Scientia Iranica E (2013) 20(6), 2201–2214

SCIENTIA IRANICA

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Scientia Iranica
Transactions E: Industrial Engineering
www.scientiairanica.com

Performance indicator for a stochastic-flow manufacturing network with reworking actions based on reliability

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Received 22 May 2012; received in revised form 9 April 2013; accepted 6 May 2013

KEYWORDS
Stochastic-Flow Manufacturing Network (SFMN);
System reliability;
Reworking;
Different success rates;
Decomposition method.

Abstract. This paper studies the performance evaluation for a manufacturing system considering reworking actions from the industrial engineering perspective. Due to failure, partial failure, and maintenance, the capacity of each machine in a manufacturing system is stochastic. Therefore, a manufacturing system is constructed as a stochastic-flow network, namely, a Stochastic-Flow Manufacturing Network (SFMN) herein. To evaluate the capability of an SFMN with reworking actions, we measure the probability that the SFMN satisfies demand, and such a probability is referred to as a system reliability. First, a decomposition method is proposed to decompose the SFMN into one general processing path and several reworking paths. Subsequently, two algorithms are designed for different network models to generate the lower boundary vector of the machine's capacity for guaranteeing that the SFMN produces sufficient products. The system reliability of SFMN is derived in terms of such a vector afterwards. According to the system reliability, the production manager may plan and adjust the production capacity in a flexible competing environment as customer demand changes.

1. Introduction

From an industrial engineering perspective, it is important to evaluate the capacity of a manufacturing system to determine the possibility of demand/order satisfaction. In other words, from the viewpoint of production management, developing a Key Performance Indicator (KPI) to measure the robustness of a manufacturing system is indeed a crucial task. The production manager could refer to the KPI for further improvement or development. To assess the capability of a manufacturing system, system reliability is one of the important performance indicators to be applied. This paper adopts a stochastic-flow network model to assess the system reliability of a manufacturing system. Relevant literature reviews, and issues to overcome are addressed in the following subsections.

1.1. Network analysis in manufacturing systems

A great deal of research [1-4] has been devoted to studying system reliability as a performance indicator for measuring the capability of a manufacturing system. Network analysis is an achievable tool that assists in evaluating the performance of a manufacturing system. That is, the manufacturing system can be modeled as a manufacturing network [3,5-9], and, thus, further analysis can be implemented based on such a network. Much research [5-7,9,10] has been devoted to performance measurement of a manufacturing system and supply chain using network analysis. Lee and Garcia-Diaz [6,7] applied a flow network approach to solve

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the grouping problem in a manufacturing system by measuring the functional similarity between machines as a performance indicator. Liste [9] designed a generic stochastic model for a supply-and-return network in a closed loop system to locate the general processing path and reworking path. Paquet et al. [10] proposed an optimization methodology to design a manufacturing network producing several products, in which the capacities of processors and workers are considered. Francas and Minner [5] utilized manufacturing network configuration in a supply chain with product rework to investigate cost and profit under different network structures. Nevertheless, this research mainly focused on cost, profit and sales for evaluating the performance of a manufacturing network without emphasizing the capability of the manufacturing network.

1.2. Stochastic-flow manufacturing network

When constructing the manufacturing system as a network, each arc can be regarded as a machine and each node denotes an inspection station following the machine. In particular, the capacity of each machine in the manufacturing network is not a fixed number and should be stochastic (i.e., multistate) due to failure, partial failure, and maintenance. Therefore, the manufacturing system is also multistate and we can treat it as a so-called stochastic-flow network [3,8,11-21], named, herein, the Stochastic-Flow Manufacturing Network (SFMN). To measure the capability of SFMN to satisfy customer requirements, Lin [3] focused on two-commodity reliability evaluation in terms of a Minimal Path (MP), in which MP is a path wherein its proper subsets are no longer paths. In Lin's work, system reliability is defined as the probability that the SFMN satisfies two-commodity demand. A great deal of research [3,8,11-21] has also been devoted to studying the system reliability of a stochastic-flow network, such as a manufacturing network [3,20], a computer network [15,17], a power supply network [16,18], a logistic network [11,14], etc., in terms of MP. In the above studies, system reliability is defined as the probability of the stochastic-flow network in satisfying demand. In this research, the demand transmitted through a network must obey flow conservation [22], which implies that no flow will be increased or decreased during transmission. More importantly, some properties related to the SFMN, such as rework and scraps, were not considered in this literature.

The success rate of each machine also influences the capability of the SFMN and leads to defective products, in which defective products would be reworked or scrapped. Thus, another important issue to be considered is how the reworking action affects the amount of output product in the SFMN. In many cases, defective products still have substantial value, e.g. caused by expensive input materials, and thus there is an incentive to rework those products into the 'as new' condition [23]. In several applications, the reworking action is implemented on the same machines. It implies that a manufacturing network would have two sources from the general processing path and the reworking path(s) for satisfying demand [23-26]. For a practical SFMN, the input flow processed by each machine would not be the same as output flow, since the success rate of each machine is considered. That is, the output products of the SFMN might be less than input raw materials. It implies that the traditional methodology for the stochastic-flow network problem could not be applied in a success rate case, due to violation of flow conservation. Moreover, based on the MP concept, an arc (machine) would not appear on the same path more than one time; otherwise it would not be an MP. However, defective WIP (work-in-process) from a machine would be reworked starting from a previous machine(s) or the same machine(s) [23,26], which would violate the basic concept of MP.

