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Delay-dependent passive analysis and control for interval stochastic time-delay systems

C. Wang[a,b], Z.W. Chen[b,*] and G.Y. Chen[c]

a. *School of Mathematics and Statistics, Huazhong University of Science and Technology, Wuhan, Hubei 430074, P.R. China.*
b. *Office of Human Resources, Huanggang Normal University, Huanggang 438000, Hubei Province, P.R. China.*
c. *School of Materials Science and Engineering, Central South University, Changsha 410083, Hunan Province, P.R. China.*

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Abstract. This paper is concerned with the problem of delay-dependent passive analysis and control for interval stochastic time-delay systems. The system matrices are assumed to be uncertain within given intervals, the time delay is a time-varying continuous function belonging to a given range, and the stochastic perturbation is in the form of a Brownian motion. By using ItÔ's differential formula and the Lyapunov stability theory, delay-dependent stochastic passive control criteria are proposed without ignoring any useful terms by considering the information of the lower bound and upper bound for the time delay. Based on the criteria obtained, a delay-dependent passive controller that ensures the stochastic passivity of the closed-loop system is presented. Then, the controller gain is characterized in terms of LMIs, which can be easily checked by resorting to available software packages. Numerical examples are given to demonstrate the effectiveness of the method.

1. Introduction

Time-delay occurs in many systems, such as manufacturing systems, telecommunications and economic systems, etc., and its existence is frequently a source of oscillation and instability. Therefore, the problem of stability analysis of time-delay systems has received considerable attention over the past decades [1-10]. It is noted that stability criteria for time-delay systems can be classified into two categories, according to their dependence on the information about the size of time delays, i.e. delay-independent criteria and delay-dependent criteria. Recently, many researchers have concentrated on the delay-dependent stability analysis of delay systems [11-14], because delay-dependent stability criteria, which make use of information on the length of delays, are generally less conservative than delay-independent ones. Especially, when the time delays are small, many researchers have concentrated on the delay-dependent stability analysis of delay systems. For example, the delay-dependent stability criterion of time-varying delay systems was discussed in [11], delay-dependent stability criterion for dynamic systems with time-varying delay and nonlinear perturbations was obtained in [12], and delay-dependent robust stabilization of uncertain stochastic systems with time-varying delays was studied in [14].

However, the range of time-varying delay considered in these papers is from 0 to an upper bound. In practice, the range of delay may vary in a range for which the lower bound is not restricted to 0. Thus, a special type of time delay in practical systems such as an interval time-varying delay, which is a time delay varying in an interval, was investigated [15-21]. In this case, it is of great significance to consider the stability of systems with interval time-varying delay, since the criteria in the previous work, not taking into

*. *Corresponding author.*
E-mail address: zhongwenchen@foxmail.com (Z.W. Chen)

account the information of the lower bound of delay, are conservative. The problem of robust H_∞ control for systems with interval time-varying delay in a range by employing the free weighting matrix method was studied in [16]. But, the results were obtained by neglecting some useful terms in the derivative of the Lyapunov functional. To derive a less conservative stability criterion, [18] concerned itself with the delay-dependent stability for systems with interval delay. The stability criteria turned out to be less conservative with fewer matrix variables than some recently reported ones [16,19].

On the other hand, interval systems have been well known for their importance in practical applications. The systems matrices are estimated only within certain closed intervals. In recent years, the stability analysis and stabilization problems of various interval systems have received considerable research attention [22-24]. Also, it is noticed that the delay-dependent technique has been applied to the analysis and synthesis of stochastic interval systems [25-26]. The exponential stability analysis problem of a class of stochastic delay interval systems was discussed using the Razumikhin method in [25]. The robust stability and stabilization problems for a class of stochastic time-delay interval systems with nonlinear disturbance, by developing delay-dependent analysis techniques, were considered in [26].

It is well known that the notion of positive realness is related to the passiveness of systems. Therefore, many results have been developed for the introduction of the notion of positive realness in system and control theory [27-31]. The objective of passive control is to design controllers such that the closed-loop system is stable and passive. By using linear matrix inequalities, the problem of passive control and the design of the observer-based passive controller for a class of nonlinear uncertain time-delay systems were dealt with in [32]. The problem of observer-based passive control of a class of uncertain linear systems with delayed state and parameter uncertainties was studied in [33]. To the best of the authors' knowledge, the delay-dependent passive control problem for stochastic interval systems with interval time-varying delay has not been adequately addressed to date, and few results have been available in the literature so far, which still remains an interesting research topic.

In this paper, we deal with the problem of delay-dependent passive control for interval stochastic time-delay systems. The main aim is to design a state-feedback controller such that the resulted closed-loop system is stochastically stable and passive. The sufficient conditions are derived by using $It\hat{o}$'s differential formula and the Lyapunov stability theory, without ignoring any useful terms, by taking into account the information of the lower bound and upper bound of delay. Based on the criteria, the proposed method of controller design is formulated in terms of LMIs, which can be easily checked by resorting to available software packages. Numerical examples are exploited to demonstrate the effectiveness of the method.

Notation. Through this paper, R^n denotes the n-dimensional Euclidean space; $R^{n\times m}$ is the set of all $n\times m$ real matrices; the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X-Y$ is positive semi-definite (respectively, positive definite); $\|\cdot\|$ stands for the Euclidean norm; the superscript "T" stands for matrix transposition; diag$\{\ldots\}$represents a block-diagonal matrix and I is the identity matrix with appropriate dimension; $(\Omega, F, \{F_t\}_{t\geq 0}\ , P)$ is a complete probability space with a filtration $\{F_t\}_{t\geq 0}$, satisfying the condition that it is right continuous, and F_0 contains all P-null sets; $L^2_{F_0}([-h,\ 0]; R^n)$ denotes the family of all F_0 -measurable $C([-h,\ 0];\ R^n)$-valued random variables $\xi = \{\xi(\theta) :\ -h \leq \theta \leq 0\}$, such that $\sup_{-h\leq\theta\leq 0} E|\xi(\theta)|^2 < \infty$; $E\{\cdot\}$ denotes the expectation operator; $L_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$; and symbol $*$ is used to denote the transposed elements in the symmetric positions of a matrix. Matrices, if the dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Problem formulation

For a pair of matrices, $A^m = [a^m_{ij}]_{n_1\times n_2}$ and $A^M = [a^M_{ij}]_{n_1\times n_2}$, satisfying $a^m_{ij} \leq a^M_{ij}$, $\forall 1 \leq i \leq n_1, 1 \leq j \leq n_2$, the interval matrix, $[A^m, A^M]$, is defined by $[A^m, A^M] = \{A = [a_{ij}]_{n_1\times n_2} : a^m_{ij} \leq a_{ij} \leq a^M_{ij}, 1 \leq i \leq n_1, 1 \leq j \leq n_2\}$. For $A_0, \Delta A \in R^{n_1\times n_2}$, any interval matrix, $[A^m, A^M]$, has a unique representation of the form $[A_0 - \Delta A, A_0 + \Delta A]$, where $A_0 = (\frac{1}{2})(A^m + A^M)$, $\Delta A = (\frac{1}{2})(A^M - A^m)$.

Consider the following stochastic interval system (Σ) with interval time-varying delay described by $It\hat{o}$'s differential equation:

$$\begin{cases} dx(t) = [Ax(t) + A_1x(t-h(t)) + Bu(t) + B_1v(t)]dt \\ \qquad\qquad + [Ex(t) + E_1x(t-h(t))]d\omega(t) \\ Z(t) = Cx(t) + Dv(t) \\ x(t) = \varphi(t), \quad \forall t \in\ [-h_2, 0] \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^p$ is the control input, $v(t) \in R^q$ is the disturbance input which belongs to $L_2[0,\infty)$, $z(t) \in R^q$ is the controlled output, and $\varphi(t)$ is a real-valued initial vector function that is continuous on the interval $[-h_2, 0]$. $\omega(t)$ is a one-dimensional Brownian motion defined on a complete probability space, $(\Omega, F, \{F_t\}_{t\geq 0}, P)$, which satisfies:

$$E\{dw(t)\} = 0, \quad E\left\{dw^2(t)\right\} = dt.$$

Furthermore, $h(t)$ is a continuous time-varying function satisfying $0 \leq h_1 \leq h(t) \leq h_2$ and $\dot{h}(t) \leq d$, where h_1, h_2 and d are constants. Here, h_1 may not be equal to 0, and, when $d = 0$, it is clear that $h_2 = h_1$. The system matrices:

$$A \in [A^m, A^M] = \{A = [a_{ij}]_{n\times n}\},$$

$$A_1 \in [A_1^m, A_1^M] = \{A_1 = [a_{1ij}]_{n\times n}\},$$

$$B \in [B^m, B^M] = \{B = [b_{ij}]_{n\times p}\},$$

$$B_1 \in [B_1^m, B_1^M] = \{B_1 = [b_{1ij}]_{n\times q}\},$$

$$E \in [E^m, E^M] = \{E = [e_{ij}]_{n\times n}\},$$

$$E_1 \in [E_1^m, E_1^M] = \{E_1 = [e_{1ij}]_{n\times n}\},$$

$$C \in [C^m, C^M] = \{C = [c_{ij}]_{n\times n}\},$$

$$D \in [D^m, D^M] = \{D = [d_{ij}]_{n\times q}\}.$$

Then, we can rewrite $A, A_1, B, B_1, E, E_1, C$ and D as follows:

$$\begin{cases} A = A_0 + \tilde{A} = A_0 + \sum_{i,j=1}^{n} e_i\tilde{a}_{ij}e_j^T, \\ \quad |\tilde{a}_{ij}| \leq |\Delta a_{ij}|, \\ A_1 = A_{10} + \tilde{A}_1 = A_{10} + \sum_{i,j=1}^{n} e_i\tilde{a}_{1ij}e_j^T, \\ \quad |\tilde{a}_{1ij}| \leq |\Delta a_{1ij}|, \\ B = B_0 + \tilde{B} = B_0 + \sum_{i=1}^{n}\sum_{j=1}^{p} e_i\tilde{b}_{ij}h_j^T, \\ \quad |\tilde{b}_{ij}| \leq |\Delta b_{ij}|, \\ B_1 = B_{10} + \tilde{B}_1 = B_{10} + \sum_{i=1}^{n}\sum_{j=1}^{q} e_i\tilde{b}_{1ij}f_j^T, \\ \quad |\tilde{b}_{1ij}| \leq |\Delta b_{1ij}|, \\ E = E_0 + \tilde{E} = E_0 + \sum_{i,j=1}^{n} e_i\tilde{e}_{ij}e_j^T, \\ \quad |\tilde{e}_{ij}| \leq |\Delta e_{ij}|, \\ E_1 = E_{10} + \tilde{E}_1 = E_{10} + \sum_{i,j=1}^{n} e_i\tilde{e}_{1ij}e_j^T, \\ \quad |\tilde{e}_{1ij}| \leq |\Delta e_{1ij}|, \\ C = C_0 + \tilde{C} = C_0 + \sum_{i,j=1}^{n} e_i\tilde{c}_{ij}e_j^T, \\ \quad |\tilde{c}_{ij}| \leq |\Delta c_{ij}|, \\ D = D_0 + \tilde{D} = D_0 + \sum_{i=1}^{n}\sum_{j=1}^{q} e_i\tilde{d}_{ij}f_j^T, \\ \quad |\tilde{d}_{ij}| \leq |\Delta d_{ij}|, \end{cases} \tag{2}$$

where $e_k \in R^n$, $h_k \in R^p$, $f_k \in R^q$ denote the column vector with the kth element being 1 and others being 0.

