{\rho_\infty a_o \nu_\infty}.$$

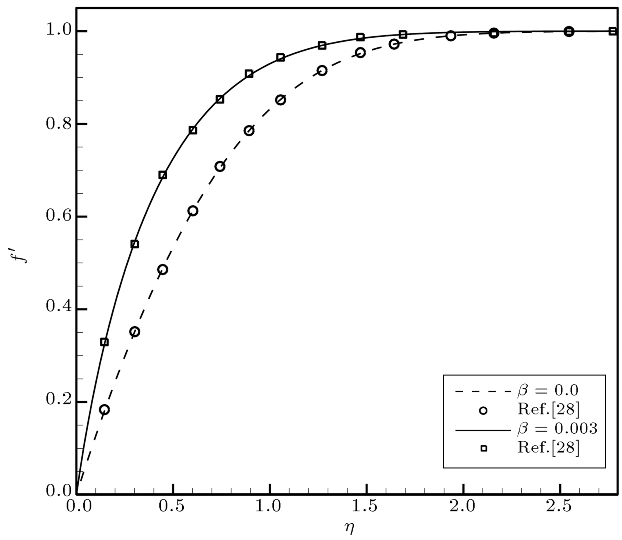

Figure 2a. Comparison of f' profiles with [28] when $T_w = 200°$C and Pr=0.7.

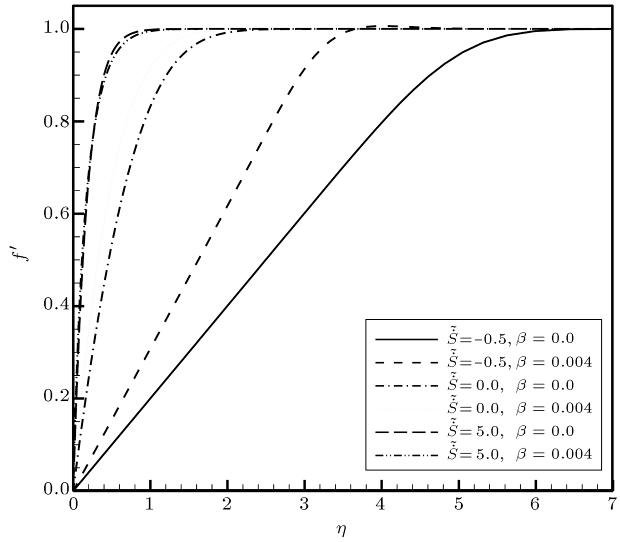

Figure 2b. Effect of β parameter on f' profiles for different values of plate velocity when $T_w = 100°$C and Pr=0.7.

5. Presentation of results

In order to validate the results obtained, f' distributions are compared with those of [28] for the case of $T_w = 200°$C and Pr=0.7. As can be seen from Figure 2(a), there is no significant difference between the results achieved and those in [28].

At first, the influences of the plate velocity on f' distributions are investigated for selected values of β number, in Figure 2(b), and wall temperature, in Figure 3. As it can be noticed from these two figures, when the plate is moving away from the incoming potential flow, i.e. with a negative value of velocity, the thickness of the velocity boundary layer is considerably higher compared to that of the plate with zero or

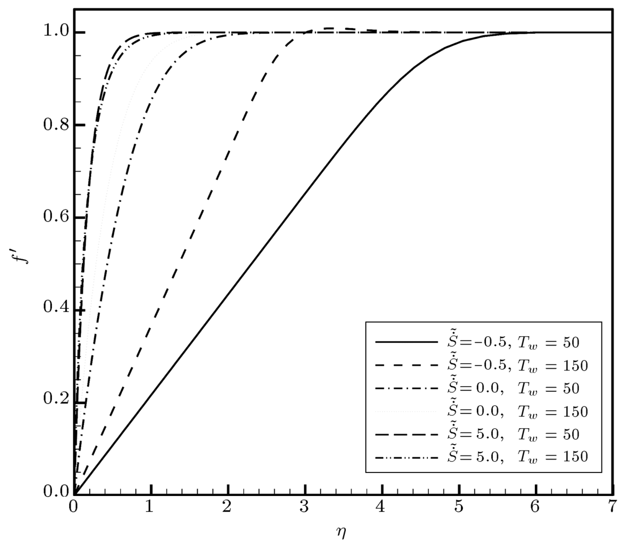

Figure 3. Effect of wall temperature on f' profiles for different values of plate velocity when $\beta = 0.003$ and Pr=0.7.

