Scientia Iranica E (2014) 21(3), 1044–1050

SCIENTIA IRANICA

Sharif University of Technology
Scientia Iranica
Transactions E: Industrial Engineering
www.scientiairanica.com

Developing functional process capability indices for simple linear profile

R. Nemati Keshteli[a], R. Baradaran Kazemzadeh[a,*], A. Amiri[b] and R. Noorossana[c]

a. *Department of Industrial Engineering, Faculty of Engineering, Tarbiat Modares University, Tehran, Iran.*
b. *Department of Industrial Engineering, Shahed University, Tehran, Iran.*
c. *Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran.*

Received 18 October 2012; received in revised form 26 July 2013; accepted 9 September 2013

KEYWORDS
Profile monitoring;
Simple linear profile;
Functional process capability index;
Specification limits;
Natural tolerance limits.

Abstract. A profile is a relation between one response variable and one or more explanatory variables that represent the quality of a product or the performance of a process. Process Capability Indices (PCI) are measured to evaluate processes in producing conforming products. All existing methods that measure process capability indices in a simple linear profile consider response variables at some levels of explanatory variable and ignore all ranges of x-values. In this paper, a functional approach is proposed to measure the process capability index of simple linear profiles in all ranges of explanatory variable. This new approach follows the traditional definition of process capability indices and leads to their accurate values for a simple linear profile. The functional approach uses a reference profile, functional specification limits and functional natural tolerance limits to present a functional form of process capability indices. This functional form results in measuring the process capability at each level of the explanatory variable in a simple linear profile, as well as the unique value of a process capability index for a simple linear profile. A comparison study using a non-conforming proportion method shows the better performance of functional process capability indices in measuring the process capability in a simple linear profile.

1. Introduction

In Statistical Process Control (SPC), we usually apply control charts to monitor the quality of a process or product by using one or more quality characteristics. There are a noticeable number of situations in which the quality of a process or product can be characterized by the relation (curve or profile) between one response variable and one or more explanatory variables rather than one or more quality characteristics in traditional SPC [1]. Profiles are commonly represented as parametric models, such as simple linear regression, multiple linear regression, polynomial regression, non-linear regression, logistic regression, circular models and cylindrical models. Various methods are presented for profile monitoring in both Phases I and II [1]. In Phase I, the goal is checking the stability of the process, as well as parameter estimation, while the aim in Phase II is detecting the shifts in the profile parameters as quickly as possible. Several researchers have studied simple linear profile monitoring (see, e.g., [2-11]). Process capability indices are used to evaluate the process performance [12,13]. Woodall [14] suggested research into assessing process capability with profile data. However, there are few papers concerning the process capability index in profiles.

*. *Corresponding author.*
E-mail addresses: r.nemati@modares.ac.ir (R. Nemati Keshteli); rkazem@modares.ac.ir (R. Baradaran Kazemzadeh); amiri@shahed.ac.ir (A. Amiri); rassoul@iust.ac.ir (R. Noorossana)

Shahriari and Sarafian [15] presented a method for a process capability index (C_{PK}) in simple linear profiles, which is the minimum value of the process capability index of the response variable at n levels of explanatory variable. Ebadi and Shahriari [16] used a multivariate process capability index for a predicted response variable to present the process capability index in a simple linear profile. Hoseinifard et al. [17] considered Gamma distribution for a non-normal response variable and then used a non-conforming proportion of the response variable to analyze the process capability of a simple linear profile. Hoseinifard and Abbasi [18] used a non-conforming proportion of a response variable to evaluate the process capability in simple linear profiles. Ebadi and Amiri [19] proposed three indices to measure process capability in a multivariate simple linear profile. In all these methods, the process capability indices are defined by the response variable at n levels of explanatory variables.

Process capability in the response variable or the predicted response variable may ignore the relationship between the response variable and explanatory variables. Hence, recently, Nemati et al. [20] proposed a functional approach to evaluate capability in a circular profile. In this paper, we develop a functional approach to define process capability indices in a simple linear profile. Using the proposed approach, all information in the entire range of the explanatory variable is utilized.

The paper is organized as follows: In Section 2, we review the existing methods for process capability assessment in simple linear profiles. In Section 3, we propose the functional approach to measure the process capability indices in simple linear profiles. Illustrative examples and comparison studies are given in Section 4 and, finally, in Section 5, we present the conclusions of the paper and some ideas for future research.

