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Economic-statistical design of adaptive X-bar control chart: a Taguchi loss function approach

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Abstract. Along with the widespread use of Taguchi methods in product design, definition of the loss function has been integrated with numerous models which require quality cost estimation. In this paper, the economic-statistical design of a variable sampling X-bar control chart is extended using the Taguchi loss function to improve chart effectiveness from a quality cost point of view. The effectiveness of the proposed schemes is evaluated by comparing optimal expected costs and statistical performance with each other and with the fixed sampling policy. Results indicate a satisfactory performance for the proposed models.

1. Introduction

Variability is an intrinsic characteristic of any production or service process. Generally speaking, process variation can be classified into two major categories of common cause and special cause. The common or chance cause variation is an inherent part of any process and can only be altered if the process nature itself is altered. On the other hand, special or assignable causes of variation are unusual disruptions in the process which should be removed immediately in order to bring the process under statistical control. The principal function of a control chart is to help the management distinguish between these two different sources of variation.

To design a control chart to monitor a process, sample size (n), sampling interval (h), and control limit coefficient (k) should be determined. A control char is referred to as Fixed Ratio Sampling (FRS), when samples of fixed size at fixed sampling intervals are obtained from the process. When any of these parameters are allowed to vary, then the control chart is referred to as an adaptive control chart. Adaptive control charts lead to improved statistical results compared to sixed ratio sampling control charts [1,2]. Variable Sample Size (VSS), Variable Sampling Interval (VSI), and Variable Sample Size and Sampling Interval (VSSI) are examples of adaptive control schemes.

Statistical and economical designs of control charts are two main approaches to design an optimum control chart. The statistical approach focuses on chart statistical performance. However, in the economic design of control charts, the emphasis is on economic issues, such as the cost associated with sampling and inspection, the cost associated with investigating an out-of-control signal and repairing the process, and the cost associated with producing nonconforming products. In order to design a control scheme with improved performance, with respect to both viewpoints, the economic-statistical design of control schemes was proposed by researchers.

Bai and Lee [3] proposed the first economic design

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of an $\overline{X}$ control chart with a VSI scheme. The economic design of a VSS Shewhart chart was developed by Park and Reynolds [4]. Economically designed VSSI Shewhart charts were considered by Das and Jain [5] and Park and Reynolds [4]. Yu and Chen [6] focused on designing a VSI $\overline{X}$ control chart in a continuous process. Lin et al. [7] and Chen and Yeh [8] extended the economic design of adaptive control charts for the case of non-normal observations. Several studies contributed to the development of the economic design of adaptive control charts [1,3,9-15].

Moreover, the Taguchi loss function, which considers loss to society as a quadratic function of product quality, has been used by many researchers to develop control schemes. This is in concordance with the main principle of six sigma methodology, where the main object is to reduce deviations around the target. Several researchers have applied the loss function approach in the economic design of control charts. Alexander et al. [16] studied an economic model for an $\overline{X}$ control chart with the Taguchi loss function. Serel and Moskowitz [17] used the Taguchi loss function to develop the joint economic design of an Exponentially Weighted Moving Average (EWMA) for mean and variance. Niaki et al. [18,19] extended the Lorenzen-Vance [20] cost function using the multivariate Taguchi loss approach. Recently, Yeong et al. [21] studied economic and economic statistical designs of the synthetic chart using loss functions.

This paper focuses on extending the $\overline{X}$ control chart with an adaptive scheme considering the Taguchi quality loss function. The remainder of this paper is organized as follows. In Section 2, the adaptive $\overline{X}$ control chart is developed. Section 3 describes the proposed algorithm. In Section 4, an illustrative example is given to evaluate the performance of the proposed methodology. Our conclusion is provided in the final section.

2. Adaptive $\overline{X}$ control chart

In an adaptive $\overline{X}$ control chart, in addition to the usual control limit with coefficient $L_{\overline{X}}$, a warning limit, denoted by $w_{\overline{X}}$, is considered. In the VSSI scheme, usually, two sampling intervals, $h_2 < h_1$, and two sample sizes, $n_1 < n_2$, are used. Sampling size and sampling interval are established based on the position of the first sampling statistic on the chart. In this regard, if the prior sample mean $(i-1)$ falls in the warning region, the chart design $(n_2, h_2, L_{\overline{X}}, w_{\overline{X}})$ should be used for the current sample point (i). Alternatively, if the prior sample point $(i-1)$ falls in the central region, the chart design $(n_1, h_1, L_{\overline{X}}, w_{\overline{X}})$ should be employed for the current sample point (i). When $n_1 = n_2 = n$ and $h_2 < h_1$, then, VSSI $\overline{X}$ simplifies to the VSI $\overline{X}$ chart. When $n_1 < n < n_2$and $h_1 = h_2 = h$, the VSSI $\overline{X}$ chart reduces to the VSS $\overline{X}$ chart. Hence, the VSSI control scheme can be defined as follows:

