Scientia Iranica B (2014) 21(1), 203–212

Sharif University of Technology
Scientia Iranica
Transactions B: Mechanical Engineering
www.scientiairanica.com

Magnetohydrodynamic flow in a permeable channel filled with nanofluid

M. Sheikholeslami[a,b] and D.D. Ganji[a,*]

a. *Department of Mechanical Engineering, Babol University of Technology, Babol, P.O. Box 484, Iran.*
b. *Department of Mechanical Engineering, University of Mazandaran, Babolsar, Iran.*

Received 8 October 2012; received in revised form 21 March 2013; accepted 21 September 2013

KEYWORDS
Nanofluid;
Laminar flow;
Permeable channel;
Uniform magnetic;
Optimal Homotopy Asymptotic Method (OHAM);
Galerkin Method (GM).

Abstract. In this paper, the problem of laminar magnetohydrodynamic nanofluid flow in a porous channel is investigated. The Optimal Homotopy Asymptotic Method (OHAM) is used to solve this problem. In order to lessen CPU time, the Galerkin method is used to minimize the residual. This investigation was compared with a numerical method (fourth-order Rungekutte method) and found to be in excellent agreement. The base fluid in the channel is water containing copper as a nanoparticle. The effective thermal conductivity and viscosity of nanofluid are calculated by the Maxwell–Garnetts (MG) and Brinkman models, respectively. The influence of the three dimensionless numbers: The nanofluid volume fraction, Hartmann number and Reynolds number, are examined. The results indicate that velocity boundary layer thickness decreases with an increase in Reynolds number and nanoparticle volume fraction, and increases as Hartmann number increases.

1. Introduction

Recently, due to the rising demands of modern technology, including chemical production, power stations, and microelectronics, there is a need to develop new types of fluid that will be more effective in terms of heat exchange performance. Nanofluids are produced by dispersing the nanometer-scale solid particles into base liquids with low thermal conductivity, such as water, Ethylene Glycol (EG), oils, etc. [1]. Khanafer et al. [2] firstly conducted a numerical investigation on heat transfer enhancement by adding nanoparticles in a differentially heated enclosure. They found that the suspended nanoparticles substantially increase the heat transfer rate at any given Grashof number. Squeezing unsteady nanofluid flow and heat transfer has been studied by Sheikholeslami et al. [3]. They showed that for a case in which two plates are moving together, the Nusselt number increases with an increase in the nanoparticle volume fraction and Eckert number, while it decreases with the growth of the squeeze number. Ashorynejad et al. [4] studied the flow and heat transfer of a nanofluid over a stretching cylinder in the presence of a magnetic field. They found that choosing copper (for small values of magnetic parameter) and alumina (for large values of magnetic parameter) leads to the highest cooling performance for this problem. The effect of a static radial magnetic field on natural convection heat transfer in a horizontal cylindrical annulus enclosure filled with nanofluid was investigated numerically by Ashorynejad et al. using the Lattice Boltzmann method [5]. They found that the average Nusselt number increases as nanoparticle volume fraction and Rayleigh number increase, while it decreases as Hartmann number increases. Sheikholeslami et al. [6] used CVFEM to simulate the effect of a magnetic field on natural convection in an inclined half-annulus enclosure filled with Cu-water

*. *Corresponding author. Tel.: +98 911 114 9475*
E-mail addresses: ddg_davood@yahoo.com (D.D. Ganji); mohsen.sheikholeslami@yahoo.com (M. Sheikholeslami)

nanofluid. Their results indicated that Hartmann number and the inclination angle of the enclosure can be considered control parameters at different Rayleigh number. Several numerical studies have been published on the modeling of natural convection heat transfer using nanofluid [7-20].

The flow problem in porous tubes or channels has received considerable attention in recent years because of its various applications in biomedical engineering; for example in the dialysis of blood in artificial kidneys, in the flow of blood in capillaries and blood oxygenators, as well as in many other engineering areas, such as the design of filters in transpiration cooling boundary layer control and gaseous diffusion. Chandran et al. [21] analyzed the effects of a magnetic field on thermodynamic flow past a continuously moving porous plate.

At the heart of different engineering sciences, most mathematical problems are modeled by ordinary or partial differential equations. There are limitations using the common perturbation method, as the basis of this method depends upon the existence of a small parameter, so, developing this method for different applications is very difficult. Recently, different methods have been introduced to eliminate the small parameter, including the homotopy perturbation method [22-25], differential transformation method [26-28], homotopy analysis method [29-32] and adomian decomposition method [33].

The Optimal Homotopy Asymptotic Method (OHAM) is a powerful method for solving nonlinear problems without depending on the small parameter. Hashemi et al. [34] used OHAM for finding the approximate solutions of a class of Volterra integral equations with weakly singular kernels. They showed that OHAM is a reliable and efficient technique for finding the solutions of weakly singular Volterra integral equations. The application of OHAM to other types of problem was introduced in [35-38]. In order to reach OHAM's constant parameters, we can use three methods: Least Square Method (LSM), Collocation Method (CM) and Galerkin Method (GM). Generally, authors used LSM in order to reach these constant parameters. The use of the least Square Method (LSM) in minimizing the residual of OHAM might increase the CPU execution time of the algorithm. So, some authors, like Ganji et al. [39], use the Galerkin technique for minimizing the residual of OHAM. Recently, several papers were published about application of new analytical and numerical methods [40-45].

In this study, laminar nanofluid flow in a semi-porous channel in the presence of a magnetic field is studied using OHAM. In order to obtain minimum residual, the Galerkin method is used. Effects of nanoparticle volume fraction, Reynolds number and Hartmann number on flow have been examined.

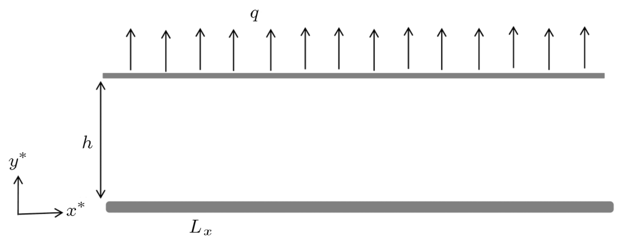

Figure 1. Schematic diagram of the system.