1.3. Issues to overcome

This paper mainly evaluates the probability that an SFMN could produce d units of a product, with reworking actions, by treating the system as a stochastic-flow network. Such a probability is named herein system reliability, implying that system reliability is a KPI to identify the possibility of SFMN satisfying the orders of the users. We consider that each machine in the SFMN has stochastic capacity and a success rate. The flow in the SFMN is defined as the input amount each machine processes per unit time. First, we concentrate on the SFMN with a single reworking action. To conquer the limitations of flow conservation and the MP concept, we propose a graphical technique to transform the manufacturing system into SFMN. According to the graph, the SFMN is decomposed into one general processing path and several reworking paths. Thus, the general processing path and reworking paths are utilized to analyze different input flows processed by each machine. An algorithm is proposed to generate the lower boundary vector, composed of the capacity of each machine, which affords to produce sufficient products satisfying demand d. In terms of such a vector, the system reliability of the SFMN can be derived. Subsequently, we extend this to the case of two reworking actions. System reliability can also be evaluated by the decomposition method, with a graphical technique and generation of a lower boundary vector. Based on the proposed solution procedure, system reliability of the SFMN with more than two reworking actions can be evaluated intuitively. The proposed models can then be easily extended to cases wherein each machine possesses a distinct success rate. From decision making and production management perspectives, system reliability can be a performance indicator for evaluating the capability of a manufacturing system. The production manager could determine if the capacity of the SFMN satisfies the customer orders/demand according to system reliability. Moreover, the production manager can plan and adjust the capacity of the SFMN with flexibility as customer demand changes.

The remainder of this paper is organized as follows. The problem description, the model construction for a single reworking action (Model I), and the algorithm to generate the lower boundary vector for d are proposed in Section 2. The model for two reworking actions (Model II) and the revised algorithm are extended in Section 3. A practical Printed Circuit Board (PCB) manufacturing system is demonstrated in Section 4 to illustrate the proposed algorithms and how system reliability may be calculated. Discussion regarding the algorithm and conclusion are summarized in Section 5.

2. Model I: SFMN with single reworking action

We focus on a flow-shop manufacturing network in which products are make-to-stock. That is, this SFMN is a high-volume system with standardized machines and processes to produce identical or highly similar products. Products produced by this manufacturing network are then provided to customers from finished products stock [27,28]. To satisfy demand, we first determine the amount of input raw material and then evaluate system reliability, in which SFMN satisfies the demand constraint. From a decision making and production management perspective, system reliability can be a performance indicator for measuring the capability of the SFMN in determining whether or not it can fulfill orders/demands from customers.

2.1. Assumptions

1. Each node (inspection station) is perfectly reliable.
2. The capacity, x_i, of each arc, a_i (machine), is a random variable according to a given probability distribution.
3. The capacities of different arcs (machines) are statistically independent.
4. No assembling action is taken and one material can produce one product.
5. Each defective WIP is reworked, at most, one time by the same machine. It implies that such a defective WIP is repaired until a usable state is achieved. If the defective WIP after reworking is still defective, it means that such a defective WIP is unrepeatable; then it is scrapped.

Vector operations are defined as follows:

$$Y \geq X \quad (y_1, y_2, \cdots, y_n) \geq (x_1, x_2, \cdots, x_n):$$

$$y_i \geq x_i \text{ for each } i = 1, 2, \cdots, n;$$

$$Y > X \quad (y_1, y_2, \cdots, y_n) > (x_1, x_2, \cdots, x_n):$$

$$Y \geq X \text{ and } y_i > x_i \text{ for at least one } i.$$

2.2. Determination of input units

In order to produce sufficient products for satisfying demand d, the input amount of raw materials should be pre-determined, based on the success rate, p, of each machine. Suppose that I units of raw material are able to produce O units of product; we attempt to obtain the relationship between I and O, fulfilling $O \geq d$. Take the simplest case, with only one machine (see Figure 1), I units of input raw material can produce $O = I \times p$ units of product. For convenience, we concentrate on cases wherein all machines possess an equal success rate, where the success rate, p, of each machine is defined as the probability of a machine operating successfully, where $0 \leq p \leq 1$. For the single machine, the relationship between I and O can be derived in terms of the success rate, p. That is, once input I is determined, output O can be easily obtained by multiplying p.

We further extend the above concept to the case of multiple machines. Let n denote the number of machines and a_i denote the ith machine. Given an SFMN with four ($n = 4$) machines (say a_1, a_2, a_3 and a_4), I units of raw material are processed through a_1, a_2, a_3 and a_4, sequentially (see Figure 2). That is, there would be I units of raw material entering a_1 and Ip units of WIP outputting from a_1. Subsequently, Ip units of WIP enter a_2 and Ip^2 units of WIP output from a_2. Without reworking, we finally have $O = Ip^4$ units of product output from a_4, since the raw materials are processed through four machines with the same success rate, p. To satisfy $O \geq d$, in this case, the input amount of raw material should be $I \geq d/p^4$ (recall that $O = Ip^4$). So far, the reworking action is not considered.

Suppose that defective WIP output from the r_1th machine can be reworked, starting from previous the k_1 machine (i.e. starting from the $(r_1 - k_1)$th machine). The special case, where k_1 is zero, implies that the defective WIP is reworked at the same machine (r_1th machine). For instance in Figure 2, defective WIP output from machine a_3 ($r_1 = 3$) is reworked, starting

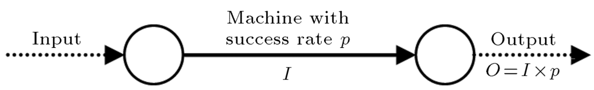

Figure 1. The input and output flow of a machine.