2. Existing process capability indices for simple linear profile

The general model of a simple linear profile is represented in Eq. (1):

$$y_{ij} = A_0 + A_1 X_{ij} + \varepsilon_{ij}, \quad i = 1, 2, ..., n \text{ and } j = 1, 2, ..., k, \tag{1}$$

where the pair observation (x_{ij}, y_{ij}) is obtained in the jth random sample in which x_{ij} is the ith design point for the explanatory variable in the jth sample. A_0 is the intercept, A_1 is the slope, and ε_{ij}s are independently normally distributed random variables with mean zero and variance equal to σ^2 [1,4].
Process capability indices are widely used in assessing process performance. Kane [21] introduced the first process capability index as C_p. This index measures the potential capability of a process with no attention to the process mean. C_p is presented in Eq. (2):

$$C_p = \frac{\text{USL} - \text{LSL}}{\text{UNTL} - \text{LNTL}} = \frac{\text{USL} - \text{LSL}}{6\sigma}, \tag{2}$$

where σ is the process standard deviation, LSL is the Lower Specification Limit and USL is the upper specification limit. UNTL is the Upper Natural Tolerance Limit (UNTL $= \mu + 3\sigma$) and LNTL is the Lower Natural Tolerance Limit (LNTL $= \mu - 3\sigma$). C_{PK} is the mostly used process capability index because it compares process dispersion and tolerance range, while considering the position of the process mean:

$$C_{PK} = \min\left\{\frac{\text{USL} - \mu}{\text{UNTL} - \mu}, \frac{\mu - \text{LSL}}{\mu - \text{LNTL}}\right\} = \min\left\{\frac{\text{USL} - \mu}{3\sigma}, \frac{\mu - \text{LSL}}{3\sigma}\right\}. \tag{3}$$

There are other types of process capability indices, such as C_{RM}, C_{PMK} and S_{PK} for univariate situations [22].

Shahriari and Sarafian [15] proposed a method for a process capability index when monitoring a simple linear profile. They considered the response variable as a quality characteristic with normal distribution and known specification limits. The C_{PK} of the response variable is calculated at each level of the explanatory variable, and, then, the minimum C_{PK} is introduced as the process capability index in a simple linear profile. Ebadi and Shahriari [16] replaced the response variable in Shahriari and Sarafian [15] by a predicted response variable at each level of the explanatory variable and then used a multivariate process capability index to measure the capability of the process. They also mentioned that the minimum C_{PK} in levels of the explanatory variable leads to an underestimated process capability index. Thus, they suggested a method, based on the Bothe [12] method, which uses a proportion of non-conforming items. Non-conforming items are the ones with one or more response variables out of the tolerance range.

Hosseinifard et al. [17] concentrated on the process capability index of a simple linear profile under the assumption of the non-normality of the response variable. They fitted a Burr XII distribution to the response variable at each level of the explanatory variable and, then, used the Clements method [23] for calculation of C_{PU}, C_{PL}, and C_{PK} for the response variable at each level of the explanatory variable. In their method, the C_{PK} of the response variable is calculated in n levels of the explanatory variable, and the minimum C_{PK} is introduced as the process capability index in the simple linear profile.

Hosseinifard and Abbasi [18] considered the response variable of a simple linear profile as a quality characteristic which has a known constant or variable specification limits. Their method to process the capability index of the response variable is based on the percentage of non-conforming items. C_P and C_{PK}, based on this method, are computed using Eqs. (4) and (5), respectively:

$$C_P = \frac{1}{6}[\phi^{-1}(1 - P_v) - \phi^{-1}(P_L)], \quad (4)$$

$$C_{PK} = \frac{1}{3}\min[\phi^{-1}(1 - P_u) - \phi^{-1}(P_L)], \quad (5)$$

where P_U is estimated using Eqs. (6) and (7):

$$P_U = 1 - \prod_{i=1}^{n} \Pr(y_{ij} < \mathrm{USL}_i), \quad (6)$$

as a result:

$$P_U = 1 - \prod_{i=1}^{n} \phi(\frac{\mathrm{USL}_i - \mu_i)}{\sigma}), \quad (7)$$

and P_L is estimated based on Eqs. (8) and (9) as follows:

$$P_L = 1 - \prod_{i=1}^{n} \Pr(y_{ij} > \mathrm{LSL}_i). \quad (8)$$

Cosequently:

$$P_L = 1 - \prod_{i=1}^{n}(1 - \phi(\frac{\mathrm{LSL}_i - \mu}{\sigma})). \quad (9)$$