2. Problem statement and mathematical formulation

The laminar two-dimensional stationary nanofluid flow in a semi-porous channel, made by a long rectangular plate with length L_x, in uniform translation in an x^*direction and an infinite porous plate, is considered. The distance between the two plates is h. We observe a normal velocity, q, on the porous wall. A uniform magnetic field, B, is assumed to be applied towards direction y^* (Figure 1). In the case of a short circuit, to neglect the electrical field and perturbations to the basic normal field and without any gravity forces, the governing equations are:

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0, \tag{1}$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{1}{\rho_{nf}}\frac{\partial P^*}{\partial x^*}+\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)-u^*\frac{\sigma_{nf}B^2}{\rho_{nf}}, \tag{2}$$

$$u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{1}{\rho_{nf}}\frac{\partial P^*}{\partial y^*}+\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right). \tag{3}$$

The appropriate boundary conditions for the velocity are:

$$y^*=0: \quad u^*=u_0^*, \quad v^*=0, \tag{4}$$

$$y^*=h: u^*=0, \quad v^*=-q. \tag{5}$$

Calculating mean velocity U by the relation:

$$y^*=0: \quad u^*=u_0^*, \quad v^*=0. \tag{6}$$

We consider the following transformations:

$$x\frac{x^*}{L_x}; \quad y=\frac{y^*}{y}, \tag{7}$$

$$u=\frac{u^*}{U}; \quad v=\frac{v^*}{q}, \quad P_y=\frac{p^*}{\rho_f.q^2}. \tag{8}$$

Then, we can consider two dimensionless numbers: The Hartman number, Ha, for the description of magnetic forces and the Reynolds number, Re, for dynamic forces:

$$\mathrm{Ha} = B^* h \sqrt{\frac{\sigma_f}{\rho_f . \upsilon_f}}, \tag{9}$$

$$\mathrm{Re} = \frac{hq}{\mu_{nf}} \rho_{nf}, \tag{10}$$

where the effective density (ρ_{nf}) is defined as [2]:

$$\rho_{nf} = \rho_f (1-\phi) + \rho_s \phi, \tag{11}$$

where ϕ is the solid volume fraction of nanoparticles. The dynamic viscosity of the nanofluids given by Brinkman [2] is:

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \tag{12}$$

the effective thermal conductivity of the nanofluid can be approximated by the Maxwell–Garnetts (MG) model as [2]:

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_s)}{k_s + 2k_f + \phi(k_f - k_s)}. \tag{13}$$

The effective electrical conductivity of nanofluid was presented by Maxwell [9] as below:

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3\left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) - \left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi}. \tag{14}$$

The thermo physical properties of the nanofluid are given in Table 1 [2]. Substituting Eqs. (6) and (10) into Eqs. (1) and (3) leads to the dimensionless equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{15}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\varepsilon^2 \frac{\partial P_y}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \frac{1}{hq}\left(\varepsilon^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - u\frac{\mathrm{Ha}^2}{\mathrm{Re}} \frac{B^*}{A^*}, \tag{16}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial P_y}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \frac{1}{hq}\left(\varepsilon^2 \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right), \tag{17}$$

where A^* and B^* are constant parameters:

$$A^* = (1-\phi) + \frac{\rho_s}{\rho_f}\phi,$$

$$B^* = 1 + \frac{3\left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) - \left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi}. \tag{18}$$

Table 1. Thermo physical properties of water and nanoparticles [9].

	ρ (kg/m^3)	k(W/m.k)	$\sigma(\Omega^{-1}\mathrm{m}^{-1})$
Pure water	997.1	0.613	0.05
Copper (Cu)	8933	401	5.96×10^7

The quantity of ε is defined as the aspect ratio between distance h and a characteristic length, L_x, of the slider. This ratio is normally small. Berman's similarity transformation is used to be free from the aspect ratio of ε:

$$v = -V(y); \quad u = \frac{u^*}{U} = u_0 U(y) + x\frac{dV}{dy}. \tag{19}$$

Introducing Eq. (19) in the second momentum equation (Eq. (17)) shows that quantity, $\partial P_y/\partial y$, does not depend on the longitudinal variable, x. With the first momentum equation, we also observe that $\partial^2 P_y/\partial x^2$ is independent of x. We omit asterisks for simplicity. Then, a separation of variables leads to:

$$V'^2 - VV'' - \frac{1}{\mathrm{Re}}\frac{1}{A^*(1-\phi)^{2.5}}V''' + \frac{\mathrm{Ha}^2}{\mathrm{Re}}\frac{B^*}{A^*}V' = \varepsilon^2 \frac{\partial^2 P_y}{\partial x^2} = \varepsilon^2 \frac{1}{x}\frac{\partial P_y}{\partial x}, \tag{20}$$

$$UV' - VU' = \frac{1}{\mathrm{Re}}\frac{1}{A^*(1-\phi)^{2.5}}[U'' - \mathrm{Ha}^2 B^*(1-\phi)^{2.5} U]. \tag{21}$$

The right-hand side of Eq. (20) is constant. So, we derive this equation with respect to x. This gives:

$$V^{IV} = \mathrm{Ha}^2 B^*(1-\phi)^{2.5} V'' + \mathrm{Re} A^*(1-\phi)^{2.5}[V'V'' - VV'''], \tag{22}$$

where primes denote differentiation with respect to y, and asterisks are omitted for simplicity. The dynamic boundary conditions are:

$$y = 0: \quad U = 1; \quad V = 0; \quad V' = 0, \tag{23}$$

$$y = 1: \quad U = 0; \quad V = 1; \quad V' = 0. \tag{24}$$

3. Fundamentals of optimal homotopy asymptotic method

The following differential equation is considered:

$$L(u(t)) + N(u(t)) + g(t) + g(t) = 0, \qquad B(u) = 0, \tag{25}$$

where L is a linear operator, τ is an independent variable, $u(t)$ is an unknown function, $g(t)$ is a known function, $N(u(\mathrm{t}))$ is a nonlinear operator and B is a boundary operator. By means of OHAM, one first constructs a set of equations:

$$(1-p)[L(\phi(\tau,p)) + g(\tau)] - H(p)[L(\phi(\tau,p)) + g(\tau) + N(\phi(\tau,p))] = 0,$$

$$B(\phi(\tau,p)) = 0, \tag{26}$$

where $p \in [0,1]$ is an embedding parameter, $H(p)$ denotes a nonzero auxiliary function for $p \neq 0$ and