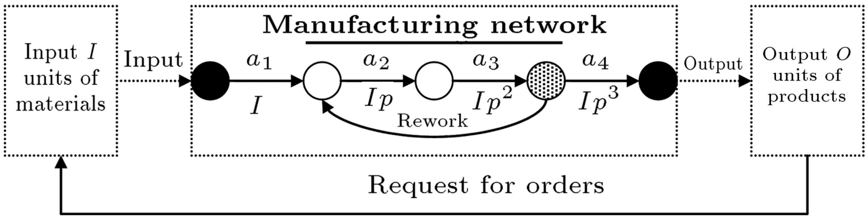

Figure 2. A manufacturing system with four machines.

from the previous ($k_1 = 1$) machine, a_2. It implies that Ip^2q units of defective WIP from a_3 would be re-input to a_2. However, the manufacturing process shown in Figure 2 is not distinguishable for input flows from the regular manufacturing process (without reworking) or the reworking process. A revised topology of the SFMN shown in Figure 3 would be able to describe the regular manufacturing process and the reworking processes more clearly. The revised topology would be helpful for further analysis. Figure 3 also shows the input flow (under each arc) for each machine.

In terms of the concept of path, a dummy-machine, say $a_2{}'$, is set to denote machine a_2 doing the reworking action. In Figure 3, the meshed node means that defective WIP (output from a_3) inspected by this inspection station can be reworked. Therefore, Ip^2q units of defective WIP from a_3 would be re-input to $a_2{}'$, where $q = 1-p$. Through the reworking process, the amount of WIP output from a_2 is Ip^3q, which would then be re-input to $a_3{}'$ (the dummy-machine for a_3). Thus, the SFMN finally has Ip^5q units of product output from $a_4{}'$ using the reworking process. With the reworking process, the total output is $O = (Ip^4 + Ip^5q)$ from both a_4 and $a_4{}'$, where the first term, Ip^4, is produced by the regular manufacturing process, while the second term, Ip^5q, is processed through the reworking process. To satisfy $O \geq d$ in the reworking case, the input amount I should be set as:

$$I \geq d/(p^4 + p^5q). \tag{1}$$

Based on the above instance, we generalize the output amount from the SFMN. Given demand, d, the number of machines, n, and success rate, p, the following lemma shows the relationship between input materials, I, and output products, O, in terms of d, n and p.

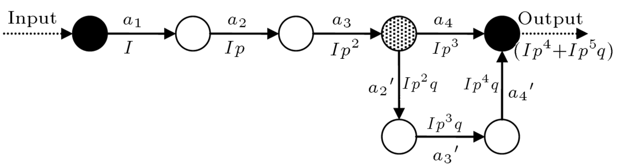

Figure 3. The revised topology for Figure 2.

Lemma 1. Assume that defective WIP output from the r_1th machine ($1 < r_1 \leq n$) is reworked, starting from the previous k_1 machine(s). Then, I units of raw material would produce (i) Ip^n units of product from the regular manufacturing process, and (ii) $Ip^{n+k_1}q$ units of product from the reworking process. (The proof is provided in Appendix A.)

Lemma 1 shows that the input determination is irrelevant with r_1. We, thus, have the output products $O = (Ip^n + Ip^{n+k_1}q)$ and it is necessary that $I \geq d/(p^n+p^{n+k_1}q)$ to obtain sufficient output, O, to satisfy demand, d. The following equation guarantees the SFMN can produce sufficient output to meet demand d:

$$I = d/(p^n + p^{n+k_1}q). \tag{2}$$

2.3. Decomposition of the SFMN

To analyze the SFMN in terms of paths, the SFMN in Figure 3 can be decomposed into two sets of paths, referred to as the general processing path, $P^{(G)}$, and the reworking path, $p^{(R|r_1,r_1-k_1)} = p^{(R|3,3-1)}$, where $(R|3, 3-1)$ denotes that defective WIP output from $a_3(r_1 = 3)$ is reworked, starting from the previous ($k_1 = 1$) machine (i.e. starting from a_2). That is, by the decomposition method, the regular manufacturing process can be seen as the general processing path, while the reworking process is seen as the reworking path. Take the same example in Subsection 2.2. For instance, set $\{a_1, a_2, a_3, a_4\}$ would be a general processing path, $P^{(G)}$, (see Figure 4). On the other hand, a path with reworking action is $\{a_1, a_2, a_3, a_2{}', a_3{}', a_4{}'\}$. In fact, no defective WIP would be processed by a_1, a_2 and a_3 and the input flow would be zero for these arcs. Thus, a_1, a_2 and a_3 can be ignored and only machines $a_2{}'$, $a_3{}'$ and $a_4{}'$ doing reworking action would be retained. Since machine ai and dummy-machine $a_i{}'$ are the same, the reworking path, $P^{(R|3,2)}$, would be $\{a_2, a_3, a_4\}$ (see Figure 5).

In order to extend the SFMN for n-machine cases, we have the following definition to decompose the

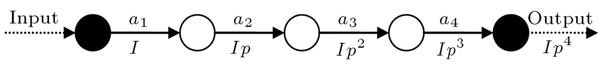

Figure 4. General processing path $P^{(G)}$ for Figure 2.

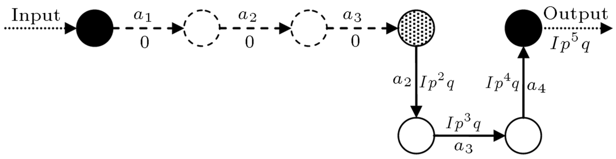

Figure 5. Reworking path $P^{(R|3,2)}$ for Figure 2.

SFMN into the general processing path, $P^{(G)}$, and the reworking path $p^{(R|r_1,r_1-k_1)}$.

Definition 1. Assume that defective WIP output from the r_1th machine $1 < r_1 \leq n$) is reworked starting from the previous k_1 machine. The general processing path is $P^{(G)} = \{a_1, a_2, \cdots, a_n\}$ and the reworking path is $P^{(R|r_1,r_1-k_1)} = \{a_{r_1-k_1}, a_{r_1-k_1+1,\cdots,a_n}\}$.

