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Consideration of transportation lags in a two-machine Flow shop scheduling problem

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Abstract. This paper considers two-machine flow shop scheduling problem while there is ineligible transportation lags in production procedure. There is one transporter to convey semi-finished jobs between machines, and another transporter to deliver finished jobs to the warehouse (customers). The problem is formulated as a Mixed Integer Linear Programming (MILP) model to minimize the makespan as an objective function. To solve the problem in an efficient way, two heuristic algorithms are also developed. Furthermore, five lower bounds are proposed and computational experiments are carried out to verify the effectiveness of the proposed lower bounds and heuristic algorithms. The results show the performance of the heuristics to deal with medium and large size problems.

1. Introduction

In most production systems with more than one machine, semi-finished jobs are transferred between machines through material handling systems such as Automated Guided Vehicles (AGVs), conveyers and robots. There are also a lot of factories in which the process of carrying finished jobs to a warehouse or delivering them to the customers is of great importance. Thus, scheduling problems which consider transportation and scheduling simultaneously are more practical than those problems that do not take the transportation into consideration [1-3]. In the last four decades, many books and numerous published papers have studied machine scheduling. However, most of studies neglect the transportation lags (see, for example [4-6]) or assume that all needed transportations can be done instantaneously (see, for example [7-10]).

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Several "logistics" and "production management" research recently focused on the coordination of machine scheduling and transportation. These studies usually divide transportation into two categories. The first category is intermediate transportation which is needed to carry jobs between machines and called as transportation Type I. The second category that named as transportation Type II is needed to deliver finished jobs to the customer or warehouse [11].

The main difference between ordinary processing machines and transporters, which influences the complexity of problems intensely, is their disability to start next job(s) after accomplishment of the previous job(s). Transporters need some time to move to the physical place of next job(s). This makes transporters quite similar to the ordinary machines which need extra setup time after each processing. Hence negligence of returning times or consideration of infinite number of transporters can be similar to neglecting the existence of transporters. The other difference is that transporters can usually carry more than one job in each trip, so they are similar to batching machines in this aspect. Some of the most recurrent and important assumptions which play an important role in applicability and complexity of the problems are: transportation type, number and capacity of transporters, transportation times, and dependency of transportation capacity and transportation times in respect of jobs.

Table 1 presents a brief comparison between our work and the previous studies by means of these criteria. Throughout the literature review, emphasis has been put upon the works of Maggu and Das [12] and Lee and Chen [11] as they discuss the important issues of integrated flow shop scheduling with transportation constraints. Maggu and Das were one of the first researchers who add transportation constraints into the machine scheduling problem. Two-machine flow shop problem is considered in their work with infinite buffer space and unlimited number of transporters. In their study, transportation times are job-dependent. The study is extended later by the same authors by adding the constraint of consecutive execution of some jobs to the problem [13]. Lee and Chen [11] present a comprehensive review on the two types of transportation. It is proved in this article that $TF_2|\text{capacity}|C_{\max}$ is strongly NP-hard when the problem consists of one transporter Type I with the capacity more than two, and also when there is one transporter Type II with the capacity of one or more than three. There are some studies (e.g. [14-17]) which consider first type of transportation integrated with scheduling problem. In the other line of investigations with transportation consideration, the studies focus on the delivery of finished jobs to customers or a warehouse with a limited number of vehicles. These researches, in other words, investigate transportation Type II (see, e.g. [2,3,18-23]).

Most of the researchers have devoted a part of their works on proving the complexity of their problems (see [13,24-26]). Hence, application of heuristic and meta-heuristic algorithms together with the exact methods is of great importance. Although, some of these studies have just proven the complexity of their problems, they have just proposed exact algorithms which are incapable of dealing with the large size problems.

It is also worthwhile to remember that all of these studies considered just one type of transportation and none dealt with both types of transportations. In this paper, we study two-machine flow shop scheduling problem which involves two types of transportation and consider makespan as our objective function. There are two transporters with different capacities; one for conveying semi-finished jobs between machines (Type I), and the other for delivering finished-jobs to the customers or a warehouse (Type II). Mathematical formulation of the problem is proposed as a Mixed Integer Linear Programming (MILP) model. Two heuristic algorithms are also developed for mid and large size problems. To evaluate the effectiveness of the heuristics, some lower bounds are also set.