$$(h_i,n_i,L_{\overline{X}},w_{\overline{X}}) = \begin{cases}(h_1,n_1,L_{\overline{X}},w_{\overline{X}}) & \text{if } \overline{X}_{i-1} \in \text{central region}\\ (h_2,n_2,L_{\overline{X}},w_{\overline{X}}) & \text{if } \bar{X}_{i-1} \in \text{warning region}\end{cases} \quad (1)$$

An important statistical measure, which determines the performance of an adaptive control chart, is the adjusted average time to signal or AATS. If the assignable cause occurs according to an exponential distribution with parameter λ, then, the expected time interval in which the process remains in control is $1/\lambda$. Hence, AATS can be defined as:

$$AATS = ATC - \frac{1}{\lambda}, \quad (2)$$

where the Average Time of Cycle (ATC) is the average time from the start of production until the first signal after the process shift. The memory less property of the exponential distribution allows the computation of the ATC using the Markov chain approach [22].

2.1. Markov chain approach

According to the VSSI scheme, at each sampling stage, one of the following transient states is met, according to the status of the process (in or out-of-control), size of the sample (small or large) and sampling frequency (short or long). The process is in state 1, if the prior sample point $(i-1)$ falls in the central region or the process is actually in-control.

State 1: Position of sample statistic is $|Z| \leq w$ and the process is in-control;

State 2: Position of sample statistic is $w < |Z| \leq k$ and the process is in-control;

State 3: Position of sample statistic is $|Z| \leq w$ and the process is out-of-control;

State 4: Position of sample statistic is $w < |Z| \leq k$ and the process is out-of-control;

State 5: Position of sample statistic is $|Z| > k$ and the process is out-of-control (true alarm state).

If the position of the sample statistic is $|Z| > k$ while the process status is out-of-control, then a true alarm is signaled and the absorbing state (State 5) is arrived. In order to model the VSSI scheme based on the Markov chain approach, transition probabilities, p_{ij} (i is the prior state and j is the current state), should be defined. The transition probability matrix, $P = [p_{ij}]$, is given as follows:

$$p = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} \\ 0 & 0 & p_{33} & p_{34} & p_{35} \\ 0 & 0 & p_{43} & p_{44} & p_{45} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (3)$$

where the transition probabilities are defined as:

$$p_{11} = \Pr(|Z| \le w|\, |Z| \le k) \times e^{-\lambda h_1},$$

$$p_{12} = \Pr(w < |Z| \le k|\, |Z| \le k) \times e^{-\lambda h_2},$$

$$p_{13} = \Pr(|Z| \le w|\, |Z| \le k) \times (1 - e^{-\lambda h_1}),$$

$$p_{14} = 1 - p_{11} - p_{12} - p_{13},$$

$$p_{21} = \Pr(|Z| \le w|\, |Z| \le k) \times e^{-\lambda h_1},$$

$$p_{22} = \Pr(w < |Z| \le k|\, |Z| \le k) \times e^{-\lambda h_2},$$

$$p_{23} = \Pr(|Z| \le w|\, |Z| \le k) \times (1 - e^{-\lambda h_1}),$$

$$p_{24} = 1 - p_{21} - p_{22} - p_{23},$$

$$p_{31} = p_{32} = 0,$$

$$p_{33} = \Pr(|Y| \le w|Y \sim N(\delta\sqrt{n_1}, 1)),$$

$$p_{34} = \Pr(w < |Y| \le k|Y \sim N(\delta\sqrt{n_1}, 1)),$$

$$p_{35} = 1 - p_{33} - p_{34},$$

$$p_{41} = p_{42} = 0,$$

$$p_{43} = \Pr(|Y| \le w|Y \sim N(\delta\sqrt{n_1}, 1)),$$

$$p_{44} = \Pr(w < |Y| \le k|Y \sim N(\delta\sqrt{n_2}, 1)),$$

$$p_{45} = 1 - p_{43} - p_{44}. \quad (4)$$

The product of the average number of visiting a transient state and the corresponding sampling interval determines the period, ATC.