μ_i and σ in Eqs. (7) and (9) are the mean and standard deviation of the response variable at different levels of the explanatory variable, respectively. LSL_i and USL_i in Eqs. (6) to (9) are upper and lower specification limits for the response variable at the ith level of the explanatory variable. They considered n fixed design points within a simple linear profile.

Shahriari and Sarafian [15] used the traditional definition of process capability indices in some designed points in order to propose the process capability index for a simple linear profile. However, choosing a minimum process capability index among designed points of the explanatory variable gives an underestimated value for the process capability index of a simple linear profile. Ebadi and Shahriari [16] modified this method using the non-conforming items method in Bothe [12]. The average of the non-conforming proportion in some levels of explanatory variable cannot propose an adequate estimate of the non-conforming proportion of the response variable over the entire range of explanatory variables. Ebadi and Shahriari [16] replaced the response variable with a predicted response variable in n design points, and used multivariate process capability indices of n predicted response variables as process capability indices of a simple linear profile. Hosseinifard et al. [17] used the minimum process capability index among designed points of the explanatory variable that gives an underestimated process capability index similar to Shahriari and Sarafian [15]. Hosseinifard and Abbasi [18] proposed a non-conforming proportion based method. Similar to Ebadi and Shahriari [16], the method by Hosseinifard and Abbasi [18] cannot recommend a suitable estimate of the non-conforming proportion of the response variable over the entire range of explanatory variables.

Ebadi and Amiri [19] proposed a process capability index for a multivariate simple linear profile based on the percentage of non-conforming items. They developed a multivariate process capability index for a multivariate simple linear profile as a second method. Finally, they used a principal component analysis to develop the third index.

In all the above mentioned methods, only the response variable at n levels of explanatory variable is considered. In this paper, we develop a method in which all ranges of the explanatory variable are used.

In the next section, a new approach is proposed to calculate the process capability index of a simple linear profile. This approach follows the traditional definition of the process capability index and proposes a functional form of process capability indices. This approach leads to a true value for the process capability index in a simple linear profile.

3. Proposed approach

The process capability index (C_p) defined in Eq. (2) is a comparison between natural tolerance limits and specification limits of a process. In a simple linear profile, $y = A_0 + A_1X$ is the reference line of the process, $a_0 + a_1x$ is the conditional mean of y in x and, so, μ is computed as follows:

$$\mu = a_0 + a_1x. \quad (10)$$

y is a normal random variable with mean of $a_0 + a_1x$ and variance of σ^2, and a_0 and a_1 are estimates of A_0 and A_1 and computed as $a_0 = (\sum_{j=1}^{k} a_{0j})/k$ and $a_1 = (\sum_{j=1}^{k} a_{1j})/k$, respectively. a_{0j} and a_{1j} are estimated intercept and slope coefficients in the jth sample profile, respectively.

The process variance (σ^2) is estimated using MSE and computed as $\mathrm{MSE} = (\sum_{j=1}^{k} \mathrm{MSE}_j)/k$, where MSE_j is the estimate of variance in the jth sample profile. Hence, we can define the UNTL and LNTL of y as Eqs. (11) and (12), respectively:

$$\mathrm{LNTL}_y = \mu + 3\sigma = a_0 + a_1x + 3\sigma, \quad (11)$$

$$\mathrm{LNTL}_y = \mu + 3\sigma = a_0 + a_1x - 3\sigma. \quad (12)$$

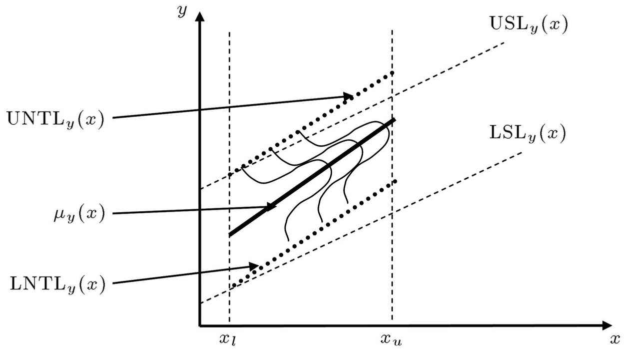

Figure 1. Functional reference line, specification limits and natural tolerance limits in simple linear profile.