$H(0) = 0, \phi(\tau, p)$ is an unknown function. Obviously, when $p = 0$ and $p = 1$, it holds that:

$$\phi(\tau, 0) = u_0(\tau), \quad \phi(\tau, 1) = u(\tau). \tag{27}$$

Thus, as p increases from 0 to 1, the solution $\phi(\tau, p)$ varies from $u_0(\tau)$ to solution $u(\tau)$, where $u_0(\tau)$ is obtained from Eq. (26) for $p = 0$:

$$L(u_0(\tau)) + g(\tau) = 0, \quad B(u_0) = 0. \tag{28}$$

We choose the auxiliary function, $H(p)$, in the form:

$$H(p) = pC_1 + p_2 C_2 + ..., \tag{29}$$

where $C_1, C_2, ...$ are constants which can be determined later.

Expanding $\phi(\tau, p)$ in a series, with respect to p, one has:

$$\phi(\tau, p, C_i) = u_0(\tau) + \sum_{k\geq 1} u_k(\tau, C_i) p_k, \quad i = 1, 2, ... \tag{30}$$

Substituting Eq. (30) into Eq. (26), collecting the same powers of p, and equating each coefficient of p to zero, we obtain a set of differential equations with boundary conditions. By solving differential equations by boundary conditions, $u_0(\tau), u_1(\tau, C_1), u_2(\tau, C_2), ...$ are obtained. Generally speaking, the solution of Eq. (25) can be determined approximately in the form:

$$\tilde{u}^{(m)} = u_0(\tau) + \sum_{k=1}^{m} u_k(\tau, C_i). \tag{31}$$

Note that the last coefficient, C_m, can be a function of τ. Substituting Eq. (28) into Eq. (25), results in the following residual:

$$R(\tau, C_i) = L(\tilde{u}^{(m)}(\tau, C_i)) + g(\tau) + N(\tilde{u}^{(m)}(\tau, C_i)). \tag{32}$$

If $R(\tau, C_i) = 0$, then $\tilde{u}^{(m)}(\tau, C_i)$ happens to be the exact solution. Generally, such a case will not arise for nonlinear problems, but we can minimize the functional by the Galerkin Method (GM):

$$w_i = \frac{\partial R(\tau, C_1, C_2, ..., C_m)}{\partial C_i}, \quad i = 1, 2, ..., m. \tag{33}$$

The unknown constants, $C_i (i = 1, 2, ..., m)$, can be identified from the conditions:

$$J(C_1, C_2) = \int_a^b w_i . R(\tau, C_1, C_2, ..., C_m) d\tau = 0, \tag{34}$$

where a and b are two values, depending on the given problem. With these constants, the approximate solution (of order m) (Eq. (31)) is well determined. It can be observed that the method proposed in this work generalizes these two methods using the special (more general) auxiliary function $H(p)$.

4. Solution with optimal homotopy asymptotic method

In this section, OHAM is applied to nonlinear ordinary differential Eqs. (21) and (22). According to the OHAM, applying Eq. (36) to Eqs. (21) and (22) gives:

$$\begin{aligned} V^{IV} &= \mathrm{Ha}^2 B^*(1-\phi)^{2.5} V'' + \mathrm{Re} A^*(1-\phi)^{2.5} \\ &\quad [V'V'' - VV'''], \\ UV' - VU' &= \frac{1}{\mathrm{Re}} \frac{1}{A^*(1-\phi)^{2.5}} \\ &\quad [U'' - \mathrm{Ha}^2 B^*(1-\phi)^{2.5} U]. \end{aligned} \tag{35}$$

We consider $V, U, H_1(p)$ and $H_2(p)$ as follows:

$$\begin{aligned} &V = V_0 + pV_1 + p^2 V_2, \quad U = U_0 + pU_1 + p^2 U_2, \\ &H_1(p) = pC_{11} + p^2 C_{12}, \quad H_2(p) = pC_{21} + p^2 C_{22}. \end{aligned} \tag{36}$$

Substituting $V, U, H_1(p)$ and $H_2(p)$ from Eq. (36) into Eq. (35), and some simplification and rearranging based on the powers of p-terms, we have:

$$\begin{aligned} p^0: \quad & V^{IV} = 0, \\ & U'' = 0, \\ & V_0(0) = 0, \quad V_0'(0) = 0, \quad V_0(1) = 0, \quad V_0'(1) = 0, \\ & U_0(0) = 1, \quad U_0(1) = 0. \end{aligned} \tag{37}$$

$$\begin{aligned} p^1: & \\ & V_1^{IV} + C_{11} V_0^{IV} - C_{11} \mathrm{Re} A^*(1-\phi)^{2.5} V_0'' V_0' \\ &\quad + C_{11} \mathrm{Re} A^*(1-\phi)^{2.5} V_0''' V_0 \\ &\quad - C_{11} \mathrm{Ha}^2 B^*(1-\phi)^{2.5} V_0'' - V_0^{IV} = 0, \\ & U_1'' - C_{21} \mathrm{Re} A^*(1-\phi)^{2.5} V_0' U_0 + C_{22} U_0'' - U_0'' \\ &\quad + C_{21} \mathrm{Re} A^*(1-\phi)^{2.5} U_0' V_0, \\ &\quad - C_{21} \mathrm{Ha}^2 B^*(1-\phi)^{2.5} U_0 = 0, \\ & V_1(0) = 0, \quad V_1'(0) = 0, \quad V_1(1) = 0, \quad V_1'(1) = 0 \\ & U_1(0) = 0, \quad U_1(1) = 0. \\ & \vdots \end{aligned} \tag{38}$$

