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On competence of vendor managed inventory in supply chains using basic mathematical inventory models

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Abstract. In this study, a two-echelon single-vendor supply chain is selected to do a cost-based comparison between short-term performances of a Vendor Managed Inventory (VMI) and a Retailer Managed Inventory (RMI). While the inventory costs include ordering and storing expenses, the rate of consumption and the price of goods are constant, the rate of production and pace of transportation are infinite and shortage is not allowed. This paper, after a comprehensive literature review, is followed by three cases of single retailer, n-retailer and two-retailer chains. Unlike the second case, in the first case, VMI shows an absolute superiority to RMI, and this is the reason for devising the third case in which a deeper analysis, including a typical performance assessment system for two-retailer chains is undertaken. The third case reveals that, although VMI is not always the better choice, under most conditions, it can be chosen as the better approach.

1. Introduction

While product manufacturing deals only with the manufacturer, logistics and inventory control are current issues of concern for everyone in the supply chain. This paper emphasizes the latter aspect, i.e. inventory control. In a traditional supply chain, each channel member operates individually with the interactions between them limited to the feed-forward flow of physical products and the feedback flow of information in the form of purchase orders and cash [1]. But, in the present competitive world, businesses should work more closely with their customers and suppliers. During the past several decades, to encounter the challenge, a number of sophisticated supply chain management initiatives have been developed that have resulted in industrial structure and improvement in firm performance. One of these initiatives is VMI, which promotes coordination and collaboration between suppliers and buyers, and is one of the best practices in enhancing supply chain integration [2-5].

The inception of VMI can be traced back to Magee [6], who studied the issue of "who should have the authority to control inventories? [7]. Since the implementation of VMI is difficult, especially when the trust between supply chain members and their actual commitments are under question, the real interest in VMI started to grow only in the 1990s [8]. However, in recent years, using information technology, such as Electronic Data Interchange or Internet based XML protocols, much of the infrastructure for VMI is economically plausible, and VMI has been experienced successfully by many well-known retailers and their first-tier suppliers. Most notably is the partnership between Wal-Mart and Proctor; and Gamble begun in 1985 [9,10]. Since then, VMI industrial applications have grown constantly [8], and companies such as Shell Chemicals, HP,

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Campbell Soup, and Johnson & Johnson have adopted it [10].

Unlike RMI with low cooperation between the players, in VMI, a downstream buyer shifts the ownership of inventories to its immediate upstream supplier and allows the supplier to access its demand information in return. In fact, the vendor decides on the appropriate inventory levels within bounds that are agreed upon in a contractual agreement between vendor and retailers [11].

Under the VMI program, the vendor has access to the retailer inventory and demand data for long-term plans. Retailers incur no ordering costs and are guarded against excessive inventory. The advantages of implementing VMI programs are very significant and can be summarized as an improvement in the dynamic performance of the supply chain [12], the higher profitability of both downstream and upstream members [13], reduced inventory costs [8,14,15], noticeable improvement of customer service levels [16,17], reduction in demand uncertainty [18,19], more efficient use of production facilities [8], and more flexibility in production planning and distribution [4]. Among all these positive points, there are also some studies that believe that supply chain integration does not necessarily result in benefits for both suppliers and buyers, and a buyer's inventory costs may be reduced only because costs are transferred to the supplier [20-23].

The main contribution of this paper is comparing VMI and RMI, quantitatively, from the perspective of their inventory expenses, using a basic mathematical inventory model in which consumption rate and good prices are constant, production rate and transportation pace are infinite and shortage is not allowed. It should be noted that the considered supply chain is a two-echelon single-vendor one.

The rest of the paper is organized as follows. Section 2 covers a broad literature review on the concept of VMI. Section 3 encompasses the main contribution of this study, and covers all the models, comparisons and experiments in three main sub-parts of a single retailer, an n-retailer and two-retail supply chains. Finally, Section 4 presents the final conclusions, along with several research areas to continue the study.

2. Literature review

There exists a large amount of literature on VMI that examines it from various aspects and which can be categorized into four categories: 1) VMI benefits, 2) VMI details, 3) VMI applicability and 4) VMI configurations. Each category highlights some kind of point. For example, the second category covers the detailed and small points of VMI, while the fourth covers structural points. It is to be noted that the categories cannot be distinguished from each other sharply and there are some overlaps, so, it is normal to see Yao et al. [23] or Lee et al. [24] in two categories.