positive value of velocity. Moreover, it can be found out that the increase of both β number from 0 to 0.004 and wall temperature from 50°C to 150°C has somehow the same effect on f' profiles whether the plate moves toward the impinging flow or away from it. The effects of enhancement of these two characterizing parameters can be more noticeably seen when the plate is moving with high negative values of velocity. As the speed of the plate approaches zero and, then, positive ones the effect of the change of β parameter and wall temperature on f' distributions decreases, gradually. It can be claimed, from these figures, that f' distributions will be, somehow, independent of β coefficient or wall temperature in physical situations in which the plate is moving toward the impinging flow with high speeds; $\tilde{\tilde{S}} = 5$ for example.

Next, in Figure 4 effects of the plate velocity along with β parameter on dimensionless distributions of velocity component in z-direction is reported. With the increase of β number, the general tendency for the absolute values of w-component is to increase for a plate receding from the main stream and to decrease for a plate advancing toward the main stream.

Distributions of the dimensionless temperature are shown for different values of dimensionless plate velocity and selected β numbers in Figure 5, and Pr numbers in Figure 6, when $T_w = 100°$C and Pr=0.7. It is revealed from these two figures that the thermal boundary layer thickness becomes smaller as the negative plate velocity tends to zero and then, afterwards, positive ones. Besides, it is understood that the increase of β parameter from 0, incompressible-stated fluid, to 0.004 has a significant effect on temperature profiles if the plate recedes from the impinging

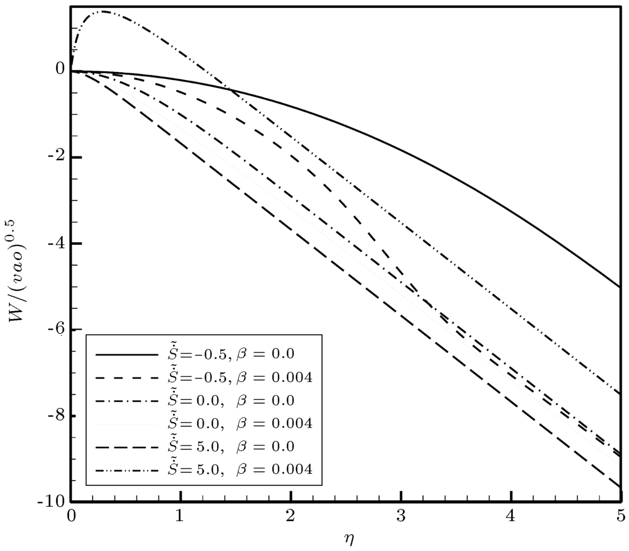

Figure 4. Effect of β parameter on dimensionless w component profiles for different values of plate velocity when $T_w = 100°$C and Pr=0.7.

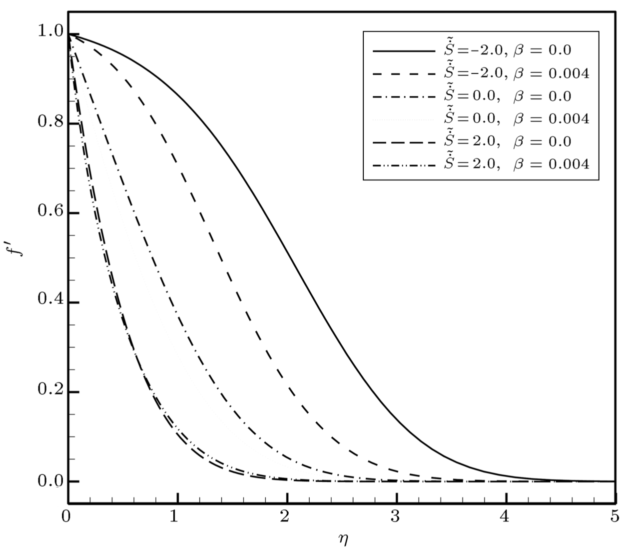

Figure 5. Effects of β parameter on θ profiles for different values of plate velocity when $T_w = 100°$C and Pr=0.7.