The further sections of the paper are organized as follows. In the next section the considered problem is described formally. Section 3 describes MILP model. Section 4 is devoted to proposed heuristic algorithms and presented lower-bounds. Computational experiments are carried out in Section 5, and finally, the paper is concluded in Section 6.

2. Definition of the problem

Classical two-machine permutation flow shop scheduling problem is considered to minimize $C_{\max}(TF_2|\ |C_{\max})$, while transportation is needed to convey semi-finished jobs between machines and also to deliver finished-jobs to the customers. According to the permutation flow shop problem, there is a set of non-preemptive "n" jobs, $j = \{1 \cdots n\}$, to be processed on two single machines, each job has one operation on each machine; and the sequence of the jobs is the same on both machines. There are two transporters to carry the jobs. One is responsible to convey semi-finished jobs from first machine to the second machine, and the other is to deliver finished jobs to the customer (warehouse). Transportation times and transporters capacities are assumed to be job-independent. Forwarding and returning times are equal for transporters. Both transporters are also available in the beginning on their aligned stations. The following notations are used for transporter "i":

(i) "capacity$_i$": The capacity of transporters (it means that the transporter "i" can transport at most "capacity$_i$" jobs at a time).

(ii) "T_i": The round trip times of transporters.

Let "C_{ji}" denotes the completion time of job "j" on the "ith" machine. The *makespan* is denoted by "$C_{\max}$" and is defined as $C_{\max} = \text{Max}_{C_{J2}}$ ($j = 1 \cdots n$). The objective function of the paper is to minimize the makespan.

It is clear that any solution should answer three questions: The first is the sequence of jobs, and the two others are the combination of jobs in transportation batches for each transporter.

3. Problem formulation

In this section, the problem is formulated as a MILP model. The parameters and the variables used in the model are as follows:

Table 1. Scheduling attributes used in this research.

	1			2			3		4		5			6			7					8	9		Considered constraints
	a	b	c	a	b	c	a	b	a	b	a	b	c	a	b	c	a	b	c	d	e		a	b	
Problem in this paper	✓			✓			✓	✓	✓				✓		✓		✓						✓	✓	
[12]	✓			✓			✓			✓				✓			✓								
[13]	✓			✓			✓			✓				✓			✓								Consecutive execution
[24]	✓			✓			✓		✓		✓			✓			✓					✓			
[15]	✓			✓			✓		✓		✓			✓			✓					✓		✓	Limited buffer space between machines
[16]	✓			✓			✓		✓		✓			✓			✓								Limited buffer space on the first machines
[17]	✓					✓	✓		✓		✓			✓			✓								
[25]	✓			✓			✓		✓		✓				✓		✓							✓	Limited buffer space on the first machines
[1]	✓			✓			✓		✓				✓												First machine is batching machine
[26]	✓			✓			✓		✓		✓					✓	✓					✓	✓		
[14]	✓			✓			✓			✓			✓	✓		✓	✓			✓		✓		✓	Limited buffer space on the first machines, first machine is batching machine
[18]		✓			✓			✓	✓				✓		✓		✓					✓	✓	✓	Raw materials must be transported form warehouse
[2]		✓	✓	✓	✓			✓		✓			✓	✓				✓							There are K customers by different locations
[19]		✓			✓			✓	✓				✓	✓					✓			✓	✓	✓	There are m customers in different locations
[20]		✓		✓				✓	✓				✓	✓			✓					✓	✓		
[22]		✓			✓			✓					✓	✓					✓						There are multiple transportation modes to select and multiple destinations
[3]		✓			✓			✓	✓		✓			✓			✓				✓	✓		✓	

1 Production system
1.a. Flowshop
1.b. Single machine
1.c. Parallel machines

2 Number of machines
2.a. One
2.b. Two
2.c. More than two

3 Transportation type
3.a. Type I
3.b. Type II

4 Number of transporters
4.a. One
4.b. Infinite

5 The capacity of the transporter
5.a. One
5.b. Infinite
5.c. Limited

6 Transportation time
6.a. Job-dependent
6.b. Job-independent
6.c. No-returning time

7 Objective function
7.a. Min makespan
7.b. Max customer satisfaction
7.c. Min sum of arrival times
7.d. Min total blocking times
7.e. Min total setup costs

