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Estimation of naturally fractured oil reservoir properties using the material balance method

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Abstract. In fractured reservoirs, a large variation of permeability due to the presence of fractures leads to changes in the production mechanism compared to conventional reservoirs. Hence, an appropriate model with the ability to describe the reservoir properly can provide a more confident prediction of its future performance. One of the features of a representative model is the number and height of the matrix blocks. The determination of these two parameters is one of the decisive steps in the calculation of an accurate amount of oil production from these reservoirs. In fact, matrix height shows its effect as a gravity force, which is one of the driving mechanisms. If the matrix height is less than the threshold height, it will have a significant influence on production. The aim of this study is, therefore, to obtain mathematical relations that are able to estimate the matrix height from material balance analysis. In this study: (a) The Havlena and Odeh straight line form of the material balance equation has been extended to analyze the behavior of naturally fractured reservoirs, and (b) Equations that can be used to estimate the matrix height are obtained for the Warren-Root and Kazemi models.

1. Introduction

Fractured reservoirs constitute a considerable percentage of hydrocarbon reservoirs in the world; it is estimated that about half of the world's oil reserves are in fractured reservoirs. A well-known oil fractured area of the world is located in the Southwest of Iran, and a substantial percentage of Iran's oil reserves are related to the fields located in this region [1]. In naturally fractured reservoirs, unlike conventional reservoirs, large variations of permeability due to the presence of fractures lead to changes in the production mechanism. The dimensions of the matrix blocks in fractured reservoirs are one of the most important factors. Yet, in most cases, it is the least known parameter and its value is frequently estimated by trial-and-error, using history matching. The aim of this study is to propose a method to find the value of these parameters from a more reliable method. Since the fractured reservoirs have more complexities [2-4], researchers use simplified models to describe these types of reservoir. The most popular of these models are Warren-Root [5] and Kazemi [6], which are both employed in this study for the description of fractured reservoirs. These two models are used to describe the reservoir structure, and the Material Balance Equation (MBE) is used to calculate the initial oil in place [7-10]. Many researchers [11-16] have conducted studies and developed various models which combine the sugar-cube model and the MBE for fractured reservoirs, as well.

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2. Material balance equation

The material balance equation is a basic method for predicting reservoir performance. This method can be used to perform the following calculations:

1) Estimation of volume in place of the initial hydrocarbons in the reservoir.
2) Prediction of future reservoir performance.
3) Estimation of ultimate recovery of hydrocarbons.

The concept of the material balance equation was first introduced by Schilthuis in 1936 [17]. In its simplest form, the general material balance equation for conventional and homogeneous reservoirs can be expressed as follows [18]:

$$\begin{aligned} &N(B_t - B_{ti}) + \frac{mB_{ti}}{B_{gi}}(B_g - B_{gi}) \\ &\quad + (1+m)B_{ti}C_e\Delta P + W_e \\ &\quad = N_p\left[B_t + (R_p - R_{si})\,B_g\right] \\ &\quad + B_wW_p - G_{inj}B_{g,inj} - W_{inj}B_{w,inj}, \end{aligned} \tag{1}$$

where:

$$B_t = B_o + (R_{si} - R_s)B_g, \tag{2}$$

$$C_e = \frac{C_wS_{wi} + C_r}{1 - S_{wi}}, \tag{3}$$

and N is the initial oil in place, N_p is the cumulative produced oil, R_p is the cumulative produced gas-oil ratio, C_r is the average rock compressibility, C_w is water compressibility and C_e is effective compressibility. Definitions of the remaining variables are given in the nomenclature.

The basic assumptions in the Material Balance Equation (MBE) are:

1) The temperature is constant and porosity is also uniform.
2) At each time of reservoir production, it is assumed that the pressure is constant everywhere in the reservoir. Another assumption, which is derived from the pressure balance assumption, is the uniformity of reservoir fluid properties all over the reservoir. Thus, any difference in pressure in different situations of the reservoir is assumed to be negligible.
3) Assumption of constant reservoir volume: in the material balance calculations, except under conditions of reservoir rock and water expansion and the conditions of the inlet water of the reservoir that are considered in the material balance equation, it is assumed that the volume is constant at different times.
4) Accurate production data.
5) Water is only in a liquid phase.

Eq. (1) is only applicable for conventional oil reservoirs (those without fractures) and is unable to describe fractured reservoirs [18].

Over the past few years, some research has been conducted to obtain the material balance equation for Naturally Fractured Reservoirs (NFR) and several models have been presented. One of the most practical forms of the MBE for NFR, which was proposed by Penuela et al. [11-12], can be written as follows:

$$\begin{aligned} &N_p\left[B_o + (R_p - R_s)\,B_g\right] + B_wW_p \\ &\quad = N_m\Bigg[B_o - B_{oi} + (R_{si} - R_s)B_g \\ &\quad + \left(\frac{C_wS_{wmi} + C_{pp,m}}{1 - S_{wmi}}\right)\Delta pB_{oi}\Bigg] \\ &\quad + N_f\Bigg[B_o - B_{oi} + (R_{si} - R_s)B_g \\ &\quad + \left(\frac{C_wS_{wfi} + C_{pp,f}}{1 - S_{wfi}}\right)\Delta pB_{oi}\Bigg], \end{aligned} \tag{4}$$

where N_m is the oil initially in place in the matrix and N_f is the oil initially in place in the fracture, $C_{pp,m}$ is matrix isothermal pore compressibility, and $C_{pp,f}$ is fracture isothermal pore compressibility.

