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Strain gradient thermoelasticity of functionally graded cylinders

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Abstract. In this paper, strain gradient thermo-elasticity formulation for axisymmetric Functionally Graded (FG) thick-walled cylinders is presented. For this purpose, the elastic strain energy density function is considered to be a function of gradient of strain tensor in addition to the strain tensor. The material properties are assumed to vary according to a power law in radial direction. Using the constitutive equations and equation of equilibrium in the cylindrical coordinates, a fourth order non-homogenous governing equation for thermo-elastic analysis of thick-walled FG cylinders subjected to thermal and mechanical loadings is obtained and solved numerically. Results show that the intrinsic length parameter affects the stress distribution in FG thick-walled cylinders greatly and increasing the intrinsic length parameter reduces the maximum radial and hoop stresses. Also, the effects of FG power indices on the radial and hoop stresses are studied.

1. Introduction

There are evidences that material responses in the elastic region are dependent on some other parameters rather than Lame parameters. Strain gradient elasticity has been recently developed to take into account this characteristic of materials response [1,2]. In strain gradient elasticity new parameters enter the constitutive equations through the elastic strain energy density function. Mindlin, in 1964 [3] used the assumption that the elastic strain energy density function can be a function of the gradient of strain tensor in addition to strain tensor. In this way, new material constants were introduced and entered into the constitutive equations.

There are many studies which have used the strain gradient elasticity in the analysis of behavior of materials [4-8]. Eshel and Rosenfeld [4] analyzed the stress concentration problem of a circular cylindrical hole in a homogenous, isotropic and centrosymmetric infinite elastic solid subjected to uniaxial tension. They showed that stress concentration depends on the radius of cavity, Poisson ratio and four new material parameters. Georgiadis et al. [5] studied free harmonic torsional motion of a homogenous half-space by strain gradient elasticity. They focused on the possibility of existence of torsional surface waves in gradient-elastic half-space. They showed the dependency of cut-off frequencies and the character of dispersion upon the size of the material unit cell. Paulino et al. [6] used the strain gradient elasticity for analysis of mode III fracture in Functionally Graded (FG) materials and used Fourier transform to solve the governing partial differential equations. They studied the stress and displacement fields near the crack and found that the stress intensity factor is dependent on intrinsic length parameters. Kong, et al. [7] studied the static and dynamic problems of Bernoulli-Euler beams analytically by using strain gradient elasticity. They considered two boundary value problems for cantilever beams and

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analyzed the size effects on the beam bending response as well as its natural frequencies.

In recent years, FG materials, in which properties are dependent on position, have attracted many researchers because of their unique features. Special characteristics of FG materials make them a potential candidate for many applications such as thick-walled cylinders. Several researchers have studied thermo-elastic deformation of FG cylinders in recent years [9-14]. Yang [9] solved the time dependent thermal stresses in FG cylinders. He found an analytical solution for thermo-elastic stresses in the materials with elastic behavior and an asymptotic solution for the materials with creep behavior. Tarn [14] considered the thermo-mechanical problem of inhomogeneous, solid or hollow circular FG cylinders. He assumed a power law dependence of the moduli and the cylinder was assumed to be subjected to an axial force and a torque at the ends. Tutuncu and Ozturk [13] studied the thermo-elasticity of FG cylinders by exponentially varying properties. They assumed that Poisson ratio is constant and Young modulus varies exponentially through the thickness. They obtained power series solutions for stresses and displacements in FG cylindrical vessels subjected to internal pressure. Jabbari et al. [11] analyzed mechanical and thermal stresses in FG hollow cylinders due to radially symmetric loads. They assumed that material properties depend on the radial position as a power function. They solved the heat conduction and Navier equations directly and obtained an analytical solution for the stress and strain components. Shao [10] considered the problem of FG hollow cylinders with finite length under mechanical and thermal loading. He used a multi-layered approach based on the theory of laminated composited and found a series solution for the components of temperature and displacement fields in the finite FG cylinders. Jabbari et al. [12] studied the axisymmetric mechanical and thermal stresses in thick short length FG cylinders. They used Fourier series to find an exact solution for the steady-state stress components in two-dimensional axisymmetric FG cylinders with material properties described with a power law.

In most of the studies on thick-walled FG cylinders, classical theory of elasticity has been employed. Thus, there is a need for a study to consider the effect of intrinsic length parameter on the thermo-elastic response of FG cylinders. In our previous work, we analytically studied the strain gradient elasticity solution for analysis of FG micro-cylinders [15]. The material properties were assumed to obey a power law in radial direction.

In this paper, axisymmetric strain gradient thermo-elasticity formulation of thick-walled FG cylinders subjected to thermal and mechanical loadings is presented. The fourth order non-homogenous governing equation is obtained and it is solved numerically together with four boundary conditions. Results are presented for different values of the intrinsic length parameter and it is compared with classical elasticity solution. Furthermore, the effects of power indices and the outer radius of the cylinder radius on the stresses and radial displacement are studied.