8 Proof of complexity

9 Solution methods
9.a. Exact algorithms
9.b. Heuristic algorithms

P_{ji}: The processing time of job "j" on machine "i"; $\begin{cases} i=1,2 \\ j=1,2,\cdots,n \end{cases}$

M: A sufficiently large positive constant

$X_{jik}=1$: If job "j", which is performed on machine "i", is assigned to the "kth" transportation batch of the transporter "i", otherwise=0

$S_{jm}=1$: If the job "j" is "mth" job in the job sequence, otherwise=0

ST_{ki}: The start time of transporting the "kth" transportation batch of transporter "i"

Start_{ji}: The start time of processing job "j" on machine "i"

Decision variables of the mathematical model are X_{jik} and S_{jm} which determine successively the combination of jobs in transportation batches, and the sequence of the jobs.

Objective function: *Minimize* $C_{\max}$

subjected to:

$$\sum_{m=1}^{n} s_{jm}=1 \qquad \begin{cases} j=1,\cdots,n \end{cases} \tag{1}$$

$$\sum_{j=1}^{n} s_{jm}=1, \qquad \begin{cases} m=1,\cdots,n \end{cases} \tag{2}$$

$$S_{j0}=0 \qquad \begin{cases} j=1,\cdots,n \end{cases} \tag{3}$$

$$\text{Start}_{j1} \geq \left(\sum_{q=0}^{m-1}\sum_{z=1}^{n} S_{zq}*P_{z1}\right) - (1-S_{jm})*M \qquad \begin{cases} j=1,\cdots,n \\ m=1,\cdots,n \end{cases} \tag{4}$$

$$ST_{i1} \geq \text{Start}_{ji}+P_{ji} - (1-x_{ji1})*M \qquad \begin{cases} j=1,\cdots,n \\ i=1,2 \end{cases} \tag{5}$$

$$ST_{ik} \geq ST_{ik-1}+T_i \qquad \begin{cases} k=2,\cdots,n \\ i=1,2 \end{cases} \tag{6}$$

$$ST_{ik} \geq \text{Start}_{ji}+P_{ji} - (1-x_{jik})*M \qquad \begin{cases} i=1,2 \\ k=2,\cdots,n \\ j=1,\cdots,n \end{cases} \tag{7}$$

$$C_{ji} \geq ST_{ik}+T_i/2 - (1-x_{jik})*M \qquad \begin{cases} i=1,2 \\ k=1,\cdots,n \\ j=1,\cdots,n \end{cases} \tag{8}$$

$$\text{Start}_{j2} \geq \text{Start}_{z2}+P_{z2}-(1-S_{zm-1})*M - (1-S_{jm})*M \qquad \begin{cases} m=1,\cdots,n \\ z=1,\cdots,n \\ j=1,\cdots,n \end{cases} \tag{9}$$

$$\text{Start}_{j2} \geq C_{j1} \qquad \begin{cases} j=1,\cdots,n \end{cases} \tag{10}$$

$$C_{\max} \geq C_{j2} \qquad \begin{cases} j=1,\cdots,n \end{cases} \tag{11}$$

$$\sum_{k=1}^{n} x_{jik}=1 \qquad \begin{cases} j=1,\cdots,n \\ i=1,2 \end{cases} \tag{12}$$

$$\sum_{j=1}^{n} x_{jik} \leq \text{Capacity}_i \qquad \begin{cases} k=1,\cdots,n \\ i=1,2 \end{cases} \tag{13}$$

$$M*\left(\sum_{j=1}^{n} x_{jik}\right) \geq \sum_{j=1}^{n} x_{jik+1} \qquad \begin{cases} i=1,2 \\ k=1,\cdots,n-1 \end{cases} \tag{14}$$

$$\sum_{j=1}^{n} x_{ji1} \geq 1 \qquad \begin{cases} i=1,2 \end{cases} \tag{15}$$

$$x_{jik}, \qquad s_{jm}=0,1 \tag{16}$$

$$ST_{jk}, \qquad C_{ji}, \qquad \text{Start}_j \geq 0. \tag{17}$$

First, second and third constraints are to determine the sequence of jobs. The forth constraint considers the starting time of job "j" at the first machine as the total processing time which is needed for completion of previous jobs on this machine. Fifth, sixth and seventh constraints are to compute the starting time of transportation for the first and second transporter. Constraint (8) demonstrates the completion transportation time of jobs on the transporters. Constraints (9) and (10) are to determine the starting time of jobs on the second machine. Constraint (11) is to give the total completion time. The constraint (12) is to guarantee that each job should be carried by transporters between machines and from second machine to the customer. Capacity constraint of each transporter is satisfied by Inequality (13). Constraints (14) and (15) ensure that each transportation batch contains at least one job. Finally, last two constraints define the range of the variables.