Throughout this paper, we shall use the following definitions for System (1).

Definition 1. The stochastic interval system (1), ($u(t) = 0, v(t) = 0$), with interval time-varying delay is said to be stochastically mean-square stable if there exists $\delta(\varepsilon) > 0$, for any $\varepsilon > 0$, satisfying $\sup_{-h\leq t\leq 0} \{E|\varphi(t)|\} < \delta(\varepsilon)$, we have:

$$E\{||x(t)||^2\} < \varepsilon,$$

which is said to be stochastically mean-square asymptotically stable if, for any initial condition, $\lim_{t\to\infty} E\{||x(t)||^2\} = 0$ holds.

Definition 2. The stochastic interval system (1), ($u(t) = 0$), with interval time-varying delay is said to be stochastically passive with dissipation rate, γ, if for any $v(t) \in L_2[0,\infty)$, under zero initial state condition, there exists $\gamma > 0$ such that:

$$E\left\{\int_0^t v^T(s)z(s)ds\right\} \geq -2\gamma E\left\{\int_0^t v^T(s)v(s)ds\right\}$$

for all $t > 0$.

The purpose of this paper is to design a memoryless state feedback controller, for the given system (1) and a prescribed dissipation rate, $\gamma > 0$, such that the corresponding closed-loop system is stochastically stable and stochastically passive with dissipation rate, γ.

3. Passivity for stochastic interval systems with interval time-varying delay

First, let us give the following lemmas, which will play an indispensable role in deriving our main results.

Lemma 1 (Schur complement). Given constant matrices, Σ_1, Σ_2 and Σ_3, with appropriate dimensions, where $\Sigma_1^T = \Sigma_1$ and $\Sigma_2^T = \Sigma_2$, then:

$$\Sigma_1 + \Sigma_3^T\Sigma_2^{-1}\Sigma_3 < 0,$$

if and only if:

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0.$$

Lemma 2 [33]. Given appropriately dimensioned matrices, ψ, H and G, with $\psi = \psi^T$, then:

$$\psi + HF(t)G + G^TF^T(t)H^T < 0,$$

holds for all $F(t)$, satisfying $F^T(t)F(t) \leq I$ if and only if for some $\varepsilon > 0$,

$$\psi + \varepsilon HH^T + \varepsilon^{-1}G^TG < 0.$$

Lemma 3 [26]. Let M_1, M_2, M_3 and $\Xi > 0$ be given constant matrices with appropriate dimensions. Then,

$$\begin{bmatrix} \Phi & A_{10}X & 0 & S_4 & F_5 & XF_1 & 0 & H^* & 0 & 0 \\ * & -(1-d)S_1-2S_5 & S_5 & S_5 & 0 & XF_2 & 0 & 0 & \tilde{H} & 0 \\ * & * & -S_2-S_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -S_3-S_4-S_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -J^* & F_3 & F_4 & 0 & 0 & \tilde{H}^* \\ * & * & * & * & * & -J & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\tilde{J} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -U & 0 & 0 \\ * & * & * & * & * & * & * & * & U^* & 0 \\ 5* & * & * & * & * & * & * & * & * & -\tilde{U}^* \end{bmatrix} < 0. \tag{8}$$

Box I

for any scalar $\varepsilon > 0$, satisfying $\varepsilon I - M_2^T \Xi M_2 > 0$, we have:

$$(M_1 + M_2M_3)^T \Xi (M_1 + M_2M_3) \leq M_1^T(\Xi^{-1} - \varepsilon^{-1}M_2M_2^T)^{-1}M_1 + \varepsilon M_3^T M_3.$$

Lemma 4 [34]. For any symmetric positive definite matrix, $R > 0$, and vector function, $x(t) : [0, h] \to \mathrm{R}^n$, such that the integrations concerned are well defined, the following inequality holds:

$$-h\int_0^h x^T(s)Rx(s)ds \leq -\int_0^h x^T(s)ds.R.\int_0^h x(s)ds.$$

For convenience, the following new state variables:

$$y(t) = Ax(t) + A_1x(t - h(t)), \tag{3}$$

$$f(t) = Ax(t) + A_1x(t - h(t)) + B_1V(t), \tag{4}$$

$$k(t)=Ax(t)+A_1x(t-h(t))+Bu(t)+B_1V(t), \tag{5}$$

and the following new perturbation variable:

$$g(t) = Ex(t) + E_1x(t - h(t)), \tag{6}$$

are defined, then System (1) can be rewritten as:

$$dx(t) = k(t)dt + g(t)d\omega(t). \tag{7}$$

3.1. Interval time-varying delay with upper and non-zero lower bounds

In the following theorem, a delay-dependent LMI approach is used to solve the passivity problem for the stochastic interval system (1), $(u(t) = 0)$, with interval time-varying delay, and the sufficient conditions are derived ensuring the solvability of the problem.

Theorem 1. Given scalars, $h_2 \geq h_1 \geq 0$, $d > 0$ and $\gamma > 0$, the stochastic interval system (1), $(u(t) = 0)$, with interval time-varying delay is stochastically passive, if there exist semi-positive definite matrices, $X \geq 0$, $S_i \geq 0$ ($i = 1, 2, \cdots, 5$), $T_j \geq 0$, $Z_j \geq 0$ ($j = 1, 2$), and positive scalars, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\eta_{kij>0}(i, j = 1, 2, \cdots, n, k = 1, 2, \cdots, 12, 14)$, $\eta_{kij} > 0$ ($i = 1, 2, \cdots, n, j = 1, 2, \cdots, q, k = 13, 15, \cdots, 19)$, such that the following linear matrix inequalities hold:

Relation (8) is shown in Box I,

$$\begin{bmatrix} -T_1 & 0 \\ 0 & -Z_1 \end{bmatrix} \leq 0, \tag{9}$$

$$\begin{bmatrix} -T_2 & 0 \\ 0 & -Z_2 \end{bmatrix} \geq 0, \tag{10}$$

where:

F_1	$[A_0^T\ A_0^T\ A_0^T\ A_0^T\ E_0^T]$,
F_2	$[A_{10}^T\ A_{10}^T\ A_{10}^T\ A_{10}^T\ E_{10}^T]$,
F_3	$[0\ \ 0\ \ B_{10}^T\ B_{10}^T\ 0]$,
F_4	$[B_{10}^T\ B_{10}^T]$,
F_5	$B_{10} - XC_0^T$,
J_1	$h_1^{-1}Z_1 - \varepsilon_1 I - \Pi_3 - \Pi_8$,
J_2	$h_{12}^{-1}Z_2 - \varepsilon_2 I - \Pi_4 - \Pi_9$,
J_3	$2h_1^{-2}X - h_1^{-2}S_4 - \Pi_5 - \Pi_{10} - \Pi_{16}$,
J_4	$h_{12}^{-2}X - h_{12}^{-2}S_5 - \Pi_6 - \Pi_{11} - \Pi_{17}$,
J_5	$X - \Pi_7 - \Pi_{12}$,
J_6	$\varepsilon_1 I - \Pi_{18}$,
J_7	$\varepsilon_2 I - \Pi_{19}$,
J^*	$\gamma I + D_0 + D_0^T - \Pi_{13} - \Pi_{14} - \Pi_{15}$,
h_{12}	$h_2 - h_1$,
J	diag $\{J_1,\ J_2,\ J_3,\ J_4,\ J_5\}$,
$\tilde{J}$	diag $\{J_6, J_7\}$,
$\tilde{H}$	$[HHHHHH]$,
H	$\underbrace{[X, ..., X]}_{n}$,

I $\quad \underbrace{[I, ..., I]}_{n}$,

$\tilde{H}^*$ $\quad [IIIII]$,

H^* $\quad [HHHHHHIH]$,

$\tilde{U}$ $\quad \text{diag}\ \{U_1\ U_3\ U_4\ U_5\ U_6\ U_7\}$,

$\tilde{U}^*$ $\quad \text{diag}\ \{U_{15}\ U_{16}\ U_{17}\ U_{18}\ U_{19}\}$,

U^* $\quad \text{diag}\ \{U_2\ U_8\ U_9\ U_{10}\ U_{11}\ U_{12}\}$,

U $\quad \text{diag}\ \{U_1\ U_3\ U_4\ U_5\ U_6\ U_7\ U_{13}\ U_{14}\}$,

U_k $\quad \text{diag}\ \{\eta_{k11},\ \ldots,\ \eta_{kn1},\ \ldots\ ,\ \eta_{k1q},\ \ldots,\ \eta_{knq}\ \}$ $(k = 13,\ 15)$,

U_k $\quad \text{diag}\ \{\eta_{k11},\ \ldots,\ \eta_{k1n},\ \ldots\ ,\ \eta_{kn1},\ \ldots,\ \eta_{knn}\ \}$ $(k = 1,\ 2,\ \cdots\ ,\ 12,\ 14)$,

U_k $\quad \text{diag}\{\eta_{k11},\ \ldots,\ \eta_{k1q},\ \ldots\ ,\ \eta_{kn1},\ \ldots,\ \eta_{knq}\ \}$ $(k = 16,\ 17,\ 18,\ 19)$,

Π_1 $\quad \sum_{i,j=1}^n \eta_{1ij}\Delta a_{ij}^2 e_i e_i^T$,

Π_2 $\quad \sum_{i,j=1}^n \eta_{2ij}\Delta a_{1ij}^2 e_i e_i^T$,

Π_3 $\quad \sum_{i,j=1}^n \eta_{3ij}\Delta a_{ij}^2 e_i e_i^T$,

Π_4 $\quad \sum_{i,j=1}^n \eta_{4ij}\Delta a_{ij}^2 e_i e_i^T$,

Π_5 $\quad \sum_{i,j=1}^n \eta_{5ij}\Delta a_{ij}^2 e_i e_i^T$,

Π_6 $\quad \sum_{i,j=1}^n \eta_{6ij}\Delta a_{ij}^2 e_i e_i^T$,

Π_7 $\quad \sum_{i,j=1}^n \eta_{7ij}\Delta e_{ij}^2 e_i e_i^T$,

Π_8 $\quad \sum_{i,j=1}^n \eta_{8ij}\Delta a_{1ij}^2 e_i e_i^T$,

Π_9 $\quad \sum_{i,j=1}^n \eta_{9ij}\Delta a_{1ij}^2 e_i e_i^T$,

Π_{10} $\quad \sum_{i,j=1}^n \eta_{10ij}\Delta a_{1ij}^2 e_i e_i^T$,

Π_{11} $\quad \sum_{i,j=1}^n \eta_{11ij}\Delta a_{1ij}^2 e_i e_i^T$,

Π_{12} $\quad \sum_{i,j=1}^n \eta_{12ij}\Delta e_{1ij}^2 e_i e_i^T$,

Π_{13} $\quad \sum_{i=1}^n \sum_{j=1}^q \eta_{13ij}\Delta b_{1ij}^2 f_j f_j^T$,

Π_{14} $\quad \sum_{i,j=1}^n \eta_{14ij}\Delta c_{ij}^2 e_i e_i^T$,

Π_{15} $\quad \sum_{i=1}^n \sum_{j=1}^q \eta_{15ij}\Delta d_{ij}^2 f_j f_j^T$,

Π_{16} $\quad \sum_{i=1}^n \sum_{j=1}^q \eta_{16ij}\Delta b_{1ij}^2 e_i e_i^T$,

Π_{17} $\quad \sum_{i=1}^n \sum_{j=1}^q \eta_{17ij}\Delta b_{1ij}^2 e_i e_i^T$,

Π_{18} $\quad \sum_{i=1}^n \sum_{j=1}^q \eta_{18ij}\Delta b_{1ij}^2 e_i e_i^T$

Π_{19} $\quad \sum_{i=1}^n \sum_{j=1}^q \eta_{19ij}\Delta b_{1ij}^2 e_i e_i^T$

and:

$$\begin{aligned}\Phi =& A_0X + XA_0^T + h_1T_1 + h_{12}T_2 + S_1 + S_2 + S_3\\ &- S_4 + \Pi_1 + \Pi_2.\end{aligned} \tag{11}$$