flow. However, the more the speed of the plate advancing toward the normal incoming flow enhances, the less important β number becomes. Also, it can be seen from Figure 6 that the increase of Pr number at any plate velocity has a considerable influence on temperature profiles and causes the thermal boundary layer thickness to decrease.

The changes of dimensionless pressure, due to the increase of β parameter and wall temperature, are, respectively, shown in Figures 7 and 8. As can be found out, the absolute values of the pressure in the vicinity of the plate receding from the incoming potential flow are considerably lower than those when the plate is moving toward the main stream. Moreover, as it is reported,

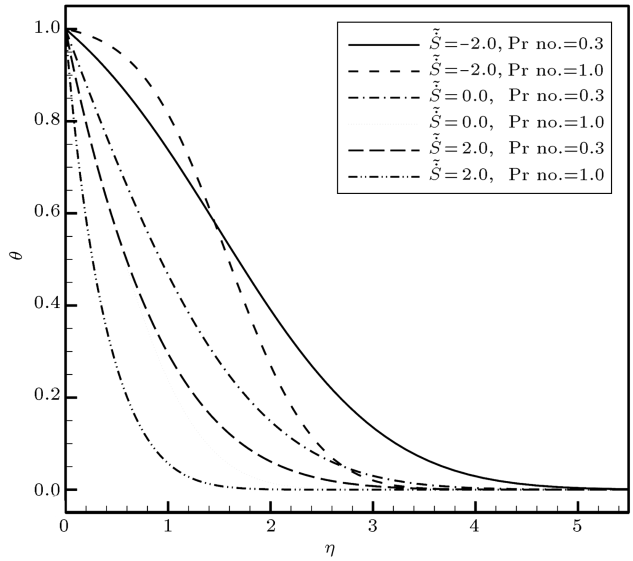

Figure 6. Effects of Pr no. on θ profiles for different values of plate velocity when $T_w = 100°$C and Pr=0.003.

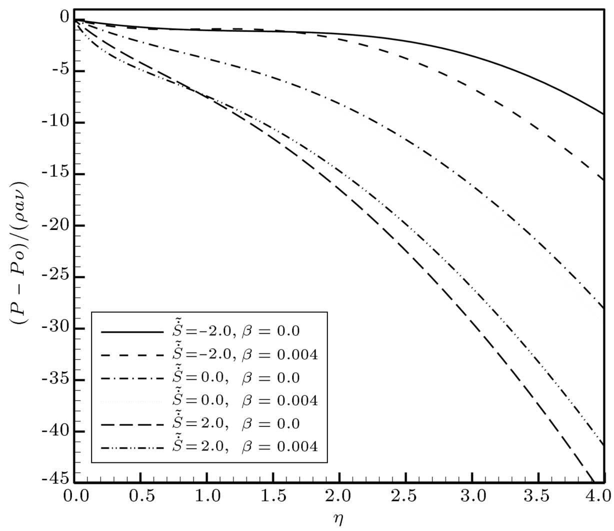

Figure 7. Effects of β parameter on dimensionless pressure distributions for different values of plate velocity when $T_w = 100°$C and Pr=0.7.

there is no noticeable change in pressure values in the region close to the plate, up to $\eta = 2$, for $\tilde{S} = -2.0$.

