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A new method based on the multi-segment decision matrix for solving decision-making problems

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Abstract. Decision-making analysis methods are employed to find the best option among feasible alternatives where an amount of alternatives versus criteria is introduced as only one value level with stationary numerical value. In real-world decision situations, the condition of multi-segment problems may exist in practice. In this paper, a new method is proposed to rank the alternatives in Multiple Criteria Decision-Making (MCDM) problems, where the amount of alternatives to the criteria can be represented by several segments. Hence, a multi-segment decision matrix can be obtained. Moreover, the proposed method, based on Simple Additive Weight (SAW), can be employed to solve the decision problems, where the amount of alternatives versus the assessment criteria at each level is introduced as a function of some parameters. These functions can be regarded as linear, exponential, and trigonometric. Finally, three real case studies are given to demonstrate the solution procedure of the proposed method, and then a sensitivity analysis for each case is reported.

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1. Introduction

Decision-making methods are utilized to find the best option among all feasible alternatives. There are many methods to solve MCDM problems. Priority-based, outranking, distance-based and mixed methods are nominated as the principal classes of the MCDM [1,2]. The SAW method is one of the most diffused approaches in MCDM problems to have been widely used in real-life situations [3,4]. This method was first utilized by Churchman and Ackoff (1945) to find the best option for portfolio problems. The SAW is applied to evaluate the methods and to identify the best one according to its simplicity and general acceptability [5]. The basic principle of this method is to calculate a weighted sum of performance ratings of alternatives versus several criteria [6,7]. The SAW is known to be the most used, intuitive, and easy [7,8]. The method includes two basic steps: first, calculating the values of all criteria to make them comparable, and second, summing up the values of all criteria for each alternative [9,10]. One important advantage of the SAW method is that the transformation of the raw data is linear. It means that the relative order of magnitude of the standardized scores remains equal [7].

In the past two decades, decision-making methods have provided a logical approach to analyze decision alternatives, where the amount of alternatives versus criteria is presented as a constant numerical value in the decision matrix. Thus, they are proposed with only one value level. To cope with many real-decision situations, the orientation of preference alternatives with respect

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to criteria may not be the same as the preference with decision-making problems. The preference is presented as one value level in every condition. In other words, the decision-making variables are adjusted according to some parameters, such as demand rate for the buyer, business volume, price, quality, and delivery lead time. In recent years, many multi-segment, Multi-Objective Decision-Making (MODM) problems have been studied, where the decision variables coefficients are applied with different contribution levels or aspiration levels are proposed as a multi-segment problem, for example the basic price of products or services often adjusted by companies to accommodate differences in customer, products, locations, etc. [11,12].

Multi-segment Multiple Attributes Decision-Making (MADM) problems, as the above-mentioned MODM problem, are available in the literature, but no methods of solving multi-segment MADM problems exist. There are many situations in which the multi-segment decision levels of the ith alternative versus the jth criterion (attribute) can be applied. For example, business volume discounts (e.g., the discounted price of allocated items to the alternatives) are applied to motivate the investors, corresponding to business volume. Producers give discounts (e.g., a reduction in the basic price of goods or services) in order to sell their products quickly and mostly give discounts to attract customers. In incremental discounts and all-unit discounts, fixed and variable purchase costs are presented as the amount of alternatives. The parameters in current methods include stationary over time, order quantity and amount of production. It is pointed out that the fixed order cost for the incremental discount can be different for each price break region; in other words, the incremental discount has several different costs [13]. The amounts of multi-segment decision matrix are in the k_{ij} amount levels for each alternative, A_i, with respect to criterion, C_j. Moreover, in some cases, some Decision Makers (DMs) believe that there may exist a situation where the amount of alternatives at each level can be presented as a function of some variables (above-denoted parameters); however, the DMs are not able to decide alternatives using current methods.

This paper proposes a new SAW method for the preference of alternatives with a multi-level decision matrix based on the above-mentioned concepts. This method is frequently applied in real-life decision-making problems. In addition, the proposed model aims to determine the preference of the options, where the amount of decision matrix is presented as a function that is changed at k levels. It means that the value of each alternative can be changed versus each criterion at k levels. The value of k for each element of the decision matrix can be different. In other words, K_{ij} levels for the ith alternative related to the jth criterion can be considered. Therefore, two main contributions in this paper are as follows:

- The proposed method leads to determine the preference of alternatives in multi-segment problems which can be easily solved, step by step.
- This method can be applied where the amount of alternatives at each level can be presented as a function of some variables. Thus, the functional amounts can be compared with each other.

Some functions are utilized in the decision-making matrix, such as linear, exponential, and trigonometric. A discount function is used in economic models. The overlapping of two curves may exist; the alternatives are ranked by considering the relation between the price break and the amount of these proposed functions. The preference of alternatives is presented in each interval as mentioned above. Therefore, the DM can prefer the options in all intervals, and then the decision-making can be done. The distance of decision making can be broken down to several intervals, according to some conditions; the functional amount of alternative A_i versus criterion C_j at the kth level, and x is the variable that depends on an amount of some parameters, such as time, order quantity, and amount of production. For alternative A_i, with respect to criterion C_j, at the kth level, x belongs to $[L_{ijk_{ij}}, U_{ijk_{ij}}]$.

In the previous methods, the DMs made decisions about the alternatives only once. However, regarding the changes, the previous methods cannot consider these changes throughout the decision-making process. Also, the value of each alternative with respect to each criterion can be a function of different variables instead of a number. In addition, since values of every column (attribute) in the decision-making matrix are normalized, all values are between 0 and 1. Thus, the criteria of the decision matrix can be taken as any various types. Further, since the area under the curve of the function is utilized for the final value of the decision matrix, this method can be employed when the DMs are not aware of the exact values of the alternatives, although they know that the values are in the $[a, b]$ interval.

There are some advantages of the multi-segment SAW method, which are provided as below:

1. The SAW is very easy and the proposed method is presented step by step; therefore, the computation process is simple and straightforward.
2. The DMs can find the best alternative in each interval form.
3. In some cases, the overlapping of two curves with each other may exist; hence, the preference of alternatives may be changed in each interval. Considering this concept, the new method proposes a

logical mathematical tool to help the DMs in order to make the best decision.

4. During this multi-segment approach, there is one parameter available to deal with decision problems. Many parameters can be introduced in real-life application, such as time, order quantity, and the amount of production.
5. This method can help the DM when the order quantity of alternatives is not exactly determinable, and the bounds of the order quantity are assigned as an interval form.
6. In this method, values of alternatives versus criteria are transformed into a dimensionless value. Thus, the final value of alternatives can be calculated, where the attributes are presented by different dimensions.

The remainder of this paper is organized as follows. The SAW method is presented in the next section. In Section 3, the procedure of ranking by the proposed SAW method with a multi-segment decision matrix is described. In Section 4, the three important functions used in the case study are described. In Section 5, three case studies, including the preference of three mechanical engines, to find the best option among three companies and suitable institutes among three alternatives, are provided. Then, the sensitivity analysis is described for each case. The last section is devoted to conclusions.

2. SAW method

Suppose a decision-making problem has n alternatives, $A_1, A_2, \cdots, A_n$ and m criteria, $C_1, C_2, \cdots, C_m$. Each alternative is evaluated with respect to the m criteria. All the alternatives' performances related to each criterion from a decision matrix are denoted by $a = [a_{ij}]_{n\times m}$. Let $W = (w_1, w_2, \cdots, w_m)$ be the relative vector presenting the criteria weights, satisfying $\sum_{j=1}^{m} w_j = 1$, then, the process of the SAW method consists of the following steps [14]:

Step 1. Construct the decision-making matrix and then normalize it. In the normalization process, the following transformations are used for each element.

$$r_{ij} = \left\{ \frac{a_{ij}}{a_j^{\max}} \middle| j \in B \right\}, \qquad i = 1, 2, \cdots, n,$$

$$r_{ij} = \left\{ \frac{a^{\min}}{a_{ij}} \middle| j \in C \right\}, \qquad i = 1, 2, \cdots, n, \qquad (1)$$

where B is the set of benefit criteria and C is the set of cost criteria.