It is obvious that UNTL and LNTL of y are two parallel lines where the distance between them is equal to 6σ. As mentioned above, μ, UNTL and LNTL of y are functions of x as $\mu_y(x) = a_0 + a_1 x, \mathrm{UNTL}_y(x) = a_0 + a_1 x + 3\sigma$ and $\mathrm{LNTL}_y(x) = a_0 + a_1 x - 3\sigma$ respectively.

Suppose that the specification limits of y are two functions of x, as obtained by Eqs. (13) and (14):

$$\mathrm{USL}_y(x) = a_{ou} + a_{1u} x, \tag{13}$$

$$\mathrm{LSL}_y(x) = a_{ol} + a_{1l} x. \tag{14}$$

A schematic representation of $\mu_y(x)$, $\mathrm{UNTL}_y(x)$, $\mathrm{LNTL}_y(x)$, $\mathrm{USL}_y(x)$ and $\mathrm{LSL}_y(x)$ is shown in Figure 1.

$\mathrm{UNTL}_y(x), \mathrm{LNTL}_y(x), \mathrm{USL}_y(x)$ and $\mathrm{LSL}_y(x)$ are functional forms of UNTL, LNTL, USL and LSL, respectively. By replacing functional forms of UNTL, LNTL, USL and LSL in the traditional index of Eq. (8), the following equation is computed:

$$C_p(x) = \frac{\mathrm{USL}_y(x) - \mathrm{LSL}_y(x)}{\mathrm{UNTL}_y(x) - \mathrm{LNTL}_y(x)} \quad x \in [x_1, x_u]. \tag{15}$$

Actually, C_P of a simple linear profile has a functional form, as presented in Eq. (15).

By using $C_P(x)$ as the process capability index of the simple linear profile, it is possible to evaluate the capability of the process at each level of x. The process capability at each level of the explanatory variable proposes detailed information of the process. However, it is necessary to have a unique value of the process capability index for a simple linear profile in all ranges of the explanatory variable to give an overall judgment about process capability. For this purpose, it is recommended to utilize the area bounded between $\mathrm{USL}_y(x)$ and $\mathrm{LSL}_y(x)$ to compute $\mathrm{USL}_y(x) - \mathrm{LSL}_y(x)$ and, also, the area bounded between $\mathrm{UNTL}_y(x)$ and $\mathrm{LNTK}_y(x)$ to compute $\mathrm{UNTL}_y(x) - \mathrm{LNTL}_y(x)$. Hence, Eq. (16) is proposed to determine a unique value for the C_p of a simple linear rofile:

$$C_P(\text{profile}) = \frac{\int_{x_l}^{x_u} [\mathrm{USL}_y(x) - \mathrm{LSL}_y(x)]dx}{\int_{x_l}^{x_u} [\mathrm{UNTL}_y(x) - \mathrm{LNTL}_y(x)]dx} \quad x \in [x_l, x_u]. \tag{16}$$

$\mathrm{UNTL}_y(x)$ and $\mathrm{LNTL}_y(x)$ are two parallel lines. We assume that $\mathrm{USL}_y(x)$ and $\mathrm{LSL}_y(x)$ are two parallel lines as $\mathrm{USL}_y(x) = a_{ou} + a_1' x$ and $\mathrm{LSL}_y(x) = a_{ol} + a_1' x$, where a_{ou}, a_{ol}, and a_1' are the intercept of $\mathrm{USL}_y(x)$, the intercept of $\mathrm{LSL}_y(x)$ and the slope of both $\mathrm{USL}_y(x)$ and $\mathrm{LSL}_y(x)$, respectively. The distance of these parallel lines can be considered as their difference. Consequently, it is possible to compute $C_P(\text{profile})$ by Eq. (17):

$$C_P(\text{profile}) = \frac{a_{ou} - a_{ol}}{6\sigma}. \tag{17}$$

Similarly, the functional form of C_{PK} is computed by Eq. (18):