The influences of β and Pr numbers on dimensionless heat transfer coefficient are investigated in Figures 9 and 10 at different dimensionless plate velocity. For a plate with a negative value of velocity, the enhancement of these two characterizing parameters does not have any significant effect on heat transfer between the plate and viscous fluid close to the plate. With the increase of the plate speed toward the impinging flow, the effects of the increase of β and Pr numbers on heat transfer become more dominated in such a way that the increase of β form 0.0 to 0.004 causes the H coefficient to decrease, and the increase of Pr number from 0.3

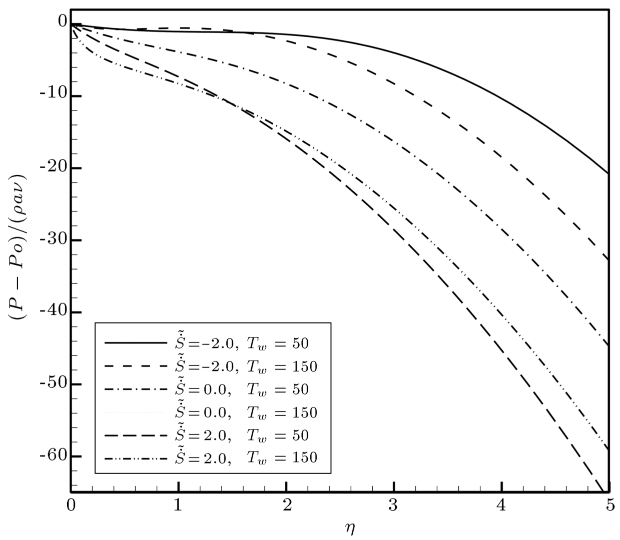

Figure 8. Effect of wall temperature on dimensionless pressure distributions for different values of plate velocity when $\beta = 0.003$ and Pr=0.7.

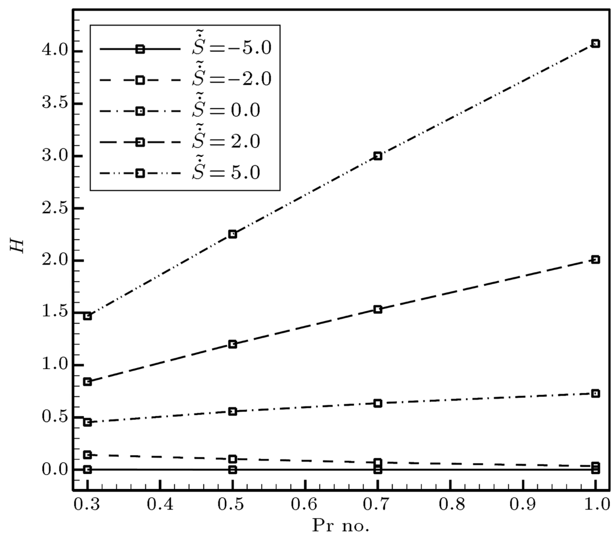

Figure 10. Effect of Pr no. on H values for different amounts of plate velocity when $T_w = 100°$C and Pr=0.7.

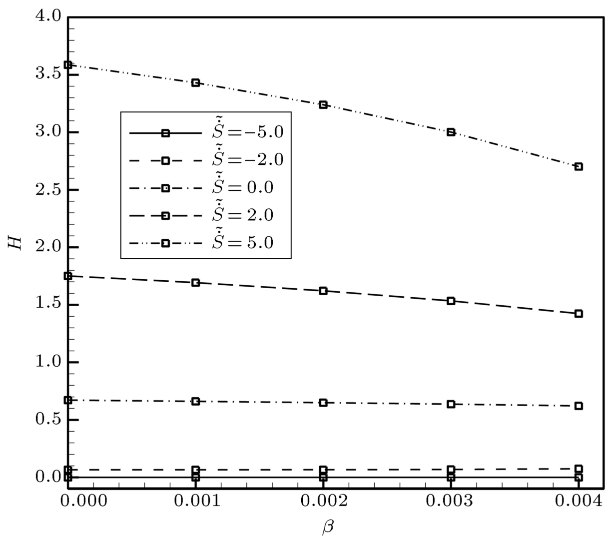

Figure 9. Effect of β parameter on H values for different amounts of plate velocity when $T_w = 100°$C and Pr=0.7.