Step 2. Consider the different importance of each criterion, $W = (w_1, w_2, \cdots, w_m)$. Then, calculate the weighted normalized matrix as:

$$v_{ij} = r_{ijk}.w_j, \qquad i = 1, 2, ..., n, \quad j = 1, 2, ..., m. \quad (2)$$

Step 3. Calculate the final evaluation value of alternatives according to the weighted normalized matrix. After calculating the final evaluation value of each alternative, the pair-wise comparison of the preference relationship between the alternatives, A_i and A_j, can be established.

$$P_i = \sum_{j=1}^{m} v_{ij}, \qquad i = 1, 2, \cdots, n, \qquad (3)$$

where P_i is the final evaluation value of alternative A_i.

3. Proposed SAW method with multi-segment decision matrix

Suppose there are n alternatives $A_i (i = 1, 2, \cdots, n)$ and m criteria $C_j (j = 1, 2, \cdots, m)$. Alternative A_i is evaluated by criterion C_j in the K_{ij} level. The multi-segment problem can be expressed in the matrix format as given in Table 1. Considering this table, f_{ijk} is the function of alternative A_i to criterion C_j in the Kth level and x is the variable that depends on an amount of some parameters, such as time, order quantity, and amount of production. For alternative A_i, with respect to criteria C_j, in the kth level, x belongs to $[L_{ijk_{ij}}, U_{ijk_{ij}}]$. In the above-mentioned context, the SAW method with multi-segment decision making is carried out in the following procedure:

Step 1. Consider all intervals, then draw out the lower bounds ($L_{ijk_{ij}}$), and upper bounds ($U_{ijk_{ij}}$) of intervals from the decision matrix. Sort all of them in ascendant order as: $a_1, a_2, \cdots, a_p$, where $a_1 < a_2 < \cdots < a_p$.

Step 2. According to the amounts in the previous step, construct the intervals as: $[a_1, a_2]$, $[a_2, a_3], \cdots, [a_{p-1}, a_p]$.

Step 3. Compare all functions of the decision-making matrix under criterion C_j, $x \in [L_i, U_i]$; $i = 1, 2, \cdots, p$.

If the intersection between functions exists, then break down intervals $[L_i, U_i]$ into several intervals.

The overlapping in every interval is only considered for the functions of one criterion. The functional amounts of alternatives versus each criterion can cross each other. In other words, the values of alternatives versus only one criterion defined as a function are compared with each other (not all criteria for the

Table 1. The multi-segment decision making problem.

Alternatives	C_1	$\cdots$	C_j	$\cdots$	C_m
A_1	$f_{111}; L_{111} \le X \le U_{111}$ $f_{112}; L_{112} \le X \le U_{112}$ $\vdots$ $f_{11k_{11}}; L_{11K_{11}} \le X \le U_{11K_{11}}$	$\cdots$	$f_{1j1}; L_{1j1} \le X \le U_{1j1}$ $f_{112}; L_{1j2} \le X \le U_{1j2}$ $\vdots$ $f_{1jk_{1j}}; L_{1jk_{1j}} \le X \le U_{1jk_{1j}}$	$\cdots$	$f_{1m1}; L_{1m1} \le X \le U_{1m1}$ $f_{1m2}; L_{1m2} \le X \le U_{1m2}$ $\vdots$ $f_{1mk_{1m}}; L_{1mk_{1m}} \le X \le U_{1mk_{1m}}$
A_2	$f_{211}; L_{211} \le X \le U_{211}$ $f_{212}; L_{212} \le X \le U_{212}$ $\vdots$ $f_{21k_{21}}; L_{21K_{21}} \le X \le U_{21K_{21}}$	$\cdots$	$f_{2j1}; L_{2j1} \le X \le U_{2j1}$ $f_{2j2}; L_{2j2} \le X \le U_{2j2}$ $\vdots$ $f_{2jk_{2j}}; L_{2jk_{2j}} \le X \le U_{2jk_{2j}}$	$\cdots$	$f_{2m1}; L_{2m1} \le X \le U_{2m1}$ $f_{2m2}; L_{2m2} \le X \le U_{2m2}$ $\vdots$ $f_{2mk_{2m}}; L_{2mk_{2m}} \le X U_{2mk_{2m}}$
	$\vdots$		$\vdots$		$\vdots$
A_i	$f_{i11}; L_{i11} \le X \le U_{i11}$ $f_{i12}; L_{i12} \le X \le U_{i12}$ $\vdots$ $f_{i1k_{i1}}; L_{i1k_{i1}} \le X \le U_{i1k_{i1}}$		$f_{ij1}; L_{ij1} \le X \le U_{ij1}$ $f_{ij2}; L_{ij2} \le X \le U_{ij2}$ $\vdots$ $f_{ijk_{ij}}; L_{ijk_{ij}} \le X \le U_{ijk_{ij}}$		$f_{im1}; L_{im1} \le X \le U_{im1}$ $f_{im2}; L_{im2} \le X \le U_{im2}$ $\vdots$ $f_{imk_{im}}; L_{imk_{im}} \le X \le U_{imk_{im}}$
	$\vdots$		$\vdots$		$\vdots$
A_n	$f_{n11}; L_{n11} \le X \le U_{n11}$ $f_{n12}; L_{n12} \le X \le U_{n12}$ $\vdots$ $f_{n1k_{n1}}; L_{n1k_{n1}} \le X \le U_{n1k_{n1}}$		$f_{nj1}; L_{nj1} \le X \le U_{nj1}$ $f_{nj2}; L_{nj2} \le X \le U_{nj2}$ $\vdots$ $f_{njk_{ij}}; L_{njk_{nj}} \le X \le U_{njk_{nj}}$		$f_{nm1}; L_{nm1} \le X \le U_{nm1}$ $f_{nm2}; L_{nm2} \le X \le U_{nm2}$ $\vdots$ $f_{nmk_{nm}}; L_{nmk_{nm}} \le X \le U_{nmk_{nm}}$

comparison). Also, the values represented as functions are normalized after the calculations. It is pointed out that in many cases, there is not any overlapping between the two functions.

Suppose that q_j intersections points exist as:

$$b_{i1}, b_{i2}, \cdots, b_{iq_j}.$$

Construct the $(q_j + 1)$ corresponding intervals as:

$$[L_i, b_{i1}], [b_{i1}, b_{i2}], \cdots [b_{iq-1}, b_{iq}], [b_{iq}, U_i].$$

Like this, construct intervals for other criteria. In addition, we have intervals as:

$$[L_1, b_{11}], [b_{11}, b_{12}], \cdots, [b_{1q_1}, U_1], \cdots,$$

$$[L_j, b_{j1}], [b_{j1}, b_{j2}], \cdots, [b_{jq_j}, U_j], \cdots,$$

$$[L_p, b_{p1}], [b_{p1}, b_{p2}], \cdots, [b_{pq_p}, U_q].$$

If P' intervals exist, then nominate them as:

$$[L'_1, U'_1], [L'_2, U'_2], \cdots, [L'_i, U'_i], \cdots, [L'_p, U'_p].$$

Step 4. Use the area under the curve of a function as an amount of alternatives versus each criterion in interval $[L'_1, U'_1]$ as:

$$\alpha_{ijk_{ij}} = \int_{L'}^{U'} f_{ijk_{ij}} dx. \tag{4}$$

Calculate all amounts of the decision matrix by Eq. (4) for all intervals. It is worth noting that the functions can be different in intervals because the decision-making process is done at each level; in other words, each interval (level) has its function and the value of the area under the curve in each level is regarded as an amount of alternatives with respect to criteria in the decision matrix.

Step 5. Calculate the normalized decision matrix. We can obtain the normalized decision matrix denoted by R as:

$$R = [r_{ijk_{ij}}],$$

$$r_{ijk} = \left(\frac{\alpha_{ijk}}{\alpha_{jk^*}}\right); \qquad j \in B,$$

$$r_{ijk} = \left(\frac{a_{jk^-}}{a_{ijk}}\right); \qquad j \in C,$$

$$a_{jk^*} = \max_i(a_{ijk}); \qquad j \in B,$$

$$a_{jk^-} = \min_i(a_{ijk}); \qquad j \in C. \tag{5}$$

Suppose that B is the set of benefit criteria and C is the cost criteria.