Solving Eqs. (37) and (38) with boundary conditions gives:

$$V_0(y) = -2y^3 + 3y^2, \qquad U_0(y) = -y + 1, \tag{39}$$

$$V_1(y) = C_{11}(0.05714285714\,\mathrm{Re}A^*(1-\phi)^{2.5}y^7 - 0.2y^6 - 0.1\mathrm{Ha}^2B^*(1-\phi)^{2.5}y^5 + 0.25\mathrm{Ha}^2B^*(1-\phi)^{2.5}y^4 - 0.3857142857\mathrm{Re}A^*(1-\phi)^{2.5}y^3 - 0.2\mathrm{Ha}^2B^*(1-\phi)^{2.5}y^3 + (0.22855714286\mathrm{Re}A^*(1-\phi)^{2.5} + 0.05)y^2),$$

$$U(y) = C_{21}(0.2\,\mathrm{Re}y^5 - 0.75\,\mathrm{Re}A^*(1-\phi)^{2.5}y^4 + \mathrm{Re}A^*(1-\phi)^{2.5}y^3 - 0.1667\mathrm{Ha}^2B^*(1-\phi)^{2.5}y^3 + 0.5\mathrm{Ha}^2B^*(1-\phi)^{2.5}y^2 - 0.45\,\mathrm{Re}A^*(1-\phi)^{2.5}y - 0.3333\,\mathrm{Ha}^2B^*(1-\phi)^{2.5}y). \quad (40)$$

The terms of $V_2(y)$ and $U_2(y)$ are too large to show graphically. Therefore, the final expression for $V(y)$ and $U(y)$ is:

$$V(y) = V_0(y) + V_1(y) + V_2(y),$$

$$U(y) = U_0(y) + U_1(y) + U_2(y). \quad (41)$$

By Substituting $V(y)$ and $U(y)$ and into Eq. (35), $R_1(\eta, C_{11}, C_{12})$ and $R_2(\eta, C_{21}, C_{22})$ are obtained. Then, J_1 and J_2 are obtained in the following manner:

$$J(C_{11}, C_{12}) = \int_a^b w_i.R_1 d\tau = 0, \quad (42)$$

$$J(C_{21}, C_{22}) = \int_a^b w_i.R_2 d\tau = 0. \quad (43)$$

Constants C_{11}, C_{12}, C_{21} and C_{22} are obtained from Eqs. (42) and (43). By substituting these constants into Eq. (41), an expression for $V(y)$ and $U(y)$ is obtained.

5. Results and discussion

In this paper, laminar nanofluid flow in a permeable channel in the presence of a uniform magnetic field is studied (Figure 1). The Optimal Homotopy Asymptotic Method (using the Galerkin method to minimize the residual) is used in order to solve this problem. The results obtained by this method were well matched with solutions obtained using a numerical method; the fourth-order Runge-kutte method, as shown in Figure 2.

The effect of the nanoparticle volume fraction on $U(y)$ is shown in Figure 3. For both cases, in

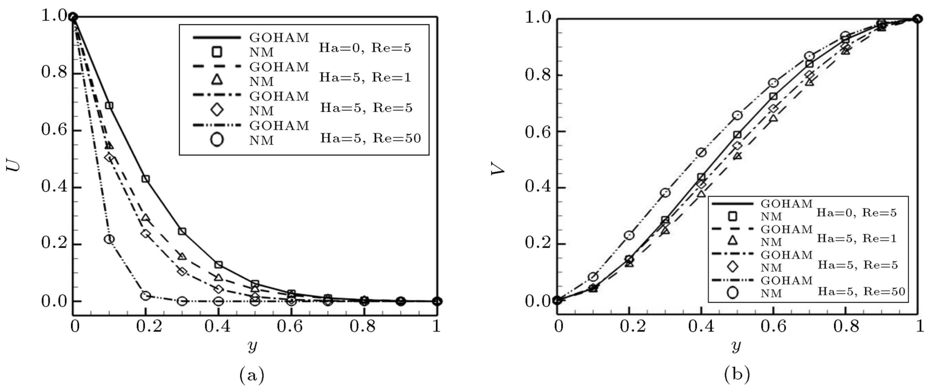

Figure 2. Comparison between the numerical results and GOHAM solution for different values of active parameters when $\phi = 0.06$.

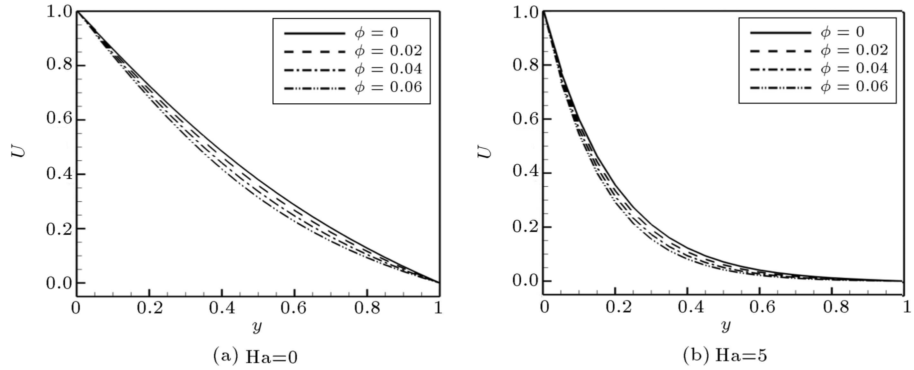

Figure 3. Effect of nanoparticle volume fraction (ϕ) on $U(y)$, when Re = 1 .