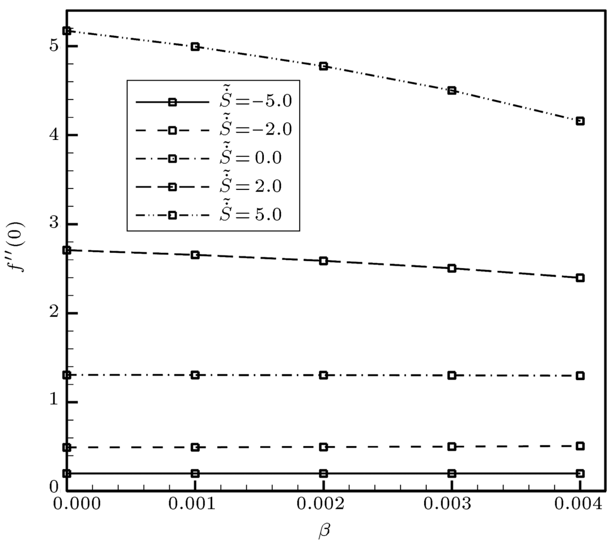

Figure 11. Effect of β parameter on shear stress for different values of plate velocity when $T_w = 100°$C and Pr=0.7.

to 1.0 brings about the increase of this coefficient. Note that with constant fluid characteristics, as the plate velocity tends to zero and then positive ones, the amount of heat transfer between the plate and viscous fluid increases noticeably. Furthermore, it is shown in Figures 11 and 12 that shear stress on a stationary plate or a plate receding from the incoming far field flow is independent of the value of β and Pr numbers. Note that if characterizing parameters are kept constant, the change of the plate velocity from negative values to positive ones has a significant effect on shear stress existing on the plate wall.

In unsteady cases, the plate can move with any arbitrary time-dependent velocity function. As the most practical example for time-dependently moving plate, the exponential function, which can be used to model a three-dimensional axisymmetric solidification problem, is taken to have the form:

$$\dot{S} = A\exp(-t), \tag{26}$$

where A is a constant. The results obtained by using the plate velocity function mentioned above are presented in Figures 13 through 19.

Velocity component in r-direction is distributed in terms of time for selected values of β number in Figure 13, and wall temperature in Figure 14. With the passage of time, the velocity and acceleration of the plate approach zero which cause the f' value to decrease. This reality is more noticeable when the

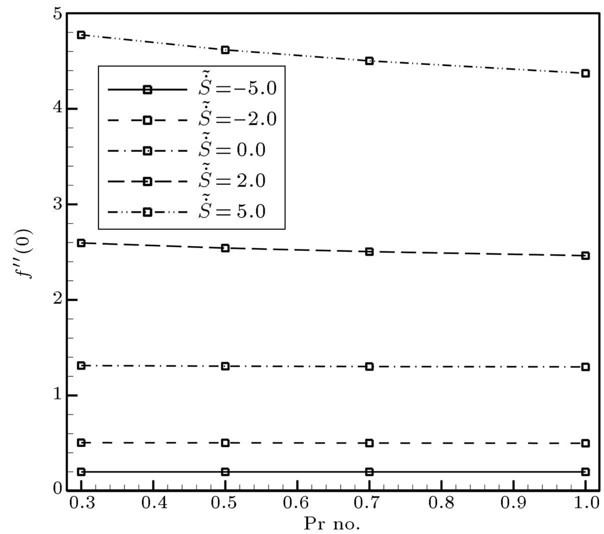

Figure 12. Effect of Pr no. on shear stress for different values of plate velocity when $T_w = 100°$C and Pr=0.7.