To distinguish the sources of input flow, we have the input flow for each machine on the paths (either the general processing path or the reworking path) as follows:

$$f_i^{(G)} = Ip^{i-1} \text{ for } i, \quad \text{such that } a_i \in p^{(G)}, \tag{3}$$

and:

$$f_i^{(R|r_1,r_1-k_1)} = Ip^{i+k_1-1}q \quad \text{for} \quad i,$$

$$\text{such that} \quad a_i \in p^{(R|r_1,r_1-k_1)}, \tag{4}$$

where $f_i^{(G)}$ is the input flow for $a_i \in P^{(G)}$ and $f_i^{(R|r_1,r_1-k_1)}$ is the input flow for $a_i \in p^{(R|r_1,r_1-k_1)}$. Both Eqs. (3) and (4) represent input flow that would be processed through each machine, a_i, where Eq. (3) is for the general processing path, $P^{(G)}$, and Eq. (4) is for the reworking path, $p^{(R|r_1,r_1-k_1)}$.

Since the amount of input raw material and the input flow for each machine are determined, it is possible to assess the capability of each machine in processing the input raw materials/WIP, or not. Furthermore, we can derive system reliability based on the decomposed paths and input flows.

2.4. Determination of capacity and evaluation of system reliability

The input raw materials/WIP processed by the ith machine, a_i, should satisfy the following constraints:

$$f_i^{(G)} + f_i^{(R|r_1,r_1-k_1)} \leq M_i. \tag{5}$$

Constraint (5) ensures that the general processing flow and the reworking flow do not exceed the maximal capacity, M_i, each machine can provide. The term, $f_i^{(G)} + f_i^{(R|r_1,r_1-k_1)}$, is further defined as the loading of each machine, say l_i, and we have the following equation:

$$l_i = f_i^{(G)} + f_i^{(R|r_1,r_1-k_1)}. \tag{6}$$

According to Assumption 2, the capacity, x_i, of each machine, a_i, is a random variable and thus the manufacturing network is stochastic. Here, c_i is the number of possible capacities of a_i and x_{ij} is the jth possible capacity of a_i, where $j = 1, 2, \cdots, c_i$. Thus, x_i takes possible values, $0 = x_{i1} < x_{i2} < \cdots < x_{ic_i} = M_i$. Under the state $X = (x_1, x_2, \cdots, x_n)$, Constraint (7) is necessary to guarantee that a_i can process the input raw materials/WIP:

$$x_i \geq l_i = f_i^{(G)} + f_i^{(R|r_1,r_1-k_1)},$$

$$\text{for} \quad i = 1, 2, \cdots, n. \tag{7}$$

Given demand d, system reliability, R_d, is the probability of the output product from the SFMN not being less than d. Thus, system reliability is $Pr\{X|V(X) \geq d\}$, where $V(X)$ is defined as the maximum output under X. It implies that each machine should provide sufficient capacity to process input raw materials/WIP and finally produce sufficient units of output product, O. However, enumerating all X, such that $V(X) \geq d$, and then combining their probabilities to derive R_d, is computationally prohibitive. The minimal capacity vector, Y, in set $\{X|V(X) \geq d\}$ is claimed to be the lower boundary vector for d. That is, Y is the lower boundary vector for d if and only if (i) $V(Y) \geq d$ and (ii) $V(Y') < d$ for any capacity vectors Y' such that $Y' < Y$. For the lower boundary vector $Y = (y_1, y_2, \cdots, y_n)$, we can calculate system reliability as:

$$\begin{aligned} R_d =& \Pr\{X|X \geq Y\} = \Pr\{X|X \geq (y_1, y_2, \cdots, y_n)\} \\ &= \Pr\{x_1 \geq y_1\} \times \Pr\{x_2 \geq y_2\} \times \cdots \\ &\times \Pr\{x_n \geq y_n\}. \end{aligned}$$

2.5. Algorithm I. For single reworking action

Algorithm I is proposed for Model I, where the SFMN has one reworking action. Given that $P^{(G)} = \{a_1, a_2, \cdots, a_n\}$ and $P^{(R|(r_1,r_1-k_1)} = \{a_{r_1-k_1}, a_{r_1-k_1+1}, \cdots, a_n\}$, the lower boundary vector for d can be derived by the following steps.boundary vector for d can be derived by the following steps:

Step 1. Determine the amount of input material by Eq. (8),

$$I = d/(p^n + p^{n+k_1}q). \tag{8}$$

Step 2. Determine input flow for each machine according to Eqs. (9) and (10):

$$f_i^{(G)} = Ip^{i-1} \text{ for } i \quad \text{such that } a_i \in P^{(G)}, \tag{9}$$

and:

$$f_i^{(R|r_1,r_1-k_1)} = Ip^{i+k_1-1}q \text{ for } i$$

$$\text{such that } \ a_i \in P^{(R|r_1,r_1-k_1)}. \tag{10}$$

Then, check that the amount of input flow fulfills the following constraint,

$$f_i^{(G)} + f_i^{(R|r_1,r_1-k_1)} \leq M_i. \tag{11}$$

Step 3. Transform input flow from the general processing path and reworking path into the machine loading vector, $L = (l_1, l_2, \cdots, l_n)$, via:

$$l_i = f_i^{(G)} + f_i^{(R|r_1,r_1-k_1)}. \tag{12}$$

Step 4. For each machine, find the smallest possible capacity, such that $x_{ij} \geq l_i > x_{i(j-1)}$. Then, $Y = (y_1, y_2, \cdots, y_n)$ is the lower boundary vector for d, where $y_i = x_{ij}$ for all i.

The exact amount of output product from this SFMN is $O = (Ip^n + Ip^{n+k_1}q)$, where I is determined by Step 1. The state, Y, determined from Step 4 is the lower boundary vector for d. The following theorem shows that capacity Y generated from the algorithm is the lower boundary vector for d.

Theorem. Capacity Y generated from the algorithm is the lower boundary vector for d. (The proof is provided in Appendix B.)