Step 6. Consider the different importance of each criterion. Then, calculate the final value of each alternative as:

$$W = (w_1, w_2, \cdots, w_m),$$

$$P_{ik} = \sum_{j=1}^{m} r_{ijk}.w_j, \qquad i = 1, 2, \cdots, n,$$

$$x \in [L_{ijk}, U_{ijk}], \tag{6}$$

where X is the argument of functions, which demonstrate the amount of variables, such as the amount of primary investment and time. The intervals are broken down according to values of X.

P_{ik} is the final evaluation value of alternative A_i in the kth level. After the calculation of the final evaluation value of each alternative, the pair-wise comparison of the preference relationship between the alternatives A_i and A_j can be established.

4. Nomination of some practical functions

In this section, we present three commonly-used functions that are employed in case studies. Then, the behavior of these functions is investigated. The functions may cross each other. Consequently, intercept points can be calculated. The functions are taken into consideration for a single variable. In other words, each function has an independent variable. The value of a variable can generally be changed during the decision making. The variable of each interval has its special function. The variables are recognized as a real variable. Therefore, the functions are presented with real outputs.

4.1. Linear function

Suppose that f_{ijk} is the functional amount of alternative A_i to criterion C_j in the kth level. There are two linear functions that are compared with each other as:

$$f_{ijk} = a_1 x + b_1, \qquad f_{i'jk} = a_2 x + b_2.$$

The comparison of the preference relationship between the above functions throughout the interval $[l_{ijk}, u_{ijk}]$ is as follows:

a) $a_1 \le a_2$, $b_1 \le b_2$ then $f_{ijk} \le f_{i'jk}$;

b) $a_1 \ge a_2$, $b_1 \ge b_2$ then $f_{ijk} \ge f_{i'jk}$;

c) $a_1 \ge a_2$, $b_1 \le b_2$ or $a_1 \le a_2$, $b_1 \ge b_2$ then f_1 and f_2 are intersected in $x_0 = \frac{b_2 - b_1}{a_1 - a_2}$ as shown in Figure 1.

Therefore, the interval $[L, U]$ is broken down into two intervals, $[L, x_0]$ and $[x_0, U]$.

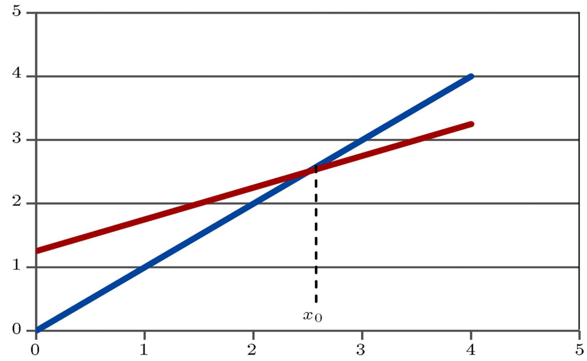

Figure 1. The comparison of the two linear functions.

4.2. Exponential function

Exponential functions are used in many economic problems. This function is introduced in mathematics as:

$$f(x) = ka^{bx+c},$$

where k, b and c are stationary coefficients.

$$f_{ijk} = k_1 a_1^{b_1 x + c_1}, \qquad f_{i'jk} = k_2 a_2^{b_2 x + c_2}.$$

If:

$$a_1, a_2 \ge 1, \qquad k_1 \ge k_2, \qquad a_1 \ge a_2, \qquad b_1 \ge b_2,$$

$$c_1 \ge c_2,$$

then:

$$f_{ijk} \ge f_{i'jk}.$$

If:

$$a_1, a_2 \le 1, \qquad k_1 \le k_2, \qquad a_1 \le a_2, \qquad b_1 \ge b_2,$$

$$c_1 \ge c_2,$$

then:

$$f_{ijk} \le f_{i'jk}.$$

In other words, there is an intersection (x_0) between f_{ijk} and $f_{i'jk}$ in the interval $[l_{ijk}\ u_{ijk}]$ as:

$$k_1 a_1^{b_1 x + c_1} = k_2 a_2^{b_2 x + c_2},$$

therefore:

$$x_0 = \frac{c_2 \ln a_2 - c_1 \ln a_1}{b_1 \ln a_1 - b_2 \ln a_2},$$

as shown in Figure 2. Also, if $a = e$, then one has:

$$x_0 = \frac{1}{a_1 - a_2}\left[\ln\left(\frac{k_1}{k_2}\right) - (b_1 - b_2)\right]. \tag{7}$$

4.3. Trigonometric function

The trigonometric function is formed as $f = k\sin(ax + b)$. Let x be an angle that terminates in any quadrant and k, a, b are stationary coefficients. This is a periodic function. The trigonometric functions may cross each other; then, intercept points are calculated.

$$f_{ijk} = k_1 \sin(a_1 x + b_1), \qquad f_{i'jk} = k_2 \sin(a_2 x + b_2).$$

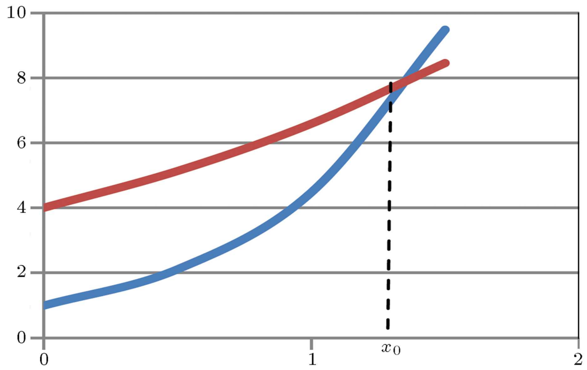

Figure 2. The comparison of the two exponential functions.

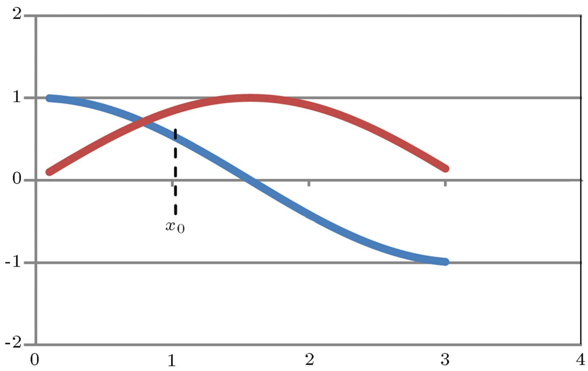

Figure 3. The comparison of the two trigonometrical functions.

This function is periodic with period $\frac{2\pi}{a}$. For interval $[-\frac{\pi}{2}, +\frac{\pi}{2}]$, the function is ascending; thus, if $k_1 \geq k_2$, $a_1 \geq a_2$, $b_1 \geq b_2$, then $f_{ijk} \geq f_{i'jk}$.

Now suppose that $x \in [+\frac{\pi}{2}, +\frac{3\pi}{2}]$. In this interval, the function can be descending. Therefore, if $a_1 \geq a_2$, $b_1 \geq b_2$, $k_1 \leq k_2$, then $f_{ijk} \leq f_{i'jk}$.

Elsewhere, we have the intersection between functions and can draw out the intersection point (x_0) from the equation:

$$k_1 \sin(a_1 x + b_1) = k_2 \sin(a_2 x + b_2),$$

as shown in Figure 3.

5. Application of the proposed method in solving problems

To demonstrate the validity and applicability of the proposed multi-segment SAW method, three illustrative cases are provided which are then solved step by step. A summarized description of alternatives and criteria is given. The sensitivity analyses are reported at the end of each case. To further illustrate, the figurations of each case are presented.

5.1. Case 1

To illustrate the above procedure, the steps of the proposed decision-making method are implemented in the application case. In this case study, the performance of three mechanical engines are estimated with respect to four criteria, including spring elasticity, price, amount of gas consumed, and speed, as shown in Figure 4. The aim is to find the best option among three alternatives. The weights of the criteria are as follows: 0.2, 0.3, 0.1 and 0.4. The performance of each alternative versus criteria is given in Table 2.

The proposed procedure, based on the conceptual model for Case 1, is as follows:

Step 1. Draw out the lower bounds $(L_{ijk_{ij}})$ and upper bounds $(U_{ijk_{ij}})$ of intervals from the decision matrix. Then, sort them in an ascendant order as: 0, 3, 6, 9 and 12.

Step 2. Construct the intervals according to the amount of the previous step as: [03], [36], [69], [912].