Yu et al. [8] believe that the first category focuses on the question of "why-VMI". For example, Kaipia et al. [25], Disney and Towill [19], Yao et al. [23] and Nagarajan and Rajagopalan [26] conclude that VMI performs better as compared to a traditional supply chain, and Vergin and Barr [27] and Lee et al. [24] conclude that VMI is becoming an effective approach for implementing the channel coordination initiative. Waller et al. [16] indicate that the VMI can improve inventory turnover and customer service levels at every stage of a supply chain. Mishra and Raghunathan [28] indicate that VMI increases the competition between sellers and so provides a perspective justifying the interest of retailers in VMI. Bertazzi et al. [29], by comparing an order-up-to policy to an order-up-to with a dump off at the last stop, find that not only does VMI lower costs over traditional methods, but also, that the latter model performs better than a pure order-up-to model. Jarugumilli and Grasman [30] provide evidence of the benefits of radio frequency identification in a VMI-controlled supply chain. Yao and Dresner [31] show that VMI may bring benefits in terms of inventory reduction to the participants. Southard and Swenseth [32] provide empirical evidence of the benefits of a technology-enabled VMI system.

As in this paper, the studies of the second category examine the factors affecting the benefits of VMI. As a matter of fact, this category of work believes in VMI as a beneficial approach for inventory management and its focus is on the next level, i.e. points and details. A good example of this category is an early study that revealed the fact that the order release policy in use with VMI influences the level of inventory required at the vendor, thus, directly affecting a supplier's inventory costs [10]. Chaouch [33] evaluated the trade-offs between inventory costs, stock-outs and shipping frequency. He found that the appropriate balance is not always intuitively clear and that businesses must try to quantitatively model their situation to obtain the optimal decision. In a more in-depth analysis, Disney and Towill [34] found that the kernel goal of VMI is achieved primarily by sharing demand and inventory information. Choi et al. [35] studied the effect of supplier service levels on customer service levels. Chen et al. [36] compared quantity-based and time-based mathematical inventory models in a VMI environment. Bernstein et al. [37] examined the effect of VMI on pricing in supply chains. Yao et al. [23] investigated the effect of supply chain parameters on cost savings. They also found that total supply chain benefits are higher if the supplier's ordering cost is small, relative to the buyer's, prior to the implementation of VMI, or large, relative to the buyer's, after the implementation of VMI. Dong and Xu [13] evaluated the short-term and long-term impact of VMI on a supply chain and showed that the buyer receives more benefits from inventory cost savings. Tan et al. [38] indicated that the optimal ordering policy is an order-up-to system based on the kind of demand information supplied by a VMI system. Pasandide et al. [39] investigated how the important supply chain parameters affect the cost savings realized from VMI.

In the above mentioned work in the second category, there is no study that compares VMI with RMI quantitatively. This is the literature gap that our paper is to fill.

VMI, in practice, possesses numerous challenges in both aspects of information sharing and coordination of processes. So, there is a stream of work on this subject that Yu et al. [8] called the "how-to" stream. On the importance of information sharing, it is enough to say that supply chains have progressed many steps ahead through information sharing [40], and the bullwhip effect can be minimized through it in the supply chain [18,41-44]. Achabal et al. [17] introduced a decision support system to deal with a VMI replenishment arrangement that can improve service levels dramatically with a nominal increase in inventory investment. Xu et al. [45] examined the impact of EDI and Internet-based technologies on the practice of VMI. Choy et al. [46] presented an intelligent supplier management tool using case-based reasoning and neural network techniques to select and benchmark suppliers. Cetinkaya and Lee [10] aimed for coordination between inventory and transportation decisions using a model, while Axsater [47] tried to solve their model. Lo and Wee [48] proposed a mathematical model on the basis of their model. Tyan and Wee [9] pointed out that aside from computer technologies, the key of implementing VMI lies in the ability of the related chain members to cooperate and to understand the flows and processes concerning their products or service deliveries. Siajadi et al. [49] introduced a new methodology to obtain the joint economic lot size for a single-vendor multi-retailer problem. Trappey et al. [50] presented an integrated business and logistics hub model, which integrates information flow and material flow. Yu et al. [8] identified the conditions under which the VMI model is favorable over the traditional chain structure, and shed light on when and why collaboration is critical for its successful, long-term implementation. Wong et al. [51] detailed how a sales rebate contract helps achieve supply chain coordination. Arora et al. [40] developed a policy analysis of the integrated inventory and logistic problem of the current world supply chain.