$$\text{ATC} = \mathbf{b}'(\mathbf{I} - \mathbf{Q})^{-\mathbf{1}}\mathbf{h}, \quad (5)$$

where I is the identity matrix of order four; $\mathbf{b}' = (p_{11}, p_{12}, p_{13}, p_{14})$ is the vector of starting probability; and Q is the transition matrix without elements associated with the absorbing state. $\mathbf{h}' = (h_1, h_2, h_1, h_2)$ is the vector of sampling intervals corresponding to the transient states. The average number of samples (ANS) in the VSSI scheme is determined as follows:

$$\text{ANS} = \mathbf{b}'(\mathbf{I} - \mathbf{Q})^{-\mathbf{1}}\eta, \quad (6)$$

where $\eta' = (n_1, n_2, n_1, n_2)$ is the vector of the sample sizes corresponding to the transition states. The expected number of false alarms per cycle is given in Eq. (7):

$$\text{ANS} = \mathbf{b}'(\mathbf{I} - \mathbf{Q})^{-\mathbf{1}}f, \quad (7)$$

where $f = (\alpha_1, \alpha_2, 0, 0)$ is the vector of false alarms probabilities in each transition state.

2.2. Taguchi loss function

In Taguchi philosophy, a loss incurs when the quality characteristic of interest deviates from its target value. The purpose of loss function is to reflect the economic loss associated with variations and deviations from the target. Variation reduction is equivalent to lower loss and higher quality. Computation of production costs based on quadratic loss function, to economically design control charts, has been suggested by many authors [16,19,23-27]. In this paper, we also use Taguchi loss function, defined as $L(X) = K(X-T)^2$, where X is a key quality characteristic, K is a positive coefficient, and T is the target. Suppose the specification limits for the quality characteristic of interest are $T \pm \Delta$ and the cost of rework or scrap for one unit of product is A. The coefficient, K, is defined as $K = \frac{A}{\Delta^2}$.

When the process is in control, the quality characteristic, X, follows a normal distribution with mean μ_0 and variance σ_0^2. It is desirable for the location of the in-control mean to coincide with the target location. However, if μ_0 is different from T, a fixed bias impacts all manufactured items. The expected quality cost per unit of product when the process is in control, J_0, is [27]:

$$J_0 = \int_{-\infty}^{+\infty} K(x-T)^2 f(x)dx = \int_{-\infty}^{+\infty} K(x - \mu_0 + \mu_0 - T)^2$$

$$f(x)dx = K[\sigma_0^2 + (\mu_0 - T)^2]. \quad (8)$$

When an assignable cause occurs, the process mean shifts to $\mu_1 = \mu_0 + \delta_0$. The expected quality cost per unit when the process is out of control, J_1, is given by:

$$J_1 = \int_{-\infty}^{+\infty} K(x - \mu_1 + \mu_1 - T)^2 f(x)dx$$

$$= \int_{-\infty}^{+\infty} K(x - \mu_0 - \delta\sigma_0 + \mu_0 + \delta\sigma_0 - T)^2 f(x)dx$$

$$= K[\sigma_0^2 + (\mu_0 - T)^2 + \delta^2\sigma_0^2 - 2\delta\sigma_0(\mu_0 - T)]. \quad (9)$$

2.3. Cost model

Following renewal reward process assumption, the expected quality cost per hour is computed as the ratio of the expected cost per cycle to the expected cycle time. A quality cycle consists of one period when the process is in-control and two periods during an out-of-control state. The expected length of a quality cycle is calculated as follows:

$$\text{ET} = \text{ATC} + T_0 \times \text{ANF} + T_1. \tag{10}$$

ET is composed of the in-control portion (including interruptions for false alarms) and the time to locate and repair the process, T_1. T_0 is the average time to search when the process is in-control.