Step 3. Compare all functions. Function $f_{112}(x) = \sqrt{2}\sin(\frac{t}{8})$ intersects $f_{212}(x) = f_{312}(x) = \sin(\frac{t}{4})$, in $= 2\pi = 6.28 \in [6, 9]$, $f_{141}(x) = f_{341}(x) = 0.5t+1$ intersect the $f_{241}(x) = t$, in $x = 2 \in [3, 6]$.

Therefore, break down the intervals [0,3] to [0,2], [2,3] and [6,9] to [6,6.28], [6.28,9]. Then, sort the new intervals as:

$$[0, 2], [2, 3], [3\ 6], [6, 6.28], [6.28, 9], [9, 12].$$

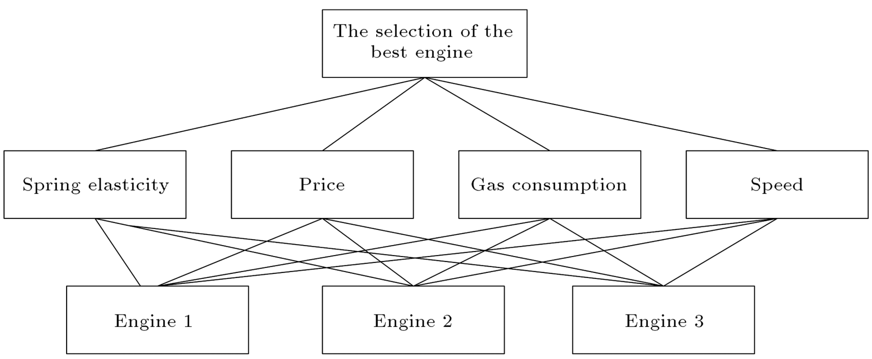

Figure 4. The selection of the best engine.

Table 2. The multi-segment decision making matrix of three engines.

Alternatives	Spring elasticity		Price		Gas consumption		Speed	
A_1	$\sin\left(\frac{t}{8}\right)$	$0 \le t \le 6$	20	$0 \le t \le 3$	$100(1-e^{-0.81t})$	$0 \le t \le 3$	$0.5t+1$	$0 \le t \le 6$
			25	$3 \le t \le 6$	$100(1-e^{-0.8t})$	$3 \le t \le 6$		
	$\sqrt{2}\sin\left(\frac{t}{8}\right)$	$6 \le t \le 12$	25	$6 \le t \le 9$	$100(1-e^{-0.82t})$	$6 \le t \le 9$	$0.8t$	$6 \le t \le 12$
			20	$9 \le t \le 12$	$100(1-e^{-0.83t})$	$9 \le t \le 12$		
A_2	$\frac{1}{2}\sin\left(\frac{t}{9}\right)$	$0 \le t \le 6$	15	$0 \le t \le 3$	$100(1-e^{-0.83t})$	$0 \le t \le 3$	t	$0 \le t \le 6$
			20	$3 \le t \le 6$	$100(1-e^{-0.81t})$	$3 \le t \le 6$		
	$\sin\left(\frac{t}{4}\right)$	$6 \le t \le 12$	24	$6 \le t \le 9$	$100(1-e^{-0.84t})$	$6 \le t \le 9$	$0.9t$	$6 \le t \le 12$
			22	$9 \le t \le 12$	$100(1-e^{-0.85t})$	$9 \le t \le 12$		
A_3	$\frac{1}{3}\sin\left(\frac{t}{10}\right)$	$0 \le t \le 6$	17	$0 \le t \le 3$	$100(1-e^{-0.79t})$	$0 \le t \le 3$	$0.5t+1$	$0 \le t \le 6$
			25	$3 \le t \le 6$	$100(1-e^{-0.78t})$	$3 \le t \le 6$		
	$\sin\left(\frac{t}{4}\right)$	$6 \le t \le 12$	22	$6 \le t \le 9$	$100(1-e^{-0.83t})$	$6 \le t \le 9$	t	$6 \le t \le 12$
			10	$9 \le t \le 12$	$100(1-e^{-0.85t})$	$9 \le t \le 12$		

Table 3. The calculated amount of under curves for three engines.

Duration	Alternatives	C_1	C_2	C_3	C_4
$0 \le t \le 2$	A_1	0.25	20	101	3
	A_2	0.11	15	102.43	2
	A_3	0.07	17	99.49	3
$2 \le t \le 3$	A_1	0.31	20	86.44	2.25
	A_2	0.135	15	87.08	2.5
	A_3	0.08	17	85.76	2.25
$3 \le t \le 6$	A_1	1.59	25	290.09	9.75
	A_2	0.715	20	290.84	13.5
	A_3	0.4	25	289.27	9.75
$6 \le t \le 6.28$	A_1	0.27	25	27.81	1.38
	A_2	0.28	24	27.83	1.55
	A_3	0.28	22	27.78	1.72
$6.28 \le t \le 9$	A_1	3.1	25	271.32	16.6
	A_2	2.5	24	271.41	18.7
	A_3	2.5	22	271.22	20.08
$9 \le t \le 12$	A_1	4	20	299.92	25.2
	A_2	1.45	22	299.94	28.35
	A_3	1.45	10	299.91	31.5

Table 4. The normalized matrix of three engines.

Duration	Alternatives	C_1	C_2	C_3	C_4
$0 \le t \le 2$	A_1	1	0.75	0.99	1
	A_2	0.44	1	1	0.67
	A_3	0.28	0.88	0.97	1
$2 \le t \le 3$	A_1	0.23	0.75	0.99	0.9
	A_2	1	1	1	1
	A_3	0.06	0.88	0.98	0.9
$3 \le t \le 6$	A_1	1	0.8	0.997	0.72
	A_2	0.45	1	1	1
	A_3	0.25	0.8	0.995	0.72
$6 \le t \le 6.28$	A_1	0.96	0.88	0.999	0.8
	A_2	1	0.92	1	0.9
	A_3	1	1	0.998	1
$6.28 \le t \le 9$	A_1	1	0.88	1	0.83
	A_2	0.81	0.92	1	0.93
	A_3	0.81	1	0.999	1
$9 \le t \le 12$	A_1	1	0.50	0.9999	0.8
	A_2	0.36	0.45	1	0.9
	A_3	0.36	1	0.9999	1

Step 4. Calculate the amounts of area under the curves by Eq. (4) as shown in Table 3. We explicitly demonstrate the calculation of this amount in Appendix A.

Step 5. Calculate the normalized decision matrix by Eq. (5) as shown in Table 4.

Step 6. Calculate the final evaluation value of alternative A_i in the kth level. After the calculation of the final evaluation value for each alternative, the pair-wise comparison of the preference relationship between the alternatives A_i, A_j can be established as provided in Table 5. In this case study, the new intervals are sorted as:

$$[0,2], [2,3], [3,6], [6,6.28], [6.28,9], [9,12].$$

When the performance of the first machine is compared with the other two machines, as given in Table 5, it is evident that the first machine has the best performance value in intervals $[0,2]$, $[3,6]$, whereas the third machine has the best performance value in intervals $[6,6.28]$, $[6.28,9]$, $[9,12]$. Moreover, the second machine

Table 5. Final value of three engines.