This equation, in addition to the aforementioned assumptions for the material balance equation in conventional reservoirs, includes the following additional assumptions:

1) The reservoir includes four components: oil, gas, water and naturally fractured rock.
2) The reservoir includes four phases: oil, gas, water and naturally fractured rock.
3) The oil component only exists in the oil phase and does not exist in the water, gas or rock phases.
4) The gas component exists free in the gas-phase and dissolved in the oil-phase.
5) The water component only exists in the water-phase.
6) The rock component exists only in the rock-phase.
7) The rock-phase is composed of two porous media which are in hydraulic communication: the secondary porosity (fractured system) and the primary porosity (matrix system).
8) The fracture and porous matrix are compressible.
9) There is no water influx, and water production is negligible.

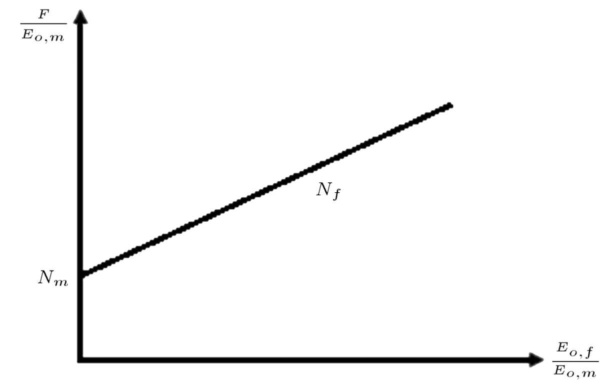


Figure 1. Material balance diagnostic curve for NFR [11].

10) There is no fluid injection (water and/or gas) into the reservoir.

Using the suggested equation of Penuela et al., we can obtain the oil initially in place for fractured reservoirs This equation is solved using the graphical method and the concept of a straight line, as expressed by Odeh and Havlena [19,20]:

$$F = N_p \left[B_o + (R_p - R_s) B_g\right] + B_w W_p, \tag{5}$$

$$E_{o,m} = B_o - B_{oi} + (R_{si} - R_s)B_g + \left(\frac{C_w S_{wmi} + C_{pp,m}}{1 - S_{wmi}}\right) \Delta p B_{oi}, \tag{6}$$

$$E_{o,f} = B_o - B_{oi} + (R_{si} - R_s)B_g + \left(\frac{C_w S_{wfi} + C_{pp,f}}{1 - S_{wfi}}\right) \Delta p B_{oi}, \tag{7}$$

$$F = N_m E_{o,m} + N_f E_{o,f}, \tag{8}$$

$$\frac{F}{E_{o,m}} = N_m + N_f \frac{E_{o,f}}{E_{o,m}}, \tag{9}$$

where F is the net fluid withdrawal, $E_{o,m}$ is the net expansion of the original oil-phase in the matrix, and $E_{o,f}$ is the net expansion of the original oil-phase in the fracture.

According to Eq. (9), a plot of $\frac{F}{E_{o,m}}$ versus $\frac{E_{o,f}}{E_{o,m}}$, on a Cartesian graph, should yield a straight line of slope N_f and intercept N_m, as shown in Figure 1.

2.1. *The material balance equation for undersaturated reservoirs as a function of the storage capacity ratio*

Recalling the general definition of the storage capacity ratio (Warren and Root), under initial reservoir conditions for an undersaturated reservoir:

$$\omega = \frac{(\varphi c_t)_f}{(\varphi c_t)_f + (\varphi c_t)_m} = \frac{(\varphi c_t)_f}{(\varphi c_t)_{f+m}}. \tag{10}$$

By substituting this equation into Eq. (8) and rearranging, the following equation will be obtained:

$$F = N\left((1-\omega)E_{o,m} + \omega E_{o,f}\right). \tag{11}$$

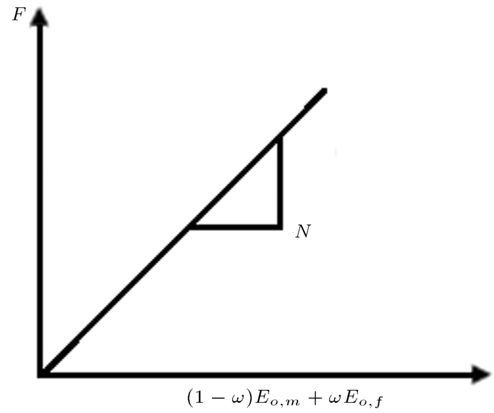


Figure 2. Material balance as a function of storage capacity ratio for a volumetric undersaturated NFR [13].

