Scientia Iranica B (2014) 21(2), 347–357

SCIENTIA IRANICA

Sharif University of Technology
Scientia Iranica
Transactions B: Mechanical Engineering
www.scientiairanica.com

Control of nonholonomic mobile manipulators for cooperative object transportation

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Received 30 July 2012; received in revised form 30 June 2013; accepted 4 November 2013

KEYWORDS
Wheeled mobile manipulators;
Cooperativer robots;
Gibbs-Appell method;
Input-output linearization;
Decentralized control;
Virtual structure.

Abstract. In this paper, a methodology for transporting objects with a group of wheeled nonholonomic mobile manipulators is presented. A full dynamic model of a mobile manipulator with a three wheeled mobile base and a three DOF manipulator is derived using the Gibbs-Appell method. Since the dynamical equations of a mobile robot are highly nonlinear, an input-output linearization technique is used to control individual robots. Transporting the object is divided into two steps. First, the robots use a decentralized behavior-based method to approach and surround the object. Then, a virtual structure method is used to control the robots to transport the object cooperatively. A numerical simulation study is performed to show the effectiveness of the control methodology. The results show that robots together are capable of approaching and grasping unknown shapes and can also manipulate objects in various ways.

1. Introduction

Multi-robot systems have been an active research area over the past few years. The main reason for this attention is the ability of these systems to perform tasks which are very difficult or impossible for a single robot to accomplish. Multi-robot systems offer other advantages also, such as increased reliability, robustness against failure and cheaper robot design.

Cooperative object transportation is an important area of research in the field of multi-robot systems. There are, in general, two approaches to transport an object by a group of robots, namely, pushing, and grasping [1].

In the first approach, robots can either push the object from one side (box pushing) [2], or surround the object and transport it by maintaining their formation (caging) [3,4]. Pushing strategies typically require decentralized control methods. In [2], four situations are defined for a box pushing problem and architecture for selecting the proper motion, which includes a situation recognizer and suitable behavior for each situation, is presented. There is no explicit communication between robots in this work. In [3], robots approach and surround the object and search for a situation called object closure, in which the object is trapped between them, and in which they can transport the object by keeping their formation and moving towards the goal. A decentralized behavior-based algorithm for transporting objects in the presence of obstacles is presented in [4]. Robots do not exchange state information and only rely on their own sensory data. Transporting a flexible object is studied in [5], where contact forces are modeled as gradients of nonlinear potential functions, and a decentralized method is used so that the agents and payload reach the same constant velocity.

In grasping approaches [6-11], robots are equipped with grippers or manipulators that can hold the object tightly during transportation. In [6], a general formula-

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tion for Multi Impedance Control is presented for distinct cooperative manipulators. In these approaches, the contact point between the robot gripper and the object is fixed. This method has the advantage of making it possible to control the motion of the object precisely. In [7], a multi-aspect performance index is presented to measure the quality of the grasp during the manipulation. In grasping strategies, all the robots may be given a predefined trajectory by an external computer to follow [8], or they may have a leader that knows the desired trajectory and goal location, and controls the follower robots [9]. A number of papers are devoted to the control of nonholonomic robots for cooperative manipulation [8,10,11]. Optimal control is used in [8] to solve the motion planning problem for different cooperative maneuvers. In [10], potential functions are used to obtain centralized control laws for nonholonomic manipulators handling a deformable object in the presence of obstacles. The leader-follower method is used in [11] for a group of specially designed robots. In this work, follower robots simply keep a constant relative distance with the object using a PI controller. In [12], two-wheeled robots with passive 2 degrees of freedom manipulators are used for cooperative transportation. Knowing the desired path for the object, the desired path for each robot is first calculated. This path is then tracked by the controller of each robot. In [13], an adaptive robust controller is designed for an interconnected system of two robots handling an object in contact with a rigid environment.

In this paper, dynamical equations of a three-wheeled mobile manipulator equipped with a 3 DOF manipulator is first derived using the Gibbs-Appell method. The input-output linearization method is used to design a controller for each individual robot. In almost all previous work on the grasping approach, robots are pre-attached to the object. The Herea decentralized method is proposed for the group of mobile manipulators to approach and grasp the object.

The outline of the paper is as follows: In Section 2, the dynamics model of a mobile manipulator is derived. In Section 3, the controller for each robot is proposed. In Section 4, the approach to cooperative manipulation is addressed. Section 5 presents the simulation results and Section 6 states a few concluding remarks.

2. Dynamic modeling of a mobile manipulator

Most research has used the Lagrangian method to derive the dynamical equations of mobile manipulators [14]. When modeling nonholonomic systems using the Lagrangian approach, it is necessary to use extra variables called Lagrange Multipliers to deal with the constraint forces. This makes the equations more complex than is needed. There are several other approaches to model nonholonomic mechanical systems, such as the Gibbs-Appell method, which results in a simpler form of dynamical equation.

Figure 1 shows the schematic top view of the mobile manipulator addressed here. As seen in the figure, the mobile manipulator consists of a three wheeled mobile platform and a 3 DOF manipulator.

In Figure 1, v is the linear velocity of the center of the front wheel, m_1 is the mass of the mobile base and I_{1c} is the mass moment of inertia of the mobile base about its center of mass, c_1. The fixed coordinate system is denoted by XYZ. $x_1y_1z_1$ is a moving coordinate system attached to the mobile base at point c_1. The position and configuration of the robot in the fixed coordinate system are determined by the point (x, y) and the angle, θ. Figure 2 shows the front

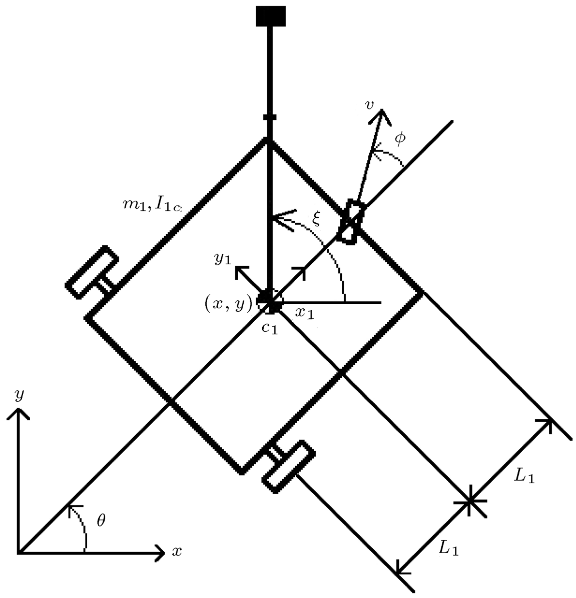

Figure 1. Top view of the mobile manipulator.