Duration	Alternatives	C_1	C_2	C_3	C_4	Final value	Rank
$0 \leq t \leq 2$	A_1	0.2	0.225	0.099	0.4	0.924	1
	A_2	0.088	0.3	0.1	0.268	0.756	3
	A_3	0.056	0.264	0.097	0.4	0.814	2
$2 \leq t \leq 3$	A_1	0.046	0.3	0.099	0.36	0.805	2
	A_2	0.2	0.225	0.1	0.4	0.925	1
	A_3	0.012	0.255	0.098	0.36	0.695	3
$3 \leq t \leq 6$	A_1	0.2	0.3	0.0997	0.288	0.8877	1
	A_2	0.09	0.24	0.1	0.4	0.83	2
	A_3	0.05 0.4	0.3	0.0995	0.288	0.7375	3
$6 \leq t \leq 6.28$	A_1	0.192	0.264	0.099	0.32	0.8759	3
	A_2	0.2	0.276	0.1	0.36	0.936	2
	A_3	0.2	0.3	0.0998	0.688	1.2878	1
$6.28 \leq t \leq 9$	A_1	0.2	0.264	0.1	0.332	0.896	3
	A_2	0.162	0.276	0.1	0.372	0.91	2
	A_3	0.162	0.3	0.0999	0.4	0.9619	1
$9 \leq t \leq 12$	A_1	0.2	0.15	0.0999	0.32	0.7699	2
	A_2	0.072	0.135	0.1	0.36	0.667	3
	A_3	0.072 1.45	0.3	0.0999	0.4	0.8719	1

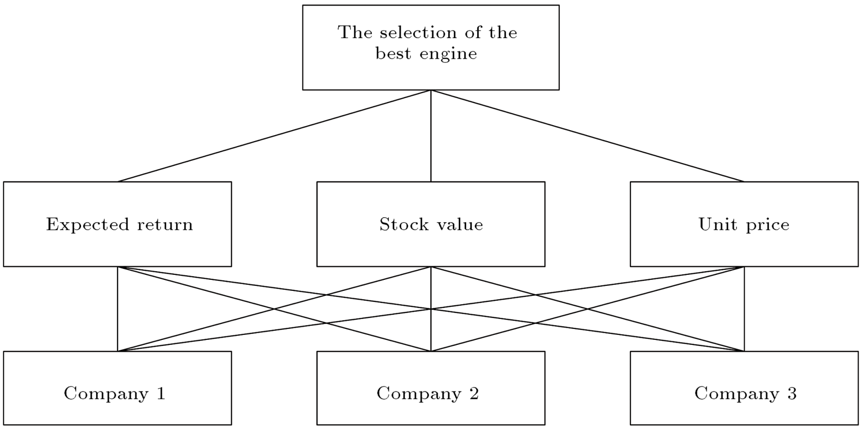

Figure 5. The selection of the best company.

has the best performance value in the interval $[2, 3]$. Therefore, the preference of alternatives depends on the amount of alternatives in each interval as given in Table 5.

5.2. Case 2

In order to invest in the stock market, it is desired to rank three companies offering their stock, according to the following criteria, as shown in Figure 5:

- Expected return for share-holder;
- Stock value at the end of maintenance period;
- Unit price.

These criteria depend on the amount of initial investment (x). The weights of the criteria are as follows: 0.5, 0.3, and 0.2. The decision matrix is given in Table 6.

The process of the proposed method in Case 2 includes the following steps:

Step 1. Introduce the lower bounds ($L_{ijk_{ij,}}$) and upper bounds ($U_{ijk_{ij}}$) of intervals and sort them as: 0, 100, 200, 600 and 1000.

Table 6. The multi-segment decision making matrix of three companies.

Alternatives	Expected	Return	Stock value		Unit price	
A_1	$0.1x$	$0 \leq t \leq 100$	xe^0	$0 \leq t \leq 200$	5000-$0.2x$	$0 \leq t \leq 100$
	$0.2x$	$100 \leq t \leq 1000$	xe^0	$200 \leq t \leq 600$	5000-$0.3x$	$100 \leq t \leq 1000$
			xe^1	$600 \leq t \leq 1000$		
A_2	$0.1x$	$0 \leq t \leq 200$	xe^0	$0 \leq t \leq 600$	6000-$0.1x$	$0 \leq t \leq 100$
	$0.3x$	$200 \leq t \leq 600$			6000-$0.2x$	$100 \leq t \leq 600$
	$0.4x$	$600 \leq t \leq 1000$	xe^1	$600 \leq t \leq 1000$	6000-$0.3x$	$600 \leq t \leq 100$
A_3	$0.2x$	$0 \leq t \leq 100$	xe^0	$0 \leq t \leq 200$	5000-$0.4x$	$0 \leq t \leq 600$
	$0.3x$	$100 \leq t \leq 1000$	xe^0	$200 \leq t \leq 1000$	5000-$0.6x$	$600 \leq t \leq 1000$

Table 7. The calculated amount of under curves for three companies.

Duration	Alternatives	C_1	C_2	C_3
$0 \leq t \leq 100$	A_1	500	9110.6	499000
	A_2	500	12298	599500
	A_3	1000	6749.3	498000
$100 \leq t \leq 200$	A_1	3000	27331.78	495500
	A_2	1500	36894	597000
	A_3	4500	20247.88	494000
$200 \leq t \leq 600$	A_1	32000	291539	1952000
	A_2	48000	393536.5	2368000
	A_3	48000	393536.5	1936000
$600 \leq t \leq 1000$	A_1	64000	1062437.42	1904000
	A_2	128000	1062437.42	2304000
	A_3	96000	787073	1808000

Table 8. The normalized matrix of three companies.

Duration	Alternatives	C_1	C_2	C_3
$0 \leq t \leq 100$	A_1	0.5	0.74	0.998
	A_2	0.5	1	0.831
	A_3	1	0.55	1
$100 \leq t \leq 200$	A_1	0.667	0.741	0.997
	A_2	0.333	1	0.827
	A_3	1	0.549	1
$200 \leq t \leq 600$	A_1	0.667	0.741	0.991
	A_2	1	1	0.818
	A_3	1	1	1
$600 \leq t \leq 1000$	A_1	0.5	1	0.95
	A_2	1	1	0.785
	A_3	0.75	0.741	1

Step 2. Construct the intervals as: $[0, 100]$, $[100, 200]$, $[200, 600]$ and $[600, 1000]$.

Step 3. Compare all functions in all intervals. The functions of this case do not cross each other.

Step 4. Calculate amounts of area under curves by Eq. (4) as given in Table 7.

Step 5. Calculate the normalized decision matrix by Eq. (5) as provided in Table 8.

Step 6. Calculate the score of each alternative. The score of each alternative is calculated by Eq. (6) as reported in Table 9. After the calculation of the final evaluation value for each alternative, alternative A_i can be compared with the others as provided in Table 9.

In this case, the new intervals are sorted as:

$$[0, 100], [100, 200], [200, 600], [600, 1000].$$

In interval $[0, 100]$, company 1 has the best rank, and companies 2 and 3 have second and third ranks, respectively. In interval $[100, 200]$, the companies are ranked as $A_2 > A_3 > A_1$. In interval $[200, 600]$, they are ranked as $A_3 > A_2 > A_1$. Moreover, companies are ranked as $A_3 > A_1 > A_2$ in the interval $[600, 1000]$ as given in Table 9.

5.3. Case 3

It is desired to evaluate three financial institutes. They receive an initial investment (t) from customers and return their investments and the cost of money (the interest rate). Now, the investor wants to select one from among these three financial firms. To do so, it is required to rank them according to the following criteria as shown in Figure 6:

- Profitability index;
- The criterion of net present value;
- The amount of amortization.

The weights of the criteria are as follows: 0.4, 0.4, and 0.2. The decision matrix is given in Table 10.

The step by step problem-solving process is proposed for the third case as follows:

Step 1. Calculate lower bounds ($L_{ijk_{ij,}}$) and upper bounds ($U_{ijk_{ij}}$) of intervals, and the results are as follows: 1, 10, 15, 20, 30.

Table 9. Final value of three companies.

Duration	Alternatives	C_1	C_2	C_3	Final value	Rank
$0 \leq t \leq 100$	A_1	0.25	0.222	0.2	0.672	3
	A_2	0.25	0.3	0.166	0.716	2
	A_3	0.5	0.165	0.2	0.865	1
$100 \leq t \leq 200$	A_1	0.334	0.222	0.199	0.755	2
	A_2	0.167	0.3	0.165	0.632	3
	A_3	0.5	0.165	0.2	0.865	1
$200 \leq t \leq 600$	A_1	0.334	0.222	0.198	0.754	3
	A_2	0.5	0.3	0.164	0.964	2
	A_3	0.5	0.3	0.2	1	1
$600 \leq t \leq 1000$	A_1	0.25	0.3	0.19	0.74	3
	A_2	0.5	0.3	0.157	0.957	1
	A_3	0.375	0.222	0.2	0.797	2

Table 10. The multi-segment decision making matrix of three institutes.