$$C_{PK}(x) = \min\left\{ \frac{\mathrm{USL}_y(x) - \mu_y(x)}{\mathrm{UNTL}_y(x) - \mu_y(x)}, \frac{\mu_y(x) - \mathrm{LSL}_y(x)}{\mu_y(x) - \mathrm{LNTL}_y(x)} \right\} \quad x \in [x_l, x_u], \tag{18}$$

where, $\mu_y(x)$ is the function of the reference line. $C_{PK}(x)$ gives the value of C_{PK} of a simple linear profile at each level of x. Eq. (19) can be used to compute a unique value for C_{PK} of a simple linear profile:

$$C_{PK}(\text{profile}) = \min\left\{ \frac{\int_{x_l}^{x_u} [\mathrm{USL}_y(x) - \mu_y(x)]dx}{\int_{x_l}^{x_u} [\mathrm{UNTL}_y(x) - \mu_y(x)]dx}, \frac{\int_{x_l}^{x_u} [\mu_y(x) - \mathrm{LSL}_y(x)]dx}{\int_{x_l}^{x_u} [\mu_y(x) - \mathrm{LNTL}_y(x)]dx} \right\}. \tag{19}$$

The process capability index (C_{PK}), when only upper or lower functional specification limits are available, can be computed by Eqs. (20) and (21), respectively:

$$C_{PK}(\text{profile}) = \frac{\int_{x_l}^{x_u} [\mathrm{USL}_y(x) - \mu_y(x)]dx}{\int_{x_l}^{x_u} [\mathrm{UNTL}_y(x) - \mu_y(x)]dx}, \tag{20}$$

$$C_{PL}(\text{profile}) = \frac{\int_{x_l}^{x_u} [\mu_y - \mathrm{USL}_y(x)(x)]dx}{\int_{x_l}^{x_u} [\mu_y(x) - \mathrm{UNTL}_y(x)]dx}. \tag{21}$$

It is obvious that μ_y is greater than $\mathrm{LNTL}_y(x)$ and less than UNTL_y in each case of a simple linear profile. However, as shown in Figure 2, there is not a predefined relationship between $\mu_y(x)$ and $\mathrm{USL}_y(x)$, or between $\mu_y(x)$ and $\mathrm{LSL}_y(x)$.

Assume, for example, $\mathrm{USL}_y(x)$ is greater than $\mu_y(x)$ in $[x_l, x_m]$ and less than $\mu_y(x)$ in $[x_m, x_u]$.

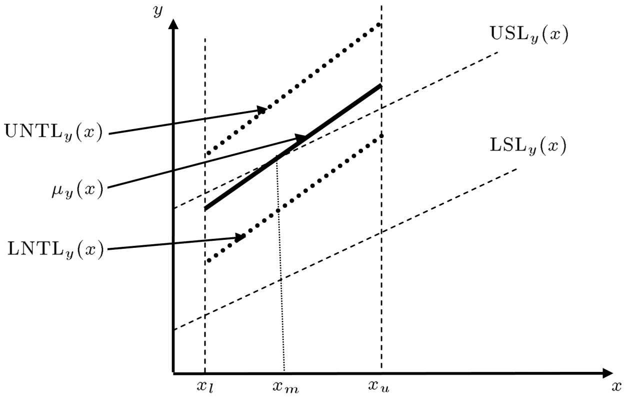

Figure 2. A special case USL$_y(x)$ is greater than $\mu_y(x)$ in $[x_1, x_m]$ and less than $\mu_y(x)$ in $[x_m, x_u]$.

Thus, The formula of C_{PK} must be changed as in Eq. (22):

$$C_{PK}(\text{profile}) = \min\left\{ \begin{array}{l} \frac{\int_{x_l}^{x_m}[\text{USL}_y(x)-\mu_y(x)]dx - \int_{x_m}^{x_u}[\mu_y(x)-\text{USL}_y(x)]dx}{\int_{x_l}^{x_u}[\text{UNTL}_y(x)-\mu_y(x)]dx}, \\ \frac{\int_{x_l}^{x_u}[\mu_y(x)-\text{LSL}_y(x)dx}{\int_{x_l}^{x_u}[\mu_y(x)-\text{LNTL}_y(x)dx} \end{array} \right\} \quad (22)$$

In fact, it is necessary that we must determine the intersection point between $\mu_y(x)$ and USL$_y(x)$. If the intersection point is inside $[x_l, x_m]$, we must use the area bounded between $\mu_y(x)$ and USL$_y(x)$ with a proper sign. This analysis must be done for $\mu_y(x)$ and LSL$_y(x)$ as well. In the next section, the performance of the proposed method is shown through some numerical examples.