In the last category of the literature, for example, Fry et al. [52] developed a model that considers a finite horizon inventory system under a specific type of contractual agreement, where the supplier must pay a penalty for falling outside the boundaries. Huang and Li [53] investigated a two-stage game mechanism in cooperative advertising models for a manufacturer-retailer supply chain. The single-vendor multi-retailer supply chain is investigated by Viswanathan and Piplani [54]. They proposed a strategy where the vendor specifies common replenishment periods. Wang et al. [55] analyzed non-cooperative behavior in a decentralized two-echelon supply chain consisting of one supplier and multiple retailers. Lee et al. [24] studied the effects of VMI configuration on newsboy environments. Shah and Goh [56] modeled the VMI problem in the context of a supply hub with a single retailer. Nachiappan and Jawahar [57] presented a two-echelon, single vendor, multiple retailer model, operating under VMI mode. Zhanga et al. [58] developed an integrated VMI model for a single vendor and multiple buyers. Darwish and Odah [11] developed a VMI model for a supply chain with a single vendor and multiple retailers that can easily describe supply chains with capacity constraints.

3. Model

The applied model in this paper is one of the oldest classical production scheduling models. The framework used is also known as the Barabas Formula. The model was developed by Harris [59] in 1913, but Wilson, a consultant who applied it extensively, is given credit for his in-depth analysis [60]. Our assumptions to run the study are as follows:

- Constant rate of consumption (the consumption of inventory is at a uniform rate throughout the cycle);
- Infinite rate of production and pace of transportation (the ordered quantity received at the time the inventory level reaches a minimum level of 0; in other words, on receipt of the material, the stock level of the inventory jumps to a maximum level);
- Constant price of goods (the price of goods is fixed in all the cycles), and unallowable state of shortage (the reorder point is set in such a manner that ordered material is received exactly when the stock level reaches the minimum stock level of 0).

In our analytical model, two control systems, VMI and RMI, on the supply chain inventory are compared. In RMI, it is assumed that, firstly, the retailer him/herself is responsible for his/her inventory. Second, the retailer ordering policy is based on the concept of Economic Order Quantity (EOQ), and lastly, because of the infinite rate of production, the supplier inventory costs consist of only ordering costs. It is to be noted that EOQ determines the optimum order quantity that a company should hold in its inventory, given a set of parameters, including demand rate, setup cost and storing cost, to minimize inventory costs. So, the EOQ assumption for RMI (in this study) is sensible and makes the results more reliable. The full equation is $\sqrt{\frac{2RA}{H}}$, where R is demand or consumption rate, A is setup or ordering cost and H is the storing cost of goods per period. However, in VMI, the supplier is responsible for the inventory control of its retailer(s), and the supplier pays their inventory cost. When there are multiple retailers, their cycle times (that is determined by the supplier) are equal in a way that minimizes the total inventory cost of the supply chain. The main parameters of this study are as follows:

A_s : Ordering cost of the supplier;

A_i : Ordering cost of the retailer, $i; i = 1, ..., n$;

T_i : Ordering cycle time of the retailer, i;

T_{Wi} : Ordering cycle time of the retailer i according to EOQ;

Q_{wi} : Ordering quantity of the retailer i according to EOQ;

T : Ordering cycle time in VMI state;

R_i : Rate of consumption of the retailer, i;

H_i : Storing cost of a good per period in warehouse of the retailer, i;

K_0 : Total inventory cost of supply chain in RMI state;

K_1 : Total inventory cost of supply chain in VMI state;

KBi_0 : Inventory cost of the retailer i in RMI state;

KS_0 : Inventory cost of the supplier in RMI state;

KBi_1 : Inventory cost of the retailer i in VMI state;

KS_1 : Inventory cost of the supplier in VMI state.

The rest of this part is organized in 3 sub-parts. Firstly, single-retailer supply chains are considered and, secondly, n-retailer ones are discussed. After coming to good conclusions about the parameters of the model in sub-part 2, in the third sub-part, to make the results more tangible, a detailed analysis is done on two-retailer supply chains.