Alternatives	Profitability	Index	Net present	Value		Amortization
A_1	$(1.2)^t$	$1 \leq t \leq 10$	$(3)^t - (2)^t$	$1 \leq t \leq 10$	$0.2t$	$1 \leq t \leq 10$
	$(1.4)^t$	$10 \leq t \leq 20$	$(4)^t - (2)^t$	$10 \leq t \leq 20$	$0.3t + \frac{3}{2}$	$10 \leq t \leq 20$
A_2	$(1.2)^t$	$1 \leq t \leq 10$	$(3)^t - (2)^t$	$1 \leq t \leq 10$	$0.1t$	$1 \leq t \leq 10$
	$(1.5)^t$	$10 \leq t \leq 30$	$(4)^t - (2)^t$	$10 \leq t \leq 30$	$0.4t$	$10 \leq t \leq 30$
A_3	$(1.1)^t$	$1 \leq t \leq 10$	$(4)^t - (2)^t$	$1 \leq t \leq 20$	$0.4t$	$1 \leq t \leq 20$
	$(1.3)^t$	$10 \leq t \leq 20$				
	$(1.4)^t$	$20 \leq t \leq 30$	$2((4)^t - (2)^t)$	$20 \leq t \leq 30$	$0.5t$	$20 \leq t \leq 30$

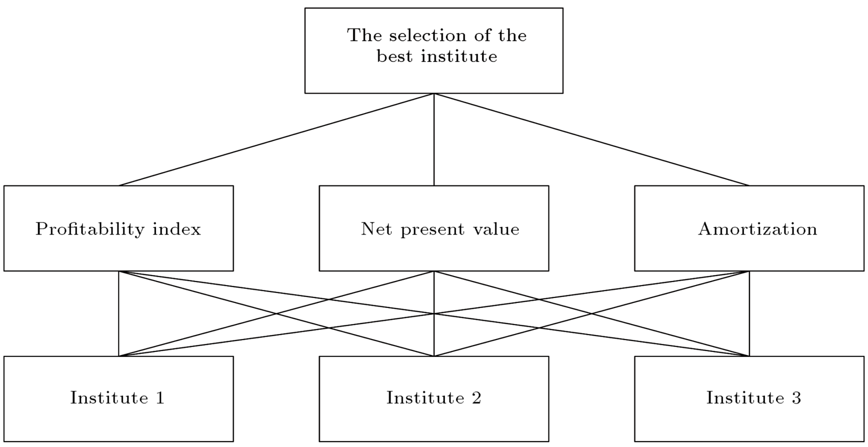

Figure 6. The selection of the best institute.

Step 2. Obtain the intervals obtained as: $[1, 10]$, $[10, 15]$, $[15, 20]$ and $[20, 30]$.

Step 3. Compare all functions. Function $f_{132}(x) = 0.3t + \frac{3}{2}$ intersects $f_{331}(x) = 0.4t$. In $x = 15 \in [10, 20]$. Therefore, break down the interval $[10, 20]$ to $[10, 15]$ and $[15, 20]$. Then, sort the new intervals as: $[1, 10]$, $[10, 15]$, $[15\ 20]$ and $[20, 30]$.

Step 4. Use Eq. (4) for calculating the amount of area under curves as given in Table 11.

Step 5. Calculate the normalized decision matrix by Eq. (5) as provided in Table 12.

Table 11. The calculated amount of under curves for three institutes.

Duration	Alternatives	C_1	C_2	C_3
$1 \leq t \leq 10$	A_1	0.12	0.42	9.9
	A_2	0.12	0.42	4.95
	A_3	0.11	0.54	19.8
$10 \leq t \leq 15$	A_1	$1.07 * 10^{-7}$	$1.36 * 10^{-3}$	26.5
	A_2	$1.75 * 10^{-7}$	$1.36 * 10^{-3}$	25
	A_3	$1.07 * 10^{-7}$	$1.36 * 10^{-3}$	25
$15 \leq t \leq 20$	A_1	$4.36 * 10^{-11}$	$4.27 * 10^{-5}$	33.75
	A_2	$9.04 * 10^{-11}$	$4.27 * 10^{-5}$	35
	A_3	$4.36 * 10^{-11}$	$4.27 * 10^{-5}$	35
$20 \leq t \leq 30$	A_2	$4.66 * 10^{-14}$	$1.37 * 10^{-6}$	100
	A_3	$6.52 * 10^{-15}$	$2.74 * 10^{-6}$	125

Table 12. The normalized matrix of three institutes.

Duration	Alternatives	C_1	C_2	C_3
$1 \leq t \leq 10$	A_1	1	0.78	0.5
	A_2	1	0.78	1
	A_3	0.92	1	0.25
$10 \leq t \leq 15$	A_1	0.611	1	0.94
	A_2	1	1	1
	A_3	0.611	1	1
$15 \leq t \leq 20$	A_1	0.48	1	1
	A_2	1	1	0.96
	A_3	0.48	1	0.96
$20 \leq t \leq 30$	A_2	1	0.5	1
	A_3	0.14	1	0.8

Step 6. Calculate the final evaluation value of each alternative. After calculation of the final evaluation value of each alternative, the pair-wise comparison of the preference relationship between the alternatives can be established as given in Table 13. In this case, the new intervals are sorted as $[1, 10]$, $[10, 15]$, $[15, 20]$, and $[20, 30]$ as given in Table 13.

The institutes are ranked in each interval. The preferences of these institutes are presented as:

$[1, 10]$, $A_3 > A_1 > A_2$,

$[10, 15]$, $A_3 > A_1 > A_2$,

$[15, 20]$, $A_2 > A_1 > A_3$,

$[20, 30]$, $A_2 > A_3$,

6. Result and discussion

In our proposed decision method, a multi-segment decision matrix is employed to determine the preference of alternatives in multi-segment problems, which can be easily solved by this method step by step. This research has conducted a performance analysis on three case studies using a multi-segment MCDM approach. In the first case study, there are four criteria for ranking the alternatives. The first criterion is the amount of spring elasticity. The rate of elasticity

is indicated by the following relation:

$$Y = a \sin(\omega t + 0), \tag{8}$$

where t is a variable referring to the time of receiving the customer order. The unit of time in this problem is a month. The amplitude A of a wave is the magnitude of the maximum displacement of the individual particles from their equilibrium position. $\omega = 2\pi/T = 2\pi f$ is the angular frequency of the wave. $\varnothing$ is called the phase constant. 0 is the initial phase of the vibrating particle (i.e., phase $t = 0$). The term $\omega t + 0$ is known as the phase of the vibrating particle.

The level of elasticity is regarded as a positive criterion, that is, more elasticity is favored. Production companies produce different springs depending on the changes of conditions, like the climate condition. The elasticity of springs produced in the first half of the year vary from those produced in the second half as depicted in Figure 7. The elasticity of the springs produced in

Table 13. Final value of three institutes.

Duration	Alternatives	C_1	C_2	C_3	Final value	Rank
$1 \leq t \leq 10$	A_1	0.4	0.312	0.1	0.812	3
	A_2	0.4	0.312	0.2	0.912	1
	A_3	0.368	0.4	0.05	0.818	2
$10 \leq t \leq 15$	A_1	0.24	0.4	0.188	0.828	3
	A_2	0.4	0.4	0.2	1	1
	A_3	0.24	0.4	0.2	0.84	2
$15 \leq t \leq 20$	A_1	0.192	0.4	0.2	0.792	2
	A_2	0.4	0.4	0.192	0.992	1
	A_3	0.192	0.4	0.192	0.784	3
$20 \leq t \leq 30$	A_2	0.4	0.2	0.2	0.8	1
	A_3	0.056	0.4	0.16	0.616	2

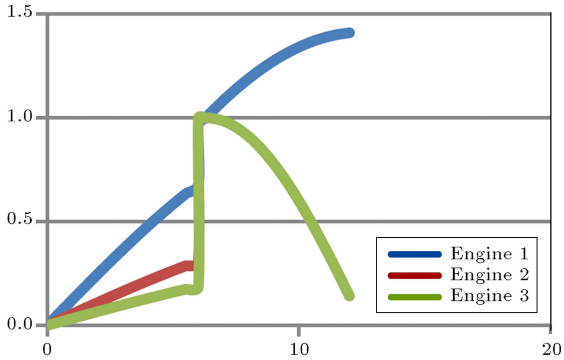

Figure 7. The amount of engines versus spring's elasticity.

the first half of the year for every company is obtained by the following relation:

$$A_1: \quad x = \sin\left(\tfrac{t}{8}\right) \qquad \omega = \tfrac{1}{8'}, \qquad T = 16\pi,$$

$$A_2: \quad x = \tfrac{1}{2}\sin\left(\tfrac{t}{9}\right) \qquad \omega = \tfrac{1}{9'}, \qquad T = 18\pi,$$

$$A_3: \quad x = \tfrac{1}{3}\sin\left(\tfrac{t}{10}\right) \qquad \omega = \tfrac{1}{10'}, \qquad T = 20\pi.$$

All three above-mentioned functions are ascending in the interval $[0, 6]$. So, the producer increases the elasticity of the produced springs through time. As indicated in Figure 7, the three functions do not cross each other in the first half of the year.