In this section, we developed the traditional C_P and C_{PK} indices to compute the capability of a process with a simple linear profile quality characteristic. Note that the interpretations of developed indices are the same as traditional ones, i.e. the proportion of non-conforming items for both the univariate and the profile quality characteristic is equal when the values of the process capability indices of both quality characteristics are the same. In other words, the relationship between the values of process capability indices for the profile quality characteristic and the proportion of non-conforming items are similar to traditional situations.

4. Illustrative examples and comparison studies

In this section, three examples are presented to evaluate the performance of the proposed method in comparison with the non-conforming proportion method.

4.1. Example 1

Hosseinifard and Abbasi [18] used the method of non-conforming items in a yogurt production process. The response variable in this case is the PH of the milk mixture and the explanatory variable is time. The range of the explanatory variable is [0,4] hours. They used two parallel lines for USL and LSL, defined as USL $= 6.1 - 0.4x$ and LSL $= 5.85 - 0.4x$. The calculated reference line is $y = 5.98 - 0.39x$, where MSE $= 0.06$. Eqs. (6) and (7) are used to determine $p_u = 0.019$ and $p_l = 1$, respectively. $C_P = 1.22$ is calculated using Eq. (4).

There are two parallel lines as specifications limits, so, Eq. (20) gives C_P as follows:

$$C_P(\text{profile}) = \frac{a_{ou} - a_{ol}}{6\sigma} = \frac{6.1 - 5.85}{6\sqrt{0.06}} = 0.17$$

The distance between the parallel lines, USL and LSL, is obviously less than the distance of the parallel lines, UNTL and LNTL. Thus, the C_P of y is less than 1 at each level of x. The outcome of this result is that C_P(profile) should be less than 1. In fact, the C_P of y at each level of x is equal to a constant value of 0.17. Consequently, it is not possible to have $C_P =$ 1.22. We can conclude that this new method leads to a more accurate value of the process capability index of a simple linear profile rather than the non-conforming method. C_{PL}(profile), C_{PU}(profile) and C_{PK}(profile) for this process are calculated as follows:

$$C_{PL}(\text{profile}) = \frac{\int_0^4[(5.98 - 0.39x) - (5.85 - 0.4x)]dx}{\int_0^4[(5.98 - 0.39x) - (5.26 - 0.39x)]dx} = 0.2,$$

$$C_{PU}(\text{profile}) = \frac{\int_0^4[(6.1 - 0.4x) - (5.98 - 0.39x)]dx}{\int_0^4[(6.72 - 0.39x) - (5.98 - 0.39x)]dx} = 0.136,$$

$$C_{PK}(\text{profile}) = \min\{C_{PL}(\text{profile}), C_{PU}(\text{profile})\} = 0.136.$$

4.2. Example 2

Hosseinifard and Abbasi [18] presented an example for evaluating the C_{PU} of a simple linear profile, where $y = 3 + 2x$, USL $= 6 + 2x$, MSE $= 1$, and the true value of C_{PU} is equal to 1. The range of x is [2,8]. They used a non-conforming proportion method to calculate the C_{PU} of a simple linear profile for three scenarios, including $n = 40, n = 100$ and $n = 1000$. The computed mean of C_{PU} in 1000 replications is 0.9851, 0.9823 and 0.9972, respectively. Simulation studies show that the standard deviation of C_{PU} decreases if we have an increase in n.

Now, we use our new method for C_{PU} assessment in this example. We can use Eq. (22) to calculate C_{PU} as follows:

$$C_{PL}(\text{profile}) = \frac{\int_2^8 [(6+2x)-(3+2x)]dx}{\int_2^8 [(6-2x)-(3-2x)]dx} = 1.$$

This shows that the new method is an accurate method, because it calculates the true value of C_{PU}. Also, the new method has no sensitivity to n. Consequently, we can conclude that the functional method is more accurate and stable rather than the non-conforming proportion method.