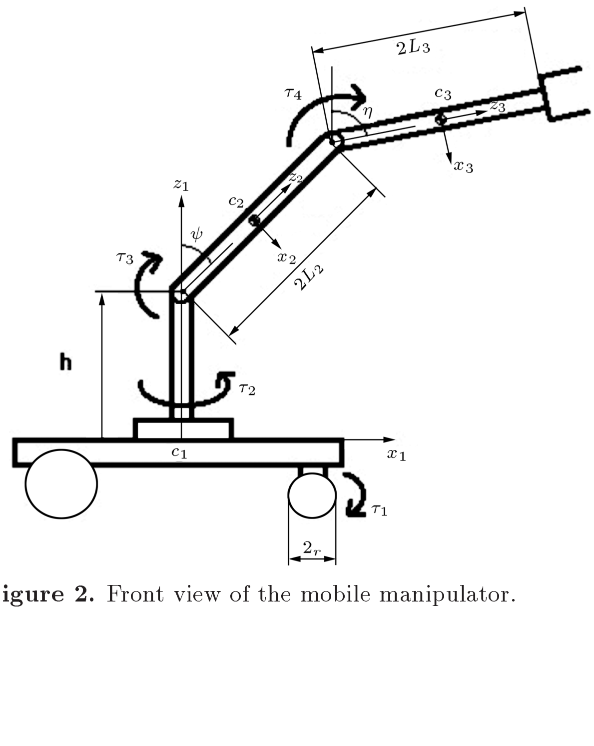

Figure 2. Front view of the mobile manipulator.

view of the robot. Coordinate systems, $x_2y_2z_2$ and $x_3y_3z_3$, are attached to the second and third arm of the manipulator at points, c_2 and c_3, respectively.

Generalized coordinate variables are chosen as $q = [\varphi, x, y, \theta, \xi, \psi, \eta]$. The input torques to the front wheel and manipulator arms are denoted by τ_is. There are two nonholonomic constraints in the location of the front and rear wheels. These constraints prevent lateral slippage of the wheels in the direction parallel to the wheel axis. Note that there are 7 generalized coordinates and 2 constraints. Therefore the system has 5 degrees of freedom, two of which are for the mobile base and three are for the manipulator. Nonholonomic constraints for the two rear wheels are exactly the same, which states that any point on the rear axis of the robot cannot move in the direction of the line that connects the two rear wheels. The equations of these nonholonomic constraints for the front and rear wheels are as follows:

$$\dot{y}\cos(\varphi+\theta) - \dot{x}\sin(\varphi+\theta) + L_1\dot{\theta}\cos(\varphi+\theta) = 0, \quad (1)$$

$$\dot{y}\cos\theta - \dot{x}\sin\theta - L_1\dot{\theta} = 0. \quad (2)$$

Linear and angular velocities of the robot, in terms of the front wheel velocity, are derived as:

$$\dot{x} = v\left(\cos\theta\cos\varphi - \frac{1}{2}(\sin\theta\sin\varphi)\right), \quad (3)$$

$$\dot{y} = v\left(\sin\theta\cos\varphi + \frac{1}{2}(\cos\theta\sin\varphi)\right), \quad (4)$$

$$\dot{\theta} = v\left(\frac{\sin\varphi}{2L_1}\right). \quad (5)$$

In order to use the Gibbs-Appell method, it is necessary to define new variables, called quasi-velocities. Here, quasi-velocities are defined as:

$$u_1 = V, \quad u_2 = \dot{\xi}, \quad u_3 = \dot{\Psi},$$

$$u_4 = \dot{\eta}, \quad u_5 = \dot{\varphi},$$

$$u_6 = \dot{y}cos(\varphi+\theta) - \dot{x}\sin(\varphi+\theta) + L_1\dot{\theta}\cos(\varphi+\theta) = 0,$$

$$u_7 = \dot{y}\cos\theta - \dot{x}\sin\theta + L_1\dot{\theta} = 0. \quad (6)$$

The last two equations are the two constraints of the system, and they are always equal to zero. They should be treated like other quasi velocities, but, since they are always zero, they do not appear in the equations. Note that the first 5 quasi-velocities must be linearly independent. The velocity of each member of the system should be written in terms of these quasi-velocities. This is possible with the aid of Eqs. (3)-(5), as follows:

$$\mathbf{V}_{c_1} = u_1\left(\cos\theta\cos\varphi - \frac{1}{2}(\sin\theta\sin\varphi)\right)\mathbf{i} + u_1\left(\sin\theta\cos\varphi + \frac{1}{2}(\cos\theta\sin\varphi)\right)\mathbf{j}. \quad (7)$$

These velocities are written in the fixed coordinate system. The angular velocities of the system should be written in terms of quasi-velocities in a similar manner.

$$\boldsymbol{\omega}_1 = (u_1\sin\xi)\mathbf{k},$$

$$\boldsymbol{\omega}_2 = (-u_3\sin\xi)\mathbf{i} + (u_3\cos\xi)\mathbf{j} + u_2\mathbf{k},$$

$$\boldsymbol{\omega}_3 = (-u_4\sin\xi)\mathbf{i} + (u_4\cos\xi)\mathbf{j} + u_2\mathbf{k}. \quad (8)$$

In order to find the dynamical equations, the Gibbs-Appell function is defined as follows:

$$S = \frac{1}{2}\sum_{i=1}^{3}(\mathbf{m}_i\dot{\mathbf{V}}_{ci}^2 + \dot{\mathbf{H}}_{ci}.\dot{\boldsymbol{\omega}}_i + \boldsymbol{\omega}_i \times \mathbf{H}_{ci}.\dot{\boldsymbol{\omega}}_i), \quad (9)$$

where, $\dot{\mathbf{H}}_{ci}$ is the angular momentum of the ith member about its center of mass, and it is written in the fixed coordinate frame, XYZ. For each manipulator arm, $\dot{\mathbf{H}}_{ci}$ is calculated by writing the angular momentum of that arm in the coordinate frame attached to it, and using transformation matrices between the coordinate frames. Now, it is possible to obtain the dynamical equations of the mobile manipulator using the Gibbs-Appell equation, $\frac{\partial S}{\partial u_i} = Q_i$, where Q_i's are generalized forces associated with quasi-velocities.