3. Model II: SFMN with two reworking actions

3.1. Determination of input flow for each machine

This section extends Model I to a case in which the SFMN has two reworking actions. Based on this section, the proposed model and algorithm can be extended to multiple reworking action cases intuitively. We illustrate Model II by another SFMN with eight ($n = 8$) machines. Suppose that defective WIP output from $a_3(r_1 = 3)$ and $a_6(r_2 = 6)$ is reworked, starting from the previous ($k_1 = k_2 = 1$) machines (see Figure 6), respectively. At first, we revise the network topology (Figure 7) to interpret the regular manufacturing process and the reworking processes. Figure 7 shows the input flow (under each arc) for each machine/dummy-machine in detail. Secondly, we can conduct the amount of output product in terms of I (see Figure 7). Thus, we have $O = (Ip^8 + Ip^9q + Ip^9q + Ip^{10}q^2)$ units of product, where terms Ip^8 and Ip^9q can be derived according to Lemma 1. The last term, $I_p^{10}q^2$, can be conducted by the following lemma.

Lemma 2. Assume that defective WIP output from the r_1th and the r_2th machines ($1 < r_1 < r_2 \leq n$) are reworked, starting from the previous k_1 and k_2 machines, respectively. Then, I units of raw material would eventually produce $Ip^{n+k_1+k_2}q^2$ units of product from the reworking path with two reworking actions. (The proof is provided in Appendix C.)

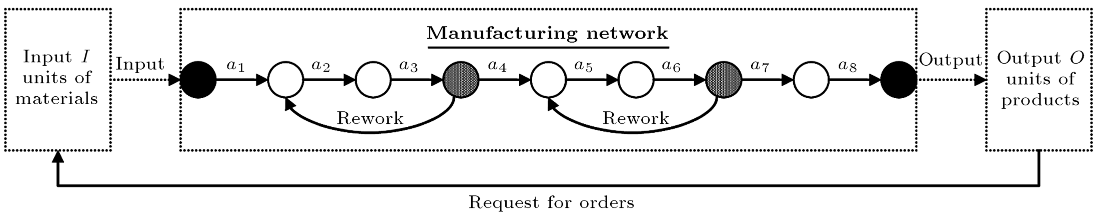

Figure 6. A manufacturing system with eight machines.

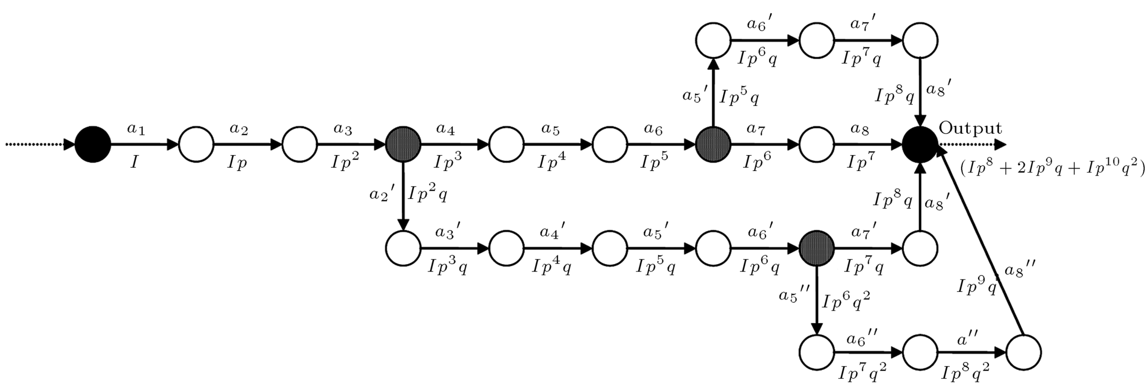

Figure 7. The revised topology for Figure 6.

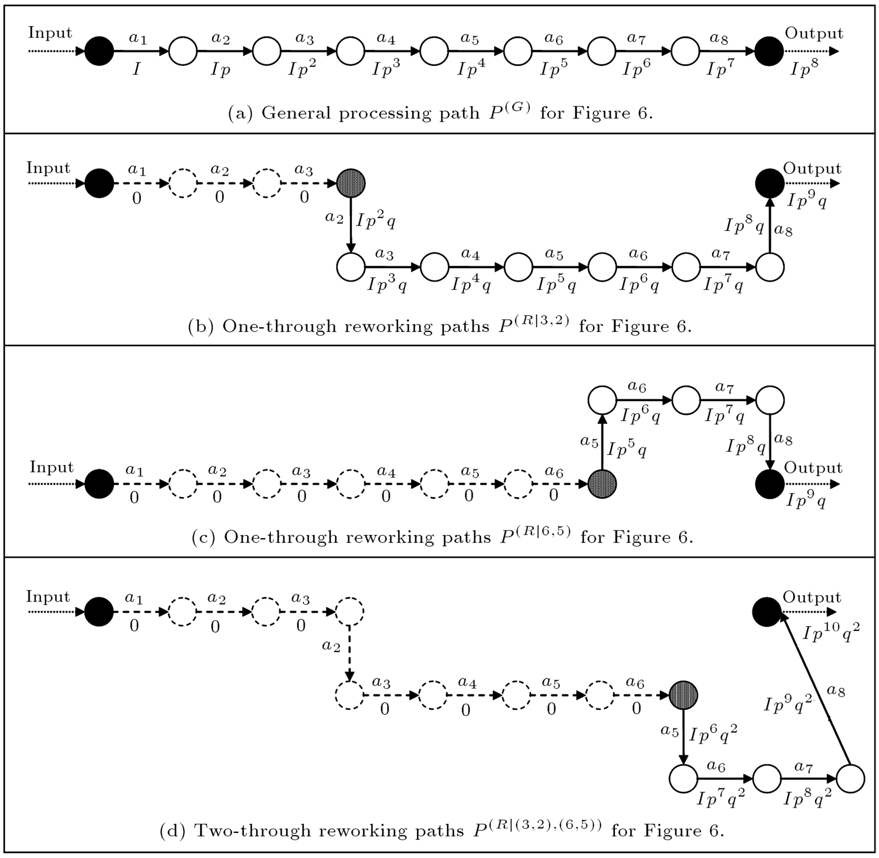

Figure 8. The decomposed paths for Figure 6.