4. Proposed heuristic algorithms

Although the MILP model provides an optimal solution, it is incapable of dealing with medium and large size problem efficiently, because the variables and constraints increase drastically when the number of jobs increases. Furthermore, as it was mentioned in Section 1, the two-machine flow shop scheduling problem with the first type of the transportation and fixed capacity is defined as a NP-hard problem. On the other hand, the problem with the second type of transportation has been also proven to be strongly NP-hard. Due to this complexity and inefficiency of the exact algorithms to solve the problem in an appropriate run time, developing heuristic algorithms seems to be inevitable.

4.1. Heuristic H_1

Heuristic H_1 is based on two well-known algorithms, "Johnson" [27] and "First Only Empty (FOE)" [28]. Johnson's rules sequence the jobs, and FOE algorithm determines the combination of each transportation batch. FOE algorithm, which is usually used for batching machines, calculates $\lceil n/\text{capacity}\rceil$ as total number of the batches. FOE also permits only the first batch to be not full and forces the other batches to contain all jobs they can. The steps of the proposed heuristic algorithm are as follows:

Step 1: Determine the sequence of jobs through the Johnson's algorithm.

Step 2: Determine the combination of transportation batches through the FOE rule by following instruction:

i. First transportation batch contains first $[n - (\lceil n/\text{capacity}\rceil - 1) * \text{capacity}]$ jobs.

ii. Next $(\lceil n/\text{capacity}\rceil - 1)$ transportation batches are all full of jobs up to the transporter's capacity.

A numerical example, described in Table 2, is used to illustrate the heuristic H_1. The parameters of the problem are set as follows:

Table 2. Processing time of the jobs.

Number of jobs	P_{j1}	P_{j2}
1	8	9
2	6	7
3	1	4
4	9	3
5	7	5

$$T_1 = 10, \qquad T_2 = 8,$$

$$\text{Capacity}_1 = 3, \qquad \text{Capacity}_2 = 2.$$

Johnson's algorithm determines 3-2-1-5-4 as sequence of the jobs. Through FOE algorithm, the first transporter carries jobs by two transportation batches ($\lceil n/\text{Capacity}_1\rceil = 2$). First batch contains first two jobs, and the other contains the rest. The second transporter conveys jobs by three batches. These batches contain one, two and two jobs, respectively. Figure 1 shows the final solution obtained by heuristic H_1 schematically. Total waiting time and makespan of this solution are, respectively, 58 and 57.

Lemma. If $C_{\text{heuristic}}$ is the makespan of the solution implementing the heuristic H_1, and C_* is the optimum solution of the problem, the maximum error of heuristic H_1 is as:

$$C_{\text{heuristic}} \leq \left(1 + \frac{2n - 2C_1}{2n - C_1} + \frac{2n - 2C_2}{2n - C_2}\right) * C_*. \quad (18)$$

Proof. If C_{Johnson} is the makespan, using Johnson's algorithm for the problem, while the transportations are neglected, then Statement (20) can be simply obtained:

$$C_{\text{heuristic}} \leq C_{\text{Johnson}} + (\lceil n/C_1\rceil - 1) * T_1 + (\lceil n/C_2\rceil - 1) * T_2 + T_1/2 + T_2/2. \quad (19)$$

On the other hand, C_* should be greater than lower bound of the problem (see Section 4.3):

$$C_* \geq C_{\text{Johnson}} + \frac{T_1}{2} + \frac{T_2}{2}. \quad (20)$$

From Statements (20) and (21) we can deduce Statement (22).

$$\frac{C_{\text{heuristic}}}{C_*} \leq 1 + \frac{(\lceil n/C_1\rceil - 1) * T_1}{C_*} + \frac{(\lceil n/C_2\rceil - 1) * T_2}{C_*}. \quad (21)$$

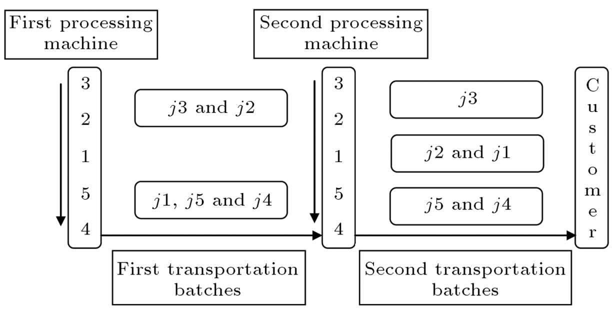

Figure 1. Solution using H_1.