The elasticity of springs produced in the second half of the year is calculated by following relations:

$$A_1: \quad x = \sqrt{2}\sin\left(\tfrac{t}{8}\right) \qquad \omega = \tfrac{1}{8'}, \qquad T = 16\pi,$$

$$A_2: \quad x = \sin\left(\tfrac{t}{4}\right) \qquad \omega = \tfrac{1}{4'}, \qquad T = 8\pi,$$

$$A_3: \quad x = \sin\left(\tfrac{t}{4}\right) \qquad \omega = \tfrac{1}{4'}, \qquad T = 8\pi.$$

As shown in Figure 7, the functions cross at the point $t = 2\pi = 6.28$. Thus, the interval in $[0, 6]$ is divided into two intervals in $[6, 6.28]$ and $[6.28, 12]$.

The second criterion of the ranking is the price of the produced engines. The engines are priced by the companies every three months. So, the price of products is fixed to the end of each session according to Table 14.

Since this criterion is negative, the less the value assigned to the alternatives, the better the given alternative. In this way;

$$0 \le t \le 3 \qquad A_2 > A_3 > A_1,$$

$$3 \le t \le 6 \qquad A_2 > A_3 = A_1,$$

$$6 \le t \le 9 \qquad A_3 > A_2 > A_1,$$

$$9 \le t \le 12 \qquad A_3 > A_1 > A_2.$$

The third criterion is the amount of gas consumed.

Table 14. The amount of engines with respect to price.

Duration	Alternatives	Price
$0 \le t \le 3$	A_1	20
	A_2	15
	A_3	17
$0 \le t \le 3$	A_1	25
	A_2	20
	A_3	25
$0 \le t \le 3$	A_1	25
	A_2	24
	A_3	22
$0 \le t \le 3$	A_1	20
	A_2	22
	A_3	10

The amount of existing gas in time t is shown by the following function:

$$y = y_{0e^{-kt}}, \tag{9}$$

where k is the coefficient of daily usage, y_0 is the amount of initial gas which equals 100 liters. The fuel tank is filled daily.

The engines produced in companies 1, 2 and 3 have different gas consumption rates in every season. In all companies, the amount of gas consumed in the second quarter of the year (summer) is less than in other seasons and in the fourth quarter (winter) is more than other seasons. As shown in Figure 8, this criterion is a negative one, that is, the less the gas consumption rate, the better.

The fourth criterion is the speed. This is a positive criterion, so the more the speed, the better. In the first half of the year, the producing companies produce engines with different speed capabilities from the second half as follows:

$$0 \le t \le 6; \qquad \begin{cases} A_1: & v = 0.5t + 1 \\ A_2: & v = t \\ A_3: & 0.5t + 1 \end{cases}$$

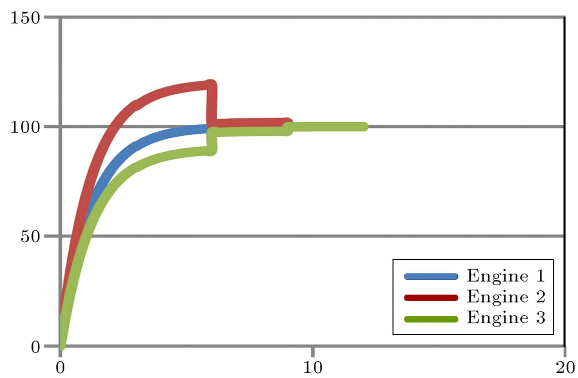

Figure 8. The amount of engines versus the amount of gas consumed.

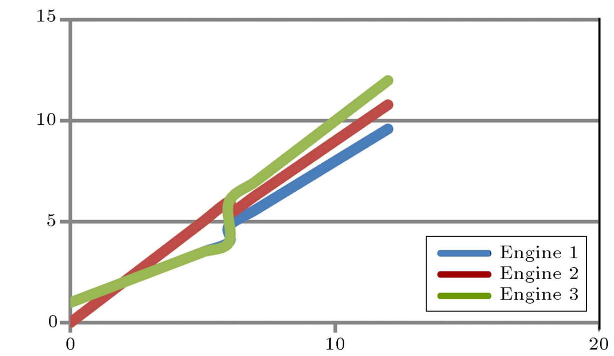

Figure 9. The amount of engines versus the speed.

$$6 \le t \le 12; \qquad \begin{cases} A_1: \ v = 0.8t \\ A_2: \ v = 0.9t \\ A_3: \ t \end{cases}$$

As shown in Figure 9, the functions cross each other at point $t = 2$. Thus, the interval $[0, 3]$ is divided into two intervals of $[0, 2]$ and $[2, 3]$. This means that in interval $[0, 2]$, the engines of company 1 are better than those of the other company, while the engines of company 2 have more speed in the interval of $[2, 3]$ as shown in Figure 9.

In the second case study, the first criterion is the expected turnover by the stockholders; the more the turnover, the better. Functions regarded as the expected turnover are ascending, that is, the more the amount of initial investment, the more the amount of turnover expected. In company A_1, if the investment is in interval $[0, 100]$, the expected turnover is 0.1 of the inventory, whereas if the investment is in interval $[100, 1000]$, the amount of turnover expected can be 0.2 of the inventory. Also, for company A_2, the amount of turnover expected is at three levels, and at two levels for company A_3, which are presented in Figure 10.

The second criterion is the stock value at the end of the maintenance period. It is a positive criterion. The functions assigned to the alternative in this criterion are ascending. Like the first criterion, the value of stock at the end of the period is proportionate to the initial investment. The future value of stock for companies A_1, A_2 and A_3 is presented at two levels. Like Figure 11, there is no overlapping of functions.

The third criterion is the price of each stock unit. Higher prices are not suitable to be invested. The functions assigned to this criterion are descending. Thus, the companies increase the discounts applied to stock prices proportionate to the number of units purchased, in order to encourage companies for the investment. For instance, if the amount of the investment is between 0 and 100 for company A_1, the investors will have a 0.2 discount. If the purchased stock units are between 100 and 1000, the amount of discount will be 0.3. Also, companies A_2 and A_3 have their regulations in

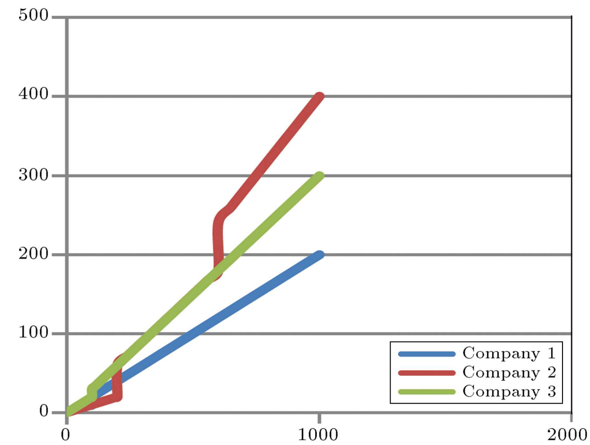

Figure 10. The values of companies versus an expected return on share-holder.

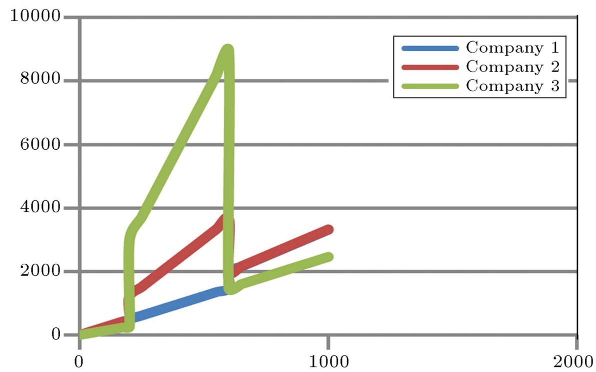

Figure 11. The values of companies versus the stock value at the end of maintenance period.