Proof. Choose the following Lyapunov-Krasovskii functional candidate for System (1):

$$\begin{aligned}V(x(t),t) =& V_1(x(t),t) + V_2(x(t),t) + V_3(x(t),t)\\ &+ V_4(x(t),t) + V_5(x(t),t),\end{aligned} \tag{12}$$

with:

$$V_1(x(t),t) = x^T(t)Px(t),$$

$$V_2(x(t),t) = \int_{t-h_1}^{t} x^T(s)Q_2x(s)ds,$$

$$\begin{aligned}V_3(x(t),t) =& \int_{t-h(t)}^{t} x^T(s)Q_1x(s)ds\\ &+ \int_{t-h_2}^{t} x^T(s)Q_3x(s)ds,\end{aligned}$$

$$\begin{aligned}V_4(x(t),t) =& \int_{-h_1}^{0}\int_{t+\theta}^{t} y^T(s)R_1y(s)dsd\theta\\ &+ \int_{-h_1}^{0}\int_{t+\theta}^{t} h_1\dot{x}^T(s), Q_4\dot{x}(s)dsd\theta,\end{aligned}$$

$$\begin{aligned}V_5(x(t),t) =& \int_{-h_2}^{h_1}\int_{t+\theta}^{t} y^T(s)R_2y(s)dsd\theta\\ &+ \int_{-h_2}^{h_1}\int_{t+\theta}^{t} h_{12}\dot{x}^T(s)Q_5\dot{x}(s)dsd\theta,\end{aligned}$$

in which P, Q_i $(i = 1,2,3,4,5)$, R_j $(j = 1,2)$ are all symmetric positive definite matrices with appropriate dimensions to be determined.

Then, the stochastic differential of $V(x(t),t)$ along System (1) with $v(t) = 0$ and $u(t) = 0$ can be obtained as follows:

$$dV(x(t),t) = LV(x(t),t)dt + 2x^T(t)Pg(t)d\omega(t), \tag{13}$$

where:

$$\begin{aligned}LV(x(t),t) =& LV_1(x(t),t) + LV_2(x(t),t) + LV_3(x(t),t)\\ &+ LV_4(x(t),t) + LV_5(x(t),t).\end{aligned} \tag{14}$$

It is easy to know that:

$$\begin{aligned}LV_1(x(t),t) =& 2x^T(t)Py(t) + g^T(t)Pg(t) = x^T(t)(PA\\ &+ A^TP)x(t) + 2x^T(t)PA_1x(t-h(t))\\ &+ [Ex(t) + E_1x(t-h(t))]^TP[Ex(t)\\ &+ E_1x(t-h(t))],\end{aligned} \tag{15}$$

$$LV_2(x(t),t) = x^T(t)Q_2x(t) - x^T(t-h_1)Q_2x(t-h_1), \tag{16}$$

$$\begin{aligned}LV_3(x(t),t) =& x^T(t)Q_1x(t)-(1-\dot{h}(t))x^T(t\\&-h(t))Q_1x(t-h(t))+x^T(t)Q_3x(t)\\&-x^T(t-h_2)Q_3x(t-h_2)\leq x^T(t)(Q_1\\&+Q_3)x(t)-(1-d)x^T(t-h(t))Q_1x(t\\&-h(t))-x^T(t-h_2)Q_3x(t-h_2), \quad (17)\end{aligned}$$

$$\begin{aligned}LV_4(x(t),t) =& h_1y^T(t)R_1y(t)-\int_{t-h_1}^{t}y^T(s)R_1y(s)ds\\&+h_1^2\dot{x}^T(t)Q_4\dot{x}(t)-\int_{t-h_1}^{t}h_1\dot{x}^T(s)Q_4\dot{x}(s)ds\\&=h_1y^T(t)R_1y(t)-\int_{t-h_1}^{t}y^T(s)R_1y(s)ds\\&-\int_{t-h_1}^{t}h_1\dot{x}^T(s)Q_4\dot{x}(s)ds+h_1^2[Ax(t)+A_1x(t\\&-h(t))]^TQ_4[Ax(t)+A_1x(t-h(t))], \quad (18)\end{aligned}$$

$$\begin{aligned}LV_5(x(t),t) =& h_{12}y^T(t)R_2y(t)-\int_{t-h_2}^{t-h_1}y^T(s)R_2y(s)ds\\&+h_{12}^2\dot{x}^T(t)Q_5\dot{x}(t)\\&-\int_{t-h_2}^{t-h_1}h_{12}\dot{x}^T(s)Q_5\dot{x}(s)ds\\&=h_{12}y^T(t)R_2y(t)-\int_{t-h_2}^{t-h_1}y^T(s)R_2y(s)ds,\\&-\int_{t-h_2}^{t-h_1}h_{12}\dot{x}^T(s)Q_5\dot{x}(s)ds\\&+h_{12}^2[Ax(t)+A_1x(t-h(t))]^TQ_5[Ax(t)\\&+A_1x(t-h(t))], \quad (19)\end{aligned}$$

Using Lemma 3 and Eq. (3), we have:

$$\begin{aligned}h_1y^T(t)R_1y(t) =& [Ax(t)+A_1x(t-h(t))]^T(h_1R_1)\\&[Ax(t)+A_1x(t-h(t))]\leq[Ax(t)\\&+A_1x(t-h(t))]^T[(h_1R_1)^{-1}\\&-\varepsilon_1I]^{-1}[Ax(t)+A_1x(t-h(t))], \quad (20)\end{aligned}$$

$$\begin{aligned}h_{12}y^T(t)R_2y(t) =& [Ax(t)+A_1x(t-h(t))]^T(h_{12}R_2)\\&[Ax(t)+A_1x(t-h(t))]\leq[Ax(t)\\&+A_1x(t-h(t))]^T[(h_{12}R_2)^{-1}\\&-\varepsilon_2I]^{-1}[Ax(t)+A_1x(t-h(t))]. \quad (21)\end{aligned}$$

For Eq. (18), by Lemma 4, we can know:

$$\begin{aligned}-\int_{t-h_1}^{t}h_1\dot{x}^T(s)Q_4\dot{x}(s)ds \leq& -\int_{t-h_1}^{t}\dot{x}^T(s)dsQ_4\\&\int_{t-h_1}^{t}\dot{x}(s)ds=-(x(t)^T-x(t\\&-h_2)^T)Q_4(x(t)-x(t-h_2)). \quad (22)\end{aligned}$$

On the other hand, we can calculate from Lemma 4 that:

$$\begin{aligned}-\int_{t-h_2}^{t-h_1}&h_{12}\dot{x}^T(s)Q_5\dot{x}(s)Q_5\dot{x}(s)ds=\\&-\int_{t-h_2}^{t-d(t)}h_{12}\dot{x}^T(s)Q_{12}\dot{x}(s)ds\\&-\int_{t-d(t)}^{t-h_1}h_{12}\dot{x}^T(s)Q_5\dot{x}(s)ds\\&\leq-\int_{t-d(t)}^{t-h_1}\dot{x}^T(s)dsQ_5\int_{t-d(t)}^{t-h_1}\dot{x}(s)ds\\&-\int_{t-h_2}^{t-d(t)}\dot{x}^T(s)dsQ_5\int_{t-h_2}^{t-d(t)}\dot{x}(s)ds\\&\leq-(x^T(t-d(t))-x^T(t-h_2))Q_5(x(t\\&-d(t))-x(t-h_2))-(x^T(t-h_1)\\&-x^T(t-d(t)))Q_5(x(t-h_1)-x(t-d(t))). \quad (23)\end{aligned}$$

Noticing that for any semi-positive matrices $W_1\geq 0$ and $W_2\geq 0$, the following equations hold:

$$h_2x^T(t)W_1x(t)-\int_{t-h_2}^{t}x^T(t)W_1x(t)ds=0, \quad (24)$$

$$h_{12}x^T(t)W_2x(t)-\int_{t-h_2}^{t-h_1}x^T(t)W_2x(t)ds=0. \quad (25)$$

Adding the left side of Eqs. (25) and (26) and substituting Eqs. (16)-(24) into Eq. (15), then, taking expectation leads to:

$$\begin{aligned}E\{LV(x(t),t)\}\leq& E\{\xi^T(t)\Sigma\xi(t)\\&+\int_{t-h_2}^{t}\xi^T(t,s)\Sigma_1\xi(t,s)ds\\&+\int_{t-h_2}^{t-h_1}\xi^T(t,s)\Sigma_2\xi(t,s)ds\}, \quad (26)\end{aligned}$$

where:

$$\Sigma_1=\begin{bmatrix}\Sigma_{11}+\tilde{\Sigma}_{11} & \Sigma_{12} & 0 & Q_4\\ * & \Sigma_{22} & Q_5 & Q_5\\ * & * & -Q_2-Q_5 & 0\\ * & * & * & -Q_3-Q_4-Q_5\end{bmatrix}, \tag{27}$$

$$\Sigma_1=\begin{bmatrix}-W_1 & 0\\ 0 & -R_1\end{bmatrix}, \tag{28}$$

$$\Sigma_2=\begin{bmatrix}-W_2 & 0\\ 0 & -R_2\end{bmatrix}, \tag{29}$$

with:

$$\xi(t)=[x^T(t),x^T(t-h(t)),x^T(t-h_1),x^T(t-h_2)]^T,$$

$$\xi(t,s)=[x^T(t),\ y^T(s)]^T,\quad W_i=PT_iP>0,\ i=1,2$$

$$\begin{aligned}\Sigma_{11}=&PA+A^TP+h_2\,W_1+h_{12}W_2+Q_1+Q_2\\ &+Q_3-Q_4,\\ \tilde{\Sigma}_{11}=&E^TPE+A^T[(h_1R_1)^{-1}-\varepsilon_1I]^{-1}A\\ &+A^T[(h_{12}R_2)^{-1}-\varepsilon_2I]^{-1}A+h_1^2A^TQ_4A\\ &+h_{12}^2A^TQ_5A,\\ \Sigma_{12}=&PA_1+E^TPE_1+A^T[(h_1R_1)^{-1}-\varepsilon_1I]^{-1}A_1\\ &+A^T[(h_{12}R_2)^{-1}-\varepsilon_2I]^{-1}A_1+h_1^2A^TQ_4A_1\\ &+h_{12}^2A^TQ_5A_1,\\ \Sigma_{22}=&-(1-d)Q_1-2Q_5+E_1^TPE_1+A_1^T[(h_1R_1)^{-1}\\ &-\varepsilon_1I]^{-1}A_1+A_1^T[(h_{12}R_2)^{-1}-\varepsilon_2I]^{-1}A_1\\ &+h_1^2A_1^TQ_4A_1+h_{12}^2A_1^TQ_5A_1.\end{aligned} \tag{30}$$

It remains to show that $\Sigma<0,\Sigma_1<0$ and $\Sigma_2<0$. Applying the Schur Complement shows that $\Sigma<0$ if and only if: (refer to Eq. (31) shown in Box II).