Therefore, a plot of F versus $(1-\omega)E_{o,m} + \omega E_{o,f}$ would yield a straight line passing through the origin with slope, as represented in Figure 2 [13].

The graphical method proposed in Eq. (11) has some advantages over the one proposed by Penuela et al. (Eq. (9)): (a) it requires less production data and (b) only one regression parameter is needed to obtain good estimates of the total original hydrocarbon in place [13].

The initial oil in place in the fractures can be estimated from:

$$N_f = \omega N. \tag{12}$$

Taking into account that:

$$N = N_f + N_m, \tag{13}$$

$$N_m = (1-\omega)N, \tag{14}$$

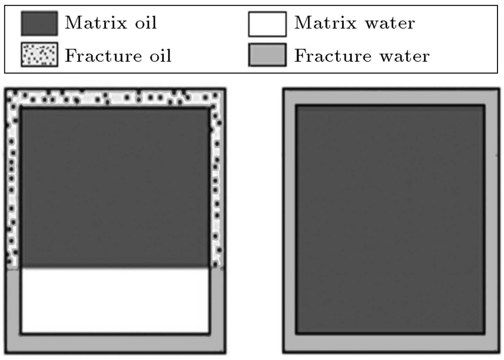
it is noteworthy that the value of ω can be found by two methods: (a) with well logging and core analysis data [1] and (b) well testing methods [21]. In this study, the value of ω is calculated using a well testing approach.

3. Acquiring other properties of fractured reservoirs from the material balance equation

The material balance equation is actually a volumetric balance, since the volume of the reservoir (which is defined by its original limits) is assumed constant. Therefore, the sum of volume changes of oil, gas, water and rock in the reservoir should be equal to zero [18, 22-24]. Hence, using this principle, this method can also predict the future performance of the reservoir. Because the size of the matrix blocks in the simulation of the fractured reservoirs (Warren-Root model and Kazemi model) is of great importance, the focus of this study is to develop a new method to find this parameter from a reservoir material balance analysis.

3.1. The significance of the size of the matrix blocks (matrix height)

In the study of fractured reservoirs and simulation by engineering software, Warren-Root and Kazemi models are more applicable than others. Thus, in this part, we state the importance of matrix height based on the Warren-Root model. A matrix is a volumetric unit that is completely surrounded by the fractured network around itself without any communication with other matrix blocks. Hence, the displacement depends only on interactions between the matrix and fracture fluids. The process of displacement in a fractured reservoir occurs when the matrix saturated by oil is partially or completely surrounded by a fluid such as water or gas. Figure 3 shows the time at which the matrix is partially or totally in the water.



(a) (b)

Figure 3. a) Matrix is partially immersed in water. b) Matrix is fully immersed in water.

A reservoir that is made up of uniform geometric matrix blocks is invaded by the expansion of a gas cap from the top and water from the bottom. One of the factors that affect the displacement is the matrix height, so that if the matrix height is lower than some threshold height (h_{Th}), it exerts a significant effect on production (displacement). This effect is so severe that production does not occur during the gravity drainage process, and is significantly reduced during the imbibition process. In fact, the matrix height shows its effects as a gravity force, which is one of two production forces in the process of gravity drainage and imbibitions; capillary force being the other one. In gravity drainage, the gravity force is the only production force, and in the imbibition process, it is one of the two production forces. This gravity force contributes to production during gravity drainage only if the matrix height is larger than h_{Th} ($h_{Th} \prec h_b$). During the imbibition process, it has a different role in production. For instance, if the matrix is fully saturated with water, the force will always increase production. Therefore, if the matrix height is greater than h_{Th}, the force of gravity is greater and production increases proportionally. However, for a matrix that is partially saturated with water, the force of gravity may have a positive or negative effect on production [1,25].

Therefore, the calculation of the matrix height is of significant importance. Figure 4 presents the effect of matrix height on imbibition and the gravity drainage process.

3.1.1. The effect of gravity force on oil recovery during the imbibition process

Figure 5 shows the effect of gravity on oil recovery for the matrix which is fully immersed in the water. As can be observed, the force of gravity has a significant

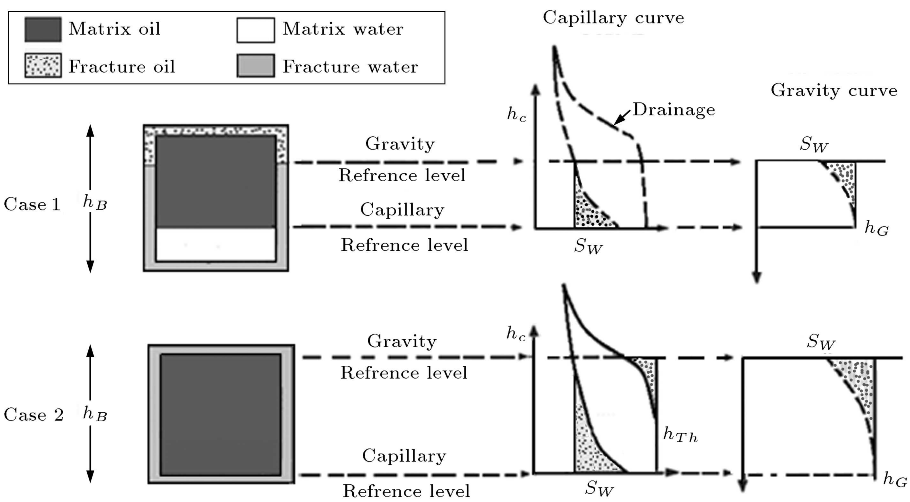


Figure 4. Effect of matrix height on imbibition and drainage processes: Case 1) matrix is partially immersed in water; and Case 2) matrix is fully immersed in water [1].

impact on the amount of oil that is recovered, which further highlights the importance of calculating the matrix height.