2. Strain gradient thermo-elasticity

2.1. Equation of heat conduction

The equation governing the temperature field in a long, thick-walled, FG cylinder in the axisymmetric condition while the temperatures at the inner and outer radii of the cylinder are T_i and T_o, respectively, is given by [11,12]:

$$\frac{d}{dr}\left(k(r)\frac{dT}{dr}\right)+\frac{k(r)}{r}\frac{dT}{dr}=0, \tag{1}$$

$$T=T_i \quad \text{at} \quad r=r_i,$$

$$T=T_o \quad \text{at} \quad r=r_o, \tag{2}$$

where T is the temperature change, $k(r)$ is thermal heat conduction coefficient, r is radial position, and r_i and r_o are the inner and outer radii of the cylinder. In this paper, it is assumed that the thermal heat conduction coefficient obeys the power law in a radial direction:

$$k(r)=k_0 r^m, \tag{3}$$

where k_0 and m are material constants. The solution of Eq. (1) is:

$$T(r)=c_1+c_2 r^{-m}, \tag{4}$$

where c_1 and c_2 are found from the boundary conditions (Eq. (2)) as:

$$c_1=\frac{T_i r_o^{-m}-T_o r_i^{-m}}{r_o^{-m}-r_i^{-m}},$$

$$c_2=\frac{T_o-T_i}{r_o^{-m}-r_i^{-m}}. \tag{5}$$

2.2. Governing equation

In the strain gradient elasticity, the elastic strain energy density function is assumed to be a function of the gradient of strain tensor, in addition to the strain tensor. Altan and Aifantis [16] presented a simplified form of the elastic strain energy density function introduced by Mindlin and Eshel [17]. Here, the same form is used to obtain the elastic strain energy density function for thick-walled FG cylinders. For a FG cylinder, elastic strain energy density function, w, can be written in the cylindrical coordinates as [15]:

$$w = \frac{1}{2}\lambda(r)(\varepsilon_{rr}(r)+\varepsilon_{\theta\theta}(r)+\varepsilon_{zz}(r))^2 + \mu(r)\left(\varepsilon_{rr}^2(r)+\varepsilon_{\theta\theta}^2(r)+\varepsilon_{zz}^2(r)\right) + \frac{1}{2}\lambda(r)l^2\left(\frac{d}{dr}\varepsilon_{rr}(r)+\frac{d}{dr}\varepsilon_{\theta\theta}(r)+\frac{d}{dr}\varepsilon_{zz}(r)\right)^2 + \mu(r)l^2\left(\left(\frac{d}{dr}\varepsilon_{rr}(r)\right)^2+\left(\frac{d}{dr}\varepsilon_{\theta\theta}(r)\right)^2 + \left(\frac{d}{dr}\varepsilon_{zz}(r)\right)^2\right), \tag{6}$$

where l is the intrinsic length parameter, ε_{rr} and $\varepsilon_{\theta\theta}$ are radial and hoop strains, and $\lambda(r)$ and μ_r are Lame parameters. In this theory, the components of Cauchy stress tensor, τ_{ij}, the double stress tensor, μ_{ijk}, the strain gradient tensor, κ_{ijk}, and total stress tensor, σ_{ij}, are, respectively, given by:

$$\tau_{ij} = \frac{\partial w}{\partial \varepsilon_{ij}}, \tag{7}$$

$$\mu_{ijk} = \frac{\partial w}{\partial \kappa_{ijk}}, \tag{8}$$

$$\kappa_{ijk} = \varepsilon_{ij,k} = \frac{1}{2}(u_{i,jk}+u_{j,ik}), \tag{9}$$

$$\sigma_{ij} = \tau_{ij} - \mu_{ijk,k}, \tag{10}$$

where u_i's are components of displacement vector. To take into account the effect of temperature change, the transformation $\varepsilon_{ij} \to \varepsilon_{ij} - \alpha T\delta_{ij}$ should be used in Eq. (6) [18] where α is the coefficient of thermal expansion and δ_{ij} is Kronecker delta, which results in:

$$w = \frac{1}{2}\lambda(r)\Big((\varepsilon_{rr}(r)-\alpha(r)T(r))\Big)+\Big(\varepsilon_{\theta\theta}(r)-\alpha(r)T(r)\Big) + \Big(\varepsilon_{zz}(r)-\alpha(r)T(r)\Big)^2+\mu(r)\Big(\varepsilon_{rr}(r)-\alpha(r)T(r)\Big)^2 + \Big(\varepsilon_{\theta\theta}(r)-\alpha(r)T(r)\Big)^2+\Big(\varepsilon_{zz}(r)-\alpha(r)T(r)\Big)^2 + \frac{1}{2}\lambda(r)l^2\left(\frac{d}{dr}(\varepsilon_{rr}(r)-\alpha(r)T(r))+\frac{d}{dr}(\varepsilon_{\theta\theta}(r) - \alpha(r)T(r))+\frac{d}{dr}(\varepsilon_{zz}(r)-\alpha(r)T(r))\right)^2 + \mu(r)l^2\Big([\frac{d}{dr}(\varepsilon_{rr}(r)-\alpha(r)T(r))]^2 + [\frac{d}{dr}(\varepsilon_{\theta\theta}(r)-\alpha(r)T(r))]^2+[\frac{d}{dr}(\varepsilon_{zz}(r)-\alpha(r)T(r))]^2 + [\frac{d}{dr}(\varepsilon_{zz}(r)-\alpha(r)T(r))]^2\Big). \tag{11}$$