The costs of producing nonconformities while in- and out-of-control, sampling and inspection costs, costs of false alarms, and locating and removing an assignable cause are elements of the expected cost per cycle. Each individual cost element is derived as follows:

$$\text{EC} = C_0(1/\lambda) + C_1 \times (\text{AATS}) + s \times \text{ANS} + f_0 \times \text{ANF} + W, \tag{11}$$

where s is the sampling cost, f_0 is the cost of false alarms, and W is the cost associated with locating and repairing the process. Moreover, C_0 and C_1 are the expected costs associated with producing non-conformities while the process is in-control and out-of-control, respectively. If p units are produced per hour, then, $C_0 = J_0 p$ and $C_1 = J_1 p$. The expected cost per hour incurred by the process can be obtained as:

$$\text{EA} = \frac{\text{EC}}{\text{ET}}. \tag{12}$$

In the economic-statistical design of an adaptive $\overline{X}$ control chart, the design vector consists of control limits, $L_{\overline{X}}$, warning lines, $w_{\overline{X}}$, sample sizes, n_1 and n_2, and sampling frequencies, h_1 and h_2. The objective is to find a design vector that minimizes EA, subject to some constraints. Hence, the optimization problem can be defined as:

$$\begin{aligned} &\text{Min} \;\; \text{EA}(n_1, n_2, h_1, h_2, L_{\overline{X}}, w_{\overline{X}}) \\ &\text{Subject to :} \\ &\text{AATS} \le \text{AATS}_M, \\ &\text{ANF} \le \text{ANF}_M, \\ &h_{\min} \le h_2 < h_1 \le h_{\max}, \\ &0 < w < L \le L_{\max}, \\ &1 \le n_1 < n_2 \le n_{\max} \text{ (integers).} \end{aligned} \tag{13}$$

In the optimization model, constraints $\text{ANF} \le \text{ANF}_M$ and $\text{AATS} \le \text{AATS}_M$ are added to form the best protection against false alarms and to detect process shifts as quickly as possible. The minimum and maximum values of possible sampling intervals between successive samples, $h_{\min}$ and $h_{\max}$, are added to keep the chart more practical. In this research, the values of $h_{\min} = 0.01$ and $h_{\max} = 8$ are used because sampling intervals less than 0.01 and greater than 8 hours may be awkward in a work shift.

3. Solution algorithm

The economic-statistical optimization model has some discrete and continuous decision variables. Hence, the Genetic Algorithm (GA), which has been used in several studies, can be considered to optimize this problem [19,27-31]. The objective of GA is to obtain a global optimum solution. GA starts to generate a new generation or population using a collection of small possible solutions in a parallel process. The quality of solutions presented by GA depends on GA parameters. GA parameters are population size (N_{pop}), crossover (CP), Number of Elite (NE), Number of Generations (GN), and mutation rate. The key parameters, which should be determined at the beginning of the algorithm and should be used while applying the algorithm, are described below.

GA starts to work using some possible initial solutions referred to as "initial population". Each population has N_{pop} chromosomes, which are produced from the solution. In this research, each chromosome consists of five genes, each of which is representative of a decision variable. The decision variable of the model includes $(n_1, n_2, h_1, h_2, L_{\overline{X}}, w_{\overline{X}})$.

The cost function is calculated for the chromosomes of each generation. The best chromosomes will be selected for crossover purposes. Each generation includes the X_{keep} superior chromosome and $N_{\text{pop}} * X_{\text{keep}}$ children, which have been generated through the crossover.

The mutation operator is employed to prevent the GA from converging into a local optimum value. Selected chromosomes for mutation are not among the best of each generation essentially. The elite of each generation will be transferred directly to the next generation to prevent losing the best chromosomes. The elitism operator is actually a method for maintaining the best chromosomes of each generation. After the mutation, for each chromosome, the cost function will be calculated. Then, the chromosomes will be ranked. The stopping criterion, the number of iterations in the algorithm, will be investigated, and the loop will continue until an optimum solution is obtained.

In the algorithm developed in this paper, mutation rate is considered a liner combination of the other parameters. The optimized combination of GA parameters is often accomplished using a trial and error method, which is a hard process, due to the multiplicity of possible states. The optimum values for four GA parameters have been determined using Taguchi orthogonal array. Many analysts in the field of Economic-Statistical Design (ESD) of control charts have already recommended using this method to determine the optimum value of GA parameters [22,29]. Nine combinations of control parameters in three difference levels will be considered by L9 orthogonal array. Table 1 shows the value for each of the parameters.