We obtain the output products, $O = (Ip^n + Ip^{n+k_1}q + Ip^{n+k_2}q + Ip^{n+k_1+k_2}q^2)$, and thus have the following equation to guarantee that I would produce sufficient output, O, meeting demand d,

$$I = d/(p^n + p^{n+k_1}q + p^{n+k_2}q + p^{n+k_1+k_2}q^2). \qquad (13)$$

To analyze the SFMN by the decomposition method, there would be four combinations, such that the paths are (i) a general processing path without reworking action, (ii) a reworking path with one reworking action starting from the $(r_1 - k_1)$th or $(r_2 - k_2)$th machine, and (iii) a reworking path with two reworking actions starting from the $(r_1 - k_1)$th and $(r_2 - k_2)$th machines. We further named situation (ii) as one-through reworking paths, $p^{(R|r_1,r_1-k_1)}$ and $p^{(R|r_2,r_2-k_2)}$, while situation (iii) is named as two-through reworking paths, $p^{(R|r_1,r_1-k_1),(r_2,r_2-k_2)}$. The output products of $P^{(G)}$, $p^{(R|r_1,r_1-k_1)}$ and $p^{(R|r_2,r_2-k_2)}$ have been determined in Lemma 1. Lemma 2 shows the output products of two-through reworking paths, $p^{(R|(r_1,r_1-k_1),(r_2,r_2-k_2))}$. For the n-machine case, each path can be obtained by the following definition.

Definition 2. Assume that defective WIP output from the r_1th and r_2th machines $(1 < r_1 < r2 \leq n)$ are reworked starting from the previous k_1 and k_2 machines, respectively. If the general processing path is given as $P^{(G)} = \{a_1, a_2, \cdots, a_n\}$, then the one-through reworking paths are $p^{(R|r_1,r_1-k_1)} = \{a_{r_1-k_1}, a_{r_1-k_1+1}, \cdots, a_n\}$ and $p^{(R|r_2,r_2-k_2)} = \{a_{r_2-k_2}, a_{r_2-k_2+1}, \cdots, a_n\}$ and two-through reworking path is $P^{(R|(r_1,r_1-k_1),(r_2,r_2-k_2))} = \{a_{r_2-k_2}, a_{r_2-k_2+1}, \cdots, a_n\}$.

From the example of Figure 6, four decomposed paths are:

$$P^{(G)} = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\},$$

$$P^{(R|3,2)} = \{a_2, a_3, a_4, a_5, a_6, a_7, a_8\},$$

$$P^{(R|6,5)} = \{a_5, a_6, a_7, a_8\},$$

and:

$$P^{(R|(3,2),(6,5))} = \{a_5, a_6, a_7, a_8\},$$

see Figure 8.

3.2. Determination of capacity

To distinguish the sources of input flow, we have the input flow for each machine on the paths as follows:

$$f_i^{(G)} = Ip^{i-1} \quad \text{for } i, \quad \text{such that} \quad a_i \in P^{(G)}, \tag{14}$$

$$f_i^{(R|r_1,r_1-k_1)} = Ip^{i+k_1-1}q \quad \text{for } i,$$

$$\text{such that} \quad a_i \in p^{(R|r_1,r_1-k_1)}, \tag{15}$$

$$f_i^{(R|r_2,r_2-k_2)} = Ip^{i+k_2-1}q \quad \text{for } i,$$

$$\text{such that} \quad a_i \in p^{(R|r_2,r_2-k_2)}, \tag{16}$$

and:

$$f_i^{(R|r_1,r_1-k_1),(r_2,r_2-k_2)} = Ip^{i+k_1+k_2-1}q^2 \text{ for } \quad i,$$

$$\text{such that} \quad a_i \in P^{(R|(r_1,r_1-k_1),(r_2,r_2-k_2))}, \tag{17}$$

where $i = 1, 2, \cdots, n$. The input flows processed by the ith machine, a_i, should satisfy the following constraints:

$$f_i^{(G)} + f_i^{(R|r_1,r_1-k_1)} + f_i^{(R|r_2,r_2-k_2)} + f_i^{(R|(r_1,r_1-k_1),(r_2,r_2-k_2))} \leq M_i, \tag{18}$$

and the loading of each machine is:

$$l_i = f_i^{(G)} + f_i^{(R|r_1,r_1-k_1)} + f_i^{(R|r_2,r_2-k_2)} + f_i^{(R|(r_1,r_1-k_1),(r_2,r_2-k_2))},$$

$$\text{for} \quad i = 1, 2, \cdots, n. \tag{19}$$

Under the state $X = (x_1, x_2, \cdots, x_n)$, Constraint (20) is necessary to guarantee that a_i can process the input raw materials/WIP:

$$x_i \geq l_i = f_i^{(G)} + f_i^{(R|r_1,r_1-k_1)} + f_i^{(R|r_2,r_2-k_2)} + f_i^{(R|(r_1,r_1-k_1),(r_2,r_2-k_2))},$$

$$\text{for } i = 1, 2, \cdots, n. \tag{20}$$

3.3. *Algorithm II. for two reworking actions*

Algorithm II extends Algorithm I to the SFMN with two reworking actions (Model II). Given that:

$$P^{(G)} = \{a_1, a_2, \cdots, a_n\},$$

$$P^{(R|r_1,r_1-k_1)} = \{a_{r_1-k_1}, a_{r_1-k_1+1}, \cdots, a_n\},$$

$$P^{(R|r_2,r_2-k_2)} = \{a_{r_2-k_2}, a_{r_2-k_2+1}, \cdots, a_n\},$$

and:

$$P^{R|(r_1,r_1-k_1),(r_2,r_2-k_2)} = \{a_{r_2-k_2}, a_{r_2-k_2+1}, \cdots, a_n\},$$

the lower boundary vector for d can be derived by the following steps.