Here, the plane strain condition is assumed ($\varepsilon_{zz} = 0$). Substituting Eq. (11) into Eqs. (7) and (8), the constitutive equations for an axisymmetric thick-walled FG cylinder are derived as:

$$\tau_{rr} = \lambda(r)\left(\varepsilon_{rr}(r)+\varepsilon_{\theta\theta}(r)-3\alpha(r)T(r)\right) + 2\mu(r)\left(\varepsilon_{rr}(r)-\alpha(r)T(r)\right), \tag{12}$$

$$\tau_{\theta\theta} = \lambda(r)\left(\varepsilon_{rr}(r)+\varepsilon_{\theta\theta}(r)-3\alpha(r)T(r)\right) + 2\mu(r)\left(\varepsilon_{\theta\theta}(r)-\alpha(r)T(r)\right), \tag{13}$$

$$\mu_{rrr} = (\lambda(r)+2\mu(r))\,l^2\frac{\mathrm{d}\varepsilon_{rr}(r)}{\mathrm{d}r}+\lambda(r)l^2\frac{\mathrm{d}\varepsilon_{\theta\theta}(r)}{\mathrm{d}r} - l^2\left(3\lambda(r)+2\mu(r)\right)\frac{\mathrm{d}\left(\alpha(r)T(r)\right)}{\mathrm{d}r}, \tag{14}$$

$$\mu_{r\theta\theta} = (\lambda(r)+2\mu(r))\,l^2\frac{\mathrm{d}\varepsilon_{\theta\theta}(r)}{\mathrm{d}r}+\lambda(r)l^2\frac{\mathrm{d}\varepsilon_{rr}(r)}{\mathrm{d}r} - l^2\left(3\lambda(r)+2\mu(r)\right)\frac{\mathrm{d}\left(\alpha(r)T(r)\right)}{\mathrm{d}r}. \tag{15}$$

It can be seen from Eqs. (12)-(15) that the relations between Cauchy stresses and strains in the strain gradient elasticity are the same as the classical theory of elasticity. Also, in the absence of the strain gradient effect ($l = 0$), the double stresses will be zero. In the cylindrical coordinates, radial and hoop strains, ε_{rr} and $\varepsilon_{\theta\theta}$, are written in terms of radial displacement, $u(r)$, as:

$$\varepsilon_{rr} = \frac{\mathrm{d}u(r)}{\mathrm{d}r}, \qquad \varepsilon_{\theta\theta} = \frac{u(r)}{r}. \tag{16}$$

The equilibrium equation in cylindrical coordinates in axisymmetric condition is:

$$\frac{\mathrm{d}\sigma_{\mathrm{rr}}}{\mathrm{d}r} + \frac{\sigma_{rr}-\sigma_{\theta\theta}}{r} = 0. \tag{17}$$

In this paper, it is assumed that Lame parameters and coefficient of thermal expansion vary along the radius according to the power law:

$$\lambda(r) = \lambda_0 r^n, \qquad \mu(r) = \mu_0 r^n, \qquad \alpha(r) = \alpha_0 r^p, \tag{18}$$

in which $\lambda_0, \mu_0, \alpha_0, p$ and n are material constants. Eqs. (10) and (12)-(16) together with equilibrium Eq. (17) result in the governing equation for an axisymmetric thick-walled FG cylinder in the strain gradient thermo-elasticity in terms of the radial displacement, $u(r)$, as:

$$[f_4(r)\frac{\mathrm{d}^4}{\mathrm{d}r^4}+f_3(r)\frac{\mathrm{d}^3}{\mathrm{d}r^3}+f_2(r)\frac{\mathrm{d}^2}{\mathrm{d}r^2}+f_1(r)\frac{\mathrm{d}}{\mathrm{d}r}+f_0(r)]\,u(r)=g(r), \tag{19}$$

where $f_i(r)$, $i=0,...,4$ and $g(r)$ are known functions and can be written as:

$$\begin{aligned}
f_0(r)=&\left(\mu_0\left(4-2n\right)+\lambda_0\left(n-2\right)\left(n-3\right)\right)l^2r^{-4}\\
&+\left(\lambda_0\,(n-1)-2\mu_0\right)r^{-2},\\
f_1(r)=&(\lambda_0+2\mu_0)(n+1)r^{-1}+\left(-\lambda_0\left(n-2\right)\left(n-3\right)\right.\\
&\left.+\mu_0\left(2n-4\right)\right)l^2r^{-3},\\
f_2(r)=&(\lambda_0+2\mu_0)+\left(2\mu_0(1-n^2)\right.\\
&\left.-\lambda_0\left(n^2+n-3\right)\right)l^2r^{-2},\\
f_3(r)=&-(\lambda_0+2\mu_0)\,(1+2n)\,l^2r^{-1},\\
f_4(r)=&-(\lambda_0+2\mu_0)l^2,\\
g(r)=&A\left[-\left(B+Cr^{-m}\right)r^{p-1}+l^2\left(D+Fr^{-m}\right)r^{p-3}\right],
\end{aligned} \tag{20}$$

where:

$$\begin{aligned}
&A=(3\lambda_0+2\mu_0)\,,\quad B=c_1\,(p+n)\,,\\
&C=c_2\,(p+n-m)\,,\\
&D=c_1\left\{p^3+(2n-3)\,p^2+\left(n^2-3n+2\right)p\right\},\\
&F=c_2\left(p^3+\left(2n-3m-3\right)p^2+\left(3m^2\right.\right.\\
&\quad\left.+\left(6-4n\right)m+n^2-3n+2\right)p-m^3\\
&\quad\left.+\left(2n-3\right)m^2+\left(-n^2+3n-2\right)m\right).
\end{aligned} \tag{21}$$

It can be seen that the governing equation for thick-walled FG cylinders is a fourth order ODE, whereas, in the classical elasticity, it is a second order ODE. Also, it is noted that in the absence of strain gradient effect ($l=0$) this fourth order ODE reduces to the second order ODE in the classical elasticity.