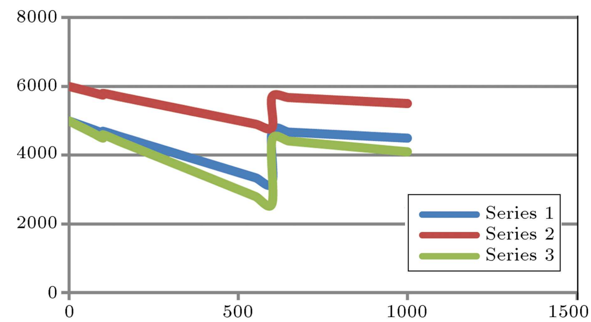

Figure 12. The values of companies versus an amount of price.

different levels. As depicted in Figure 12, there is no overlapping of functions in the interval of 0 and 1000 in the third criteria. As observed in the process of solving the second case, the ranking is done in every interval.

In the third case study, the first criterion of the decision-making process is the profitability. The profitability varies according to the initial investment in different institutes. In institute A, if initial investment is 1 to 10 units, the profitability can equal $(1.2)^t$, whereas, if the initial investment (t) is 10 to

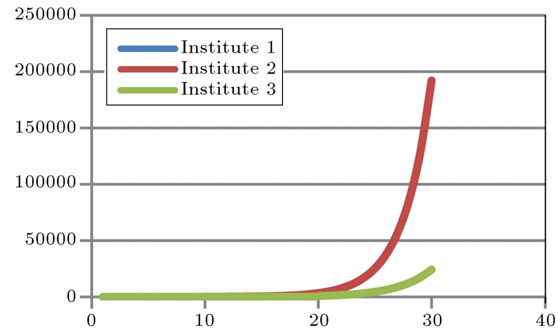

Figure 13. The values of institutes versus profitability index.

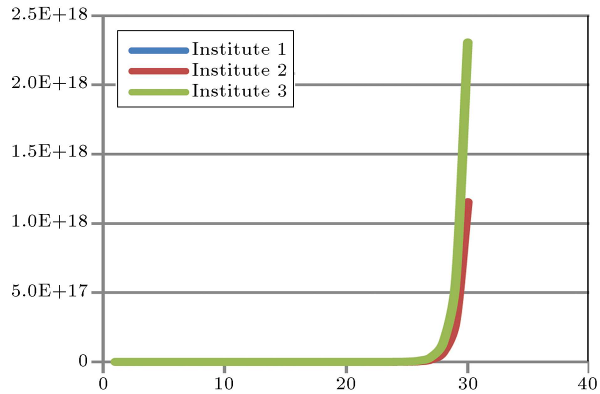

Figure 14. The values of institutes versus the criterion of net present value.

20, the profitability can be $(1.4)^t$ to encourage more investment. The maximum amount of investment in institute A_1 is 20 units, while in institutes A_2 and A_3 it is possible to invest up to 30 units. The functions of this criterion do not cross each other as shown in Figure 13.

The second criterion is the net present value. This is a positive criterion; in other words, the more the net present value, the better. Like Figure 14, there is no overlapping of functions.

The third criterion is the amount of amortization. Since this criterion is a negative one, the less the value assigned to the alternatives, the better the given alternative. The functions cross each other at point $t = 15$. Thus, interval $[10, 20]$ is divided into two intervals $[10, 15]$ and $[15, 20]$ as shown in Figure 15.

Based on the results of the analysis, some essential findings are discussed as follows. Because the SAW method is very easy and the proposed method is provided step by step, the computation process is simple and straightforward. Moreover, the DM can find the best alternative in each interval. In some cases, an overlapping of the curves of the functions may exist. Hence, the preference of alternatives may be changed in each interval. Considering this concept, the new method is proposed as a logical mathematical tool to help the DM in order to make the best decision.

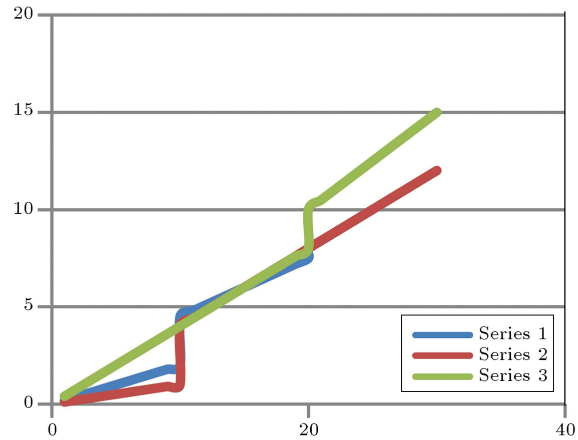

Figure 15. The values of institutes versus the amount of amortization.

During this multi-segment approach, one parameter alone is not taken into consideration to deal with the complex decision problems. Many parameters can be introduced, such as time, order quantity, and amount of production. This method can assist the DM when the order quantity of alternatives is not exactly provided, and then the bounds of the order quantity are assigned as an interval. In this method, the values of alternatives with respect to criteria are transformed into the dimensionless value. Thus, the final value of alternatives can be calculated where the criteria are presented with different dimensions. For example, in the first case study, three mechanical engines are estimated with respect to four criteria, including spring elasticity, price, amount of gas consumed, and speed. When comparing the performance of the first machine with the other two machines, it can be observed that the first machine has the best performance value in intervals $[0, 2]$ and $[3, 6]$, whereas the third machine has the best performance value in intervals $[6, 6.28]$, $[6.28, 9]$ and $[9, 12]$. Moreover, the second machine has the best performance value in interval $[2, 3]$. Hence, the preference of alternatives depends on the amount of alternatives in each interval. In the second and third cases, by considering the preference criteria, the rankings of alternatives are different in each interval where each interval has the corresponding ranking. This leads to a competitive advantage because other methods cannot consider these factors and only present the preference without considering environmental conditions.

As mentioned in Section 1, all response spaces are considered. The presented method is a decision-making method. The main difference from other methods is that the proposed decision-making method can change under different conditions, according to the complexity of the decision problems in a real-world situation. The previous methods have provided only one ranking throughout the decision-making process and the decision matrix has a fixed value, whereas in this method, besides changing the decision-matrix in different levels, the values can be properly regarded as a function. For example, in the first case, instead of presenting one ranking in general ($[0, 12]$), the decision-making process is done by considering the values of functions related to each alternative in each of the corresponding intervals (i.e., $[0, 2]$, $[2, 3]$, $[3, 6]$, $[6, 6.28]$, $[6.28, 9]$ and $[9, 12]$).

7. Conclusion

In this paper, a new SAW method was proposed to solve problems in which an amount of alternatives, with respect to criteria, is presented in several levels. The proposed method led to preference alternatives in multi-segment problems which can be easily solved by the step by step method. Considering the fact that in real-world situations, the value of alternatives is not stationary under every condition, the proposed method can be applied to deal with problems wherein the data of the decision matrix is introduced as a functional amount, and can be dependent on some parameters at every level. Therefore, the functional amounts were employed and compared to rank the alternatives in real-life situations. The area under curves was used in order to calculate the elements of the matrix. Because the assessment value changed under different conditions for the complex decision problem, this method has high accuracy in determining the preference of all options. Moreover, this method can help the decision maker when the order quantity is not determinable and the bounds of the order quantity are assigned as an interval. Hence, this method is applied to a greater number of issues in order to deal with real-world decision problems under multiple criteria. Finally, to show the validity and applicability of the proposed SAW method, based on a multi-segment decision making matrix, three illustrative cases were provided. Then, the sensitivity analysis was described in detail. The weight of criteria was changeless during the problem-solving, whereas the weight of criteria can be represented in several segments. Also, the weight can be represented as a function of some parameters. This subject is recommended for further research in discrete decision-making problems.

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Appendix

Here, the calculation style is described for the amount of area under curve for alternative A_1 versus criterion

C_1 in the first level. Like this, other amounts can be calculated. According to Table 2, we have:

$$f(t) = \sin\left(\frac{t}{8}\right),$$

$$a_{111} = \int_0^2 \sin\frac{t}{8}dt = 0.25, \qquad 0 \leq t.$$

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