Step 1. Determine the amount of input material by Eqs. (21)-(25),

$$I = d/(p^n + p^{n+k_1}q + p^{n+k_2}q + p^{n+k_1+k_2}q^2. \tag{21}$$

Step 2. Determine the input flows for each machine according to Eqs. (22)-(25),

$$f_i^{(G)} = Ip^{i-1} \text{ for } i, \quad \text{such that } a_i \in P^{(G)}, \tag{22}$$

$$f_i^{R|r_1,r_1-k_1} = Ip^{i+k_1-1}q \quad \text{for} \quad i,$$

$$\text{such that} \quad a_i \in P^{(R|r_1,r_1-k_1)}, \tag{23}$$

$$f_i^{R|r_2,r_2-k_2} = Ip^{i+k_2-2}q \quad \text{for } i,$$

$$\text{such that} \quad a_i \in P^{(R|r_2,r_2-k_2)}, \tag{24}$$

$$f_i^{R|(r_1,r_1-k_1)(r_2,r_2-k_2)} = Ip^{i+k_1+k_2-1}q^2 \quad \text{for } i,$$

$$\text{such that} \quad a_i \in P^{(R|(r_1,r_1-k_1),(r_2,r_2-k_2))}. \tag{25}$$

Then, check that the amount of input flow fulfils the following constraint:

$$f_i^{(G)} + f_i^{(R|r_1,r_1-k_1)} + f_i^{(R|r_2,r_2-k_2)} + f_i^{(R|(r_1,r_1-k_1),(r_2,r_2-k_2))} \leq M_i. \tag{26}$$

Step 3. Transform input flow from general processing path and the reworking paths into the machines' loading vector, $L = (l_1, l_2, \cdots, l_n)$, via:

$$l_i = f_i^{(G)} + f_i^{(R|r_1,r_1-k_1)} + f_i^{(R|r_2,r_2-k_2)} + f_i^{(R|(r_1,r_1-k_1),(r_2,r_2-k_2))}. \tag{27}$$

Step 4. For each machine, find the smallest possible capacity, such that $x_{ij} \geq l_i > x_{i(j-1)}$. Then, $Y = (y_1, y_2, \cdots, y_n)$ is the lower boundary vector for d, where $y_i = x_{ij}$ for all i. The exact amount of output product from this SFMN is:

$$O = (Ip^n + Ip^{n+k_1} + Ip^{n+k_2}q + Ip^{n+k_1+k_2})q^2),$$

where I is determined by Step 1. The state Y determined from Step 4 is the lower boundary vector for d. The situation with more than two reworking actions can be derived as well.

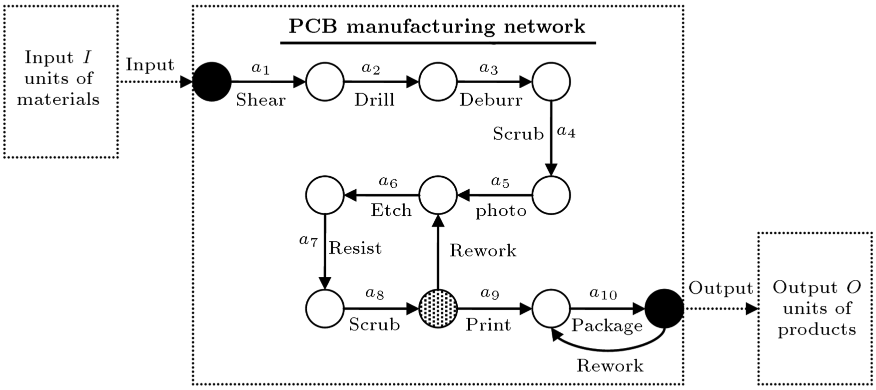

Figure 9. A PCB manufacturing network with 10 machines.

4. Numerical example

In this example, a typical PCB manufacturing system is utilized to demonstrate an SFMN with two reworking actions. For single sided board manufacturing, the input raw material is a board with a thin layer of copper foil. For different product types, the manufacturing processes and sequences may be different. Generally, the regular manufacturing process of PCB starts from shearing (a_1), in which the board is cut to a specific size. Subsequently, an automated drilling machine (a_2) drills holes through the board for mounting electronic components. After drilling, the deburring machine (a_3) removes copper particles from the board and then the scrubbing machine (a_4) is for cleaning the board. Following cleaning, the photo imaging machine (a_5) creates the circuit pattern on the board. By chemical etching (a_6) and resist stripping (a_7), copper that is not part of the circuit pattern is removed. Once again, the scrubbing machine (a_8) is used for cleaning the chemicals and resistance on the board. After scrubbing, the legend printing process (a_9) puts on the required logos or letters. Finally, the regular manufacturing process is finished by packaging (a_{10}). Thus, the PCB manufacturing system is described as a PCB manufacturing network with ten machines ($n = 10$), as shown in Figure 9.

In the PCB manufacturing network, some defective WIP output from $a_8(r_1 = 8)$ is reworked, starting from the previous two ($k_1 = 2$) machines (i.e. starting from a_6). The defective product output from $a_{10}(r_2 = 10)$ can be reworked by the same machine ($k_2 = 0$). To analyze the PCB manufacturing network in terms of paths, it is divided into one general processing path:

$$P^{(G)} = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\},$$

and three reworking paths:

$$P^{(R|r_1,r_1-k_1)} = P^{(R|8,8-2)} = \{a_6, a_7, a_8, a_9, a_{10}\},$$

$$P^{(R|10,10)} = \{a_{10}\},$$

and $P^{(R|(8,6),(10,10))} = \{a_{10}\}$.