2.3. Boundary conditions

Let the FG cylinder be subjected to boundary pressures and on its inner and outer surfaces, respectively. So, boundary conditions for the proposed problem can be given by [19]:

$$\begin{aligned}
&\left\{\tau_{rr}-l^2\left[\frac{d^2\tau_{rr}}{dr^2}+\frac{1}{r}(\frac{d\tau_{rr}}{dr}-\frac{d\tau_{\theta\theta}}{dr})-\frac{2}{r^2}(\tau_{rr}-\tau_{\theta\theta})\right]\right\}_{r=r_i}=-P_i,\\
&\left\{\tau_{rr}-l^2\left[\frac{d^2\tau_{rr}}{dr^2}+\frac{1}{r}(\frac{d\tau_{rr}}{dr}-\frac{d\tau_{\theta\theta}}{dr})-\frac{2}{r^2}(\tau_{rr}-\tau_{\theta\theta})\right]\right\}_{r=r_o}=-P_o,\\
&l^2\mu_{rrr}|_{r=r_i}=0,\\
&l^2\mu_{rrr}|_{r=r_o}=0.
\end{aligned} \tag{22}$$

The prescribed double stress traction is considered zero on both the inner and outer surfaces. It can be seen that in the absence of the strain gradient effect ($l=0$), boundary conditions Eq. (22) reduce to the boundary conditions in the classical theory of elasticity.

3. Numerical results and discussion

3.1. Verification of the solution

The governing equation (Eq. (19)) together with four boundary conditions (Eq. (22)) has been solved numerically and the results are presented in this section. The numerical solution used in this problem is a combination of the base scheme (trapezoid), and a method enhancement scheme (Richardson extrapolation) with the traprich method that have been coded in the MAPLE software. For the detailed discussion on the numerical method, we refer to [20,21]. For verification of the numerical solution, a homogenous thick-walled cylinder is considered with the inner and outer radii, $r_i=1\,\mu$m and $r_o=5\,\mu$m, respectively. It is assumed that the cylinder is subjected to boundary pressures, $P_i=10\,$MPa and $P_o=0\,$MPa, at the inner and outer surfaces, respectively. Gao and Park [19] presented an analytical solution for elastic deformation of homogenous thick-walled cylinders using the strain gradient elasticity. A plot of the radial stress for the homogenous cylinder along the radius using the solution presented by Gao and Park and our solution is presented in Figure 1 with the parameter values: $l=0.5\,\mu$m, $E=135\,$GPa and $\nu=0.3$. Also, the results obtained from the classical theory of elasticity are presented in this figure for comparison. It can be seen that the results are in good agreement with those obtained from the analytical solution presented by Gao and Park [19].

3.2. Case studies

In this section the effect of the intrinsic length parameter as well as FG parameters on thermo-elastic

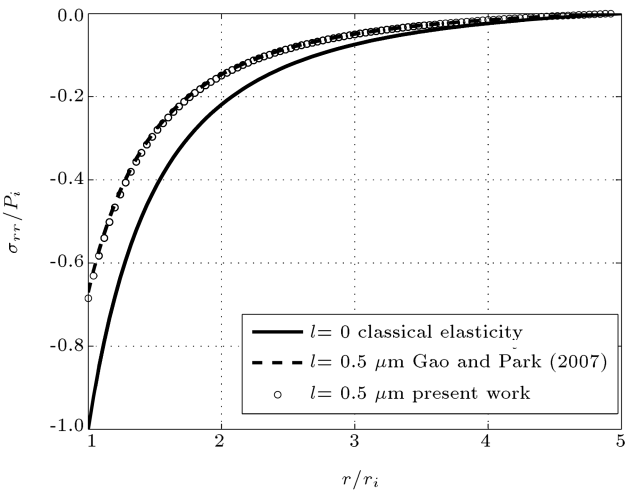

Figure 1. Comparison of non-dimensional radial stress along the radius for a homogenous thick-walled cylinder.

Table 1. Parameter values used in the numerical examples.

$l(\mu$m)	n	m	p	$r_o(\mu$m)
Variable	0.4	0.6	0.8	5
0.1	**Variable**	0.6	0.8	5
0.1	0.4	**Variable**	0.8	5
0.1	0.4	0.6	**Variable**	5
0.1	0.4	0.6	0.8	**Variable**

deformation of thick-walled FG cylinders are discussed. To this aim, the intrinsic length parameter, power indices and outer radius of the cylinder are varied according to Table 1. Lame parameters and the coefficient of thermal expansion at the inner radius are assumed to be $\lambda_i = 1200\,\text{MPa}$, $\mu_i = 800\,\text{MPa}$ and $\alpha_i = 1.6 \times 10^{-6}\,(1/\text{K})$. The following geometrical, thermal and mechanical parameters have been used in the numerical examples $T_i = 300\,\text{K}$, $T_o = 400\,\text{K}$, $r_i = 1\,\mu\text{m}$, $r_o = 5\,\mu\text{m}$, $P_i = 10\,\text{MPa}$ and $P_o = 0\,\text{MPa}$.