The algorithm iterated three times ($Y1$,$Y2$, and $Y3$) for each of the levels. Then the results are obtained from running the algorithm for 27 times. Since the objective function for the problem is a minimizing one, the signal-to-noise ratio (SN), defined as below, is calculated to evaluate the results of the experiments.

$$\text{SN} = -10\log\left(\frac{1}{r}\sum_{i=1}^{r} Y_i^2\right). \tag{14}$$

In Eq. (14), r is the number of replicates for each level. Solution values for each level of parameters and the SN value for each level is tabulated in Table 2. The sum of the SN ratio for each level of the GA parameters is shown in Table 3. The optimized combination of the levels of four GA parameters, $N_{\text{pop}} = 500$, CF = 0.5, NE = 6 and NG = 100, are recommended, based on the maximum of SN for each level.

4. Numerical analysis

To illustrate the application of the developed adaptive $\overline{X}$ control chart, numerical analysis has been undertaken. Logical ranges for each of the control chart parameters: Sampling size, sampling interval, and control limit coefficient range, have been considered to be [1,30],[0.1,8] and [1,5], respectively. Cost and process parameters are as follows: Sampling cost is $s = 5\$$, cost of detecting a reasonable deviation is $W = 1000\$$, cost of false signal is $f_0 = 1500$, average time to search for a false signal is $T_0 = 5$ hour, and the average time to detect the deviations and modify the process is $T_1 = 2$ hour; process mean increases by 1.5 standard deviation ($\delta = 1.5$).

Also, the quadratic loss function coefficient is $K = 1$, the actual process average is equal to the characteristic's target value, and the process variance is $\sigma_0^2 = 1$. The average process in-control period is

Table 1. Levels for each of the model parameters.

Parameter	Range	Level 1	Level 2	Level 3
Population size (N_{pop})	100-900	100	500	900
Crossover Fraction (CF)	0.1-0.9	0.1	0.50	0.90
Number of Elites (NE)	4-10	4	6	10
Number of Generations (NG)	50-150	50	100	150

Table 2. The objective values for each level of GA parameters.

Runs	N_{pop}	CF	NE	NG	$Y1$	$Y2$	$Y3$	SN
1	100	0.10	2	50	121.848	123.106	123.106	-41.7760
2	100	0.50	6	100	121.848	123.106	121.848	-41.7463
3	100	0.90	10	150	123.106	123.106	123.106	-41.8056
4	500	0.10	6	150	121.848	121.848	121.848	-41.7164
5	500	0.50	10	50	121.848	121.848	121.848	-41.7164
6	500	0.90	2	100	121.848	121.848	121.848	-41.7164
7	900	0.10	10	100	121.848	121.848	121.848	-41.7164
8	900	0.50	2	150	121.848	121.848	121.848	-41.7164
9	900	0.90	6	50	121.848	121.848	121.848	-41.7164

Table 3. The sum of SN ratio for each level of GA parameters.

	N_{pop}	CF	NE	NG
Level 1	-125.3279	-125.2088	-125.2088	-125.2088
Level 2	-125.1491*	-125.1790*	-125.1790*	-125.1790*
Level 3	-125.1491*	-125.2383	-125.2383	-125.2383

* largest sum of SN ratio for each parameter within different levels [33].

equal to 100. If the production rate is equal to Pr = 100, the production cost for each defective product, while the process is under in-control and out-of-control conditions, will be equal to $C_0 = 100J_0$ and $C_1 = 100J_1$, respectively.

In the proposed model, solutions are evaluated based on the objective function of quality costs. The minimum cost solution for fixed ratio sampling is $(n, h, L_{\overline{X}}) = (4, 8.0, 2.15)$, with the cost equal to \$119.50, where the average number of false signals is equal to $\text{ANF}_{\text{Classic}} = 0.375$, and the adjusted average time to signal is equal to $\text{AATS}_{\text{Classic}} = 60.0$ hours. However, the VSSI scheme solution vector is $(n_1, n_2, h_1, h_2, L_{\overline{X}}, w_{\overline{X}})$. The optimum economic costs achieved by solution vector $(n_1, n_2, h_1, h_2, L_{\overline{X}}, w_{\overline{X}}) = (1, 22, 8.00, 8.00, 2.43, 1.30)$, which will result in a cost equal to \$118.60. In this scheme, $\text{ANF}_{\text{VSSI}} = 0.183$ and $\text{AATS}_{\text{VSSI}} = 46.55$ hours are obtained. The optimum VSI scheme is equal to $(n_1, n_2, h_1, h_2, L_{\overline{X}}, w_{\overline{X}}) = (4, 4, 8.00, 0.10, 2.19, 1.75)$. This scheme results in a cost equal to \$119.45, $\text{ANF}_{\text{VSI}} = 0.361$, and $\text{AATS}_{\text{VSI}} = 56.69$ hours. On the other hand, the optimum economic cost of the VSS scheme achieved by the solution vector $(n_1, n_2, h_1, h_2, L_{\overline{X}}, w_{\overline{X}}) = (1, 23, 8.00, 8.00, 2.44, 1.23)$ is \$118.60 per hour, with $\text{ANF}_{\text{VSS}} = 0.177$ and $\text{AATS}_{\text{VSS}} = 46.63$ hours.