3.2. The proposed method

In recent years, numerous studies have been conducted on fractured reservoir simulations (see, for example [26-31]). In this study, using the original oil in place, N, calculated from the material balance equation, and other concepts of reservoir engineering, we obtain the relations that estimate matrix height for the models of Warren-Root and Kazemi [5,6].

3.2.1. Warren-Root model

This model is one of the most comprehensive models for fractured reservoirs and its application has been largely developed. In this model, the reservoir rock is divided into blocks of rock (matrix) by two series of parallel plates perpendicular to each other (fractures) surrounding the matrix blocks like a network. This model assumes that the reservoir consists of n matrix blocks with equal length, width and height a and fractures with width equal to b, according to Figure 6. Using the following definitions:

$$\nu_f = \frac{\text{Pore volume in the fractures}}{\text{Total volume of the fractures}},$$

$$\varphi_f = \frac{\text{Total volume of the fractures}}{\text{Reservoir volume}},$$

$$\nu_m = \frac{\text{Total volume of the matrix blocks}}{\text{Reservoir volume}},$$

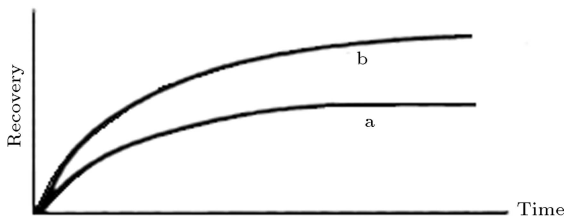


Figure 5. a) Oil recovery, counting only capillary force. b) Oil recovery, including both capillary and gravity forces [1].

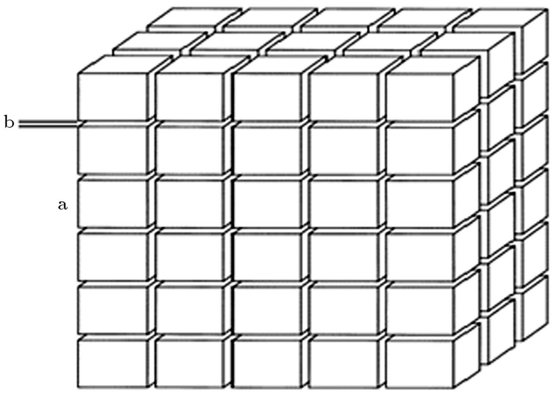


Figure 6. Warren-Root schematic model [5].

$$\varphi_m = \frac{\text{Pore volume in the matrix blocks}}{\text{Total volume of the matrix blocks}},$$

we can write:

$$\varphi_t = \varphi_f \times \nu_f + \varphi_m \times \nu_m. \tag{15}$$

According to assumptions $\nu_m \simeq 1$, $\nu_f \simeq 1$, Eq. (15) is modified as follows [32]:

$$\varphi_t = \varphi_f + \varphi_m. \tag{16}$$

Hence, the total amount of oil initially in place N can be demonstrated as follows:

$$N = \frac{\left((\varphi_t \times V_b) \times \left(1 - S_{wi(f,m)}\right)\right)}{5.615}, \tag{17}$$

where V_b is the bulk volume of the reservoir, φ_t is the average porosity, and $S_{wi(f,m)}$ is the average initial water saturation in the NFR system. In the Warren and Root model V_b is defined as follows:

$$V_b = n(a+b)^3, \tag{18}$$

where a is the cube's length, b is the fracture height and n is the number of matrix blocks in the reservoir. So, by combining Eqs. (17) and (18):

$$N = \frac{n(a+b)^3 \times \varphi_t \times \left(1 - S_{wi(f,m)}\right)}{5.615}. \tag{19}$$

The original oil in place in the fractures in the whole reservoir, N_f, can be found from:

$$N_f = \frac{(V_b - V_m) \times (1 - S_{wfi})}{5.615}, \tag{20}$$

where V_M is the total volume of matrix blocks and S_{wfi} is the initial water saturation in the fractured system. In the Warren and Root model, V_m is defined as follows:

$$V_m = na^3. \tag{21}$$

By combining Eqs. (18), (20) and (21) the following is obtained:

$$N_f = \frac{n\left((a+b)^3 - a^3\right) \times (1 - S_{wfi})}{5.615}$$
$$= \frac{n\left(3a^2b + 3ab^2 + b^3\right) \times (1 - S_{wfi})}{5.615}, \tag{22}$$

and by combining Eqs. (19) and (22), it can be shown that:

$$\frac{N}{N_f} = \frac{n(a+b)^3 \times \varphi_t \times \left(1 - S_{wi(f,m)}\right)}{n\left(3a^2b + 3ab^2 + b^3\right) \times (1 - S_{wfi})}, \tag{23}$$

which results in the following equation:

$$a^3 + \left(3b - \frac{3Nb\,(1 - S_{wfi})}{\varphi_t N_f \left(1 - S_{wi(f,m)}\right)}\right) a^2$$