The distribution of non-dimensional radial displacement as well as non-dimensional radial and hoop stresses along the cylinder radius for different values of the intrinsic length parameter, l, are presented in Figures 2-4, respectively. It is understood that by decreasing l to zero, the strain gradient solution approaches the classical elasticity solution. Furthermore, it can be seen that by increasing l, the difference between the strain gradient solution and that of the classical elasticity raises, rapidly. It is interesting to see that the difference between the stresses and radial displacement for different values of l at smaller values of r/r_i is more than the difference between the results at larger values of r/r_i. For example, the difference between σ_{rr}/P_i for $l = 0.3\,\mu$m and for $l = 0$ at $r/r_i = 1$ is 42%, whereas, there is

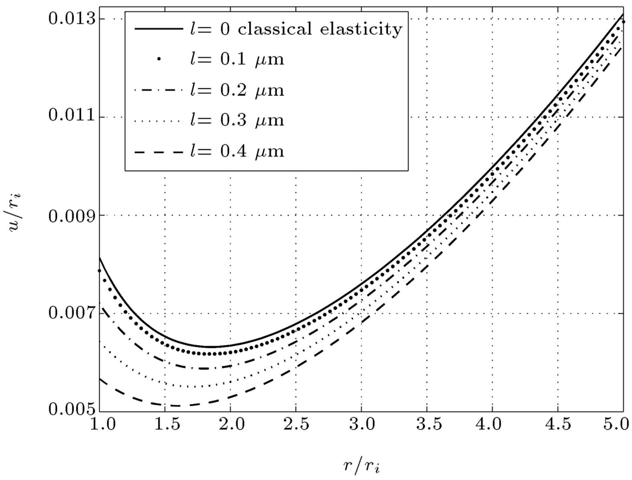

Figure 2. Radial displacement along the FG cylinder thickness for different values of the intrinsic length parameter, l.

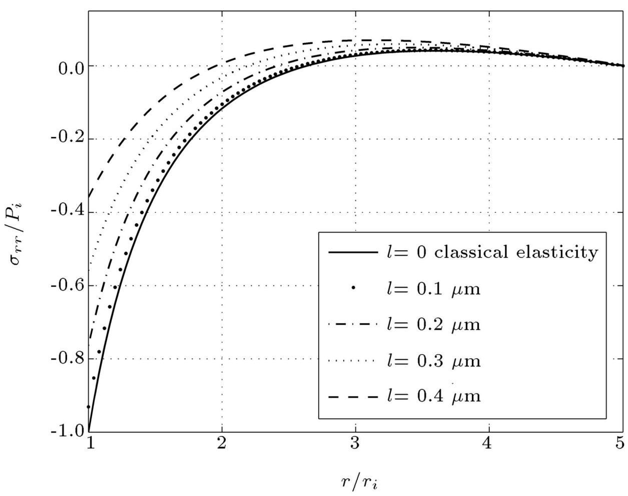

Figure 3. Radial stress along the FG cylinder thickness for different values of the intrinsic length parameter, l.

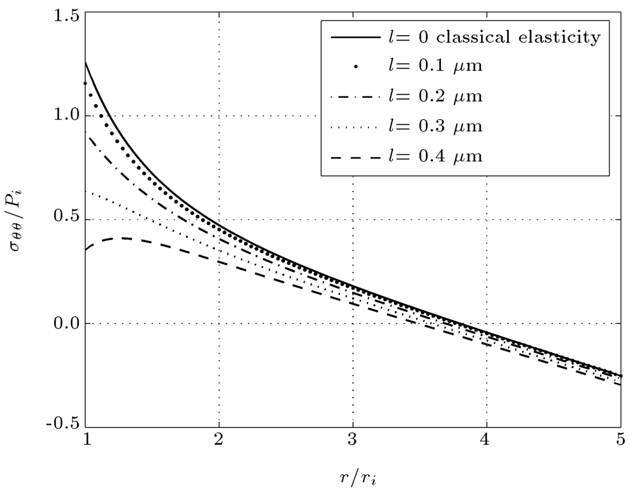

Figure 4. Hoop stress along the FG cylinder thickness for different values of the intrinsic length parameter, l.

almost no difference between these values at $r/r_i = 5$ which shows that the intrinsic length parameter has more effect at smaller dimensions. Also, from the plotted results we see that increasing l decreases the absolute value of maximum radial displacement as well as radial and hoop stresses. For example, there is 72% difference between the maximum values of $\sigma_{\theta\theta}/P_i$ for $l = 0.3\,\mu$m and $l = 0\,\mu$m. In addition, it can be observed that for different values of l, radial and hoop stresses always attain their maximum values at the inner surface. So, it is concluded that intrinsic length parameter has a significant effect on the displacement and stress distributions in the thick-walled FG cylinders.