There are also two other lower bounds for the problem:

$$C_* \geq (\lceil n/C_1 \rceil - 1/2) * T_1 + T_2/2, \quad (22)$$

$$C_* \geq (\lceil n/C_2 \rceil - 1/2) * T_2 + T_1/2. \quad (23)$$

By modifying Statements (23) and (24), and applying them in Statement (22), Statement (25) can be obtained.

$$\frac{C_{\text{heuristic}}}{C_*} \leq 1 + \frac{(\lceil n/C_1 \rceil - 1) * T_1}{(\lceil n/C_1 \rceil - 1) * T_1 + \frac{T_1+T_2}{2}} + \frac{(\lceil n/C_2 \rceil - 1) * T_2}{(\lceil n/C_2 \rceil - 1) * T_2 + \frac{T_1+T_2}{2}}. \quad (24)$$

By subtracting $\frac{T_2}{2}$ from the second term's denominator of the right side of Statement (25), and $\frac{T_1}{2}$ from the third term's denominator, Inequality (26) can be deduced.

$$\frac{C_{\text{heuristic}}}{C_*} \leq 1 + \frac{(\lceil n/C_1 \rceil - 1) * T_1}{(\lceil n/C_1 \rceil - 1) * T_1 + \frac{T_1}{2}} + \frac{(\lceil n/C_2 \rceil - 1) * T_2}{(\lceil n/C_2 \rceil - 1) * T_2 + \frac{T_2}{2}}, \quad (25)$$

$$\frac{C_{\text{heuristic}}}{C_*} \leq 1 + \frac{(\lceil n/C_1 \rceil - 1)}{(\lceil n/C_1 \rceil - 1/2)} + \frac{(\lceil n/C_2 \rceil - 1)}{(\lceil n/C_2 \rceil - 1/2)}, \quad (26)$$

$$\frac{C_{\text{heuristic}}}{C_*} \leq 1 + \frac{2n - 2C_1}{2n - C_1} + \frac{2n - 2C_2}{2n - C_2}. \quad (27)$$

4.2. Heuristic H_2

Despite the pros of the heuristic H_1 (developed based on two optimal algorithms, simplicity, requiring short CPU run time and qualified solutions in large size problems), there are some deficiencies in solving small and mid-size problems (see Section 5.1). Heuristic H_1 neglects transportation times during solving procedure, and transporters have to wait too long for the completion of their batches; whereas these transporters can start transporting before their capacities are full, in order to save time. Based on this fact, second heuristic method is suggested.

In heuristic H_2, Johnson's algorithm determines the sequence of jobs. Then a heuristic method determines the combination of jobs in transportation batches to reduce waiting time in transporters. Unlike the heuristic H_1 algorithm, transportation times are considered during batching process in heuristic H_2.

Let "$L_z = l$", if job z be lth job in the sequence, and B_{Ki} be a non-empty set of jobs, this set of jobs must be transported by Kth batch on transporter "i". Heuristic batching method forces transporters to wait for the next job when: batch "K" is not full, and the total waiting time of the transporter is not more than "$h * T_1$", where "h" is the "Efficiency Multiplier" (this is named "h" because it affects the efficiency of second heuristic algorithm). Heuristic H_2 limits total waiting time of transporters regarding transportation times and "h". During the algorithm, "h" will modify between [0.1, 6] in each iteration to obtain the best solution. For each "h" we repeat our batching method, calculate its corresponding makespan, and report the best solution as an output of H_2.

By applying heuristic H_2 to previous example, the makespan reduces about 10 percent. We also have huge reduction (34 unit times) in total waiting time of the transporters by applying second heuristic method. Figure 2 depicts the obtained solution by heuristic H_2 schematically.