The equations of motion are obtained as:

$$\frac{\partial S}{\partial \dot{u}_1} = \frac{\tau_1}{r} - \frac{\tau_2}{2L_1\sin\varphi},$$

$$\frac{\partial S}{\partial \dot{u}_2} = \tau_2,$$

$$\frac{\partial S}{\partial \dot{u}_3} = \tau_3 - \tau_4 - m_3gL_2\sin\Psi - 2m_3gL_2\sin\Psi,$$

$$\frac{\partial S}{\partial \dot{u}_4} = \tau_4 - m_3gL_3\sin\eta. \quad (10)$$

Note that the inertia of the wheels is ignored. Thus, the equation for u_5 is zero. The equations can be shown in the matrix form as follows:

$$\mathbf{M}(q)\dot{\mathbf{u}} + \mathbf{n}(q,u) = \mathbf{A}_1\boldsymbol{\tau}_1, \quad (11)$$

where $\mathbf{M}$ is a 4×4 matrix, $\mathbf{A}_1$ is a 4×4 matrix, $\mathbf{n}$ is a 4×1 matrix, and $\boldsymbol{\tau}_1 = [\tau_1, \tau_2, \tau_3, \tau_4]^T$.

The equations are written in the state space form by choosing the state vector $\mathbf{X}$ $[V, \dot{\xi}, \dot{\Psi}, \dot{\eta},$

$\varphi, x, y, \theta, \xi, \psi, \eta]^T$, and using the two constraint equations (equations $u_6 = 0$ and $u_7 = 0$), as follows:

$$\begin{bmatrix} M(q) 0_{4\times 7} \\ 0_{7\times 4} I_{7\times 7} \end{bmatrix} \dot{\mathbf{X}} + \mathbf{N}(X) = \mathbf{A}\boldsymbol{\tau}, \tag{12}$$

where $\mathbf{N}$ is an 11×1 matrix, $\mathbf{A}$ is an 11×5 matrix and $\boldsymbol{\tau} = [\tau_1, \tau_2, \tau_3, \tau_4, \tau_5]^T$ is the system imput vector. τ_5 is the rate of change of the steering angle, φ. In fact, the equations are rearranged, so that $\dot{\varphi}$ appears as an input.

3. Controller design

In order to design a controller for an individual robot, the dynamical equations should be written in the so called control affine form as:

$$\dot{\mathbf{X}} = \mathbf{f}(x) + \mathbf{g}(x)\boldsymbol{\tau},$$

$$\mathbf{Y} = \mathbf{h}(X), \tag{13}$$

where:

$$\mathbf{f}(X) = -\begin{bmatrix} M(q) 0_{4\times 7} \\ 0_{7\times 4} \mathbf{I}_{7\times 7} \end{bmatrix}^{-1} \mathbf{N}(X),$$

$$\mathbf{g}(X) = \begin{bmatrix} M(q) 0_{4\times 7} \\ 0_{7\times 4} \mathbf{I}_{7\times 7} \end{bmatrix}^{-1} \mathbf{A}. \tag{14}$$

It is well known that nonholonomic systems, such as mobile robots cannot be stabilized by a continuous state feedback (see, for example [15]). To overcome this problem, both non-smooth [16] and time-varying feedbacks [17] have been proposed. However, for the purpose of transporting objects, it is only necessary to control the position of the end-effector. Therefore, by choosing a proper output vector, it is possible to use the input-output linearization method. The output equation is chosen as:

$$\begin{aligned} \mathbf{Y} =& [x + (L_2 \sin\Psi + L_3 \sin\eta)\cos\xi, y \\ &+ (L_2 \sin\Psi + L_3 \sin\eta)\sin\xi, \xi, \Psi, \eta]^T. \end{aligned} \tag{15}$$

The first two rows in Eq. (15) show the (x, y) coordinates of the end effector. Note that the number of rows of the output must be the same as the number of inputs. The procedure is to differentiate the output equation until the inputs appear. The *Lie derivate* of a scalar function, $h(x)$, with respect to a vector field, $\mathbf{f}(x)$, is defined as:

$$L_f h = \nabla h \mathbf{f}. \tag{16}$$

Suppose that r_i is the smallest number of differentiations for the ith row of the output vector until, at least, one of the inputs appear. Differentiating r_i times yields:

$$y_i^{(r_i)} = L_f^{r_i} h_i + \sum_{j=1}^{m} L_{gj} L_f^{r_i - 1} h_i \tau_i. \tag{17}$$

So that, for at least one j, we have $L_{gj} L_f^{r_i-1} h_i(x) \neq 0$. In Eq. (17), m is the number of rows, and y_i is the ith member of the output vector. Writing the equations in the matrix form yields:

$$\begin{bmatrix} y_1^{(r_1)} \\ \cdots \\ \cdots \\ y_i^{(r_m)} \end{bmatrix} = \begin{bmatrix} L_f^{r_1} h_1(X) \\ \cdots \\ \cdots \\ L_f^{r_m} h_m(X) \end{bmatrix} + \mathbf{E}(\mathbf{X}). \tag{18}$$

In Eq. (18), $\mathbf{E}$ is called the decoupling matrix, and it is defined obviously. Nonsingularity of matrix $\mathbf{E}$ is the necessary condition to use the input-output linearization method. Using an input transformation:

$$\boldsymbol{\tau} = \mathbf{E}^{-1} \begin{bmatrix} v_1 - L_f^{r_1} h_1 \\ \cdots \\ \cdots \\ v_m - L_f^{r_m} h_m \end{bmatrix} \tag{19}$$

yields m equations of the simple form:

$$y_i^{(r_i)} = v_i \quad (i = 1, ..., m), \tag{20}$$

where v_i is a new artificial input. When $u_1 = v$ is equal to zero, matrix E becomes singular. As a result, the control algorithm must prevent V from becoming absolute zero. This can be done by resetting the value of v to a small threshold when it falls below that threshold. In the case of the mobile manipulator studied here, differentiating each row of the output vector (Eq. (15)) twice and using an input transformation similar to Eq. (19), with $r_i = 2$, results in a simple equation of the form:

$$y_i = v_i \quad (i = 1, ..., 5). \tag{21}$$

Now, using a control law of the form:

$$v_i = y_{id} - 2k[\dot{y}_i - \dot{y}_{id}] - k^2[y_i - y_{id}], \tag{22}$$

where k is an arbitrary constant, leads to exponentially stable dynamics for the tracking error ($e = y_i - y_{id}$). In Eq. (22), k determines the location of the poles of the transformed system. Increasing k increases the distance of system poles from the origin, thus, increasing the response speed of the system. As a result, choosing a desired k depends on the desired response properties of the system, and it is done by trial and error. In other words, it is possible to control the end effector to track a desired path asymptotically. The actual torque inputs are calculated using Eq. (19), with $r_i = 2$. A complete discussion of the input-output linearization method is found in [18].