the presence and absence of a magnetic field, velocity boundary layer thickness decreases with an increase in nanoparticle volume fraction. Also, it can be seen that increasing nanoparticle volume fraction leads to a decrease in the values of $U(y)$ and this decrement is more sensible in the absence of a magnetic field. Figure 4 shows the effects of various values of Hartmann number on $V(y)$ and $U(y)$. Generally, when the magnetic field is imposed on the enclosure, the velocity field is suppressed owing to the retarding effect of the Lorenz force. For low Reynolds number, as Hartmann number increases $V(y)$ decreases for $y > y_m$, but, the opposite trend is observed for $y < y_m$; y_m is a meeting point at which all curves join together. When Reynolds number increases, this meeting point shifts to the solid wall and it can be seen that $V(y)$

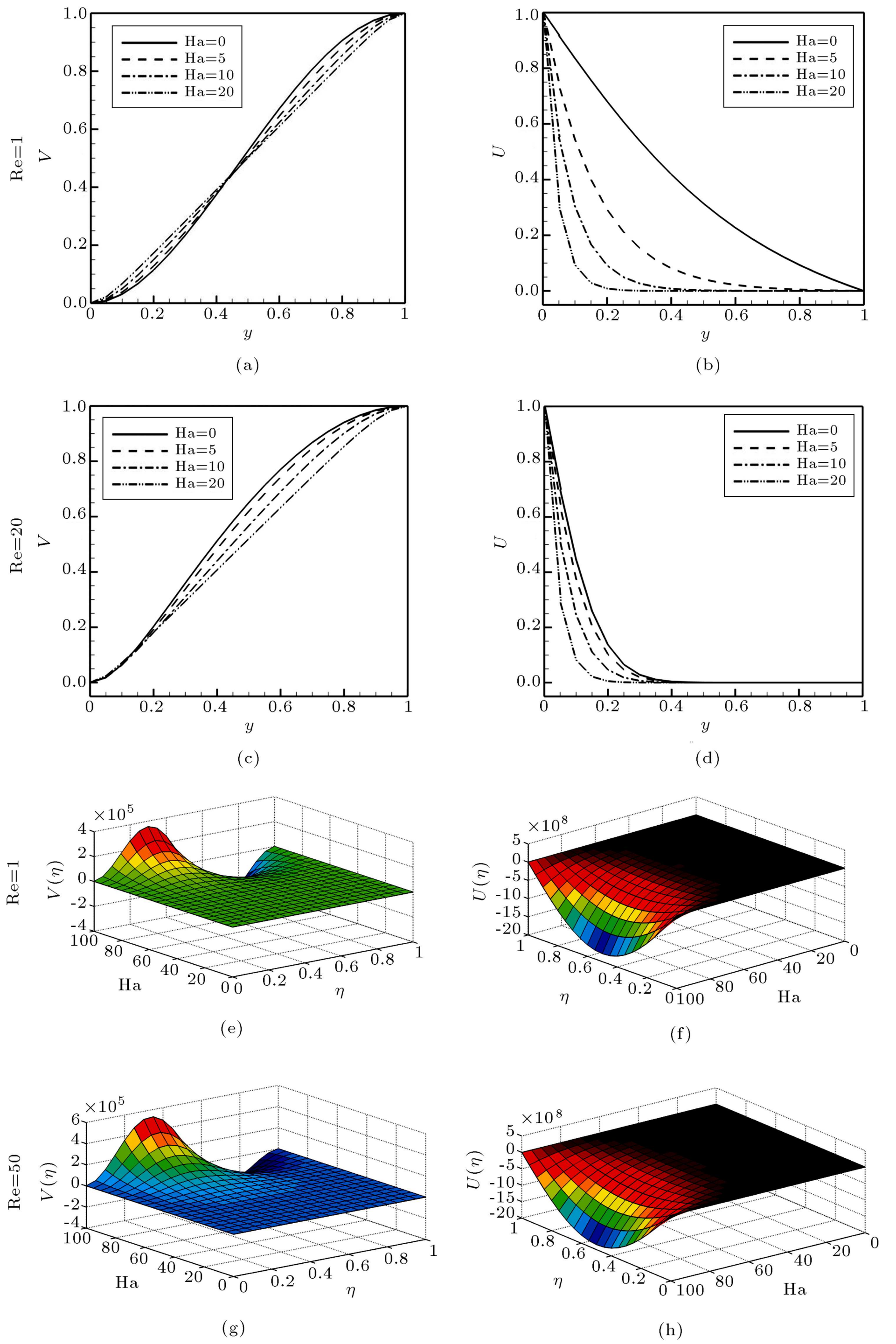

Figure 4. Effect of various values of Hartmann numbers (Ha) on $V(y)$ and $U(y)$ when $\phi = 0.06$.

decreases with an increase in Hartmann number. As Hartmann number increases, $U(y)$ decreases for all values of Reynolds number. Besides, this figure shows that this change is more pronounced for low Reynolds numbers.

Figure 5 shows the effects of various values of Reynolds number (Re) on $V(y)$ and $U(y)$. It is worth mentioning that the Reynolds number indicates the relative significance of the inertia effect compared to the viscous effect. Thus, the velocity profile decreases as Re increases, and in turn, increasing Re leads to an increase in the magnitude of the skin friction coefficient. With increasing Reynolds number, $V(y)$ and $U(y)$ increase. These effects become less at higher Hartmann numbers because of the retarding flow owing to Lorenz forces. Also, it shows that increasing