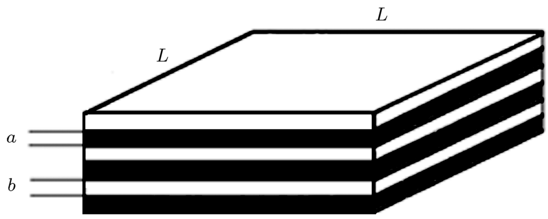


Figure 7. Schematic of Kazemi model [6].

$$+\left(3b^2-\frac{3Nb^2\left(1-S_{wfi}\right)}{\varphi_t N_f\left(1-S_{wi(f,m)}\right)}\right)a$$

$$+\left(b^3-\frac{Nb^3\left(1-S_{wfi}\right)}{\varphi_t N_f\left(1-S_{wi(f,m)}\right)}\right)=0. \quad (24)$$

The real positive root of this equation shows the matrix dimension. In most cases, there is one real root (see Appendix A), where a is the matrix dimension in the Warren-Root model obtained by using the material balance parameters, N and N_f.

3.2.2. Kazemi model

This model assumes that the reservoir is made up of n matrix blocks (or slabs) and horizontal fractures, according to Figure 7. The volume of each matrix is aL^2 and the volume for each fracture is bL^2 in which a is the height of the matrix blocks, L is the length and width of the matrix blocks and fractures, and b is the height of the fracture [6]. (The reservoir is assumed to have n fracture layers.)

Here, Eq. (17) can be used again for the Kazemi model:

$$V_b=n(a+b)L^2, \quad (25)$$

where a is the slab height, L is the slab's length and width, and n is the number of matrix slabs in the reservoir. So, by combining Eqs. (17) and (25) the following is obtained:

$$N=\frac{n(a+b)L^2\times\varphi_t\times\left(1-S_{wi(f,m)}\right)}{5.615}. \quad (26)$$

Again if N_f shows the original oil in place in the fractures in the whole reservoir, N_f can be found from:

$$N_f=\frac{(V_b-V_m)\times(1-S_{wfi})}{5.615}, \quad (27)$$

where V_m is the total volume of the matrix in slabs and S_{wfi} is the initial water saturation in the fractured system. In the Kazemi model, V_m is defined as follows:

$$V_m=naL^2. \quad (28)$$

By combining equations 25, 27 and 28 we obtain:

$$N_f=\frac{n\left((a+b)L^2-aL^2\right)\times(1-S_{wfi})}{5.615}$$

$$=\frac{nbL^2\times(1-S_{wfi})}{5.615}, \quad (29)$$

and, by combining Equations 26 and 29, it can be shown that:

$$\frac{N}{N_f}=\frac{n(a+b)L^2\times\varphi_t\times\left(1-S_{wi(f,m)}\right)}{nbL^2\times(1-S_{wfi})}, \quad (30)$$

which results in the following equation:

$$a=\frac{bN\times(1-S_{wfi})}{\varphi_t N_f\left(1-S_{wi(f,m)}\right)}-b, \quad (31)$$

where is the matrix height in the Kazemi model obtained by using the material balance parameters, N and N_f.

4. Results and discussions

In order to ensure the accuracy of the results of the obtained relations, two undersaturated synthetic reservoir models are considered. The first reservoir, called reservoir 1, is simulated using the Kazemi method, and the second one, called reservoir 2, is simulated using the Warren-Root method. In the first reservoir, the fractures are horizontal (slab model), while in the second one, the fractures are horizontal and vertical. The schematic plots of these reservoirs are depicted in Figures 8 and 9. The first reservoir is composed of 15

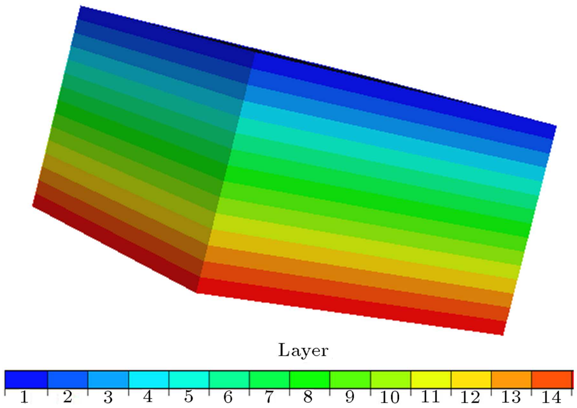


Figure 8. Schematic plot of reservoir 1.