4.3. Example 3

In this subsection, we present an example in which the reference profile line has an intersection with the upper specification line. Suppose there is a simple linear profile, where USL $= 6 + x$, LSL $= 2 + x$, $y = 4 + 1.5x$, and MSE $= 1$. Suppose [2,8] is the range of x values. The goal is determination of C_{PK}. There is an intersection point between USL $= 6 + x$ and $y = 4 + 1.5x$, that is, $x = 4$. In fact, USL $= 6 + x$ is greater than $y = 4 + 1.5x$ in [2,4], but it is less than $y = 4 + 1.5x$ in [4,8]. $C_{PL}(\text{profile}), C_{PU}(\text{profile})$, and $C_{PK}(\text{profile})$ are calculated as follows:

$$C_{PL}(\text{profile}) = \frac{\int_4^8 [(4+1.5x)-(2+x)]dx}{\int_2^8 [(4-1.5x)-(1+1.5x)]dx} = 3.33,$$

$$C_{PU}(\text{profile}) =$$

$$\frac{\int_2^4 [(6+x)-(4+1.5x)]-dx \int_4^8 [(4+1.5x)-(6+x)]dx}{\int_2^4 [(7+1.5x)-(4+1.5x)]dx}$$

$$= -1.83,$$

$$C_{PK}(\text{profile}) =$$

$$\min\{C_{PL}(\text{profile}), C_{PU}(\text{profile})\} = 0.183.$$

It is obvious that when μ is greater than USL in a range of x-values, we should expect negative values of the process capability index.

5. Conclusion

In this paper, a functional approach is proposed to determine the process capability indices of a simple linear profile. This new approach defined the process capability indices as a function of the explanatory variable. These indices can compute the capability of the process at each level of the explanatory variable. In addition, the area bounded between the mean, specification limits and natural tolerance limits is used to compute a unique value for the process capability indices of a simple linear profile. By using this concept, traditional C_P and C_{PK} were generalized to compute the process capability indices of a simple linear profile. The performance of the proposed method is evaluated through three numerical examples. The results show the more accurate performance of the proposed method in comparison with the traditional non-conforming proportion method. The proposed functional approach can be developed for other types of profile, such as polynomial profiles and multiple linear regression profiles, for future research.

Acknowledgement

The authors are grateful to the anonymous reviewer and Prof. S.T.A. Niaki, the editor of Transaction E: Industrial Engineering, for their invaluable comments which led to improvements in the paper.

References

1. Noorossana, R., Saghaie, A. and Amiri, A. *Statistical Analysis of Profile Monitoring*, John Wiley and Sons, Inc (2011).
2. Mestek, O., Pavlik, J. and Suchánek, M. "Multivariate control charts: Control charts for calibration curves", *Journal of Analytical Chemistry*, **350**(6), pp. 344-351 (1994).
3. Stover, F.S. and Brill, R.V. "Statistical quality control applied to ion chromatography calibrations", *Journal of Chromatography*, **804**(1), pp. 37-43(1998).
4. Kang, L. and Albin, S.L. "On-Line monitoring when the process yields a linear profile", *Journal of Quality Technology*, **32**(4), pp. 418-426 (2000).
5. Kim, K., Mahmoud, M.A. and Woodall, W.H. "On the monitoring of linear profiles", *Journal of Quality Technology*, **35**(3), pp. 317-328 (2003).
6. Mahmoud, M.A., Parker, P.A., Woodall, W.H. and Hawkins, D.M. "A change point method for linear profile data", *Quality and Reliability Engineering International*, **23**(2), pp. 247-268 (2007).
7. Zou, C., Zhang, Y. and Wang, Z. "Control chart based on change-point model for monitoring linear profiles", *IIE Transactions*, **38**(12), pp. 1093-1103 (2006).
8. Croarkin, C. and Varner, R. "Measurement assurance for dimensional measurements on integrated-circuit photo masks", *NBS Technical Note* 1164, U.S. Department of Commerce, Washington, D.C., USA (1982).
9. Wang, K. and Tsung, F. "Using profile monitoring techniques for a data-rich environment with huge sample size", *Quality and Reliability Engineering International*, **21**(7), pp. 677-688 (2005).
10. Gupta, S., Montgomery, D.C. and Woodall, W.H. "Performance evaluation of two methods for online

monitoring of linear calibration profiles", *International Journal of Production Research*, **44**(10), pp. 1927-1942 (2006).