Comparison of the adaptive schemes with the FRS scheme shows the efficiency of the proposed model. Cost reduction due to using the VSSI plan is equal to $\frac{119.50-118.60}{119.50} = 0.8\%$. Furthermore, the average number of false signals has remarkably decreased to $\text{ANF}_{\text{VSSI}} = 0.183$ from $\text{ANF}_{\text{Classic}} = 0.375$ (51.20 % improvement). The $\text{AATS}_{\text{Classic}} = 60.0$ hours has reduced to $\text{AATS}_{\text{VSSI}} = 46.55$ hours, which indicates 22.42% improvement. For this process shift size, the VSS scheme performs as well as the VSSI scheme, in terms of both cost and statistical measures. The VSI scheme shows better performance compared to the FRS scheme, but the difference is not significant. Since a control chart is designed to detect a variety of shift sizes in the process mean, other shift sizes need to be investigated and this will be examined in the next section.

5. Sensitivity analysis

Investigation of the effects of changes in the estimated values of model parameters has been recommended in previous studies [19,32]. In this section, the effect of the process mean shift of sizes $\delta \in \{0,5,1,0,1,5,2,0,2,5,3,0\}$ will be evaluated.

The optimum solutions of adaptive schemes for different process mean shift sizes are tabulated in Tables 4 to 6. As shown in Table 4, the corresponding cost for small process mean shifts is less than the cost value for larger shift values. However, the proposed model offers better AATS for larger shift values, which will result in early identification of assignable causes.

Also, the optimum solutions of the classic scheme are evaluated for different process mean shift sizes. The effect of process mean shift on the optimum solution of the classic scheme is summarized in Table 7. The performance of each adaptive scheme when the process is facing different shift sizes can be concluded by comparing these results. To better understand the difference in each scheme, the cost and AATS of each scheme are compared in Figures 1 and 2, respectively.

Table 4. Effect of process mean shift on the optimum solution of the VSI scheme.

δ	n_1	n_2	h_1	h_2	$L_{\overline{X}}$	$w_{\overline{X}}$	ANF	AATS	EA
0.5	4	4	8.00	0.10	2.19	1.75	0.361	56.69	119.46
1.0	7	7	8.00	0.10	2.79	1.34	0.078	5.87	120.14
1.5	4	4	4.53	0.10	3.08	1.38	0.054	2.88	120.78
2.0	3	3	2.86	0.10	3.32	1.62	0.035	1.67	121.46
2.5	2	2	1.86	0.10	3.39	1.65	0.040	1.10	121.85
3.0	2	2	1.52	0.10	3.61	2.05	0.021	0.83	123.09

Table 5. Effect of process mean shift on the optimum solution of the VSS scheme.

δ	n_1	n_2	h_1	h_2	$L_{\overline{X}}$	$w_{\overline{X}}$	ANF	AATS	EA
0.5	1	22	8.00	8.00	2.43	1.30	0.183	46.55	118.60
1.0	5	11	7.73	0.10	3.01	1.39	0.040	6.22	119.65
1.5	3	6	3.80	0.10	3.33	1.51	0.026	2.70	120.14
2.0	2	4	2.28	0.10	3.51	1.61	0.021	1.56	120.56
2.5	1	3	1.25	0.10	3.70	1.61	0.019	1.02	120.77
3.0	1	2	1.07	0.10	3.64	1.68	0.028	0.75	121.28

Table 6. Effect of process mean shift on the optimum solution of the VSSI scheme.

δ	n_1	n_2	h_1	h_2	$L_{\overline{X}}$	$w_{\overline{X}}$	ANF	AATS	EA
0.5	1	23	8.00	8.00	2.44	1.32	0.177	46.63	118.60
1.0	8	14	8.00	8.00	2.51	1.57	0.145	7.43	121.77
1.5	6	9	5.06	5.06	2.75	2.02	0.116	3.49	123.12
2.0	4	6	3.05	3.05	2.95	2.23	0.103	2.00	123.96
2.5	3	4	2.13	2.13	3.10	2.36	0.089	1.31	124.59
3.0	2	3	1.41	1.41	3.19	2.33	0.101	0.92	124.90

Table 7. Effect of process mean shift on the optimum solution of classic scheme.