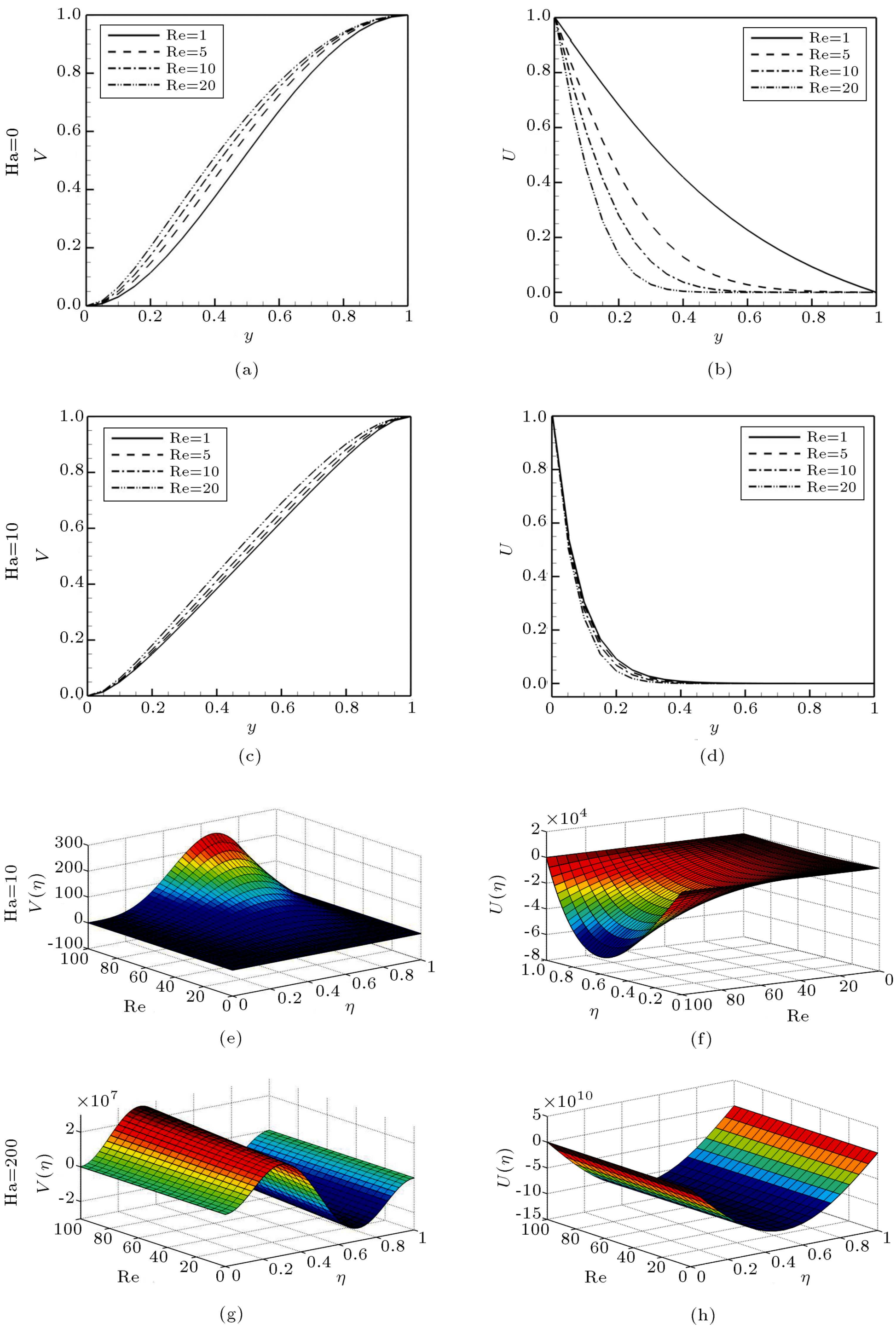

Figure 5. Effects of various values of Reynolds numbers (Re) on $V(y)$ and $U(y)$, when$\phi = 0.06$.

Hartmann number leads to an increase in the curve of the velocity profile.

6. Conclusion

In this paper, laminar MHD nanofluid flow in a semi-porous channel is studied. The Optimal Homotopy Asymptotic Method is used to solve governing equations. In order to lessen CPU time, the Galerkin method is used to minimize the residual. It can be found that this method is a powerful approach to solving this problem. The effects of active parameters on flow are examined. The results indicate that velocity boundary layer thickness decreases with an increase in Reynolds number and nanoparticle volume fraction, and it increases as Hartmann number increases. Also, it is discovered that the effect of Reynolds number on flow becomes less for higher values of Hartmann number.

Nomenclature

A^*, B^*	Constant parameter
P	Fluid pressure
q	Normal velocity of porous wall
x_k	General coordinates
f	Velocity function
$\bar{k}$	Fluid thermal conductivity
n	Power law index in temperature distribution
Re	Reynolds number
Ha	Hartmann number
u, v	Dimensionless components velocity in x and y directions, respectively
u^*, v^*	Velocity components in x and y directions respectively
x, y	Dimensionless horizontal, vertical coordinates respectively
x^*, y^*	Distance in x, y directions parallel to the plates

Greek symbols

υ	Kinematic viscosity
σ	Electrical conductivity
ε	Aspect ratio (h/L$_x$)
μ	Dynamic viscosity
ρ	Fluid density