In small and mid-size problems, considering transportation times in the batching procedure reduces waiting time on both transporters intensely. Consequently the obtained makespan by heuristic H_2 is closer to the optimum solution in comparison with heuristic H_1 (it can be inferred from the computational results, Sections 5.1 and 5.2).

4.3. Proposed lower bounds

An optimal solution is not obtainable for most of problems in appropriate time. Hence to assess the performance of the proposed heuristic algorithms, five lower bounds are developed. These lower bounds are so simple to be proven. The proposed lower bounds are defined as follows:

- By assuming that each individual machine is the bottleneck of the system and there are no idle time in the transporters and the other machine, the first two lower bounds can be defined as the sum of the total processing time of jobs on the machine, plus the minimum processing time on the other machine plus one way transportation time for each transporter.

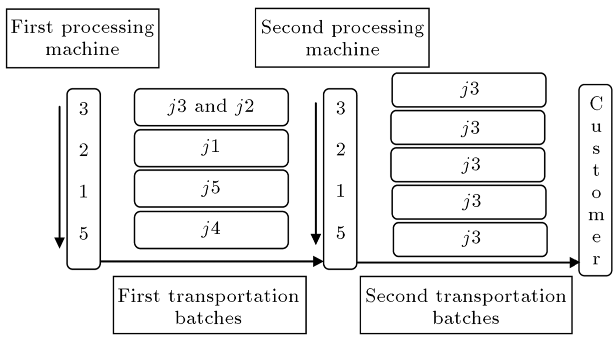

Figure 2. Solution of H_2.

$$lb_i = \sum_{j=1}^{n} P_{ji} + \min_{j,k \neq i} P_{jk} + T_1/2 + T_2/2,$$

$$i = 1, 2. \quad (28)$$

- The next two lower bounds are based on the assumption that the bottleneck is one of the transporters and it is defined as summation of: the minimum required time for transporting all jobs by this transporter, the minimum processing times on each machines, and one way transportation of the other transporter.

$$lb_{i+2} = \lceil n/C_1 \rceil * T_i - T_i/2 + \min_j P_{j1} + \min_j P_{j2} + T_{k \neq i}/2 \qquad i = 1, 2. \quad (29)$$

- The last lower bound is based on the Johnson's algorithm which gives an optimum solution for the classical two-machine flow shop problem. This lower bound assumes that the transportation lags have no effect on the makespan and is defined as below:

$$lb_5 = C_{\text{Johnson}} + T_1/2 + T_2/2. \quad (30)$$

The lower bound that is used to evaluate the performance of heuristic algorithms is given by the following statement:

$$lb = \max_k lb_k \qquad k = 1 \cdots 5. \quad (31)$$

5. Computational experiments

A number of computational experiments were carried out to verify the effectiveness of lower bounds, MILP model (presented in Section 3) and proposed heuristic algorithms. Lingo 8 software is used to apply B&B to obtain optimum solution of the MILP model with small size problems. The heuristic algorithms were also coded in MATLAB R2006b and executed on the computer with 3MB RAM and 2.2GH CPU. For all instances the processing times are randomly generated by $U \approx (0, 100)$. The following equation calculates the error as a gap between heuristic solutions and lower bound:

$$e = (C_{\text{heuristic}} - lb)/lb. \quad (32)$$

5.1. *Small size instances*

To evaluate the effectiveness of MILP model and proposed lower bounds, five different set of parameters are randomly generated, see Table 3. Lingo 8 were not capable to solve problems with more than 5 jobs in appropriate CPU run time, so we fixed problem sizes to 5, and generated 6 instances for each set. Other parameters are also randomly generated:

$$\text{Capacity}_i \approx U(2, 8), \qquad T_i \approx U(20, 100).$$

CPU run time for heuristics was negligible.

Table 3 illustrates that lower bounds are close to optimal solutions, which assure effectiveness of the proposed lower bounds. Furthermore the superiority of the Heuristic H_2 in comparison with H_1 is obviously observable. H_2 try to reduce waiting times for transporters (and consequently for the whole system), so this superiority over the other heuristic was predictable.