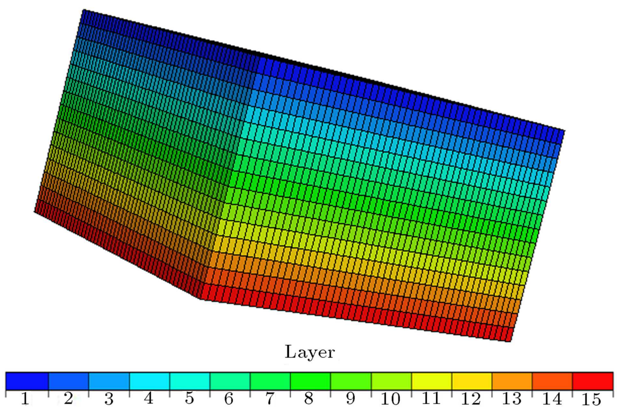


Figure 9. Schematic plot of reservoir 2.

Table 1. Reservoir information.

Reservoir and fluid parameter	Value
Initial pressure (p_i)	4039 psia
Initial water saturation in the matrix system (S_{wmi})	20%
Initial water Saturation in the fractured system (S_{wfi})	20%
Average initial water saturation in the NFR system ($S_{wi(f,m)}$)	20%
Water compressibility (c_w)	3×10^{-6}psi^{-1}
Isothermal pore compressibility (c_{pp})	1×10^{-5}psi^{-1}
Matrix porosity (φ_m)	25%
Fracture porosity of reservoir 1 (φ_f)	0.015%
Fracture porosity of reservoir 2 (φ_f)	0.045%
Storativity ratio of reservoir 1 (ω)	6.3×10^{-4}
Storativity ratio of reservoir 2 (ω)	1.85×10^{-3}
Matrix permeability (K_m)	1 md
Fracture permeability (K_f)	300 md
Length and width of reservoir (L)	1000 ft
Height of oil containing reservoir (h)	240 ft
Fracture height (b)	0.003 ft

Table 2. Production data of reservoir 1 (Kazemi model).

Pressure (psi)	Cumulative oil produced (STB)	Cumulative water produced (STB)	Oil formation volume factor (bbl/STB)
4035.921	320.0	45.9	1.250784
4004.893	3200.0	847.2	1.250971
3970.729	6416.8	1653.9	1.251176
3903.84	12841.3	3062.3	1.251577
3768.604	25906.3	5786.0	1.252388
3504.781	51235.5	11217.3	1.253971
3196.79	80546.6	17698.6	1.255819
3033.553	96000.0	21179.3	1.256799
2862.398	112128.0	24855.9	1.257826
2692.108	128115.8	28539.8	1.258847

layers and the second is formed of 50 matrix blocks in the x direction, 50 matrix blocks in the y direction and 30 matrix blocks in the z direction. The primary information of the reservoirs is presented in Table 1 and the production data of reservoirs 1 and 2 are shown in Tables 2 and 3, respectively.

Using Eqs. (5)-(7) and the data from Tables 1 to 3, the parameters F, $E_{o,m}$ and $E_{o,f}$ are calculated for each step, respectively (in undersaturated reservoirs: $R_p = R_s$). Since the fracture pore volume compressibility is supposed to be equal to the matrix pore volume compressibility, $E_{o,m}$ will be equal to $E_{o,f}$. The results for reservoir 1 (Kazemi model) and reservoir 2 (Warren-Root model) are summarized in Tables 4 and 5, respectively.

Table 3. Production data of reservoir 2 (Warren-Root model).

Pressure (psi)	Cumulative oil produced (STB)	Cumulative water produced (STB)	Oil formation volume factor (bbl/STB)
4036.131	320.0	16.3	1.250783
4000.079	4055.4	455.8	1.251000
3972.541	6891.7	824.1	1.251165
3945.522	9660.5	1194.8	1.251327
3877.572	16583.2	2150.3	1.251735
3721.529	32349.6	4475.8	1.252671
3562.719	48260.4	6966.7	1.253624
3400.637	64319.8	9671.7	1.254596
3240.037	80083.2	12485.7	1.255560
3076.708	96000.0	15450.2	1.256540

Table 4. Results of the calculations for reservoir 1 (Kazemi model).

$\Delta P = P_i - P$ (psi)	$(1-\omega)E_{o,m} + \omega E_{o,f}$ (bbl/STB)	F (bbl)
3.2	6.93E-05	447.7
34.2	0.00075	4877.8
68.4	0.0015	9736.3
135.2	0.002968	19233.7
270.5	0.005936	38418.8
534.3	0.011726	75829.6
842.3	0.018486	119425.8
1005.5	0.022069	142520.3
1176.7	0.025825	166701.2

Now, as discussed earlier, in the plot of F versus $(1-\omega)E_{o,m} + \omega E_{o,f}$, the slope of the straight line is equal to the oil initially in place (N). The plot of $(1-\omega)E_{o,m} + \omega E_{o,f}$ versus for reservoirs 1 and 2 are shown in Figures 10 and 11. The slopes of these straight lines give the values of oil originally in place for reservoirs 1 and 2, as 6439566 STB and 6463268 STB, respectively.