Subscripts

∞	Condition at infinity
nf	Nanofluid
f	Base fluid
s	Nano-solid-particles

References

1. Wang, X.Q. and Mujumdar, A.S. "Heat transfer characteristics of nanofluids: a review", *International Journal of Thermal Science*, **46**(1), pp. 1-19 (2007).
2. Khanafer, K., Vafai, K. and Lightstone, M. "Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids", *Int. J. Heat Mass Transfer*, **46**, pp. 3639-3653 (2003).
3. Sheikholeslami, M., Ganji, D.D. and Ashorynejad, H.R. "Investigation of squeezing unsteady nanofluid flow using ADM", *Powder Technology*, **239**, pp. 259-265 (2013).
4. Ashorynejad, H.R., Sheikholeslami, M., Pop, I. and Ganji, D.D. "Nanofluid flow and heat transfer due to a stretching cylinder in the presence of magnetic field", *Heat Mass Transfer*, **49**, pp. 427-436 (2013).
5. Ashorynejad, H.R. Mohamad, A.A. and Sheikholeslami, M. "Magnetic field effects on natural convection flow of a nanofluid in a horizontal cylindrical annulus using Lattice Boltzmann method", *International Journal of Thermal Sciences*, **64**, pp. 240-250 (2013).
6. Sheikholeslami, M., Gorji-Bandpy, M., Ganji, D.D., Soleimani, S. "Effect of a magnetic field on natural convection in an inclined half-annulus enclosure filled with Cu-water nanofluid using CVFEM", *Advanced Powder Technology*, doi.org/10.1016/j.apt.2013.01.012 (2013).
7. Sheikholeslami, M., Gorji-Bandpy, M., Ganji, D.D. and Soleimani, S. "MHD natural convection in a nanofluid filled inclined enclosure with sinusoidal wall using CVFEM", *Neural Comput & Applic*, DOI 10.1007/s00521-012-1316-4 (2013).
8. Sheikholeslami, M. and Ganji, D.D. "Heat transfer of Cu-water nanofluid flow between parallel plates", *Powder Technology*, **235**, pp. 873-879 (2013).
9. Sheikholeslami, M., Gorji-Bandpy, M., Ganji, D.D., Soleimani, S. and Seyyedi, S.M. "Natural convection of nanofluids in an enclosure between a circular and a sinusoidal cylinder in the presence of magnetic field", *International Communications in Heat and Mass Transfer*, **39**, pp. 1435-1443 (2012).
10. Soleimani, S., Sheikholeslami, M., Ganji, D.D. and Gorji-Bandpay, M. "Natural convection heat transfer in a nanofluid filled semi-annulus enclosure", *International Communications in Heat and Mass Transfer*, **39**, pp. 565-574 (2012).
11. Sheikholeslami, M., Gorji-Bandpay, M. and Ganji, D.D. "Magnetic field effects on natural convection around a horizontal circular cylinder inside a square enclosure filled with nanofluid", *International Communications in Heat and Mass Transfer*, **39**, pp. 978-986 (2012).
12. Sheikholeslami, M., Gorji-Bandpy, M. and Ganji, D.D. "Numerical investigation of MHD effects on Al_2O_3-water nanofluid flow and heat transfer in a semi-annulus enclosure using LBM", *Energy*, **60**, pp. 501-510 (2013).

13. Sheikholeslami, M., Bani Sheykholeslami, F., Khoshhal, S., Mola-Abasia, H., Ganji, D.D. and Rokni, Houman, B. " Effect of magnetic field on Cu-water nanofluid heat transfer using GMDH-type neural network", *Neural Comput & Applic*, DOI 10.1007/s00521-013-1459-y (2013).

14. Sheikholeslami, M., Gorji-Bandpy, M., Pop, I. and Soleimani, S. "Numerical study of natural convection between a circular enclosure and a sinusoidal cylinder using control volume based finite element method", *International Journal of Thermal Sciences*, **72**, pp. 147-158 (2013).

15. Sheikholeslami, M., Gorji-Bandpy, M., Seyyedi, S.M., Ganji, D.D., Rokni, H. B. and Soleimani, S. "Application of LBM in simulation of natural convection in a nanofluid filled square cavity with curve boundaries", *Powder Technology*, **247**, pp. 87-94 (2013).

16. Sheikholeslami, M., Gorji-Bandpy, M. and Soleimani, S. "Two phase simulation of nanofluid flow and heat transfer using heatline analysis", *International Communications in Heat and Mass Transfer*, http://dx.doi.org/10.1016/j.icheatmasstransfer. 2013.07.006 (2013).

17. Sheikholeslami, M., Gorji-Bandpy, M. and Ganji, D.D. "Natural convection in a nanofluid filled concentric annulus between an outer square cylinder and an inner elliptic cylinder", *Scientia Iranica, Transactions B: Mechanical Engineering*, **20**(4), pp. 1-13 (2013).

18. Sheikholeslami, M., Hatami, M. and Ganji, D.D. "Analytical investigation of MHD nanofluid flow in a semi-porous channel", *Powder Technology*, **246**, pp. 327-336 (2013).

19. Sheikholeslami, M., Gorji-Bandpy, M., Ganji, D.D. and Soleimani, S. "Natural convection heat transfer in a cavity with sinusoidal wall filled with CuO-water nanofluid in presence of magnetic field", *Journal of the Taiwan Institute of Chemical Engineers*, http://dx.doi.org/10.1016/j.jtice.2013.04.019 (2013).

20. Sheikholeslami, M., Gorji-Bandpy, M. and Domairry, G. "Free convection of nanofluid filled enclosure using Lattice Boltzmann Method (LBM)", *Appl. Math. Mech. -Engl. Ed.*, **34** (7), pp. 1-15 (2013).

21. Chandran, P., Sacheti, N.C. and Singh, A.K. "Hydromagnetic flow and heat transfer past a continuously moving porous boundary", *International Communications of Heat and Mass Transfer*, **23**, pp. 889-898 (1996).

22. He, J.H. "Homotopy perturbation technique", *Computer Methods in Applied Mechanics and Engineering*, **178**, pp. 257-262 (1999).

23. Sheikholeslami, M., Ashorynejad, H.R., Ganji, D.D. and Kolahdooz, A. "Investigation of rotating MHD viscous flow and heat transfer between stretching and porous surfaces using analytical method", *Hindawi Publishing Corporation Mathematical Problems in Engineering*, **2011**, Article ID 258734, 17 pages, doi:10.1155/2011/258734.

24. Sheikholeslami, M., Ashorynejad, H.R., Ganji, D.D., Yildirim, A. "Homotopy perturbation method for three-dimensional problem of condensation film on inclined rotating disk", *Scientia Iranica, B*, **19**(3), pp. 437-442 (2012).

25. Sheikholeslami, M., Ganji, D.D., Rokni, H.B. "Nanofluid flow in a semi-porous channel in the presence of uniform magnetic field", *IJE TRANSACTIONS C*, **26**(6), pp. 653-662 (2013).

26. Domairry, D., Sheikholeslami, M., Ashorynejad, H.R., Subba Reddy Gorla, R. and Khani, M. "Natural convection flow of a non-Newtonian nanofluid between two vertical flat plates", *Proc IMechE Part N:J Nanoengineering and Nanosystems*, **225**(3), pp. 115-122 (2012). DOI: 10.1177/1740349911433468.