3.1. Single-retailer supply chains

Independent of retailer numbers and whether the system is VMI or RMI, the total inventory cost of the supply chain is a summation of the inventory expenses of the retailers and supplier, which, in a single-retailer state, are as Eqs. (1) and (2), respectively;

$$KB1 = \frac{A_1}{T_1} + \frac{R_1 H_1 T_1}{2}, \tag{1}$$

$$KS = \frac{A_S}{T_1}. \tag{2}$$

In RMI, the retailer determines the ordering cycle time by Eq. (3):

$$T_1 = T_{W1} = \frac{Q_{W1}}{R} = \sqrt{\frac{2A_1}{R_1 H_1}}. \tag{3}$$

So, K_0^* is calculated, as shown by Eq. (4):

$$K_0^* = (A_S + A_1)\sqrt{\frac{R_1 H_1}{2A_1}} + \sqrt{\frac{R_1 H_1 A_1}{2}}. \tag{4}$$

In VMI, the supplier determines the ordering cycle time in such a way as to minimize the total inventory cost of the supply chain (Eq. (5)) that is incurred completely by it.

$$K_1 = \frac{A_S + A_1}{T} + \frac{R_1 H_1 T}{2}. \tag{5}$$

After Eq. (5), we come to Eq. (6), from which the optimum cycle time and then the optimum inventory cost are achieved as Eqs. (7) and (8), respectively:

$$\frac{\partial K_1}{\partial T} = -\frac{A_S + A_1}{T^2} + \frac{R_1 H_1}{2} = 0, \tag{6}$$

$$T^* = \sqrt{\frac{2\left(A_S + A_1\right)}{R_1 H_1}}, \tag{7}$$

$$K_1^* = \sqrt{2 R_1 H_1 \left(A_S + A_1\right)}. \tag{8}$$

By a simple comparison between Eq. (4) and Eq. (8), it can be proved that always $K_1^* \leq K_0^*$. In other words, in two-echelon, single-supplier, single-retailer supply chains, RMI is always more expensive than VMI.

3.2. n-retailer supply chains

Here, an analysis, like the previous sub-part but with n retailers, is undertaken. On the basis of what has been discussed, in the traditional state, when there are n retailers, for the total inventory cost of the supply chain, we have Eq. (9):

$$K_0 = \sum_{i=1}^{n} \frac{A_S + A_i}{T_i} + \sum_{i=1}^{n} \frac{R_i H_i T_i}{2}, \tag{9}$$

and, since $T_i = T_{iW} = \sqrt{\frac{2A_i}{R_i H_i}}$, K_0^* is calculated by Eq. (10):

$$K_0^* = A_S \sum_{i=1}^{n} \sqrt{\frac{R_i H_i}{2A_i}} + \sum_{i=1}^{n} \sqrt{2 R_i H_i A_i}. \tag{10}$$

In the VMI state, the total cost is obtained by Eq. (11):

$$K_1=\frac{A_S+\sum_{i=1}^{n}A_i}{T}+\frac{T}{2}\sum_{i=1}^{n}R_iH_i, \tag{11}$$

and, since the supplier wants the minimum of K_1, Eq. (12) is necessary to calculate the optimum ordering cycle time, T^*, as shown by Eq. (13):

$$\frac{\partial K_1}{\partial T}=-\frac{A_S+\sum_{i=1}^{n}A_i}{T^2}+\frac{1}{2}\sum_{i=1}^{n}R_iH_i=0, \tag{12}$$

$$T^*=\sqrt{\frac{2\left(A_S+\sum_{i=1}^{n}A_i\right)}{\sum_{i=1}^{n}R_iH_i}}. \tag{13}$$

Eq. (14) is the result of substituting T^* of Eq. (13) into Eq. (11), as follows:

$$K_1^*=\frac{A_S+\sum_{i=1}^{n}A_i}{\sqrt{\frac{2\left(A_S+\sum_{i=1}^{n}A_i\right)}{\sum_{i=1}^{n}R_iH_i}}}+\frac{\sqrt{\frac{2\left(A_S+\sum_{i=1}^{n}A_i\right)}{\sum_{i=1}^{n}R_iH_i}}}{2}$$

$$\times\sum_{i=1}^{n}R_iH_i=\sqrt{2\left(A_S+\sum_{i=1}^{n}A_i\right)\times\sum_{i=1}^{n}R_iH_i}. \tag{14}$$