4.1. Matrix height in the Kazemi model (reservoir 1)

According to the data from Table 1 and using Eqs. (12), (14) and (31), the value of parameters N_f, N_m and a are calculated as 4057 STB, 6435509 STB and 19.03 ft, respectively. While using volumetric calculations, the original oil in place of this reservoir is

Table 5. Results of the calculations for reservoir 2 (Warren-Root model).

$\Delta P = P_i - P$ (psi)	$(1-\omega)E_{o,m} + \omega E_{o,f}$ (bbl/STB)	F (bbl)
2.9	6.42E-05	417.1
39.0	0.000855	5543.9
66.5	0.00146	9473.6
93.6	0.002053	13322.1
161.5	0.003544	22977.9
317.6	0.006969	45144.7
476.4	0.010454	67693.4
638.4	0.014012	90681.3
799.0	0.017536	113440.7

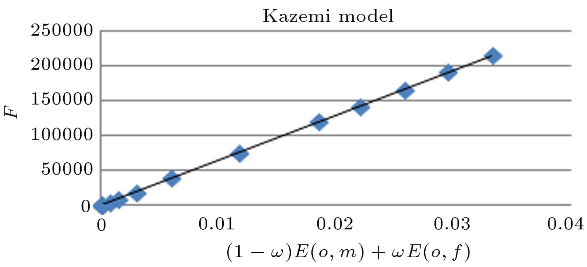


Figure 10. F versus $(1-\omega)E_{o,m} + \omega E_{o,f}$ curve for reservoir 1.

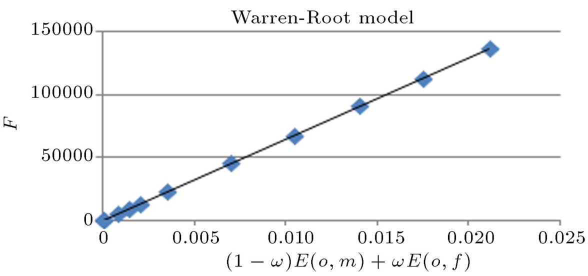


Figure 11. F versus $(1-\omega)E_{o,m} + \omega E_{o,f}$ curve for reservoir 2.

6648698 STB. The proposed material balance method has predicted this value by an error of 3.1% and, while the value of the height of slabs (in the Kazemi model) for the reservoir is 20 ft, the obtained value by the proposed equation (19.03 ft) shows 4.85% error.

4.2. The matrix dimensions in the Warren-Root model (reservoir 2)

According to the data from Table 1 and using Eqs. (12), (14) and (24), parameters N_f, N_m and a are calculated as 11957 STB, 6451311 STB and 19.4 ft, respectively. Using volumetric calculations, the original oil in place of this reservoir is 6656902 STB. The proposed material balance method has predicted this value by an error of 2.9% and, while the value of the matrix block size (in the Warren-Root model) for this reservoir is 20 ft, the obtained value by the proposed equation (19.4 ft) shows 3.0% error.

In general, calculation of matrix height in fractured reservoirs can be explained in these steps:

1. Calculation of F, $E_{o,m}$ and $E_{o,f}$ using production data and PVT data;
2. A plot of F versus $(1-\omega)E_{o,m} + \omega E_{o,f}$;
3. Calculation of the amount of original oil in place (N), which is equal to the slope of the straight line resulting from the plot of F versus $(1-\omega)E_{o,m} + \omega E_{o,f}$;
4. Calculation of matrix height in Kazemi and Warren-Root models using the proposed equations.

The summary of the results for the two cases (shown in Table 6) shows that the proposed method can predict the parameters required to create an appropriate model for fractured reservoirs. Despite the importance of these parameters (the slab height in the Kazemi model and matrix block size in the Warren-Root model) in simulation of fractured reservoirs, they are always considered as uncertain values, whose values, in most cases, are found by history matching methods. The proposed method can solve this problem by suggestion of a realistic value for these parameters, which will lead to a better reservoir model for simulation of the behavior of fractured reservoirs [33].

Also, other properties of the fractured reservoir, such as number of matrix blocks and Linear Fracture Density (LFD) [1,34] for both reservoirs, can be estimated, using these parameters, by the following equations:

$$n_f = \frac{h}{a}, \tag{32}$$

Table 6. Comparison of the calculated results.

Reservoir	Parameter	Simulator	Material balance equation	Error
Reservoir 1 (Kazemi Model)	N, STB	6648698	6439566	3.1%
	a, ft	20	19.03	4.85%
Reservoir 2 (Warren-Root Model)	N, STB	6656902	6463268	2.9%
	a, ft	20	19.4	3.0%

$$\text{LFD} = \frac{n_f}{h}. \tag{33}$$

5. Conclusions

1. A new method for calculation of the average matrix block size of fractured reservoirs was proposed.
2. The results obtained from simulation reveal the accuracy of the new relations for calculating the matrix height in the models of Warren-Root and Kazemi.
3. The height of the matrix in gravity drainage and imbibition, when treated as a gravity force, plays an important role, especially in the imbibition process.
4. When the matrix blocks possess large sizes or capillary forces are negligible, we can estimate the height of the matrix using the proposed relations.
5. If the percentage of error in the calculation of the oil initially in place (N) is low, the proposed relations have more accuracy and, by using these relations, a reservoir engineer can predict the amount of oil recovery in naturally fractured reservoirs.