In addition, let $P=X^{-1},Q_i=PS_iP>0$ $(i=1,\ 2,\ 3,\ 4,\ 5),R_j=Z_j^{-1}(j=1,2)$, pre- and post-multiplying (31) by diag$\{X,\ X,\ X,\ X,\ I,\ I,\ I,\ I,\ I\}$ result in Relation (32), shown in Box III, with:

$$\Sigma_{11}^*=AX+XA^T+h_1T_1+h_{12}T_2+S_1+S_2+S_3-S_4.$$

Similarly, pre- and post-multiplying $\Sigma_1<0$ and $\Sigma_2<0$ by diag$\{X,Z\}$ yields:

$$\Sigma_1^*=\begin{bmatrix}-T_1 & 0\\ 0 & -Z_1\end{bmatrix}<0,\quad \Sigma_2^*=\begin{bmatrix}-T_2 & 0\\ 0 & -Z_2\end{bmatrix}<0. \tag{33}$$

Note that we use the shorthand $\Re_n[(R_1)_{i_1,j_1},\ (R_2)_{i_2,j_2},\cdots,(R_r)_{i_r,j_r}]$ to represent a nth-order block square matrix, whose all nonzero blocks are the $i_1\ j_1$th block R_1, the $i_2\ j_2$th block $R_2,\ \cdots,$ the $i_r\ j_r$th block R_r, and all other blocks are zero matrices. Then, matrix Ξ can be rearranged in Eq. (34) as shown in Box IV.

It follows from Lemma 2 and Eqs. (2) that, for any real scalars $\eta_{1ij}>0\,(i,j=1,2,\cdots,n)$, the following holds:

$$\begin{aligned}\Psi_1=&[X,0,0,0,0,0,0]^T[(\tilde{A})^T,0,0,0,0,0,0]\\ &+[(\tilde{A})^T,0,0,0,0,0,0]^T[X,0,0,0,0,0,0]\\ &=\sum\nolimits_{i,j=1}^n\{[X,0,0,0,0,0,0]^T[(e_i\tilde{a}_je_j^T)^T,0,0,0,0,0,0]\\ &+[(e_i\tilde{a}_{ij}e_j^T)^T,0,0,0,0,0,0]^T[X,0,0,0,0,0,0]\}\\ &=\sum\nolimits_{i,j=1}^n\{[e_j^TX,0,0,0,0,0,0]^T[(e_i\tilde{a}_{ij})^T,0,0,0,0,0,0]\\ &+[(e_i\tilde{a}_{ij})^T,0,0,0,0,0,0]^T[e_j^TX,0,0,0,0,0,0]\}\\ &\le\sum\nolimits_{i,j=1}^n\{\eta_{1ij}^{-1}[e_j^TX,0,0,0,0,0,0]^T[e_j^TX,0,0,0,0,0,0]\\ &+\eta_{1ij}\Delta a_{ij}^2[(e_i)^T,0,0,0,0,0,0]^T[e_i^T,0,0,0,0,0,0]\}\\ &=\Re_7[(\Pi_1)_{1,1}]+[H,0,0,0,0,0,0]^TU_1^{-1}[H,0,0,0,0,0,0]\\ &=\Re_7[(\Pi_1+H^TU_1^{-1}H)_{1,1}].\end{aligned}$$

$$\begin{bmatrix}\Sigma_{11} & PA_1 & 0 & Q_4 & A^T & A^T & A^T & A^T & E^T\\ * & -(1-d)Q_1-2Q_5 & Q_5 & Q_5 & A_1^T & A_1^T & A_1^T & A_1^T & E_1^T\\ * & * & -Q_2-Q_5 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -Q_3-Q_4-Q_5 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & \varepsilon_1I-(h_1R_1)^{-1} & 0 & 0 & 0 & 0\\ * & * & * & * & * & \varepsilon_2I-(h_{12}R_2)^{-1} & 0 & 0 & 0\\ * & * & * & * & * & * & -(h_1^2Q_4)^{-1} & 0 & 0\\ * & * & * & * & * & * & * & -(h_{12}^2Q_5)^{-1} & 0\\ * & * & * & * & * & * & * & * & -P^{-1}\end{bmatrix}<0 \tag{31}$$

Box II

$$\Xi=\begin{bmatrix}\Sigma_{11}^{*} & A_1X & 0 & S_4 & XA^T & XA^T & XA^T & XA^T & XE^T\\ * & -(1-d)S_1-2S_5 & S_5 & S_5 & XA_1^T & XA_1^T & XA_1^T & XA_1^T & XE_1^T\\ * & * & -S_2-S_5 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -S_3-S_4-S_5 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & \varepsilon_1 I-h_1^{-1}Z_1 & 0 & 0 & 0 & 0\\ * & * & * & * & * & \varepsilon_2 I-h_{12}^{-1}Z_2 & 0 & 0 & 0\\ * & * & * & * & * & * & -h_1^{-2}Q_4^{-1} & 0 & 0\\ * & * & * & * & * & * & * & -h_{12}^{2}Q_5^{-1} & 0\\ * & * & * & * & * & * & * & * & -X\end{bmatrix}<0. \tag{32}$$

Box III

$$\Xi=\begin{bmatrix}\Sigma_{110}^{*} & A_1X & 0 & S_4 & XA^T & XA^T & XA^T & XA^T & XE^T\\ * & -(1-d)S_1-3S_5 & 2S_5 & S_5 & XA_1^T & XA_1^T & XA_1^T & XA_1^T & XE_1^T\\ * & * & -S_2-2S_5 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -S_3-S_4-S_5 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & \varepsilon_1 I-h_1^{-1}Z_1 & 0 & 0 & 0 & 0\\ * & * & * & * & * & \varepsilon_2 I-h_{12}^{-1}Z_2 & 0 & 0 & 0\\ * & * & * & * & * & * & -h_1^{-2}Q_4^{-1} & 0 & 0\\ * & * & * & * & * & * & * & -h_{12}^{-2}Q_5^{-1} & 0\\ * & * & * & * & * & * & * & * & -X\end{bmatrix}$$

$$+\Re_9[(\tilde{\Sigma}_{11}^{*})_{1,1}]_{1,1}+\Re_9[(\tilde{A}_1X)_{1,2},(X(\tilde{A}_1)^T)_{2,1}]+\Re_9[(X(\tilde{A})^T)_{1,5},(\tilde{A}X)_{5,1}]+\Re_9[(X(\tilde{A})^T)_{1,6},(\tilde{A}X)_{6,1}]$$

$$+\Re_9[(X(\tilde{A})^T)_{1,7},(\tilde{A}X)_{7,1}]+\Re_9[(X(\tilde{A})^T)_{1,8},(\tilde{A}X)_{8,1}]+\Re_9[(X(\tilde{E})^T)_{1,9},(\tilde{E}X)_{9,1}]$$

$$+\Re_9[(X(\tilde{A}_1)^T)_{2,5},(\tilde{A}_1X)_{5,2}]+\Re_9[(X(\tilde{A}_1)^T)_{2,6},(\tilde{A}_1X)_{6,2}]+\Re_9[(X(\tilde{A}_1)^T)_{2,7},(\tilde{A}_1X)_{7,2}]$$

$$+\Re_9[(X(\tilde{A}_1)^T)_{2,8},(\tilde{A}_1X)_{8,2}]+\Re_9[(X(\tilde{E}_1)^T)_{2,9},(\tilde{E}_1X)_{9,2}]$$

$$=\Psi_0+\Psi_1+\Psi_2+\Psi_3+\Psi_4+\Psi_5+\Psi_6+\Psi_7+\Psi_8+\Psi_9+\Psi_{10}+\Psi_{11}+\Psi_{12}, \tag{34}$$

where:

$$\Sigma_{110}^{*}=A_0X+XA_0^T+h_1T_1+h_{12}T_2+S_1+S_2+S_3-S_4,\quad \tilde{\Sigma}_{11}^{*}=\tilde{A}X+X\tilde{A}^T.$$

Box IV

Π_1, U_1 and H were defined in Eq. (11).

Similarly, for any scalars, $\eta_{kij}>0$ ($i,j=1,2,\cdots,n,k=2,3,\cdots,12$), we have:

$$\Psi_2\le\Re_9[(\Pi_2)_{2,2}]$$
$$+[0,H,0,0,0,0,0]^TU_2^{-1}[0,H,0,0,0,0,0]$$
$$=\Re_9[(\Pi_2)_{1,1}+(H^TU_2^{-1}H)_{2,2}],$$

$$\Psi_3\le\Re_9[(\Pi_3)_{5,5}]$$
$$+[H,0,0,0,0,0,0]^TU_3^{-1}[H,0,0,0,0,0,0]$$
$$=\Re_9[(\Pi_3)_{5,5}+(H^TU_3^{-1}H)_{1,1}],$$

$$\Psi_4\le\Re_9[(\Pi_4)_{6,6}]$$
$$+[H,0,0,0,0,0,0]^TU_4^{-1}[H,0,0,0,0,0,0]$$
$$=\Re_9[(\Pi_4)_{6,6}+(H^TU_4^{-1}H)_{1,1}],$$

$$\Psi_5\le\Re_9[(\Pi_5)_{7,7}]$$
$$+[H,0,0,0,0,0,0]^TU_5^{-1}[H,0,0,0,0,0,0]$$
$$=\Re_9[(\Pi_5)_{7,7}+(H^TU_5^{-1}H)_{1,1}],$$

$$\Psi_6\le\Re_9[(\Pi_6)_{8,8}]+[H,0,0,0,0,0,0]^TU_6^{-1}[H,0,0,0,0,0,0]$$
$$=\Re_9[(\Pi_6)_{8,8}+(H^TU_6^{-1}H)_{1,1}],$$

$$\tilde{\Xi}=\begin{bmatrix}\tilde{\Sigma}_1 & A_{10}X & 0 & S_4 & XA_0^T & XA_0^T & XA_0^T & XA_0^T & XE_0^T & \tilde{H} & 0\\ * & -(1-d)S_1-2S_5 & S_5 & S_5 & XA_{10}^T & XA_{10}^T & XA_{10}^T & XA_{10}^T & XA_{10}^T & 0 & \tilde{H}\\ * & * & -S_2-S_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -S_3-S_4-S_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & -J_1 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & -J_2 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & -\tilde{J}_3 & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & * & -\tilde{J}_4 & 0 & 0 & 0\\ * & * & * & * & * & * & * & * & -J_5 & 0 & 0\\ * & * & * & * & * & * & * & * & * & -\tilde{U} & 0\\ * & * & * & * & * & * & * & * & * & * & -U^*\end{bmatrix}$$

where: $\tilde{J}_3=h_1^{-2}XS_4^{-1}X-\Pi_5-\Pi_{10}$, $\tilde{J}_4=h_{12}^{-2}XS_5^{-1}X-\Pi_6-\Pi_{11}$, and $J_1,J_2,J_5,\tilde{H},\tilde{U},U^*$ were defined in Eq. (11) with $\tilde{\Sigma}_1=\Sigma_{110}^*+\Pi_1+\Pi_2$.