4. Cooperative control of robots

The task of transporting an object is divided into two parts. First, robots should approach and surround the object. In other words, they should make a proper formation around the object. The second part is to maintain the formation tightly to transport the object. There are several methodologies to control the formation of mobile robots. The main formation control frameworks are the behavior based, potential fields, leader-follower, graph-theoretic, and virtual structure approaches. In behavior-based approaches [14,19], each robot has several desired behaviors, and the control action of each member in a particular situation is a suitable combination of these behaviors. Potential fields are often used to generate formations by forcing the members to maintain relative distances with each other [20,21]. In leader-follower methods, some robots are designated as leaders and others are designated as followers, and follower robots try to maintain a desired distance from their leader [22,23]. Control graphs are sometimes used in formation control methodologies to determine robot behavior [24]. Finally, in the virtual structure method, the formation is considered to be a rigid body. This rigid structure is then moved in a desired manner, and the robots are controlled to track their positions within the structure [25,26].

Here, a behavior-based method for the first step of transportation (approaching and grasping the object) is used. It is assumed that each robot has sensors that can obtain the position of the closest point of the object, in a non-rotating coordinate frame, attached to its mobile base. Therefore, it is possible to obtain $y-y_d$ and $x-x_d$ in the control law (Eq. (22)) at any moment. Clearly, the position of the global coordinate frame does not affect the output of the controller, since only the relative distance of the robot and desired position appears in Eq. (22).

If $\dot{y}_d = \dot{x}_d = y_d = x_d = 0$, the robot can move toward the object using the controller designed in the previous section. Robots should be able to avoid obstacles while they are moving toward their goal position. Figure 3 shows the mobile robot encountering an obstacle. In this situation, if the distance between the robot and the obstacle becomes less than r, the desired point for the robot to follow (y_d and x_d) will be switched from the goal position to a point located at distance a from the current location of the robot, in the direction tangent to a circle drawn from the obstacle with radius of r (point 1 in Figure 3). Eq. (23) shows the new coordinate of the desired point that the robot should follow when facing an obstacle:

$$x_d = -a\sin\beta, \quad y_d = a\cos\beta. \tag{23}$$

When the robot comes out of the circle, the desired point will be switched back to the goal position. In

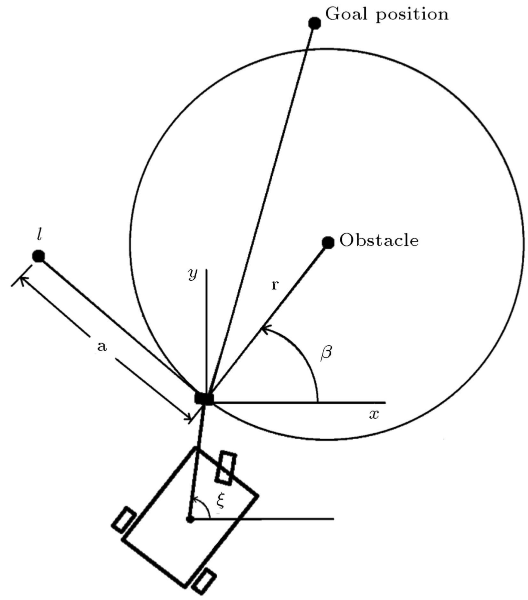

Figure 3. Robot avoiding an obstacle.

the meantime, the ξ angle of the arm should converge to the angle β in Figure 3. The robot arm moves in such a way that the base of the robot moves away from the obstacle. However, the safe radius, r, in which the robot detects the obstacle, is chosen large enough to prevent colliding with the base in any situation. In this way, the robot can find its way around the obstacle toward the goal position. Other non-point obstacles could be made up by putting together a number of point obstacles. The robot considers the closest point of the obstacle for its calculations. By using this method, robots can pass an obstacle by moving parallel to its perimeter.

The first step for transporting an object by a group of mobile manipulators is to approach and surround it. A decentralize control method is used for this step. It is assumed that the shape of the object is not known. It is only necessary to know the relative size of the object to decide how many robots are needed to surround the object properly. One of the robots is considered to be the leader of the group. Figure 4 shows the leader with a follower near the object. The leader approaches the closest point of the object. Equations for the desired point for the leader to follow are:

$$X_{d\_\text{Leader}} = X_{\text{Closest point of the object}},$$

$$y_{d\_\text{Leader}} = y_{\text{Closest point of the object}}. \tag{24}$$

The follower also approaches the object to a specific distance, then starts to move parallel to the perimeter of the object until its end effector is located at distance R from the leader's end effector. Since the robots initially approach the object and then start to move parallel to its perimeter, they do not have to know the shape of the object in advance. As a result, they can

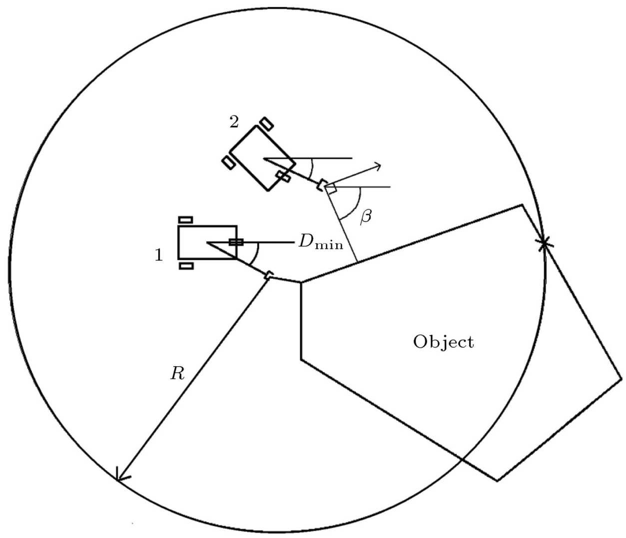

Figure 4. Two robots approaching the object.

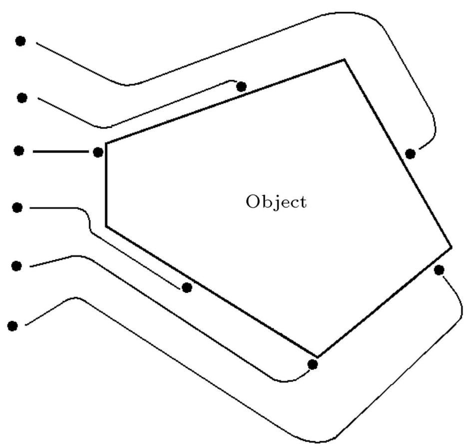

Figure 5. Surrounding the object with a group of robots.

surround object with unknown shapes. Equations of the desired point for the follower robots are:

If "Distance from object" > $D_{\min}$

$$X_{d\_\text{follower}} = X_{\text{Closest point of the object to the follower}},$$

$$y_{d\_\text{follower}} = y_{\text{Closest point of the object to the follower}}.$$

Else:

$$X_{d\_\text{follower}} = -a\sin\beta, \quad y_{d\_\text{follower}} = a\cos\beta. \tag{25}$$

The angles of the robot arms are controlled to become perpendicular to the perimeter of the object, as they move to their final positions.