The variations of non-dimensional radial displacement as well as non-dimensional radial and hoop stresses along the cylinder thickness for different values of the power index n are shown in Figures 5-7, respectively. From the plotted results, we see that the power index n has a significant effect on the stress distribution along the radial direction. Also, as n increases from 0 to 3, the maximum radial displacement decreases first and then increases. It can be seen that as n increases, the maximum radial stress increases sharply and the radius, in which the maximum radial stress occurs, moves to larger values. Furthermore, by increasing n, the maximum hoop stress increases rapidly, too. So, it is understood that the power index n can significantly affect the radial displacement as well as radial and hoop stresses. Figures 8-10 show the non-dimensional radial displacement as well as non-dimensional radial and hoop stresses along the radial direction for different values of the power index m. It can be seen from these figures that the value of m does not have much effect on the results for selected values of m. Figures 11-13 show the non-dimensional radial displacement, radial stress and hoop stress distributions through the thickness for different

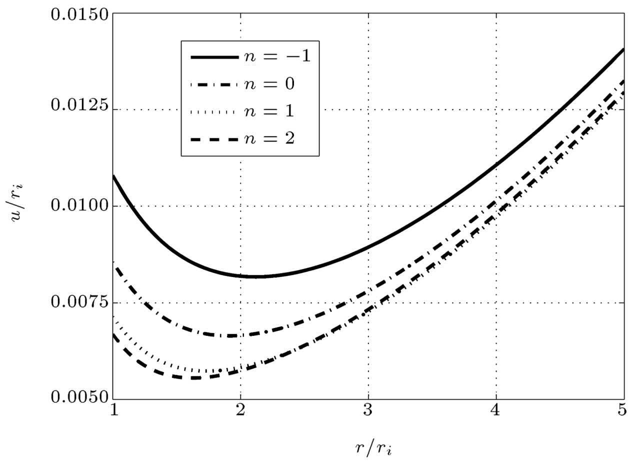

Figure 5. Radial displacement along the FG cylinder thickness for different values of the power index, n.

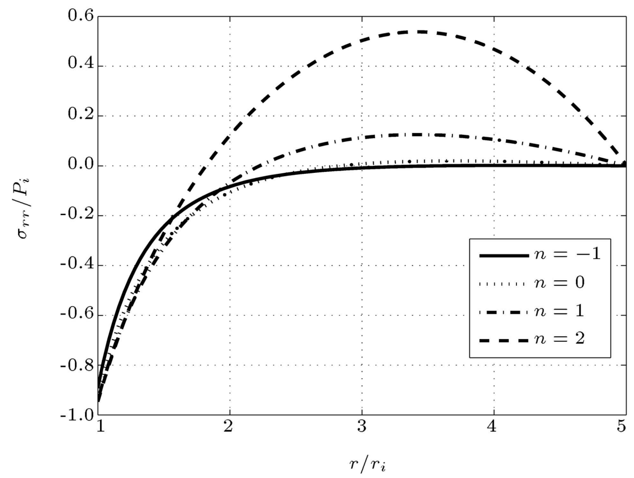

Figure 6. Radial stress along the FG cylinder thickness for different values of the power index, n.

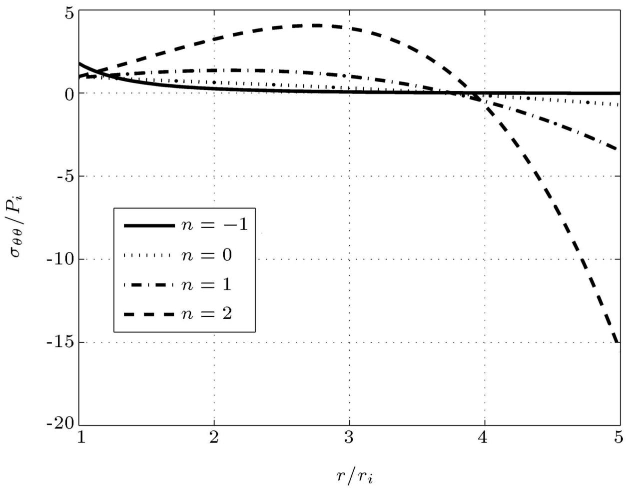

Figure 7. Hoop stress along the FG cylinder thickness for different values of the power index, n.

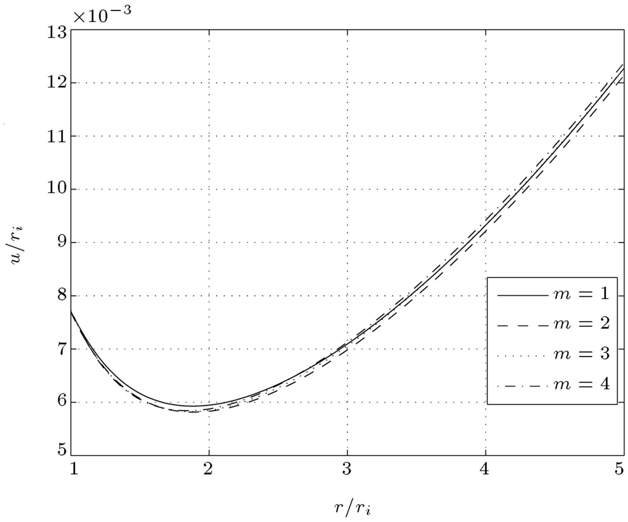

Figure 8. Radial displacement along the FG cylinder thickness for different values of m.