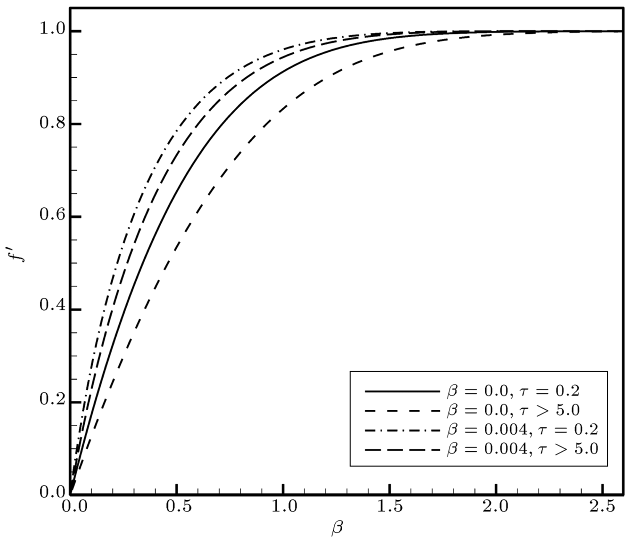

Figure 13. Effect of β parameter on f' profiles at different times when $T_w = 125°$C and Pr=0.7.

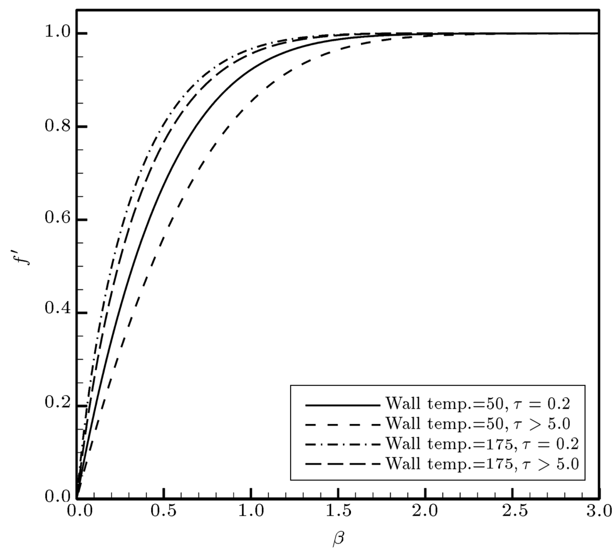

Figure 14. Effect of wall temperature on f' profiles at different times when $\beta = 0.003$ and Pr=0.7.

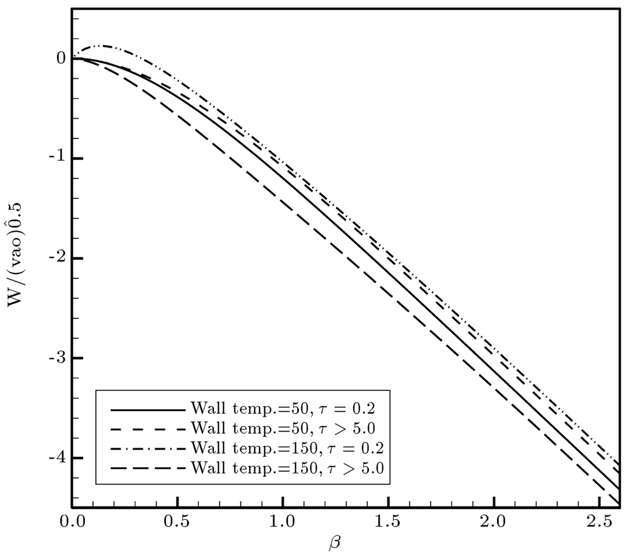

Figure 15. Effects of wall temperature on dimensionless form of w component distributions at different times when $\beta = 0.003$ and Pr=0.7.

density variations with respect to temperature are negligible, which are evident for the cases of $\beta = 0$ in Figure 13 and a wall temperature of 50°C in Figure 14. It is worth mentioning that for an incompressible fluid with $\beta = 0$, the increase of time, which makes the plate velocity and acceleration close to zero, approaches the f' profile to that of Homann flow, which is captured for $\tau > 5.0$ in Figure 13.