Box V

$$\begin{aligned}\Psi_7 &\le \Re_9[(\Pi_7)_{9,9}]\\ &\quad+[H,0,0,0,0,0,0]^TU_7^{-1}[H,0,0,0,0,0,0]\\ &=\Re_9[(\Pi_7)_{9,9}+(H^TU_7^{-1}H)_{1,1}],\\ \Psi_8 &\le \Re_9[(\Pi_8)_{5,5}]\\ &\quad+[0,H,0,0,0,0,0]^TU_8^{-1}[0,H,0,0,0,0,0]\\ &=\Re_9[(\Pi_8)_{5,5}+(H^TU_8^{-1}H)_{2,2}],\\ \Psi_9 &\le \Re_9[(\Pi_9)_{6,6}]\\ &\quad+[0,H,0,0,0,0,0]^TU_9^{-1}[0,H,0,0,0,0,0]\\ &=\Re_9[(\Pi_9)_{6,6}+(H^TU_9^{-1}H)_{2,2}],\\ \Psi_{10} &\le \Re_9[(\Pi_{10})_{7,7}]\\ &\quad+[0,H,0,0,0,0,0]^TU_{10}^{-1}[0,H,0,0,0,0,0]\\ &=\Re_9[(\Pi_{10})_{7,7}+(H^TU_{10}^{-1}H)_{2,2}],\\ \Psi_{11} &\le \Re_9[(\Pi_{11})_{8,8}]\\ &\quad+[0,H,0,0,0,0,0]^TU_{11}^{-1}[0,H,0,0,0,0,0]\\ &=\Re_9[(\Pi_{11})_{8,8}+(H^TU_{11}^{-1}H)_{2,2}],\\ \Psi_{12} &\le \Re_9[(\Pi_{12})_{9,9}]\\ &\quad+[0,H,0,0,0,0,0]^TU_{12}^{-1}[0,H,0,0,0,0,0]\\ &=\Re_9[(\Pi_{12})_{9,9}+(H^TU_{12}^{-1}H)_{2,2}].\end{aligned}$$

where $\Pi_2,\Pi_3,\Pi_4,\Pi_5,\Pi_6,\Pi_7,\Pi_8,\Pi_9,\Pi_{10},\Pi_{11},\Pi_{12},U_2,U_3,U_4,U_5,U_6,U_7,U_8,U_9,U_{10},U_{11},U_{12}$ were defined in Eq. (11).

According to the Schur Complement Lemma, $\Xi<0$ is equivalent to:

$$\tilde{\Xi}<0, \tag{35}$$

where: (please refer to Box V).

Condition $\tilde{\Xi}<0$ still cannot be implemented using standard numerical software, due to the existence of the terms $XS_4^{-1}X$ and $XS_5^{-1}X$. By noticing that $S_4\ge 0$ and $S_5\ge 0$, we have $(S_4-X)S_4^{-1}(S_4-X)\ge 0$ and $(S_5-X)S_5^{-1}(S_5-X)\ge 0$ which is equivalent to:

$$\begin{aligned}&-XS_4^{-1}X\le S_4-2X,\\ &-XS_5^{-1}X\le S_5-2X.\end{aligned} \tag{36}$$

By combining Relations (35) and (36), we readily obtain the LMI by Eq. (37) as shown in Box VI.

Obviously, Condition (8) results in Relation (37). Because Conditions (9) and (10) are satisfied, we can get $E\{LV(x(t),t)\}<0$, which indicates that the stochastic interval system (1) ($v(t)=0$, $u(t)=0$), with interval time-varying delay, is stochastically mean-square asymptotically stable.

Next, consider the stochastic passivity for stochastic interval system (1) ($u(t)=0$) with interval time-varying delay; we modify the Lyapunov-Krasovskii functional candidate (12), as:

$$\begin{aligned}\tilde{V}(x(t),t)=&V_1(x(t),t)+V_2(x(t),t)+V_3(x(t),t),\\ &+\tilde{V}_4(x(t),t)+\tilde{V}_5(x(t),t),\end{aligned} \tag{38}$$

$$\begin{bmatrix} \tilde{\Sigma}_1 & A_{10}X & 0 & S_4 & XA_0^T & XA_0^T & XA_0^T & XA_0^T & XE_0^T & \tilde{H} & 0 \\ * & -(1-d)S_1-2S_5 & S_5 & S_5 & XA_{10}^T & XA_{10}^T & XA_{10}^T & XA_{10}^T & XA_{10}^T & 0 & \tilde{H} \\ * & * & -S_2-S_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -S_3-S_4-S_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -J_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -J_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -J_3 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -J_4 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -J_5 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\tilde{U} & 0 \\ * & * & * & * & * & * & * & * & * & * & -U^* \end{bmatrix} < 0, \tag{37}$$

J_3, J_4 were defined in Eq. (11).

Box VI

with:

$$V_1(x(t),t) = x^T(t)Px(t),$$

$$V_2(x(t),t) = \int_{t-h_1}^{t} x^T(s)Q_2x(s)ds,$$

$$V_3(x(t),t) = \int_{t-h(t)}^{t} x^T(s)Q_1x(s)ds + \int_{t-h_2}^{t} x^T(s)Q_3x(s)ds,$$

$$V_4(x(t),t) = \int_{-h_1}^{0}\int_{t+\theta}^{t} f^T(s)R_1f(s)dsd\theta + \int_{-h_1}^{0}\int_{t+\theta}^{t} h_2\dot{x}^T(s)Q_4\dot{x}(s)dsd\theta,$$

$$V_5(x(t),t) = \int_{-h_2}^{h_1}\int_{t+\theta}^{t} f^T(s)R_2f(s)dsd\theta + \int_{-h_2}^{h_1}\int_{t+\theta}^{t} h_{12}\dot{x}^T(s)Q_5\dot{x}(s)dsd\theta,$$

with $v(t) \neq 0$, it can be derived by $It\hat{o}$'s differential formula that:

$$d\tilde{V}(x(t),t) = L\tilde{V}(x(t),t)dt + 2x^T(t)Pg(t)d\omega(t), \tag{39}$$

Using Lemma 3 and Eq. (3), we have:

$$\begin{aligned} h_1f^T(t)R_1f(t) &= [Ax(t)+A_1x(t-h(t)) \\ &\quad + B_1v(t)]^T(h_1R_1)[Ax(t)+A_1x(t-h(t)) \\ &\quad + B_1v(t)] \leq [Ax(t)+A_1x(t-h(t))]^T[(h_1R_1)^{-1} \\ &\quad - \varepsilon_1 I]^{-1}[Ax(t)+A_1x(t-h(t))] \\ &\quad + \varepsilon_1^{-1}v^T(t)B_1^TB_1v(t), \end{aligned}$$

$$\begin{aligned} h_{12}f^T(t)R_2f(t) &= [Ax(t)+A_1x(t-h(t)) \\ &\quad + B_1v(t)]^T(h_{12}R_2)[Ax(t)+A_1x(t-h(t)) \\ &\quad + B_1v(t)] \leq [Ax(t)+A_1x(t \\ &\quad - h(t))]^T[(h_{12}R_2)^{-1}-\varepsilon_2 I]^{-1}[Ax(t)+A_1x(t \\ &\quad - h(t))] + \varepsilon_2^{-1}v^T(t)B_1^TB_1v(t). \end{aligned}$$

Similar to the above progress, for:

$$\zeta(t) = [x^T(t), x^T(t-h(t)), x^T(t-h_1), x^T(t-h_2), v^T(t)]^T,$$

$$\zeta(t,s) = [x^T(t),\ f^T(s)]^T,$$

we can obtain that:

$$EL\tilde{V}(x(t),t) \leq E\{\zeta^T(t)\Theta\zeta(t) + \int_{t-h_1}^{t} \zeta^T(t,s)\Sigma_1\zeta(t,s)ds + \int_{t-h_2}^{t-h_1} \zeta^T(t,s)\Sigma_2\zeta(t,s)ds\}, \tag{40}$$

where:

$$\Theta = \begin{bmatrix} \Sigma_{11}+\tilde{\Sigma}_{11} & \Sigma_{12} & 0 & Q_4 & PB_1+\Theta_{15} \\ * & \Sigma_{22} & Q_5 & Q_5 & \Theta_{25} \\ * & * & -Q_2-Q_5 & 0 & 0 \\ * & * & * & -Q_3-Q_4-Q_5 & 0 \\ * & * & * & * & \Theta_{55} \end{bmatrix}, \tag{41}$$

with:

$$\Theta_{15} = h_1^2 A^T Q_4 B_1 + h_{12}^2 A^T Q_5 B_1,$$

$$\Theta_{25} = h_1^2 A_1^T Q_4 B_1 + h_{12}^2 A_1^T Q_5 B_1,$$

$$\Theta_{55} = (\varepsilon_1^{-1} + \varepsilon_2^{-1}) B_1^T B_1 + h_1^2 B_1^T Q_4 B_1 + h_{12}^2 B_1^T Q_5 B_1,$$

and matrix elements $\Sigma_{11}, \tilde{\Sigma}_{11}, \Sigma_{12}, \Sigma_{22}$ are defined in the proof process of Theorem 1.

Then, let $F(t) = L\tilde{V}(x(t),t) - 2v^T(t)z(t) - \gamma v^T(t)v(t)$. Taking expectation leads to:

$$E\{F(t)\} \leq E\{\zeta^T(t)\Omega\zeta(t) + \int_{t-h_1}^{t} \zeta^T(t,s)\Sigma_1\zeta(t,s)ds + \int_{t-h_2}^{t-h_1} \zeta^T(t,s)\Sigma_2\zeta(t,s)ds\},$$

where:

$$\Omega = \begin{bmatrix} \Sigma_{11}+\tilde{\Sigma}_{11} & \Sigma_{12} & 0 & Q_4 & PB_1 - C^T + \Theta_{15} \\ * & \Sigma_{22} & Q_5 & Q_5 & \Theta_{25} \\ * & * & -Q_2-Q_5 & 0 & 0 \\ * & * & * & -Q_3-Q_4-Q_5 & 0 \\ * & * & * & * & -\gamma I - D - D^T + \Theta_{55} \end{bmatrix}. \quad (42)$$

After some manipulations, using contragradient transformation and the Schur Complement Lemma, the inequality $\Omega < 0$ can be shown to be equivalent to $\Gamma < 0$, where: (please refer to Box VII).

On the other hand, from the above proving procedures, by applying the Schur Complement to Relation (8), after tedious but straightforward calculation, this results in:

$$\Gamma < 0, \quad \Sigma_1 < 0, \quad \Sigma_2 < 0.$$

Therefore, we can conclude that $E\{F(t)\} < 0$.

Consider zero initial state conditions, for all $t > 0$; we can obtain:

$$\begin{aligned} 2E\{\int_0^t v^T(s)z(s)ds\} &= E\{\int_0^t [L\tilde{V}(x(s),s) \\ &- F(s) - \gamma v^T(s)v(s)]ds\} \geq E\{\int_0^t [L\tilde{V}(x(s),s) \\ &- \gamma v^T(s)v(s)]ds\} = E\{\tilde{V}(x(t),t)\} - E\{V_1(0)\} \\ &- \gamma E\{\int_0^t v^T(s)v(s)ds\} \geq -\gamma E\{\int_0^t v^T(s)v(s)ds\}, \end{aligned}$$

and it follows that the stochastic interval system (1) ($u(t) = 0$) with interval time-varying delay is stochastically passive.

Remark 1. Theorem 1 is delay-dependent, which is generally less conservative than delay-independent results. Moreover, Theorem 1 is applicable to d, not necessarily restricted to being less than 1, as in many works on delay systems using the Lyapunov-Krasovskii approach. The relaxation of the condition brought about the use of Lemma 4 and the exploitation of $h(t)$, $h_2 - h(t)$ and $h(t) - h_1$.