27. Joneidi, A.A., Ganji, D.D. and Babaelahi, M. "Differential Transformation method to determine fin efficiency of convective straight fins with temperature dependent thermal conductivity", *International Communications in Heat and Mass Transfer*, **36**, pp. 757-762 (2009).

28. Sheikholeslami, M., Ashorynejad, H.R., Barari, A., Soleimani, Soheil. "Investigation of heat and mass transfer of rotating MHD viscous flow between a stretching sheet and a porous surface", *Engineering Computations*, **30**(3), pp. 357-378 (2013).

29. Xinhui, S., Liancun, Zh., Xinxin, Zh., Jianhong, Y. "Homotopy analysis method for the heat transfer in a asymmetric porous channel with an expanding or contracting wall", *Applied Mathematical Modelling*, **35**, pp. 4321-4329 (2011).

30. Xinhui, S., Liancun, Zh., Xinxin, Zh., Xinyi, S. "Homotopy analysis method for the asymmetric laminar flow and heat transfer of viscous fluid between contracting rotating disks", *Applied Mathematical Modelling*, **36**, pp. 1806-1820 (2012).

31. Xinhui, S., Liancun, Zh., Xinxin, Zh., Min, L., Jianhong, Y. and Ying, Ch. "Multiple solutions for the laminar flow in a porous pipe with suction at slowly expanding or contracting wall", *Applied Mathematics and Computation*, **218**, pp. 3515-3521 (2011).

32. Sheikholeslami, M., Ashorynejad, H.R., Domairry, G. and Hashim, I. "Flow and heat transfer of Cu-water nanofluid between a stretching sheet and a porous surface in a rotating system", *Hindawi Publishing Corporation Journal of Applied Mathematics,* Article ID 421320, 19 pages, doi:10.1155/2012/421320 (2012).

33. Sheikholeslami, M., Ganji, D.D., Ashorynejad, H.R. and Rokni, H.B. "Analytical investigation of Jeffery-Hamel flow with high magnetic field and nano particle by Adomian decomposition method", *Appl. Math. Mech.-Engl. Ed.*, **33**(1), pp. 1553-1564 (2012).

34. Hashmi, M.S., Khan, N. and Iqbal, S. "Numerical solutions of weakly singular Volterra integral equations using the optimal homotopy asymptotic method", *Computers & Mathematics with Applications,* **64**(6), pp. 1567-1574 (2012).

35. Haq, S. and Ishaq, M. "Solution of strongly nonlinear ordinary differential equations arising in heat transfer with optimal homotopy asymptotic method", *International Journal of Heat and Mass Transfer*, **5**, pp. 5737-5743 (2012).

36. Sheikholeslami, M., Ashorynejad, H.R., Domairry, D. and Hashim, I. "Investigation of the laminar viscous flow in a semi-porous channel in the presence of uniform magnetic field using optimal homotopy asymptotic method", *Sains Malaysiana*, **41**(10), pp. 1177-1229 (2012).

37. Herisanu, N. and Marinca, V. "Accurate analytical solutions to oscillators with discontinuities and fractional-power restoring force by means of the optimal homotopy asymptotic method", *Comput. Math. Appl.*, **60**, pp. 1607-1615 (2010).

38. Joneidi, A.A., Ganji, D.D. and Babaelahi, M. "Micropolar flow in a porous channel with high mass transfer", *International Communications in Heat and Mass Transfer*, **36**, pp. 1082–1088 (2009).

39. Ganji, D.D., Sheikholeslami, M., Barzegar, G.H. and Eftari, M. "Application of optimal homotopy asymptotic method to solve MHD Jeffery-Hamel flows in non-parallel walls by using Galerkin method", *World Applied Sciences Journal*, **16** (Special Issue of Applied Math), pp. 75-81 (2012).

40. Sheikholeslami, M. and Ganji, D.D. "Numerical investigation for two phase modeling of nanofluid in a rotating system with permeable sheet", *Journal of Molecular Liquids*, **194**, pp. 13-19 (2014).

41. Sheikholeslami, M., Gorji-Bandpy, M. and Ganji, D.D. "Lattice Boltzmann method for MHD natural convection heat transfer using nanofluid", *Powder Technology*, **254**, pp. 82-93 (2014).

42. Sheikholeslami, M. and Ganji, D.D. "Three dimensional heat and mass transfer in a rotating system using nanofluid", *Powder Technology*, **253**, pp. 789-796 (2014).

43. Sheikholeslami, M., Gorji-Bandpy, M., Ganji, D.D. and Soleimani, S. "Heat flux boundary condition for nanofluid filled enclosure in presence of magnetic field", *Journal of Molecular Liquids*, **193**, pp. 174-184 (2014).

44. Hatami, M., Sheikholeslami, M. and Ganji, D.D. "Laminar flow and heat transfer of nanofluid between contracting and rotating disks by least square method", *Powder Technology*, **253**, pp. 769-779 (2014).

45. Sheikholeslami, M., Ganji, D.D., Gorji-Bandpy, M. and Soleimani, S. "Magnetic field effect on nanofluid flow and heat transfer using KKL model", *Journal of the Taiwan Institute of Chemical Engineers*, http://dx.doi.org/10.1016/j.jtice.2013.09.018 (2014).

Biographies

Mohsen Sheikholeslami Kandelousi received his BS degree in Mechanical Engineering from Mazandaran University, Iran, and his MS degree (Energy Conversion) from Babol University of Technology, Iran, where he is currently a PhD degree student of Mechanical Engineering. His research interests include CFD, mesoscopic modeling of fluid flow using LBM and Monte Carlo methods, and applications of nonlinear science in mechanical engineering.

Davood Domiri Ganji received his PhD degree in Mechanical Engineering from Tarbiat Modares University, Tehran, Iran, and is currently Associate Professor of Mechanical Engineering at Babol Noshirvani University of Technology, Iran. His research interests include nonlinear science in science and engineering, inverse problems and spray modeling.