11. Saghaei, A., Mehrjoo, M. and Amiri, A. "A cusum-based method for monitoring simple linear profiles", *The International Journal of Advanced Manufacturing Technology*, **45**(11), pp. 1252-1260 (2009).
12. Bothe, D.R. *Measuring Process Capability*, McGraw-Hill, New York (1997).
13. Pearn, W.L. and Kotz, S. *Encyclopedia and Handbook of Process Capability Indices: A Comprehensive Exposition of Quality Control Measures*, World Scientific, Singapore (2006).
14. Woodall, W.H. "Current research on profile monitoring", *Producao*, **17**(3), pp. 420-425 (2007).
15. Shahriari, H. and Sarafian, M. "Assessment of process capability evaluation in linear profiles", *Proceeding of the 6th International Industrial Engineering Conference*, Tehran, Iran (2009).
16. Ebadi, M. and Shahriari, H. "A process capability index for simple linear profile", *The International Journal of Advanced Manufacturing and Technology*, **64**(5-8), pp. 857-865 (2012).
17. Hosseinifard, S.Z., Abbasi, B. and Abdollahian, M. "Performance analysis in non-normal linear profiles using gamma distribution", *2010 Eighth International Conference on Information Technology: New Generations*, pp. 603- 607 (2011).
18. Hosseinifard, S.Z. and Abbasi, B. "Evaluation of process capability indices of linear profiles", *International Journal of Quality and Reliability Management*, **29**(2), pp. 162-176 (2012).
19. Ebadi, M. and Amiri, A. "Evaluation of process capability in multivariate simple linear profiles", *Scientia Iranica*, **19**(6), pp. 1960-1968 (2012).
20. Nemati, R., Kazemzadeh, R.B., Amiri, A. and Noorossana, R. "Functional process capability indices for circular profile", *Published online in Quality and Reliability Engineering International* (2013). DOI: 10.1002/qre.1515
21. Kane, V.E. "Process capability indices", *Journal of Quality Technology,* **18**(1), pp. 41-52 (1986).
22. Kotz, S. and Johnson, N.L., *Process Capability Indices*, Chapman and Hall, London (1991).
23. Clements, J.A. "Process capability calculations for non-normal distributions", *Quality Progress*, **22**(9), pp. 95-97 (1989).

Biographies

Ramezan Nemati Keshteli holds BS and MS degrees in Industrial Engineering from Khajeh Nasir University of Technology, Iran, and Isfahan University of Technology, Iran, respectively, and a PhD degree in Industrial Engineering from Tarbiat Modares University, Tehran, Iran. His research interests include statistical quality control and artificial intelligence.

Reza Baradaran Kazemzadeh received a BS degree in Operations Research, an MS degree in Industrial Engineering, and a PhD degree in Technology Management from the Bradford University, UK. He is currently Associate Professor at Tarbiat Modares University, Tehran, Iran. His research interests include quality engineering and management, profile monitoring, applied multivariate statistics, and operations management. His publications have appeared in Quality & Quantity, the European Journal of Operational Research, Communications in Statistics, and the International Journal of Advanced Manufacturing Technology.

Amirhossein Amiri holds BS, MS, and PhD degrees in Industrial Engineering from Khajeh Nasir University of Technology, Iran University of Science and Technology, and Tarbiat Modares University, Tehran, Iran, respectively. He is currently Assistant Professor at Shahed University, Tehran, Iran, and is a member of the Iranian Statistical Association. His research interests include statistical quality control, profile monitoring, and Six Sigma.

Rassoul Noorossana received a BS degree in Engineering from Louisiana State University, USA, in 1983, and MS and PhD degrees in Engineering Management and Statistics from the University of Louisiana in 1986 and 1990, respectively. He is currently Professor of Statistics at Iran University of Science and Technology, Tehran, Iran. His primary research interests include statistical process control, process optimization, Six Sigma, and statistical analysis. He is Editor of the Journal of Industrial Engineering International and serves on the editorial review board of many other journals. He is member of the founding committee for the Iranian Society for Quality, a senior member of the American Society for Quality, the Iranian Society for Quality, the Iranian Statistical Association, and the Industrial Engineering Society.