Remark 2. When estimating $LV(x(t),t)$, we have not introduced any free weighting matrices, as [34], thus, making Theorem 1 only involve the matrix variables in the Lyapunov functional. From a mathematical point of view, it is simple.

Remark 3. It is worth mentioning that a much tighter bounding technology for cross terms is adopted in the proof of Theorem 1. To reduce the conservatism $-\int_{t-h_2}^{t-h_1} h_{12}\dot{x}^T(s)Q_5\dot{x}(s)ds$ is not simply enlarged as $-\int_{t-d(t)}^{t-h_1} h_{12}\dot{x}^T(s)Q_5\dot{x}(s)ds$, but $-\int_{t-h_2}^{t-d(t)} h_{12}\dot{x}^T(s)Q_5\dot{x}(s)ds$ is retained as well. Furthermore, the latter is not over bounded with $-(h_2 - d(t))\int_{t-h_2}^{t-d(t)} h_{12}\dot{x}^T(s)Q_5\dot{x}(s)ds$, but rather $-(d(t) -$

$$\Gamma =$$

$$\begin{bmatrix} \Sigma_{11} & A_1X & 0 & S_4 & B_1 - XC^T & XA^T & XA^T & XA^T & XA^T & XE^T & 0 & 0 \\ * & -(1-d)S_1 - 2S_5 & S_5 & S_5 & 0 & XA_1^T & XA_1^T & XA_1^T & XA_1^T & XE_1^T & 0 & 0 \\ * & * & -S_2 - S_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -S_3 - S_4 - S_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma I - D - D^T & 0 & B_1^T & B_1^T & 0 & 0 & B_1^T & B_1^T \\ * & * & * & * & * & \varepsilon_1 I - h_1^{-1}Z_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \varepsilon_2 I - h_{12}^{-1}Z_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -h_1^{-2}Q_4^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -h_{12}^{-2}Q_5^{-1} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -X & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix}$$

Box VII

$h_1)\int_{t-h_2}^{t-d(t)} h_{12}\dot{x}^T(s)Q_5\dot{x}(s)ds$ is taken into account. Therefore, the passivity criteria derived here are expected to be less conservative. Further, Lemma 4 is a more general and tighter bounding technology for dealing with cross terms.

3.2. Time-varying delay with upper and zero lower bounds

Theorem 1 considers the case of $h_1 \le h(t) \le h_2$. If we do not consider the lower bound of the delay, i.e. $0 \le h(t) \le h_2$, we can draw the following corollary.

Corollary 1. Given scalars $h_2 > 0$, $d > 0$ and $\gamma > 0$, the stochastic interval system (1) ($u(t) = 0$), with interval time-varying delay is stochastically passive, if there exist positive definite matrices, $X > 0$, $S_i > 0$ $(i = 1, 3, 5), T_1 > 0, Z_1 > 0$, and positive scalars, $\varepsilon_1 > 0$ $\eta_{kij} > 0$ $(i, j = 1, 2, \cdots, n, k = 1, 2, \cdots, 8, 10)$, $\eta_{kij} > 0$ $(i = 1, 2, \cdots, n, j = 1, 2, \cdots, q,\ k = 9, 11, 12, 13)$, such that Relation (9) and linear matrix inequality (43) shown in Box VIII, hold:

Proof. For $h_1 = 0$ and $h_{12} = h_2 - h_1 = h_2$, choose $V_1(x(t), t)$, $V_3(x(t), t)$ and $V_5(x(t), t)$ and remove $V_2(x(t), t)$ and $V_4(x(t), t)$ from Eq. (12). Construct $\zeta^T(t) = [x^T(t), x^T(t - h(t)), x^T(t - h_2)]$. According to Lemma 4, we have:

$$-\int_{t-h_2}^{t} h_2\dot{x}^T(s)Q_5\dot{x}(s)ds \le -[x(t-h(t)) - x(t - h_2)]^T Q_5[x(t-h(t)) - x(t-h_2)] - [x(t) - x(t-h(t))]^T Q_5[x(t) - x(t-h(t))].$$

Then, the following proof is similar to that for Theorem 1, and is omitted here.

3.3. Time-invariant delay

If the time-delay is time invariant, e.g., $h(t) \equiv h$ and $\dot{h}(t) = 0$, then we have the following corollary

Corollary 2. Consider the stochastic interval time-varying delay system (1) ($u(t) = 0$) with $h(t) \equiv h$ and $\dot{h}(t) = 0$. Given scalars $h_2 > 0$ and $\gamma > 0$, the system is stochastically passive for any time-delay satisfying $0 \le h \le h_2$, if there exist positive definite matrices, $X > 0$, $S_3 > 0$, $S_5 > 0, T_1 > 0, Z_1 > 0$, and positive scalars $\varepsilon_1 > 0$, $\eta_{kij} > 0$ $(i, j = 1, 2, \cdots, n, k = 1, 2, \cdots, 8, 10)$, $\eta_{kij} > 0$ $(i = 1, 2, \cdots, n, j = 1, 2, \cdots, q, k = 9, 11, 12, 13)$, such that Relation (9) and the linear matrix inequality (45) shown in Box IX hold.

We omit the same matrices expression as in Corollary 1.

Proof. In Eq. (44), we set $d = 0$. According to Lemma 4, we re-arrange some items of the equations, and then the LMI which is expressed by Relation (45) can be deduced. We complete the proof.

4. Design of the passive controller for stochastic interval systems with interval time-varying delay

Applying Theorem 1 in this section, we aim to propose a design procedure for a stochastic passive controller that can achieve passivity of the closed-loop stochastic interval system with interval time-varying delay. Again, a delay-dependent LMI technique will be used in order to obtain a less conservative condition. The main result is given in the following theorem.

Theorem 2. Given scalars $h_2 \ge h_1 \ge 0, d > 0$ and $\gamma > 0$. If there exist matrix Y, positive definite matrices, $X > 0$, $S_i > 0$ $(i = 1, 2, \cdots, 5), T_j > 0, Z_j > 0$ $(j = 1, 2)$, and positive scalars, $\varepsilon_1 > 0, \varepsilon_2 > 0, \eta_{kij>0}$ $(i, j = 1, 2, \cdots, n, k = 1, 2, \cdots, 12, 14), \eta_{kij} > 0$ $(i = 1, 2, \cdots, n, j = 1, 2, \cdots, q, k = 13, 15, \cdots, 19), \eta_{kij} > 0$ $(i = 1, 2, \cdots, n, j = 1, 2, \cdots, p, k = 20, \cdots, 24)$, such that Relations (9) and (10) and the LMI, shown in Box X, hold:

Then, with the stochastically passive controller given by:

$$u(t) = Kx(t), \quad K = YX^{-1}, \tag{47}$$

the closed-loop system is stochastically stable and stochastically passive. We omit the same matrices expression as Theorem 1.

Proof. Substituting Eq. (47) into System (1) yields the closed-loop system:

$$dx(t) = [(A + BK)x(t) + A_1x(t - h(t)) + B_1v(t)]dt + [Ex(t) + E_1x(t - h(t))]d\omega(t). \tag{48}$$

The Lyapunov-Krasovskii functional candidate is chosen as:

$$V(x(t), t) = x^T(t)Px(t) + \int_{t-h(t)}^{t} x^T(s)Q_1x(s)ds + \int_{t-h_1}^{t} x^T(s)Q_2x(s)ds + \int_{t-h_2}^{t} x^T(s)Q_3x(s)ds + \int_{-h_1}^{0}\int_{t+\theta}^{t} h_1\dot{x}^T(s)Q_4\dot{x}(s)dsd\theta$$

$$\begin{bmatrix} \Phi & A_{10}X+S_5 & 0 & F_4 & XF_1 & 0 & H^* & 0 & 0 \\ * & -(1-d)S_1-2S_5 & -S_5 & 0 & XF_2 & 0 & 0 & \tilde{H} & 0 \\ * & * & -S_3-S_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -J^* & F_3 & B_{10}^T & 0 & 0 & \tilde{H}^* \\ * & * & * & * & -J & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -J_4 & 0 & 0 & 0 \\ * & * & * & * & * & * & -U & 0 & 0 \\ * & * & * & * & * & * & * & -U^* & 0 \\ * & * & * & * & * & * & * & * & -\tilde{U}^* \end{bmatrix} < 0, \tag{43}$$

where:

$$F_1=[A_0^TA_0^TE_0^T],\quad F_2=[A_{10}^TA_{10}^TE_{10}^T],\quad F_3=[0B_{10}^T0],\quad F_4=B_{10}-XC_0^T,\quad J_4=\varepsilon_1I-\Pi_{13},$$

$$J_1=h_2^{-1}Z_1-\varepsilon_1I-\Pi_3-\Pi_6,\quad J_2=2h_2^{-2}X-h_2^{-2}S_4-\Pi_4-\Pi_7-\Pi_{12},\quad J_3=X-\Pi_5-\Pi_8,$$

$$J^*=\gamma I+D_0+D_0^T-\Pi_9-\Pi_{10}-\Pi_{11},\quad \tilde{H}^*\quad [III],\quad H=[\underbrace{X,\cdots,X}_{n}],\quad I=[\underbrace{I,\cdots,I}_{n}],$$

$$\tilde{H}=[HHHH],\quad H^*=[HHHHIH],\quad \tilde{U}^*=\mathrm{diag}\{U_{11},U_{12},U_{13}\},\quad J=\mathrm{diag}\{J_1,J_2,J_3\},$$

$$U^*=\mathrm{diag}\{U_2,U_6,U_7,U_8\},\quad U=\mathrm{diag}\{U_1U_3U_4U_5U_9U_{10}\},$$

$$U_k=\mathrm{diag}\{\eta_{k11},\ldots,\eta_{k1n},\ldots,\eta_{kn1},\ldots,\eta_{knn}\}(k=1,2,\cdots,8,10),$$

$$U_k=\mathrm{diag}\{\eta_{k11},\ldots,\eta_{kn1},\ldots,\eta_{k1q},\ldots,\eta_{knq}\}(k=9,11),$$

$$U_k=\mathrm{diag}\{\eta_{k11},\ldots,\eta_{k1q},\ldots,\eta_{kn1},\ldots,\eta_{knq}\}(k=12,13),$$

$$\Pi_1=\sum_{i,j=1}^n\eta_{1ij}\Delta a_{ij}^2e_ie_i^T,\quad \Pi_2=\sum_{i,j=1}^n\eta_{2ij}\Delta a_{1ij}^2e_ie_i^T,\quad \Pi_3=\sum_{i,j=1}^n\eta_{3ij}\Delta a_{ij}^2e_ie_i^T,$$

$$\Pi_4=\sum_{i,j=1}^n\eta_{4ij}\Delta a_{ij}^2e_ie_i^T,\quad \Pi_5=\sum_{i,j=1}^n\eta_{5ij}\Delta e_{ij}^2e_ie_i^T,\quad \Pi_6=\sum_{i,j=1}^n\eta_{6ij}\Delta a_{1ij}^2e_ie_i^T,$$

$$\Pi_7=\sum_{i,j=1}^n\eta_{7ij}\Delta a_{1ij}^2e_ie_i^T,\quad \Pi_8=\sum_{i,j=1}^n\eta_{8ij}\Delta e_{1ij}^2e_ie_i^T,\quad \Pi_9=\sum_{i=1}^n\sum_{j=1}^q\eta_{9ij}\Delta b_{1ij}^2f_jf_j^T,$$

$$\Pi_{10}=\sum_{i,j=1}^n\eta_{10ij}\Delta c_{ij}^2e_ie_i^T,\quad \Pi_{11}=\sum_{i=1}^n\sum_{j=1}^q\eta_{11ij}\Delta d_{ij}^2f_jf_j^T,\quad \Pi_{12}=\sum_{i=1}^n\sum_{j=1}^q\eta_{12ij}\Delta b_{1ij}^2e_ie_i^T,$$

$$\Pi_{13}=\sum_{i=1}^n\sum_{j=1}^q\eta_{13ij}\Delta b_{1ij}^2e_ie_i^T,\quad \Phi=A_0X+XA_0^T+h_2T_1+S_1+S_3-S_5+\Pi_1+\Pi_2. \tag{44}$$