Nomenclature

B_o	Oil formation volume factor, bbl/STB
B_{oi}	Initial oil formation volume factor, bbl/STB
B_w	Water formation volume factor, bbl/STB
B_{winj}	Injected water formation volume factor, bbl/STB
B_g	Gas formation volume factor, bbl/Scf
B_{gi}	Initial gas formation volume factor, bbl/Scf
B_{ginj}	Injected gas formation volume factor, bbl/Scf
B_t	Two phase formation volume factor, bbl/STB
B_{ti}	Initial two phase formation volume factor, bbl/STB
b	Fracture height, ft
C_e	Effective compressibility, psi^{-1}
C_r	Average rock compressibility, psi^{-1}
C_w	Water compressibility, psi^{-1}
$C_{pp,f}$	Fracture isothermal pore compressibility, psi^{-1}
$C_{pp,m}$	Matrix isothermal pore compressibility, psi^{-1}
Δp	Change in average reservoir pressure ≈ Change in effective stress, psi
$E_{o,m}$	Net expansion of the original oil-phase in the matrix system, bbl/STB
$E_{o,f}$	Net expansion of the original oil-phase in the fracture system, bbl/STB
F	Net fluid withdrawal, bbl
G_{ing}	Cumulative gas injected, Scf
m	Ratio of the initial gas cap volume to the initial oil volume
h	Height of oil containing reservoir, ft
h_{Th}	Threshold height, ft
h_B	Matrix height, ft
k_m	Matrix permeability, md
k_f	Fracture permeability, md
LFD	Linear Fracture Density
L	Length and width, ft
N_p	Cumulative produced oil, STB
N	Initial oil in place, STB
N_m	Original oil in place in the matrix, STB
N_f	Original oil in place in the fractures, STB
n	Number of matrix blocks in the reservoir
n_f	Number of fractures
p	Pressure, psi
p_i	Initial pressure, psi
R_{soi}	Initial solution gas-oil ratio, Scf/STB
R_s	Solution oil-gas ratio, Scf/STB
R_p	Cumulative produced gas-oil ratio, Scf/STB
S_{wi}	Initial water saturation, dimensionless
S_{wfi}	Initial water saturation in the fractured system, dimensionless
S_{wmi}	Initial water saturation in the matrix system, dimensionless
$S_{wi(f,m)}$	Average initial water saturation in the NFR system, dimensionless
V_b	Bulk volume of reservoir, ft^3
V_m	Total volume of matrix blocks, ft^3
v_f	Ratio of pore volume in the fractures to the total volume of the fractures
v_m	Ratio of the total of the matrix blocks to the reservoir volume
W_p	Cumulative water production, STB
W_e	Cumulative water influx, bbl
W_{inj}	Cumulative water injected, STB
φ_f	Fracture porosity, fraction
φ_m	Matrix porosity, fraction
φ_t	Average porosity, fraction

ω Storage capacity ratio, dimensionless

Subscripts

f	Fracture
$f+m$	Total NFR system (fracture + matrix)
f,m	NFR system (fracture, matrix)
g	Gas phase
i	Initial value
inj	Injected
m	Matrix
o	Oil phase
w	Water phase

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Appendix A

Solution of cubic equations:

$$Z^3 + a_1 Z^2 + a_2 Z + a_3 = 0.$$

Let:

$$Q = \frac{3a_2 - a_1^2}{9},$$

$$J = \frac{9a_1 a_2 - 27a_3 - 2a_1^3}{54},$$

$$D = Q^3 + J^2.$$

If $D \succ 0$, the equation has only one real root:

$$Z_1 = \sqrt[3]{(J + \sqrt{D})} + \sqrt[3]{(J - \sqrt{D})} - \frac{a_1}{3}.$$

If $D \prec 0$, the equation has three real roots:

$$Z_1 = 2\sqrt{-Q}\cos\left(\frac{\theta}{3}\right) - \frac{a_1}{3},$$

$$Z_2 = 2\sqrt{-Q}\cos\left(\frac{\theta}{3} + 120^\circ\right) - \frac{a_1}{3},$$

$$Z_3 = 2\sqrt{-Q}\cos\left(\frac{\theta}{3} + 140^\circ\right) - \frac{a_1}{3},$$

where:

$$\theta = \cos^{-1}\left(\frac{J}{\sqrt{-Q^3}}\right).$$

If $D = 0$, the equation has three real roots, at least two of which are equal:

$$Z_1 = 2\sqrt[3]{J} - \frac{a_1}{3}, \qquad Z_2 = Z_3 = -\sqrt[3]{J} - \frac{a_1}{3}.$$

Biographies

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