Other follower robots can be added, as shown in Figure 5. In this way, the group can surround the object by any suitable number of robots.

The second step for transporting the object is maintaining the formation generated in the previous step. The formation must be solid during the motion.

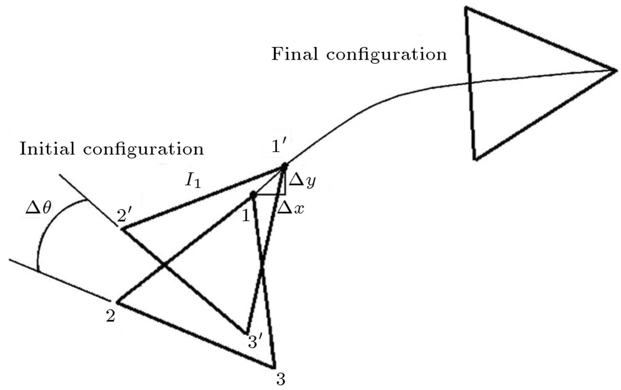

Figure 6. Virtual structure generated by three robots.

For this purpose, a virtual structure method is used. In this method, the initial formation is treated as a solid body. It is assumed that the leader robot knows the goal position and orientation of the object. Therefore, in each time step, the leader robot moves a specific distance towards the goal position. The initial formation or the virtual structure is also moved, according to the leader's motion. Figure 6 shows the virtual structure generated by three robots and moved parallel to the ground. Points 1, 2 and 3 show the positions where the end effectors of the robots grasp the object. A non-rotating coordinate frame is attached to the end effector of the leader and it is assumed that the followers can obtain their positions in this coordinate frame. As the virtual structure moves, the new positions of the end effectors of the followers are calculated and transmitted to each follower by the leader. Then, the follower robots move toward their new positions. The following equations are used to calculate the desired position of the leader and followers during the transportation of the object, for the leader and follower number 2:

$$x_{d\_\text{Leader}} = \Delta x, \quad y_{d\_\text{Leader}} = \Delta y,$$

$$\xi_{d\_\text{Leader}} = \xi_0 - \Delta\theta,$$

$$(\xi_0 \text{ is the initial value of } \xi \text{ of the Leader}) \tag{26}$$

$$x_{d\_\text{follower2}} = \Delta x + l_1\cos(\theta + \Delta\theta),$$

$$y_{d\_\text{follower2}} = \Delta y + l_1\sin(\theta + \Delta\theta). \tag{27}$$

The angles of the robot arms should also change, according to the motion of the virtual structure.

5. Simulation results

In this section, the simulation results for different group tasks are presented. First, the task of approaching the object by a group of robots is considered (Table 1 shows the dimensions and other parameters of the robot). For

Table 1. Parameters of the robot

Parameter	Value
m_1	5 kg
m_2	2 kg
m_3	2 kg
$L_1 = L_2 = L_3$	40cm
r	5 cm
h	30 cm
I_{c1}	0.1 kg-m^2

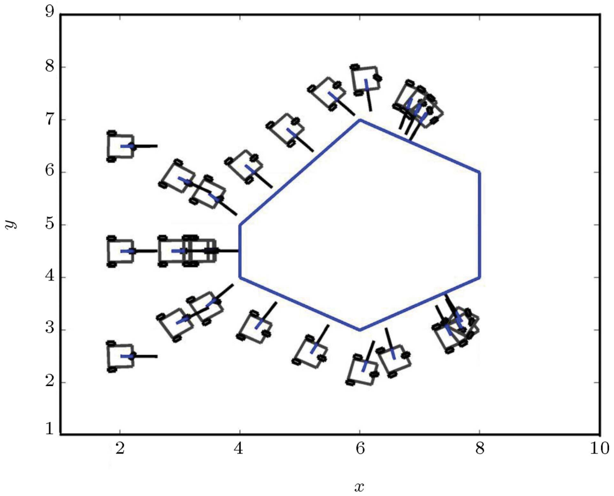

Figure 7. Simulation results for three robots approaching and surrounding a polygon.

the first simulation, approaching an arbitrary polygon object with three robots is considered. Figure 7 shows the simulation results for three robots approaching and surrounding the object. According to this figure, at the beginning of the motion, each robot moves towards the closest point of the object. The follower robots, 2 and 3, then move parallel to the polygon edge until their end effectors reach a specific distance from the leader's end effector.

Figure 8 shows the control inputs for the task of approaching and surrounding the object. The two other inputs, τ_3 and τ_4, are constant in this operation. Figure 9 shows the front wheel velocity (v) and steering angle (φ) of the robots.

For the the second simulation, transporting an object parallel to the ground with three robots is considered. Figure 10 shows the task of transporting the object, with three robots making a triangular formation. The triangle depicted in the figure is the virtual structure. Robot 1 is the leader and the other robots are followers. In each time step, the contact point of robot 1 and the virtual structure is displaced 1.5 cm in x and y directions, and the virtual structure is rotated 0.5 degrees. The elevation of the object is considered to be constant in this task.