5.2. *Large size instances*

In this section, we consider different sets of parameters, and ran 100 times the algorithms for random problems for each set of parameters. Problem size altered as: $n =$ 10, 15, 20, 30, 50, 100, and 200; transportation times as: $T_1 = T_2 = 45$, $T_1 = 2T_2 = 90$, and $T_2 = 2T_1 = 90$; and finally the capacities as: $C_1 = C_2 = 3$, $C_1 = 2C_2 = 6$, and $C_2 = 2C_1 = 6$. Table 4 shows the average results briefly. CPU run times were close to zero for all problems so we did not report them in Table 4. For almost all sets of parameters we see increase in the efficiency of both heuristics by increasing the size of problems. On the other, performance of both heuristics decreased by an increase in transportation times. We can also see a decrease in efficiency of heuristic H_1 when capacity of the first transporter increased where it shows little effect on the accuracy of heuristic H_2. The superiority of H_2 on H_1 is also clearly inferable for almost all sets of parameters.

6. Conclusion

In this paper, a two-machine flow shop problem was studied while there are ineligible transportation lags in production procedure. Two types of transportations were considered: one for transferring semi-finished

Table 3. Small size instances.

Instance number	$T1$	$T2$	$C1$	$C2$	Lower bound	B&B Value	B&B CPU(s)	H_1 Value	H_1 e%	H_2 Value	H_2 e %
1	30	74	7	4	352.0	352.0	125.00	571.2	62.2	359.8	2.2
2	25	66	5	7	384.0	384.0	90.16	675.5	75.9	390.4	1.6
3	63	45	8	7	389.2	394.9	1236.6	614.9	55.7	413.9	4.8
4	84	39	6	2	386.8	401.6	1059.6	620.3	54.4	415.4	3.4
5	89	25	3	6	349.8	377.5	1579	463.7	22.8	388.5	2.9

Table 4. Mid and large size instances with different set of parameters.

0 − 100		$T_1 = T_2 = 45$		$T_1 = 2 * T_2 = 90$		$T_2 = 2 * T_1 = 90$	
C_1, C_2	n	Ave E H_1 %	Ave E H_2 %	Ave E H_1 %	Ave E H_2 %	Ave E H_1 %	Ave E H_2 %
$C_1 = C_2 = 3$	10	7.80	2.24	10.08	5.78	11.04	6.36
	15	3.75	1.65	4.52	3.38	6.88	4.82
	20	2.11	1.65	3.54	2.08	4.71	4.60
	30	1.82	1.46	3.06	1.46	4.53	4.22
	50	1.51	1.27	3.54	1.21	4.26	4.15
	100	1.73	0.89	4.79	1.11	5.26	5.13
	200	1.61	1.03	4.94	1.18	5.44	5.38
$C_1 = 2 * C_2 = 6$	10	34.84	3.05	31.84	7.87	35.95	7.18
	15	20.63	2.68	17.95	5.64	24.25	7.04
	20	11.35	2.48	10.16	4.52	15.74	6.46
	30	5.37	1.68	5.50	3.67	8.92	5.15
	50	2.20	1.68	2.80	1.59	5.84	5.83
	100	1.48	1.10	1.77	1.25	5.28	5.18
	200	1.40	1.24	1.91	1.03	5.52	5.59
$C_2 = 2 * C_1 = 6$	10	7.54	2.29	9.36	4.94	8.68	3.19
	15	3.55	1.74	3.72	2.47	3.67	1.86
	20	1.53	1.00	2.80	1.48	1.80	1.31
	30	1.06	0.57	2.24	0.84	1.49	1.01
	50	0.72	0.25	3.24	0.37	1.32	0.94
	100	1.05	0.18	4.89	0.30	1.60	0.94
	200	1.04	0.20	5.32	0.22	1.58	0.97

jobs between machines (transportation Type I), and the other for delivering finished jobs to the customers (transportation Type II). The capacity of transporters and transportation times were assumed to be predetermined and job-independent. There were neither buffers nor priority constraints. The problem is formulated as MILP model. B&B algorithm, using Lingo 8 software, is used to solve MILP model, but it shows inefficiency in solving the problems with more than 5 jobs in desired times. Due to this incapability, two heuristic algorithms are developed to deal with large size problems. Furthermore a set of lower bounds is introduced in order to evaluate the effectiveness of heuristics. The results of computational experiments show that the two proposed heuristics are capable to produce good solutions in comparison with the lower bounds. Further extensions to this research can add more machines to this problem. Considering multiple customer locations be could also considered in order to make the problem more practical. Transportation times and capacity can be also assumed to be job-dependent to increase the reality of the problem.

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