Box VIII

$$+\int_{-h_1}^0\int_{t+\theta}^t k^T(s)R_1k(s)dsd\theta$$

$$+\int_{-h_2}^{h_1}\int_{t+\theta}^t h_{12}\dot{x}^T(s)Q_5\dot{x}(s)dsd\theta+$$

$$+\int_{-h_2}^{h_1}\int_{t+\theta}^t k^T(s)R_2k(s)dsd\theta.$$

Similar to the proof of Theorem 1, which is equivalent to replacing A with $A+BK$ in Relation (40), we have:

$$ELV(x(t),t)\le E\{\zeta^T(t)\tilde{\Sigma}\zeta(t)$$

$$+\int_{t-h_2}^t\eta^T(t,s)\Sigma_1\eta(t,s)ds$$

$$+\int_{t-h_2}^{t-h_1}\eta^T(t,s)\Sigma_2\eta(t,s)ds\},$$

$$\begin{bmatrix} A_0X+XA_0^T+h_2T_1+S_3-S_5+\Pi_1+\Pi_2 & A_{10}X+S_5 & F_4 & XF_1 & 0 & H^* & 0 & 0 \\ * & -S_3-S_5 & 0 & XF_2 & 0 & 0 & \tilde{H} & 0 \\ * & * & -J^* & F_3 & B_{10}^T & 0 & 0 & \tilde{H}^* \\ * & * & * & -J & 0 & 0 & 0 & 0 \\ * & * & * & * & -J_4 & 0 & 0 & 0 \\ * & * & * & * & * & -U & 0 & 0 \\ * & * & * & * & * & * & -U^* & 0 \\ * & * & * & * & * & * & * & -\tilde{U}^* \end{bmatrix} < 0. \qquad (45)$$

Box IX

$$\begin{bmatrix} \tilde{\Sigma}_1 & A_{10}X & 0 & S_4 & F_5 & XF_1+Y^TB_0^T & 0 & H^* & 0 & 0 & \tilde{Y}^T \\ * & -(1-d)S_1-2S_5 & S_5 & S_5 & 0 & XF_2 & 0 & 0 & \tilde{H} & 0 & 0 \\ * & * & -S_2-S_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -S_3-S_4-S_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -J^* & F_3 & F_4 & 0 & 0 & \tilde{H}^* & 0 \\ * & * & * & * & * & -J & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\tilde{J} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -U & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -U^* & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\tilde{U}^* & 0 \\ * & * & * & * & * & * & * & * & * & * & -V \end{bmatrix} < 0, \qquad (46)$$

where:

$$V=\text{diag}\{U_{20}U_{21}U_{22}U_{23}U_{24}\}, \qquad \tilde{Y}=[Y^TY^TY^TY^TY^T]^T,$$

$$U_k=\text{diag}\{\eta_{k11},\dots,\eta_{k1p},\dots,\eta_{kn1},\dots,\eta_{knp}\}(k=20,\cdots,24), \qquad Y=[\underbrace{Y^T,\cdots,Y^T}_{n}]^T,$$

$$B_0^T=[B_{10}^TB_{10}^TB_{10}^TB_{10}^T0], \qquad \Pi_{20}=\sum\nolimits_{i=1}^n\sum\nolimits_{j=1}^p\eta_{20ij}\Delta b_{ij}^2e_ie_i^T, \qquad \Pi_{21}=\sum\nolimits_{i=1}^n\sum\nolimits_{j=1}^p\eta_{21ij}\Delta b_{ij}^2e_ie_i^T,$$

$$\Pi_{22}=\sum\nolimits_{i=1}^n\sum\nolimits_{j=1}^p\eta_{22ij}\Delta b_{ij}^2e_ie_i^T, \qquad \Pi_{23}=\sum\nolimits_{i=1}^n\sum\nolimits_{j=1}^p\eta_{23ij}\Delta b_{ij}^2e_ie_i^T, \qquad \Pi_{24}=\sum\nolimits_{i=1}^n\sum\nolimits_{j=1}^p\eta_{24ij}\Delta b_{ij}^2e_ie_i^T,$$

$$\Psi=A_0X+XA_0^T+B_0Y+Y^TB_0^T+h_1T_1+h_{12}T_2+S_1+S_2+S_3-S_4+\Pi_1+\Pi_2+\Pi_{20}.$$

Box X

where Σ_1 and Σ_2 are defined in Eqs. (28) and (29), $\eta(t,s)=[x^T(t),k^T(s)]^T$, and $\tilde{\Sigma}$ is derived from Θ in Eq. (41) by replacing A with $A+BK$. Similarly, we have:

$$E\{F(t)\}\leq E\{\zeta^T(t)\tilde{\Omega}\zeta(t)+\int_{t-h_2}^{t}\eta^T(t,s)\Sigma_1\eta(t,s)ds+\int_{t-h_2}^{t-h_1}\eta^T(t,s)\Sigma_2\eta(t,s)ds\},$$

where $\tilde{\Omega}$ is derived from Ω in Eq. (42) by replacing A with $A+BK$.

Along a similar line to that in the proof of Theorem 1, we can know from Relations (9), (10), (46) and the expression of K in Eq. (47), that $\tilde{\Omega}<0$, $\Sigma_1<0, \Sigma_2<0$ and, therefore, $E\{F(t)\}<0$, which implies that the resulted closed-loop system is stochastically passive with dissipation rate $\gamma > 0$. The proof is complete.

Similar to Section 3, when $h_1 = 0$, Theorem 2 reduces to the following corollary.

Corollary 3. The closed-loop system (48) is stochastically stable and stochastically passive for given $h_2 > 0$, $d > 0$, $h_1 = 0$ and $\gamma > 0$ if there exist matrix Y, positive definite matrices, $X > 0$, $S_i > 0$ $(i = 1, 3, 5), T_1 > 0, Z_1 > 0$, and positive scalars, $\varepsilon_1 > 0, \eta_{kij} > 0$ $(i, j = 1, 2, \cdots, n, k = 1, 2, \cdots, 8, 10), \eta_{kij} > 0$ $(i = 1, 2, \cdots, n, j = 1, 2, \cdots, q, k = 9, 11, 12, 13), \eta_{kij} > 0$ $(i = 1, 2, \cdots, n, j = 1, 2, \cdots, p, k = 14, 15, 16)$, such that Relation (9) and linear matrix inequality (49), as shown in Box XI, hold:

Then, with the stochastically passive controller given by:

$$u(t) = Kx(t), \quad K = YX^{-1},$$

we omit the same matrices expression as Corollary 1.

When the information of the time derivative of delay is zero, that is $h(t) \equiv h$ and $\dot{h}(t) = 0$, by eliminating S_1 and re-arranging some items of Relation (49), we have the following result from Corollary 3.

Corollary 4. The closed-loop system (48) is stochastically stable and stochastically passive for given $d = 0$, $0 \le h \le h_2$ and $\gamma > 0$ if there exist matrix Y, positive definite matrices, $X > 0, S_i > 0\,(i = 3, 5), T_1 > 0, Z_1 > 0$, and positive scalars $\varepsilon_1 > 0, \varepsilon_3 > 0, \eta_{kij} > 0\,(i, j = 1, 2, \cdots, n, k = 1, 2, \cdots, 8, 10), \eta_{kij} > 0\,(i = 1, 2, \cdots, n, j = 1, 2, \cdots, q, k = 9, 11, 12, 13), \eta_{kij} > 0\,(i = 1, 2, \cdots, n, j = 1, 2, \cdots, p, k = 14, 15, 16)$, such that Relation (9) and the linear matrix inequality, shown in Box XII, hold.

Then, with the stochastically passive controller given by:

$$u(t) = Kx(t), K = YX^{-1},$$

we omit the same matrices expression as Corollary 3.

Remark 4. When $h_1 = 0$, Theorem 1 yields Corollary 1 and Theorem 2 provides Corollary 3. When $h(t) = 0$, Theorem 1 results in Corollary 2 and Theorem 2 leads to Corollary 4. The time-invariant system investigated in the above Corollary 2 and Corollary 4 is also considered in [34]. Compared with the free matrix method in [34], our method uses fewer variables while giving less conservative results. This will be discussed in detail in the following section.

$$\begin{bmatrix} \Lambda_1 & A_{10}X + S_5 & 0 & F_4 & XF_1 + Y^TB_0^T & 0 & H^* & 0 & 0 & Y^T & Y^T & Y^T \\ * & -(1-d)S_1 - 2S_5 & S_5 & 0 & XF_2 & 0 & 0 & \tilde{H} & 0 & 0 & 0 & 0 \\ * & * & -S_3 - S_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -J^* & F_3 & B_{10}^T & 0 & 0 & \tilde{H}^* & 0 & 0 & 0 \\ * & * & * & * & -J & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -J_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -U & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -U^* & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\tilde{U}^* & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -U_{14} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -U_{15} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & U_{16} \end{bmatrix} < 0 \tag{49}$$

where:

$$B_0^T = [B_{10}^T B_{10}^T 0], \quad U_k = \mathrm{diag}\{\eta_{k11}, \ldots, \eta_{k1p}, \ldots, \eta_{kn1}, \ldots, \eta_{knp}\}(k = 14, 15, 16),$$

$$J = \mathrm{diag}\{J_1 - \Pi_{15}, J_2 - \Pi_{16}, J_3\}, Y = [\underbrace{Y^T, \ldots, Y^T}_{n}]^T,$$

$$\Pi_{14} = \sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{p} \eta_{14ij} \Delta b_{ij}^2 e_i e_i^T, \quad \Pi_{15} = \sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{p} \eta_{15ij} \Delta b_{ij}^2 e_i e_i^T, \quad \Pi_{16} = \sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{p} \eta_{16ij} \Delta b_{ij}^2 e_i e_i^T,$$

$$\Lambda = A_0X + XA_0^T + B_0Y + Y^TB_0^T + h_1T_1 + S_1 + S_3 - S_4 + \Pi_1 + \Pi_2 + \Pi_{14}$$

Box XI

$$\begin{bmatrix} \tilde{A}_1 & A_{10}X+S_5 & F_4 & XF_1+Y^TB_0^T & 0 & H^* & 0 & 0 & Y^T & Y^T & Y^T \\ * & -S_3-S_5 & 0 & XF_2 & 0 & 0 & \tilde{H} & 0 & 0 & 0 & 0 \\ * & * & -J^* & F_3 & B_{10}^T & 0 & 0 & \tilde{H}^* & 0 & 0 & 0 \\ * & * & * & -J & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -J_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -U & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -U^* & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\tilde{U}^* & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -U_{14} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -U_{15} & 0 \\ * & * & * & * & * & * & * & * & * & * & -U_{16} \end{bmatrix} < 0$$

where:

$$\tilde{\Lambda} = A_0X + XA_0^T + B_0Y + Y^TB_0^T + h_1T_1 + S_3 - S_4 + \Pi_1 + \Pi_2 + \Pi_{14}.$$

Box XII

5. Numerical examples

In this section, we use examples and compare our results with previous ones to show the effectiveness and flexibility of the theory obtained in the previous section.