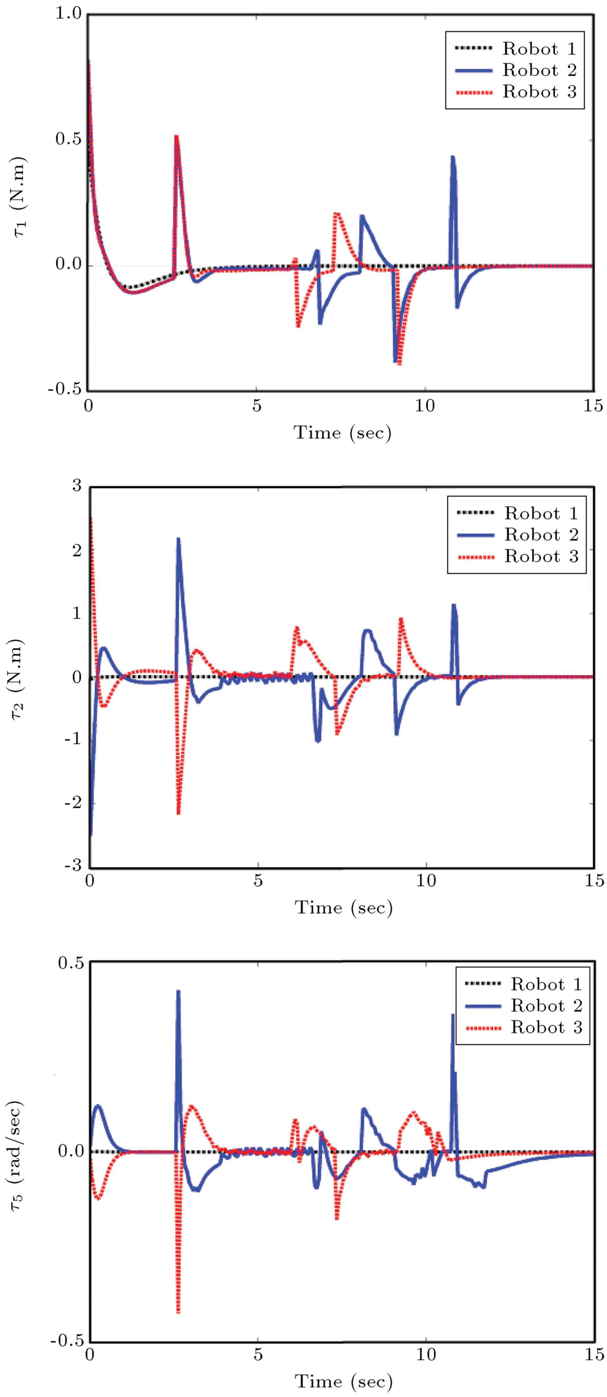

Figure 8. Control inputs for approaching and surrounding the object.

The control inputs for the task of transporting the object parallel to the ground are shown in Figure 11. The front wheel velocities and steering angles of the robots for this task are shown in Figure 12.

In the previous example, the object was transported parallel to the ground, but, it is possible to manipulate the object in any desired manner. As an example, the task of moving a triangle from a horizontal to a vertical position is simulated here.

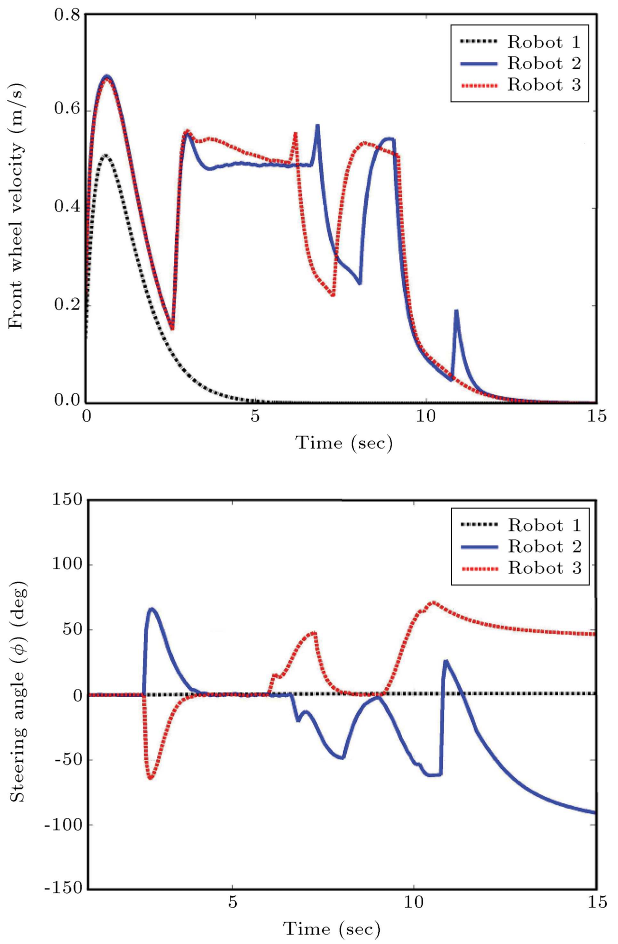

Figure 9. Front wheel velocity (v) and steering angle (φ) for approaching and surrounding the object.

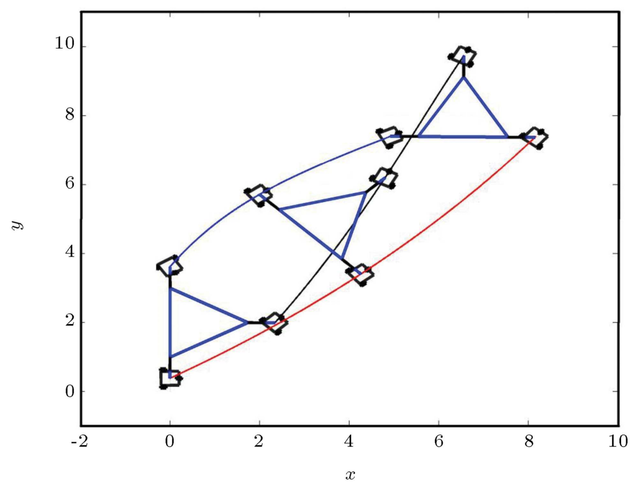

Figure 10. Transporting the object parallel to the ground with three robots.

Figure 13 shows the sequence of this operation by three robots. The robot which raises the edge of the triangle in Figure 13 is the leader, which knows the desired motion of the object. The leader calculates the position of each robot's end effector to produce the