δ	n	h	$L_{\overline{X}}$	ANF	AATS	EA
0.5	4	8.00	2.15	0.375	60.00	119.50
1.0	10	8.00	2.49	0.153	6.74	122.06
1.5	6	5.07	2.69	0.139	3.53	123.25
2.0	4	3.06	2.90	0.121	2.02	124.07
2.5	3	2.13	3.07	0.099	1.32	124.66
3.0	2	1.41	3.14	0.121	0.93	125.06

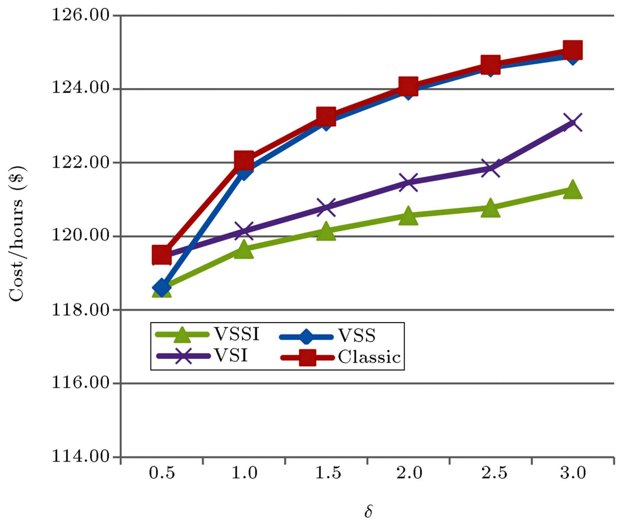

Figure 1. Comparison of cost in adaptive and classic schemes.

As shown in Figures 1 and 2, when a small shift (i.e. 0.5 standard deviation) incurs in the process, VSSI and VSS perform better compared to VSI and classic schemes. However, when the process is faced with moderate or large shift sizes (i.e. 1 or 2 standard deviation), VSSI and VSI schemes are always superior to VSS and classic schemes, in terms of both cost and AATS.

6. Conclusions

In this research, adaptive $\overline{X}$ control charts are developed to monitor process mean, while process operating costs and deviation from the target are considered

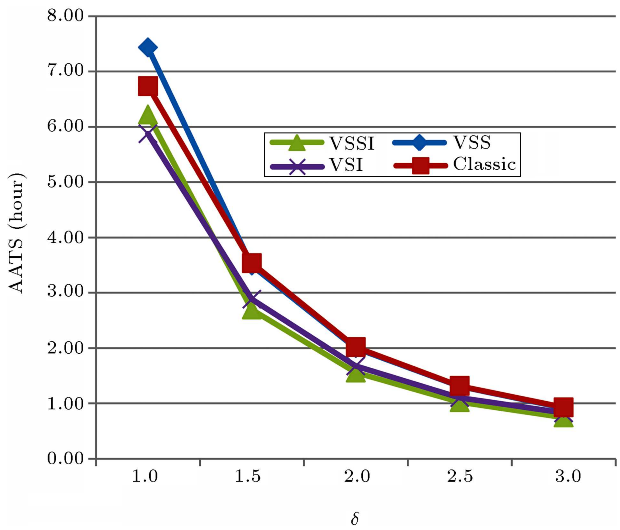

Figure 2. Comparison of AATS in adaptive and classic schemes.

simultaneously. The relationship between process monitoring costs and deviations from the designed target value is incorporated in the model considering Taguchi loss function. Adaptive schemes, consisting of VSS, VSI, and VSSI schemes, are compared with the classic FRS scheme. Evaluation of the optimum solutions shows that shift size in the process mean influences expected cost, as well as adjusted average time to signal. The proposed adaptive schemes remarkably improve both quality cost and alarm rates. Sensitivity analyses of the proposed model show that VSSI and VSS perform better in comparison to VSI and classic schemes when the chart is optimized for identifying small shifts in the process. However, VSSI and VSI schemes are always better than VSS and classic schemes when the process is facing moderate or large shift sizes. Hence, one can conclude that the proposed adaptive schemes are superior to the FRS scheme, in both aspects of process monitoring costs and statistical measures.

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