5.1. Example 1

Consider the stochastic time-delay system with the following parameters:

$$A = \begin{bmatrix} -0.9 & -2.2 \\ 2.2 & -2 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0 \\ -2.3 & 0 \end{bmatrix}, \quad B_1 \begin{bmatrix} 0.99 \\ 1 \end{bmatrix},$$

$$B = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.55 & 0.55 \\ 0.55 & 0.55 \end{bmatrix},$$

$$C = [0 \ \ 2], \quad D = 1.05, \quad E = \alpha I. \qquad (50)$$

In order to compare our results with those in [34], we choose a simple system as in [34]. The results are shown in Tables 1 and 2 in terms of different passivity performance γ.

Case 1. The system in [34] is a special case of the stochastic interval system (1) with interval time-varying delay. If the system matrices are not interval matrix, but known constant matrix, and $h(t) \equiv h$, $\dot{h}(t) = 0$, then, System (1) reduces to the systems as in [34].

Case 2. For $\alpha = -0.45$ and different values of γ, we apply Theorem 1 in [34] and Corollary 2 to calculate the maximal allowable value, h, that guarantees the stochastical passivity of the autonomous system ($B = [0 \ \ 0]^T$), of Eqs. (50). Table 1 illustrates the numerical results for different γ, respectively. It can be seen from

Table 1. Maximum upper bound of h with different values of γ.

$\alpha = -0.45$					
γ	0	0.2	0.5	0.8	1
Theorem 1 in [34]	0.4258	0.435	0.4478	0.4602	0.4678
Corollary 2	0.4934	0.499	0.506	0.5117	0.515

Table 2. Maximum upper bound of h and passive controller gain with different values of γ.

$\alpha = 0.55$			
γ	0	0.5	1
Theorem 2 in [34]	0.6151	0.6158	0.6162
Feed-back gain	[9.4585 − 23.7871]	[9.4294 − 23.8194]	[9.3676 − 23.797]
Corollary 4	0.6749	0.6758	0.6764
Feed-back gain	[−0.6796 − 2.2529]	[−0.6848 − 2.2362]	[−0.6881 − 2.2252]

Table 1 that the maximum allowable delay, h, increases as γ increases. In addition, it is easy to see that our proposed passivity criteria give less conservative results than those in [34].

Case 3. For $\alpha = 0.5$, $\gamma = 0$, the autonomous system of Eqs. (50) is not stochastically passive. According to Corollary 4, we can calculate that the close-loop system (50) is stochastically passive. That is to say, under the passive controller, $u(t) = Kx(t)$, the closed-loop system (50) is stochastically stable and stochastically passive with dissipation, γ.

Table 2 lists the results of the maximum allowable delay bounds and the passive feed-back controller gain derived from Corollary 4 and Theorem 2 in [34]. It is seen from Table 2 that the results obtained from our method are less conservative than those in [34].

5.2. Example 2

To demonstrate the effectiveness and passive control results obtained in this paper, let us consider the following stochastic interval time-varying delay system (1) with:

$$A^M = \begin{bmatrix} -7.9 & 2 \\ 0.1 & 2.1 \end{bmatrix}, \quad A^m = \begin{bmatrix} -8.1 & 2 \\ -0.1 & 1.9 \end{bmatrix},$$

$$B^M = \begin{bmatrix} 4.1 & 0 \\ -2 & 6.1 \end{bmatrix}, \quad B^m = \begin{bmatrix} 3.9 & 0 \\ -2 & 5.9 \end{bmatrix},$$

$$A_1^M = \begin{bmatrix} 3.1 & 0 \\ 0 & 2.1 \end{bmatrix}, \quad A_1^m = \begin{bmatrix} 2.9 & 0 \\ 0. & 1.9 \end{bmatrix},$$

$$B_1^M = \begin{bmatrix} 2.1 & 0 \\ 0 & 3.1 \end{bmatrix}, \quad B_1^m = \begin{bmatrix} 1.9 & 0 \\ 0 & 2.9 \end{bmatrix},$$

$$E^M = \begin{bmatrix} 1.1 & 0.3 \\ 0.2 & 1.7 \end{bmatrix}, \quad E^m = \begin{bmatrix} 0.9 & -0.3 \\ -0.2 & 1.3 \end{bmatrix},$$

$$E_1^M = \begin{bmatrix} 1.1 & 0.1 \\ 0.2 & 2.1 \end{bmatrix}, \quad E_1^m = \begin{bmatrix} 0.9 & -0.1 \\ -0.2 & 1.9 \end{bmatrix},$$

$$C^M = \begin{bmatrix} 1.1 & 0 \\ 0 & -1.4 \end{bmatrix}, \quad C^m = \begin{bmatrix} 0.9 & 0 \\ 0 & -1.8 \end{bmatrix},$$

$$D^M = \begin{bmatrix} 2 & 0 \\ 0 & 3.2 \end{bmatrix}, \quad D^m = \begin{bmatrix} 1.6 & 0 \\ 0 & 2.8 \end{bmatrix}.$$

Suppose we know that $h_1 = 0.2$, $d = 0.2$ and $\gamma = 0.9$. Using Matlab LMI control Toolbox to solve the LMIs (9), (10) and (46), we obtain the maximum allowable bound of the upper time-delay, as $h_2 = 0.5660$. Hence, we have the conclusion that, under the passive feedback controller $u(t) = Kx(t)$, the considered system, with $0.2 \leq h(t) \leq 0.5660$, is stochastically passive.

The solution of the LMIs (9), (10) and (46) in the case of $h_2 = 0.5660$ are given as follows:

$$X = \begin{bmatrix} 0.9077 & -0.0011 \\ -0.0011 & 1.0087 \end{bmatrix}, Z_1 = \begin{bmatrix} 16401 & 1420 \\ 1420 & 16563 \end{bmatrix},$$

$$Z_2 = \begin{bmatrix} 25295 & 2474 \\ 2474 & 25585 \end{bmatrix}, S_1 = \begin{bmatrix} 4.4318 & -0.1549 \\ -0.1549 & 2.0599 \end{bmatrix},$$

$$S_2 = \begin{bmatrix} 0.0130 & -0.0101 \\ -0.0101 & 0.0215 \end{bmatrix}, S_3 = \begin{bmatrix} 0.0135 & -0.0098 \\ -0.0098 & 0.0206 \end{bmatrix},$$

$$S_4 = \begin{bmatrix} 0.0094 & -0.0071 \\ -0.0071 & 0.0158 \end{bmatrix}, S_5 = \begin{bmatrix} 0.0035 & -0.0026 \\ -0.0026 & 0.0060 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} 0.0675 & -0.0532 \\ -0.0532 & 0.1116 \end{bmatrix}, T_2 = \begin{bmatrix} 0.0371 & -0.0291 \\ -0.0291 & 0.0612 \end{bmatrix},$$

$$Y = \begin{bmatrix} -1.0796 & -0.4947 \\ -0.4428 & -2.4298 \end{bmatrix}, K = \begin{bmatrix} -1.1900 & -0.4917 \\ -0.4907 & -2.4094 \end{bmatrix},$$

$$\varepsilon_1 = 31274, \quad \varepsilon_2 = 26820,$$

$$U_1 = \mathrm{diag}(12.5840,\ 26214,\ 16.4602,\ 16.4602),$$

$$U_2 = \mathrm{diag}(13.9995,\ 26231,\ 26231,\ 19.2736),$$

$$U_3 = \mathrm{diag}(26235,\ 26214,\ 26234,\ 26234),$$

$$U_4 = \mathrm{diag}(26217,\ 26214,\ 26216,\ 26216),$$

$$U_5 = \mathrm{diag}(68.7038,\ 26214,\ 90.6871,\ 90.6871),$$

$$U_6 = \mathrm{diag}(1707094,\ 26214,\ 22.7152,\ 22.7152),$$

$$U_7 = \mathrm{diag}(4.1417,\ 1.3654,\ 2.9554,\ 2.9554),$$

$$U_8 = \mathrm{diag}(26252,\ 26231,\ 26231,\ 26252),$$

$$U_9 = \mathrm{diag}(26234,\ 26231,\ 26231,\ 26233),$$

$$U_{10} = \mathrm{diag}(75.1389,\ 26231,\ 26231,\ 98.5753),$$

$$U_{11} = \mathrm{diag}(19.6683,\ 26231,\ 26231,\ 25.1669),$$

$$U_{12} = \mathrm{diag}(4.6222,\ 4.6222,\ 3.2975,\ 6.6675),$$

$$U_{13} = \mathrm{diag}(48.7256,\ 26218,\ 26218,\ 46.5574),$$

$$U_{14} = \mathrm{diag}(46.6156,\ 26214,\ 26214,\ 20.5723),$$

$$U_{15} = \mathrm{diag}(9.0414,\ 26086,\ 26086,\ 8.6126),$$

$$U_{16} = \mathrm{diag}(36.5380,\ 26086,\ 26086,\ 51.9269),$$

$$U_{17} = \mathrm{diag}(7.2950,\ 26086,\ 26086,\ 9.6476),$$

$$U_{18} = \mathrm{diag}(26100,\ 26086,\ 26086,\ 26100),$$

$$U_{19} = \mathrm{diag}(26086,\ 26086,\ 26086,\ 26087),$$

$$U_{20} = \mathrm{diag}(26.2988,\ 26841,\ 26841,\ 34.1442),$$

$$U_{21} = \mathrm{diag}(26862,\ 26841,\ 26841,\ 26861),$$

$$U_{22} = \mathrm{diag}(26844,\ 26841,\ 26841,\ 26843),$$

$$U_{23} = \mathrm{diag}(130.5014,\ 26841,\ 26841,\ 167.3216),$$

$$U_{24} = \mathrm{diag}(36.6324,\ 26841,\ 26841,\ 46.5514).$$

According to Theorem 2, the problem of a passive control for a stochastic interval system with interval time-varying delay is solvable. With the designed controller gain, K, the closed-loop system is stochastically passive.

6. Conclusion

In this paper, the delay-dependent passive control problem has been investigated for stochastic interval systems with interval time-varying delay. The effects of both variable ranges of interval time-varying delay and interval matrices are taken into account. A delay-dependent LMI approach has been developed to derive sufficient conditions under which the corresponding closed-loop system is stochastically stable and stochastically passive with dissipation, γ. Based on these conditions, a memoryless feedback passive controller is proposed. Numerical examples are provided to demonstrate the effectiveness of the results obtained.

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Biographies

Cheng Wang was born in Huanggang, China, in 1980. She received an MS degree from Wuhan University of Technology, China, in 2006, and is now a postdoctoral fellow in the School of Automation at Huazhong University of Science and Technology, and Associate Professor of Huanggang Normal University, China. Her research interests include stability theory and stochastic systems.

Zhongwen Chen was born in Huanggang, China, in 1965. He received his BS degree, in 1987, from Huanggang Normal University, China, his MS degree from Peking University, China, in 1990, and a PhD degree from Wuhan University of Technology, China, in 2011. He is currently Professor in Huanggang Normal University. His current research interests include system control and optimization, and management decision.

Genyu Chen was born in Huanggang, China, in 1993. He is currently an undergraduate student in the School of Materials Science and Engineering at Central South University, China. His current research interests include the stability of the material Processing System.