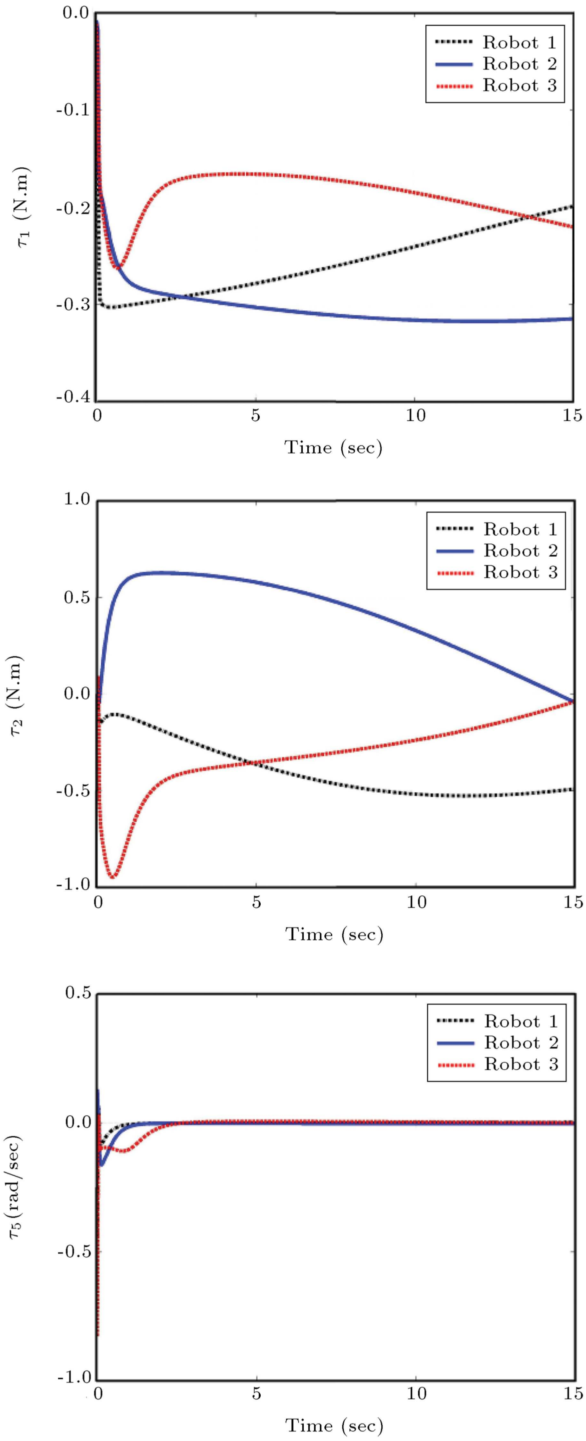

Figure 11. Control inputs for transporting the object.

desired motion, and transmits this information to each follower.

The method presented here for approaching and transporting the object is scalable, i.e., the number of robots could be increased without affecting the performance of the method. To show this, a simulation with five robots is undertaken. The first part of Figure 14 shows the robots approaching and surrounding

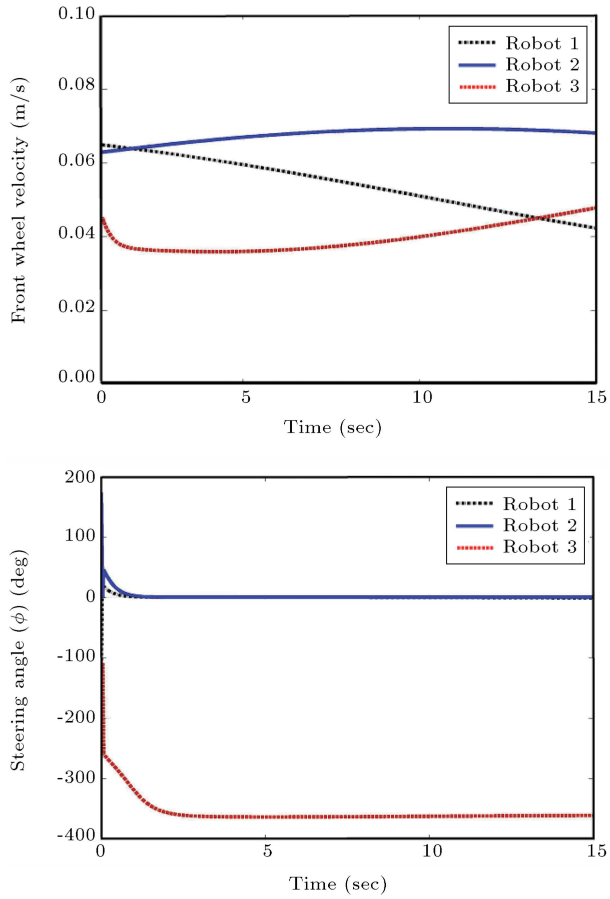

Figure 12. Front wheel velocity and steering angle for transporting the object.

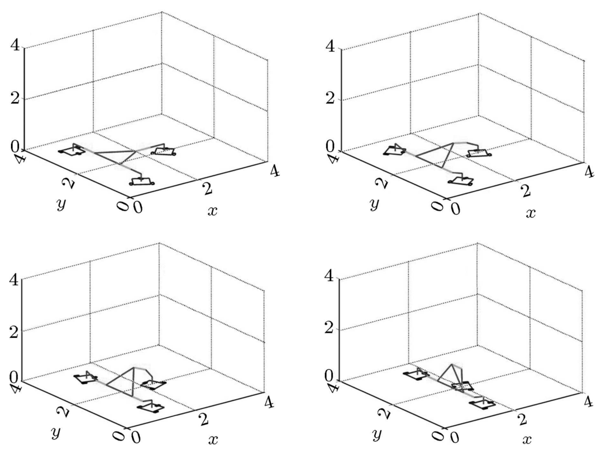

Figure 13. Moving an object from horizontal position to vertical position.

a circular object, and the second part shows the robots moving the object to the goal position. It is seen that the task is preformed successfully with five robots.

6. Conclusions

In this paper, a control methodology for transporting an object with a group of nonholonomic mobile manipulators was presented. The dynamic model of the robot was derived using the Gibbs-Appell method. Input-output linearization was applied to control the individual robots. Simulations show the stability of the internal dynamics of the control system, and, therefore, the controller has an acceptable performance. The method by which the robots approach an object makes it possible to grasp objects with various shapes without any initial information about the shape and location of the object. Using the virtual structure method for transporting and manipulating objects, it is possible to perform different tasks with a very simple algorithm.

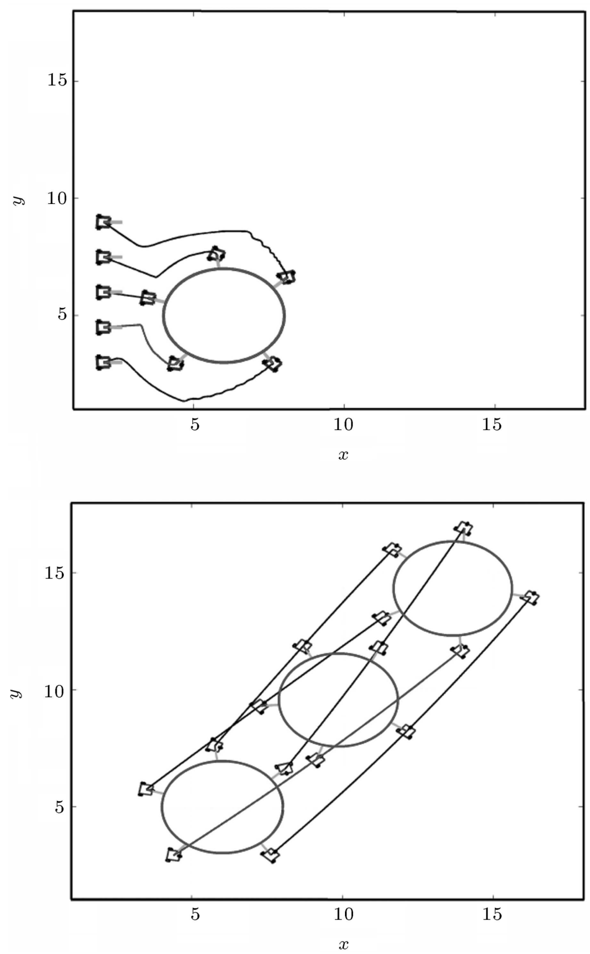

Figure 14. Grasping and transporting a circular object.