In Figure 15, one can see the sample results of w-component distributions with respect to time and selected wall temperatures. The effect of passage of time on distributions of dimensionless heat transfer coefficient and shear stress in a wide range of wall temperature is reported in Figures 16 and 17. As it is revealed, the increase of time, firstly, causes the decrease in the amounts of both heat transfer coefficient and shear stress. This trend is continued to reach a constant value at steady state conditions. It should be noted that the more the wall temperature, the less H and shear stress will be at any selected time.

Later on, the variations of dimensionless temperature and pressure versus time are shown in Figures 18 and 19 for selected Pr numbers, 0.3 and 1, respectively. As the velocity and acceleration of the plate vanish, the increase in temperature and decrease in the absolute value of pressure are captured. Besides, the enhancement of Pr number causes the thermal boundary layer thickness to decrease (Figure 18), however, it does not have any considerable effect on pressure distributions (Figure 19).

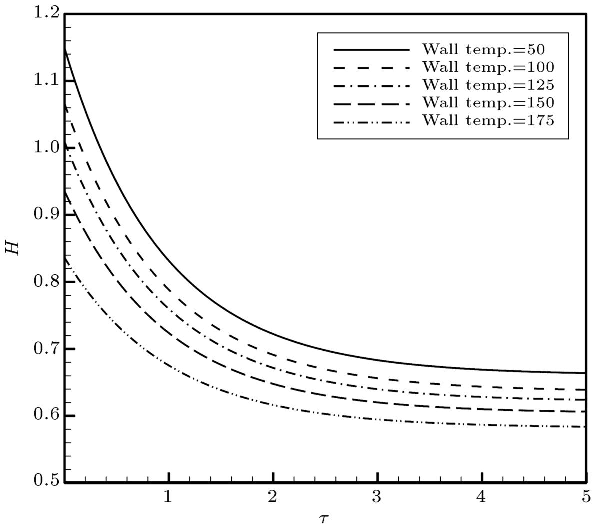

Figure 16. Distributions of dimensionless heat transfer coefficient at unsteady procedure for different values of wall temperature when $\beta = 0.003$ and Pr=0.7.

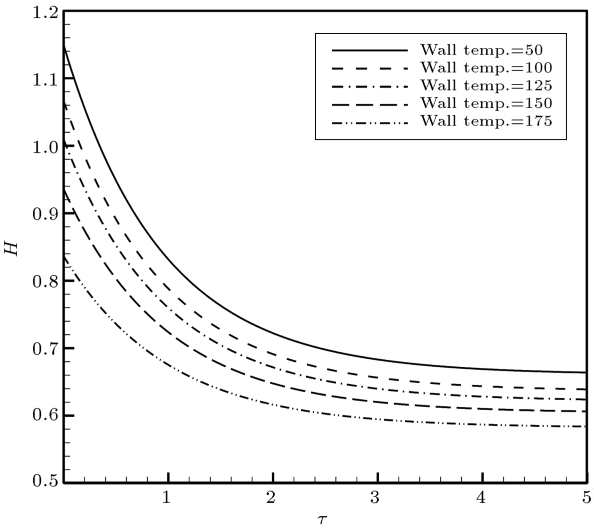

Figure 17. Distributions of shear stress on the plate in unsteady procedure for different values of wall temperature when $\beta = 0.003$ and Pr=0.7.

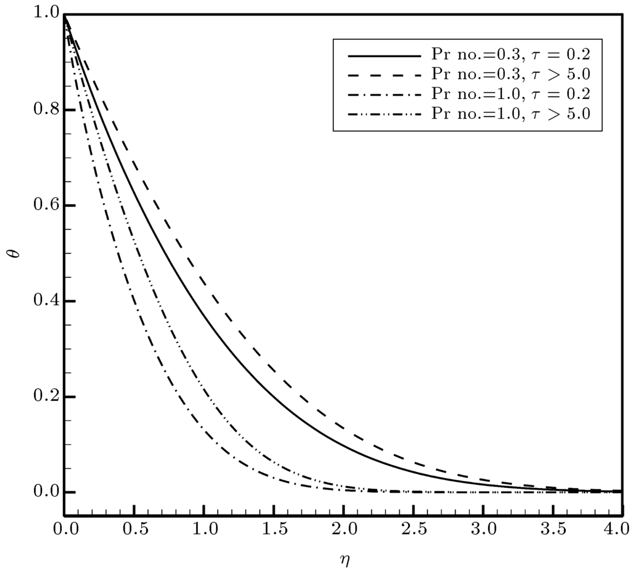

Figure 18. Effect of Pr on θ profiles at different times when $T_w = 125°$C and $\beta = 0.003$.