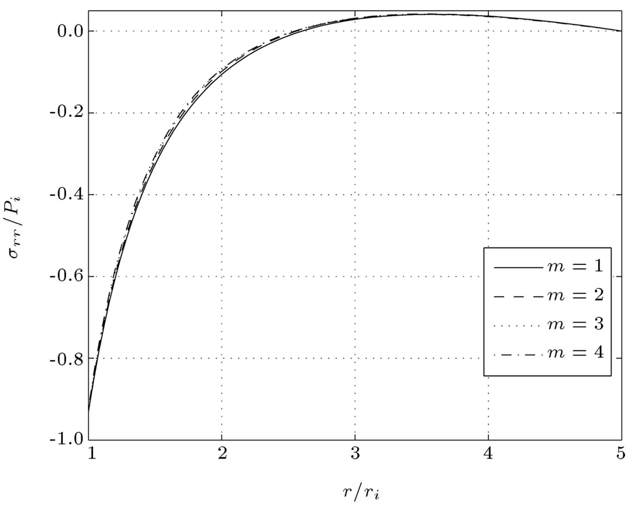

Figure 9. Radial stress along the FG cylinder thickness for different values of m.

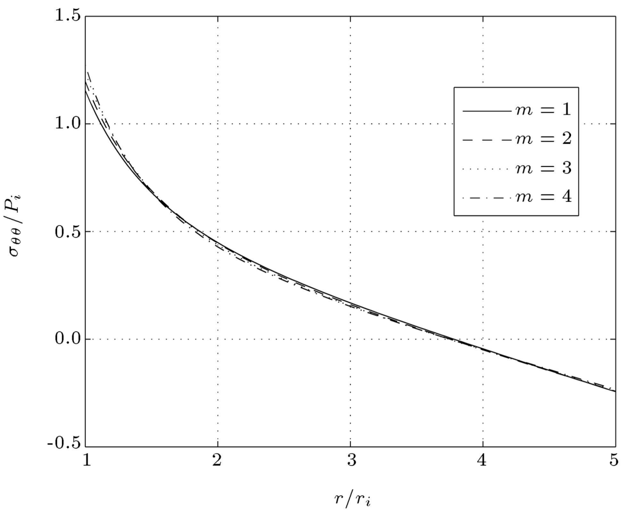

Figure 10. Hoop stress along the FG cylinder thickness for different values of m.

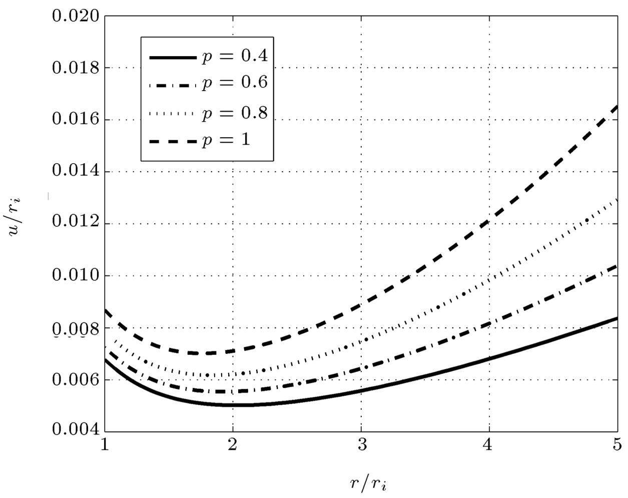

Figure 11. Radial displacement along the FG cylinder thickness for different values of p.

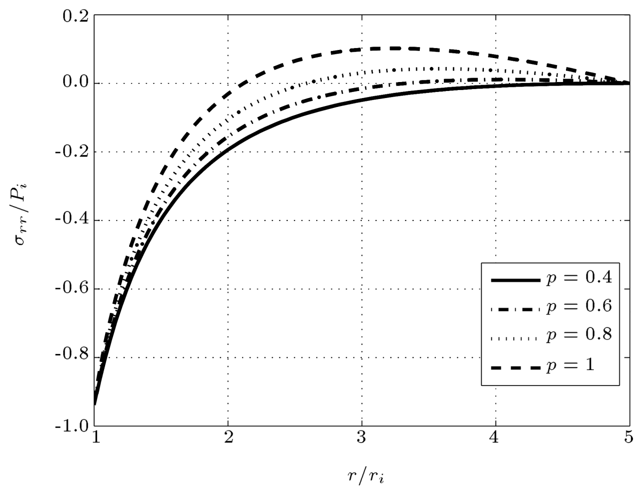

Figure 12. Radial stress along the FG cylinder thickness for different values of p.

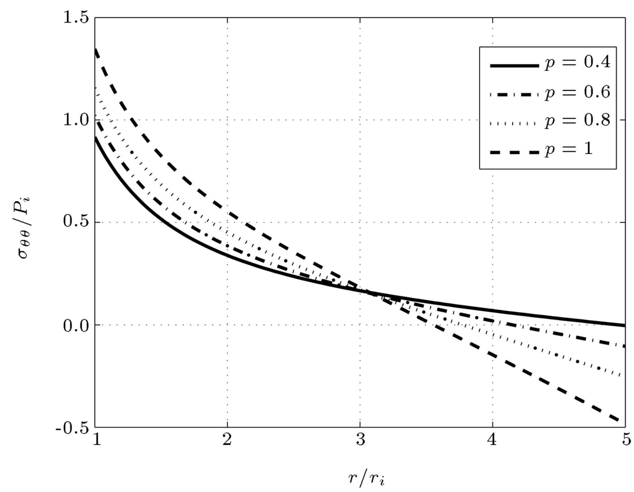

Figure 13. Hoop stress along the FG cylinder thickness for different values of p.

values of power index p, respectively. It can be seen that p has an important effect on the results. It is noted that increasing p increases the maximum radial displacement and hoop stress whereas it has no effect on the maximum radial stress.