After calculation of K_0^* and K_1^*, as discussed in the above lines, investigation of the conditions in which $K_1^*\leq K_0^*$ composes the remainder of this sub-part. First, $(K_1^*)^2$ and $(K_0^*)^2$ are calculated as Eqs. (15) and (16), respectively:

$$(K_1^*)^2=2(A_S+\sum_{i=1}^{n}A_i)\times\sum_{i=1}^{n}R_iH_i, \tag{15}$$

$$(K_0^*)^2=A_S^2(\sum_{i=1}^{n}\sqrt{\frac{R_iH_i}{2A_i}})^2+(\sum_{i=1}^{n}\sqrt{2R_iH_iA_i})^2$$

$$+2A_S(\sum_{i=1}^{n}\sqrt{\frac{R_iH_i}{2A_i}})$$

$$(\sum_{i=1}^{n}\sqrt{2R_iH_iA_i}), \tag{16}$$

if $(K_1^*)^2\leq(K_0^*)^2$, there are Inequalities (17) and then (18):

$$2A_S(\sum_{i=1}^{n}R_iH_i)+2(\sum_{i=1}^{n}A_i)(\sum_{i=1}^{n}R_iH_i)$$

$$\leq A_S^2(\sum_{i=1}^{n}\sqrt{\frac{R_iH_i}{2A_i}})^2+(\sum_{i=1}^{n}\sqrt{2R_iH_iA_i})^2$$

$$+2A_S(\sum_{i=1}^{n}\sqrt{\frac{R_iH_i}{2A_i}})(\sum_{i=1}^{n}\sqrt{2R_iH_iA_i}), \tag{17}$$

$$A_S^2(\sum_{i=1}^{n}\sqrt{\frac{R_iH_i}{2A_i}})^2+A_S(2(\sum_{i=1}^{n}\sqrt{\frac{R_iH_i}{2A_i}})(\sum_{i=1}^{n}\sqrt{2R_iH_iA_i})$$

$$-2\sum_{i=1}^{n}R_iH_i)+((\sum_{i=1}^{n}\sqrt{2R_iH_iA_i})^2$$

$$-2(\sum_{i=1}^{n}A_i)(\sum_{i=1}^{n}R_iH_i))\geq 0. \tag{18}$$

Lemma

If $a>0$, the multinomial ax^2+bx+c is positive under the following conditions:

$$\begin{cases}x\leq\frac{-b-\sqrt{b^2-4ac}}{2a}\text{ or }x\geq\frac{-b+\sqrt{b^2-4ac}}{2a} & \text{if } b^2-4ac\geq 0.\\ \text{always} & \text{if } b^2-4ac<0.\end{cases}$$

On the basis of the lemma, b^2-4ac for Inequality (18) is calculated by Relation (19):

$$4\left(\sum_{i=1}^{n}R_iH_i\right)^2+8\left(\sum_{i=1}^{n}\sqrt{\frac{R_iH_i}{2A_i}}\right)\left(\sum_{i=1}^{n}R_iH_i\right)$$

$$\times\left(\left(\sum_{i=1}^{n}\sqrt{\frac{R_iH_i}{2A_i}}\right)\left(\sum_{i=1}^{n}A_i\right)\right.$$

$$\left.-\sum_{i=1}^{n}\sqrt{2R_iH_iA_i}\right). \tag{19}$$

Obviously, Relation (19) is not always negative (the second condition of the lemma) and is positive when Inequality (20) is met:

$$\left(\sum_{i=1}^{n}\sqrt{\frac{R_iH_i}{2A_i}}\right)\left(\sum_{i=1}^{n}A_i\right)\geq\sum_{i=1}^{n}\sqrt{2R_iH_iA_i}. \tag{20}$$

Therefore, since Inequality (18) is not always met, it can be concluded that if $n\geq 2$, RMI sometimes is the cheaper system. However, according to the lemma, VMI is still the superior choice if Inequality (21) or (22), shown in Box I, is met. According to Inequality

$$AS \geq \frac{2(\sum_{i=1}^{n} R_iH_i - (\sum_{i=1}^{n} \sqrt{\frac{R_iH_i}{2A_i}})(\sum_{i=1}^{n} \sqrt{2R_iH_iA_i}))}{2(\sum_{i=1}^{n} \sqrt{\frac{R_iH_i}{2A_i}})^2}$$

$$+\frac{\sqrt{(2(\sum_{i=1}^{n} \sqrt{\frac{R_iH_i}{2A_i}})(\sum_{i=1}^{n} \sqrt{2R_iH_iA_i}) - 2\sum_{i=1}^{n} R_iH_i)^2 - 4(\sum_{i=1}^{n} \sqrt{\frac{R_iH_i}{2A_i}})^2((\sum_{i=1}^{n} \sqrt{2R_iH_iA_i})^2 - 2(\sum_{i=1}^{n} A_i)(\sum_{i=1}^{n} R_iH_i))}}{2(\sum_{i=1}^{n} \sqrt{\frac{R_iH_i}{2A_i}})^2}. \tag{21}$$