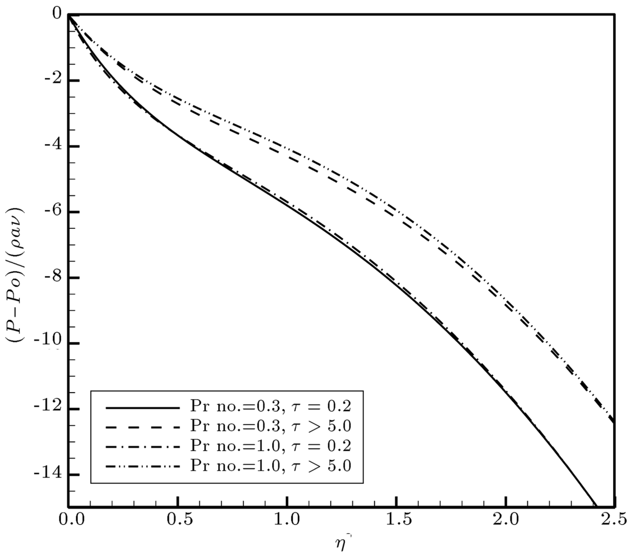

Figure 19. Effect of Pr on pressure distributions at different times when $T_w = 125°$C and $\beta = 0.003$.

6. Conclusions

A similarity solution for the problem of unsteady three-dimensional axisymmetric stagnation-point flow and heat transfer of a viscous, compressible fluid on an accelerated flat plate has been obtained in this paper when an external low Mach number flow with strain rate a impinges on this plate. Firstly, the introduced similarity transformations are used to reduce the unsteady Navier-Stokes and energy equations to a coupled system of nonlinear ordinary differential equations. The density of the fluid changes because of the temperature difference existing between the plate and incoming far field flow. A Boussinesq approximation is used to take the density variations into account. The results achieved in this paper were presented for a wide range of parameters characterizing the problem including volumetric expansion coefficient (β), wall temperature, Prandtl number and plate velocity. The solutions obtained show that the thickness of velocity and thermal boundary layer for a plate receding from the impinging flow is much more than those when the plate moves toward the incoming potential flow. Moreover, it was shown that the increase of β and Pr numbers does not have any significant effect on dimensionless heat transfer coefficient and shear stress at steady state conditions. However, the passage of time causes H and $\tilde{\sigma}$ to decrease for an exponentially moving plate.

Nomenclature

$a(t)$	Time-dependent flow strain rate
a_o	Flow strain rate at time = 0
c	Density ratio
f, g	Similarity functions
h	Local heat transfer coefficient
H	Dimensionless heat transfer coefficient
k	Thermal conductivity of the fluid
p	Pressure
P	Dimensionless pressure
Pr	Prandtl number
$S, \dot{S}, \ddot{S}$	Displacement, velocity and acceleration of the plate, respectively, in z-direction
$\tilde{S}, \dot{\tilde{S}}, \ddot{\tilde{S}}$	Dimensionless displacement, velocity and acceleration of the plate, respectively, in z-direction
T	Temperature
u, w	Velocity components near the plate in x and z directions
U, W	Potential region velocity components in x and z directions
r, z	Cylindrical coordinates

Greeks

β	Volumetric expansion coefficient
η	Similarity variable
μ	Dynamic viscosity
θ	Dimensionless temperature
σ	Shear stress
τ	Dimensionless time
ζ	Variable $(z - S_{(t)})$
ξ	Dimensionless x axis
ρ	Density
ν	Kinematic viscosity

Subscripts

o	Stagnation point
w	Wall
∞	Infinite

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