To study the effect of outer radius of the cylinder on the results, r_o has been changed from $2\,\mu$m to $5\,\mu$m and the results are shown in Figures 14-16. It can be seen from Figure 14 that as r_o increases, the maximum radial displacement decreases until $r_o = 4\,\mu$m and then, increasing r_o to $5\,\mu$m increases the maximum radial displacement. Also, it can be seen that in the cylinders with larger outer radii, the results show less length dependency and follow the same pattern. The same behavior can be observed from Figure 16 for the hoop stresses. Figure 15 shows the radial stress distribution along the radial direction for different values of r_o. It can be seen that the outer radius does not have any effect on the maximum radial stress.

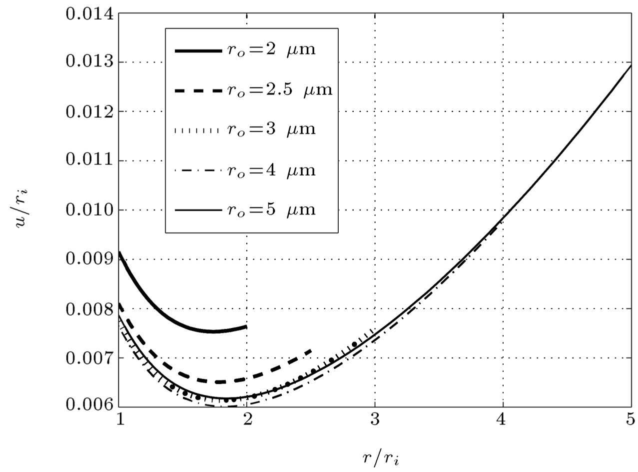

Figure 14. Radial displacement along the FG cylinder thickness for different values of outer radius, r_o.

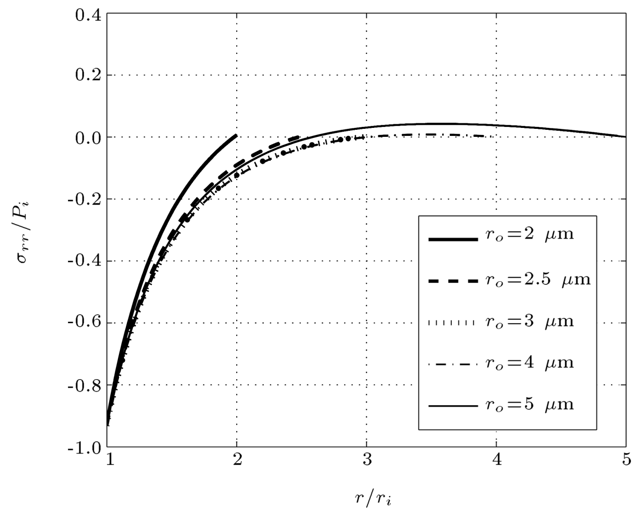

Figure 15. Radial stress along the FG cylinder thickness for different values of outer radius, r_o.

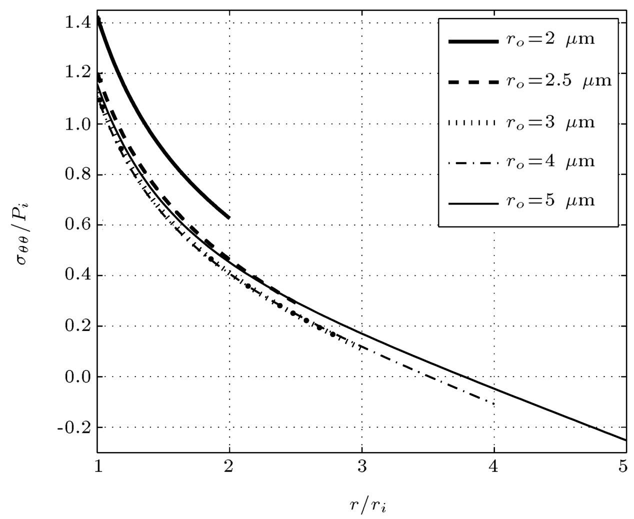

Figure 16. Hoop stress along the FG cylinder thickness for different values of outer radius, r_o.

4. Summary and conclusion

Strain gradient thermo-elasticity formulation for an axisymmetric thick-walled FG cylinders is presented. The power law of distribution is assumed for variation of material properties in radial direction. A fourth order governing equation is obtained in strain gradient thermo-elasticity and it is solved numerically together with four boundary conditions. Results show that the intrinsic length parameter has a significant effect on the stress distribution of thick-walled FG cylinders and increasing the intrinsic length parameter reduces the maximum radial and hoop stresses, rapidly. Also, it is shown that the power index m does not have much effect on the results whereas the power indices n and p have significant effect on the results. In addition, it is shown that by increasing n the maximum radial and hoop stresses increases, rapidly. It is noted that increasing p increases the maximum radial displacement and hoop stress, whereas it has no effect on the maximum radial stress. Furthermore, it is shown that increasing the outer radius of the FG cylinder affects the distribution of the radial and hoop stresses.

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