$$AS \leq \frac{2(\sum_{i=1}^{n} R_iH_i - (\sum_{i=1}^{n} \sqrt{\frac{R_iH_i}{2A_i}})(\sum_{i=1}^{n} \sqrt{2R_iH_iA_i}))}{2(\sum_{i=1}^{n} \sqrt{\frac{R_iH_i}{2A_i}})^2}$$

$$-\frac{\sqrt{(2(\sum_{i=1}^{n} \sqrt{\frac{R_iH_i}{2A_i}})(\sum_{i=1}^{n} \sqrt{2R_iH_iA_i}) - 2\sum_{i=1}^{n} R_iH_i)^2 - 4(\sum_{i=1}^{n} \sqrt{\frac{R_iH_i}{2A_i}})^2((\sum_{i=1}^{n} \sqrt{2R_iH_iA_i})^2 - 2(\sum_{i=1}^{n} A_i)(\sum_{i=1}^{n} R_iH_i))}}{2(\sum_{i=1}^{n} \sqrt{\frac{R_iH_i}{2A_i}})^2}. \tag{22}$$

Box I

$$A_S \geq \frac{-2\sqrt{R_1H_1R_2H_2}\left(\sqrt{\frac{A_2}{A_1}} + \sqrt{\frac{A_1}{A_2}}\right) + 2\sqrt{R_1H_1R_2H_2\left(\frac{A_1}{A_2} + \frac{A_2}{A_1}\right) + \frac{R_1^2H_1^2A_2}{A_1} + \frac{R_2^2H_2^2A_1}{A_2}}}{\frac{R_1H_1}{A_1} + \frac{R_2H_2}{A_2} + 2\sqrt{\frac{R_1H_1R_2H_2}{A_1A_2}}}. \tag{24}$$

Box II

(23), the bound expressed by Inequality (22) is always negative and, so, infeasible in the real world:

$$\left(\sum_{i=1}^{n} R_iH_i - \left(\sum_{i=1}^{n} \sqrt{\frac{R_iH_i}{2A_i}}\right)\left(\sum_{i=1}^{n} \sqrt{2R_iH_iA_i}\right)\right) \leq 0. \tag{23}$$

although we come to the single inequality of (21) and the comparison is finished, but it is impossible to infer any managerial insight about VMI and RMI. For example, when $n = 1$, it was concluded that always VMI is superior, but, what about multiple-retailer supply chains?! How often and under what conditions is VMI better?! To answer such questions, a deeper and more practical analysis (as an example) is done in the following sub-part.

3.3. An investigation of two-retailer supply chains

On the basis of Inequality (21), when there are two retailers, VMI is better than RMI if the situation of Inequality (24), shown in Box II, is met. The inequality configuration implies that unlike single-retailer supply chains, there are situations in which RMI has better performance than VMI.

3.3.1. Performance assessment system

The structure of the assessment system, to develop 10000 different samples that cover all the probable situations of the real world, even the very rare ones, is on the basis of six parameters of R_1, R_2, H_1, H_2, A_1 and A_2, as shown by Table 1. For example, there are samples in which demand, storing cost and ordering cost of one retailer is 2000 times bigger than the other retailer. It is nearly impossible in the real world that two retailers who work with the same supplier have such immense differences. Naturally, consideration of these unusual situations increases the reliability of our results and conclusions.

Hereafter, in this paper, the right hand side of Inequality (24) is called the Critical Amount (CA), i.e. VMI has a better performance if the ordering cost of the supplier is greater than CA. So, after calculation

Table 1. The parameters that are building blocks of 10000 samples.

Parameters	Amounts
R_1	50, 500, 5000, 50000, 500000
R_2	100, 1000, 10000, 100000, 1000000
H_1	0.5, 5, 50, 500, 5000
H_1	1, 10, 100, 1000, 10000
A_1	10, 100, 1000, 10000
A_2	20, 200, 2000, 20000

Table 2. The system of performance assessment.