In this paper, unlike many previous works, the dynamics of the mobile manipulator is included. However, the mass or dynamics of the object are neglected. Future work includes adding the inertia effects of the object to the study.

References

1. Gross, R., Mondada, F. and Dorigo, M. "Transport of an object by six pre-attached robots interacting via physical links", *IEEE International Conference*

on Robotics and Automation, Orlando, FL, USA, pp. 1317-1323 (2006).

2. Yamada, S. and Saito, J. "Adaptive action selection without explicit communication for multi-robot box-pushing", *IEEE Transactions on Systems, Man, and Cybernetics, Part C: Applications and Reviews*, **31**(3), pp. 398-404 (2001).

3. Wang, Z.D. and Kumar, V. "Object closure and manipulation by multiple cooperating mobile robots", *IEEE International Conference on Robotics and Automation*, pp. 394-399 (2002).

4. Fink, M., Hsieh, A. and Kumar, V. "Multi-robot manipulation via caging in environments with obstacles", *IEEE International Conference on Robotics and Automation*, pp. 19-23 (2008).

5. Bai, H. and Wen, J.T. "Cooperative load transport: A formation-control perspective", *IEEE Transactions on Robotics*, **26**(4), pp. 742-750 (2010).

6. Moosavian, S.A.A. and Papadopoulos, E. "Cooperative object manipulation with contact impact using multiple impedance control", *International Journal of Control, Automation, and Systems*, **8**(2), pp. 314-327 (March 2010).

7. Cheraghpour, F., Moosavian, S.A.A., and Nahvi, A. "Multi-aspect grasp index for robotic arms", *International Journal of Science & Technology, Scientia Iranica*, Sharif University of Technology, **18**(2), pp. 222-230 (April 2011).

8. Desai, J., Wang, C.C., Zefran, M. and Kumar, V. "Motion planning for multiple mobile manipulators", *IEEE International Conference on Robotics and Automation*, pp. 2073-2078 (1996).

9. Sugar, T.G. and Kumar, V. "Control of cooperating mobile manipulators", *IEEE Transactions on Robotics and Automation*, **18**(1), pp. 94-103 (2002).

10. Tanner, H.G. and Loizou, S.G. "Nonholonomic navigation and control of cooperating mobile manipulators", *IEEE Transactions on Robotics and Automation*, **19**(1), pp. 53-64 (2003).

11. Yang, X., Watanabe, K., Izumib, K. and Kiguchib, K. "A decentralized control system for cooperative transportation by multiple nonholonomic mobile robots", *International Journal of Control*, **77**(10), pp. 949-963 (2004).

12. Abou-Samah, M., Tang, C.P., Bhatt, R.M. and Krovi, V. "A kinematically compatible framework for cooperative payload transport by nonholonomic mobile manipulators", *Autonomous Robots*, **21**(3), pp. 227-242 (2006).

13. Li, Z. Li, J. and Kang, Y. "Adaptive robust coordinated control of multiple mobile manipulators interacting with rigid environments", *Automatica*, **46**(12), pp. 2028-2034 (2010).

14. Antonelli, G. Arrichiello, F. Chakraborti, S. and Chiaverini, S. "Experiences of formation control of multi-robot systems with the Null-space-based behavioral control", *IEEE International Conference on Robotics and Automation*, pp. 1068-1073 (2007).

15. Samson, C. "Control of chained systems application to path following and time-varying point-stabilization of mobile robots", *IEEE Transactions on Automatic Control*, **40**(1), pp. 64-77 (1995).

16. Canudas, C. and Socdrdalen, J. "Exponential stabilization of mobile robots with nonholonomic constraints", *IEEE Transactions on Automatic Control*, **37**(11), pp. 1791-1797 (1991).

17. Samson, C. "Velocity and torque feedback control of a nonholonomic cart", *International Workshop on Adaptive and Nonlinear Control: Issues in Robotics*, pp. 125-151 (1990).

18. Slotine, J.J. and Li, W. *Applied Nonlinear Control*, pp. 207-271, Prentice-Hall (1991).

19. Balch, T. and Arkin, R.C. "Behavior-based formation control for multirobot teams", *IEEE Transactions on Robotics and Automation*, **14**(6), pp. 926-939 (1998).

20. Cao, Z., Xie, L., Zhang, B., Wang, S. and Tan, M. "Formation constrained multi-robot system in unknown environments", *Proceedings of IEEE International Conference on Robotics and Automation*, pp. 735-740 (2003).

21. Reif, J.H. and Wang, H. "Social potential fields: A distributed behavioral control for autonomous robots", *Proceedings of the Workshop on Algorithmic Foundations of Robotics*, pp. 331-345 (1995).

22. Leonard, N.E. and Fiorelli, E. "Virtual leaders, artificial potentials and coordinated control of groups", *40th IEEE Conference on Decision and Control*, pp. 2968-2973 (2001).

23. Fredslund, J. and Mataric, M.J. "A general algorithm for robot formations using local sensing and minimal communication", *IEEE Transactions on Robotics and Automation*, **18**(5), pp. 837-846 (2002).

24. Desai, J.P. "Modeling multiple teams of mobile robots: A graph theoretic approach", *IEEE/RSJ International Conference on Intelligent Robots and Systems*, pp. 381-386 (2001).

25. Tan, K.H. and Lewis, M.A. "Virtual structures for high-precision cooperative mobile robotic control", *IEEE/RSJ International Conference on Intelligent Robots and Systems*, pp. 132-139 (1996).

26. Lewis, M.A. and Tan, K.H. "High precision formation control of mobile robots using virtual structures", *Autonomous Robots*, **4**(4), pp. 387-403 (1997).

Appendix

Matrices in Eq. (11) have the following form:

$$A_1 = \begin{bmatrix} \frac{1}{r} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} M_{1,1} & M_{1,2} & M_{1,3} & M_{1,4} \\ M_{2,1} & M_{2,2} & 0 & 0 \\ M_{3,1} & 0 & M_{3,3} & M_{3,4} \\ M_{4,1} & 0 & M_{4,3} & M_{4,4} \end{bmatrix}$$

$$n = \begin{bmatrix} n_{1,1} \\ \vdots \\ n_{1,4} \end{bmatrix}$$

where $M_{i,j}$'s and $n_{i,j}$'s are functions of the system generalized coordinates.

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