Condition	**Assessment**
CA $< \frac{1}{2}(\text{Max}(A_1, A_2))$	Very good
$\frac{1}{2}(\text{Max}(A_1, A_2)) \leq$ CA $<$ Max(A_1, A_2)	Good
Max$(A_1, A_2) \leq$ CA $< \frac{3}{2}(\text{Max}(A_1, A_2))$	Average
$\frac{3}{2}(\text{Max}(A_1, A_2)) \leq$ CA $< 2(\text{Max}(A_1, A_2))$	Bad
CA $\geq 2(\text{Max}(A_1, A_2))$	Very bad

of CA for each sample, and the bigger amount it gets, VMI will be considered less practical, and vice-versa. In other words, there is an inverse relation between CA and VMI effectiveness. But, when is CA considered big and when small?

On the basis of the fact that the ordering cost of a supplier is usually greater than its retailer [61], a system is designed to assess the magnitude of CA. The system assesses VMI in 5 levels of very good, good, average, bad and very bad according to Table 2.

Since, in VMI, the supplier coordinates the inventory decisions of the chain and pays for the associated costs, the more similar the retailers of the chain are, the more economic VMI is. So, if, among the 10000 generated samples, we consider the ones in which $\frac{R_2}{R_1} = 20000$, to conclude about VMI efficiency versus RMI, we have been very strict regarding VMI, as under the mentioned conditions, the dissimilarity between the retailers from the perspective of their demands is maximum, which can cause an inappropriate situation for implementing VMI. Among 400 samples, in which $\frac{R_2}{R_1} = 20000$, 50 samples are selected randomly, and then, the associated CA is calculated to come to the following results, according to our performance assessment system: 52% very good, 28% good, 20% average and zero for the Bad and Vary Bad levels. In the state of Very Good for many of the samples, $\frac{A_S}{Max\,(A_1, A_2)}$ is far less than 0.5, which is really promising for the new system.

3.3.2. *The trend of CA*

After a comprehensive and detailed review of the generated samples, it is revealed that CA is directly a function of only three factors of A_1, A_2 and $\frac{R_2 H_2}{R_1 H_1}$. For

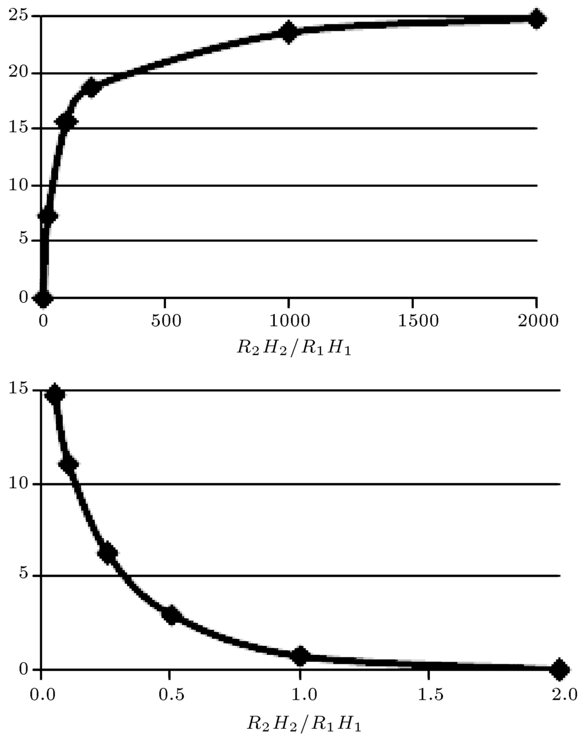

Figure 1. The trend of CA for different amounts of $\frac{R_2 H_2}{R_1 H_1}$ when $A_1 = 10$ and $A_2 = 20$.

example, for a particular amount of $\frac{R_2 H_2}{R_1 H_1}$ (independent of the individual amounts of R_1, R_2, H_1 and H_2), if A_1 and A_2 are multiplied by n, the associated CA will also multiplied by n. So, to explore the trends of CA under different situations, it seems wise to categorize them on the basis of different combinations of A_1 and A_2 (according to Table 1), which results in 16 categories of charts. In the first category $A_1 = 10$, $A_2 = 20$, in the second category $A_1 = 10$, $A_2 = 200$, and so on. In each category, two curves are devised, while, for both of them, the x-axis and y-axis cover $\frac{R_2 H_2}{R_1 H_1}$ and CA, respectively. In the upper curve, $\frac{R_2 H_2}{R_1 H_1} \geq \frac{A_2}{A_1}$ and, in the other, $\frac{R_2 H_2}{R_1 H_1} \leq \frac{A_2}{A_1}$. As a sample, Figure 1 shows one of these charts in which its behavior is very similar to the others.

The figure also implies the amount of $\left(\frac{R_2 H_2}{R_1 H_1}\right)^*$ in which CA gets its minimum amount, zero. Obviously, if CA $= 0$, VMI is absolutely the better choice. After a simple survey on the charts, it is understood that $\left(\frac{R_2 H_2}{R_1 H_1}\right)^* = \frac{A_2}{A_1}$; but, can it be proved mathematically? Or can it be justified conceptually why in the implementation of VMI, the best situation is achieved if $\frac{R_2 H_2}{R_1 H_1} = \frac{A_2}{A_1}$? The mathematical proof is as follows: First, the CA is divided by $R_1 H_1$ to get Eq. (25) shown in Box III. Second, on the basis of Eq. (25), the equation of $\frac{\partial (CA)}{\partial \left(\frac{R_2 H_2}{R_1 H_1}\right)} = 0$ is achieved. Third, and finally after dealing with some computational complexities, it will be proved.

$$\mathrm{CA}=\frac{-2\left(\sqrt{\frac{A_2}{A_1}}+\sqrt{\frac{A_1}{A_2}}\right)\sqrt{\frac{R_2H_2}{R_1H_1}}+\sqrt{\left(\frac{A_1}{A_2}\right)\frac{R_2^2H_2^2}{R_1^2H_2^2}+\left(\frac{A_1}{A_2}+\frac{A_2}{A_1}\right)\frac{R_2H_2}{R_1H_1}+\frac{A_2}{A_1}}}{\left(\frac{1}{A_2}\right)\frac{R_2H_2}{R_1H_1}+\frac{2}{\sqrt{A_1A_2}}\sqrt{\frac{R_2H_2}{R_1H_1}}+\frac{1}{A_1}} \tag{25}$$

Box III

For the conceptual justification, it should be noted that in a traditional inventory control system, we have Eq. (26):

$$\frac{T_2}{T_1}=\sqrt{\frac{R_1H_1}{R_2H_2}}\sqrt{\frac{A_2}{A_1}}. \tag{26}$$

So, if $\frac{R_2H_2}{R_1H_1}=\frac{A_2}{A_1}$, then, $\frac{R_2H_2}{R_1H_1}\times\frac{A_1}{A_2}=1$, and, then, $\frac{T_2}{T_1}=1$. Obviously, when the ordering cycle times of the retailers are equal, they are, in fact, one soul in two bodies and, apparently, there is an excellent opportunity for improvement of the supply chain; a fact that VMI discovers and takes much advantage from.

4. Conclusions

This study can be summarized in three terms of "traditional supply chain", "new supply chain" and "inventory control". While the main difference between traditional and new supply chains is their system of inventory control, the main contribution of this paper is a cost-based comparison between them, with simplifying assumptions of 1) constant rate of consumption, 2) infinite rate of production, 3) infinite pace of transportation, 4) constant price of goods and 5) unallowable state of shortage. It is to be noted that, despite the fact that most VMI advantages are achieved over the long term, here, the focus is on short term economic advantages.

The comparisons of the study can be divided into three main cases, on the basis of number of retailers in the supply chain, with the following results:

1. For single-retailer supply chains, VMI always performs better.
2. For two-retailer supply chains, the results are not absolute, like a single-retailer state, and the superiority of VMI is dependent on the supplier ordering costs and CA, in such a way that, if $A_s <$ CA, VMI will be proposed and, otherwise, not. In this part, to make the results more tangible, 10000 samples are produced, solved and analyzed with an innovative system of performance assessment. According to the system, VMI, in 52%, 28% and 20% of cases performs very well, well, and ordinarily, respectively, with no bad or very bad performances. Moreover, it is seen that VMI can cause a considerable reduction in inventory costs of the supply chain.
3. For n retailers, the general formula to calculate CA is developed.

Finally, it is concluded that with any number of retailers in the supply chain, two factors make the advantages of VMI more powerful. The first factor includes more coordination between retailers from the perspective of inventory control parameters (this is exactly the reason for the fact that in two-retailer supply chains, CA is a function of $\frac{R_2H_2}{R_1H_1}$, which is a gauge to measure the level of coordination between the retailers), and the second factor is the large amounts of A_s.

The current study can be continued, for example, as a feasibility study of VMI for higher numbers of retailers, like 3, or analyzing the problem after removing one of the simplifying assumptions, such as certainty in demand, constant price, constant rate of consumption and so on.

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