The same success rate, $p = 0.98$, for each machine and the capacity distribution of each machine is given in Table 1. Although the production process is based on a real PCB manufacturing system, the machine data shown in Table 1 is hypothetical to demonstrate the proposed algorithm. In practice, once real data is obtained by historical records or specifications, system reliability can be derived in a similar manner. Then, we can generate the lower boundary vector for $d = 200$, as follows:

Step 1. Determine the amount of input material:

$$I = d/(p^n + p^{n+k_1}q + p^{n+k_2}q + p^{n+k_1+k_2}q^2)$$

$$=200/(0.98^{10} + 0.98^{10+2} \times 0.02 + 0.98^{10+0}$$

$$\times 0.022 + 0.98^{10+2+0} \times 0.02^2) = 235.455.$$

Step 2. Determine the input flow for each machine according to Eqs. (22)-(25). The input flow of each machine is shown in rows 2 to 6 of Table 2.

Step 3. Transform input flow from the general processing path and reworking paths into the machines' loading vector, as follows:

$$l_1 = f_1^{(G)} + f_1^{(R|8,6)} + f_1^{(R|10,10)} + f_1^{(R|(8,6),(10,10))}$$

$$= 235.455 + 0 + 0 + 0 = 235.455;$$

$$l_2 = f_2^{(G)} + f_2^{(R|8,6)} + f_2^{(R|10,10)} + f_2^{(R|(8,6),(10,10))}$$

$$= 230.746 + 0 + 0 + 0 + 230.746;$$

$$\vdots$$

$$l_{10} = f_{10}^{(G)} + f_{10}^{(R|8,6)} + f_{10}^{(R|10,10)} + f_{10}^{(R|(8,6),(10,10))}$$

$$= 196.310 + 3.771 + 3.926 + 0.075 = 204.082.$$

Table 1. The machine data of example.

Machine	Capacity[a]	Probability	Machine	Capacity	Probability
				0	0.002
	0	0.005		120	0.003
a_1	300	0.005	a_6	240	0.005
	600	0.990		360	0.015
				480	0.975
				0	0.002
	0	0.001		60	0.002
a_2	300	0.001	a_7	120	0.005
	600	0.998		180	0.005
				240	0.010
				300	0.976
				0	0.001
	0	0.002		60	0.001
a_3	250	0.003	a_8	120	0.001
	500	0.995		180	0.012
				240	0.015
				300	0.970
	0	0.005		0	0.002
a_4	250	0.010	a_9	120	0.003
	500	0.985		240	0.995
	0	0.001			
	120	0.001		0	0.010
a_5	240	0.002	a_{10}	120	0.015
	360	0.003		240	0.975
	480	0.993			

[a]: Units of raw materials/WIP that each machine can process per unit time.

Table 2. The results of Step 2 for example.

Input flow	i = 1	2	3	4	5
$f_i^{(G)}$	235.4550	230.7459	226.1310	221.6084	217.1762
$f_i^{(R\|8,6)}$	0.0000	0.0000	0.0000	0.0000	0.0000
$f_i^{(R\|10,10)}$	0.0000	0.0000	0.0000	0.0000	0.0000
$f_i^{(R\|(8,6),(10,10))}$	0.0000	0.0000	0.0000	0.0000	0.0000
$\sum f_i$	235.455	230.746	226.131	221.608	217.176
	6	**7**	**8**	**9**	**10**
$f_i^{(G)}$	212.8327	208.5760	204.4045	200.3164	196.3101
$f_i^{(R\|8,6)}$	4.0881	4.0063	3.9262	3.8477	3.7707
$f_i^{(R\|10,10)}$	0.0000	0.0000	0.0000	0.0000	3.9262
$f_i^{(R\|(8,6),(10,10))}$	0.0000	0.0000	0.0000	0.0000	0.0754
$\sum f_i$	216.921	212.582	208.331	204.164	204.082

We obtain that:

$$L=(l_1,l_2,\cdots,l_{10})$$
$$=(235.455, 230.746, 226.131, 221.608, 217.176,$$
$$216.921, 212.582, 208.331, 204.164, 204.082).$$

Step 4. Find the lower boundary vector, Y, such that $Y \geq L$. For instance, the possible capacity states of a_1 are $\{0, 300, 600\}$, and $y_1 = 300$ is the smallest capacity satisfying $l_1 = 235.455$. Thus,

$$Y=(y_1,y_2,\cdots,y_{10})=(300, 300, 250, 250, 240, 240,$$
$$240, 240, 240, 240)$$

is the lower boundary vector for $d = 200$.

The exact amount of output product from this SFMN is approximating:

$$O = 235.455 \times (0.98^{10} + 0.98^{10} + 2 \times 0.02 + 0.98^{10+0} \times 0.02 + 0.98^{10+2+0} \times 0.022) = 200.0008.$$

System reliability, R_{200}, can be derived as follows:

$$\begin{aligned} R_{200} &= \Pr\{O \geq d\} = \Pr\{O \geq 200\} \\ &= \Pr\{X|X \geq (300, 300, 250, 250, 240, 240, 240, 240, 240, 240)\} \\ &= \Pr\{x_1 \geq 300\} \times \Pr\{x_2 \geq 300\} \times \cdots \times \Pr\{x_{10} \geq 240\} \\ &= 0.995 \times 0.999 \times \cdots \times 0.975 = 0.92350. \end{aligned}$$

5. Discussion

This subsection first addresses the benefit of the proposed algorithms compared to complete enumeration. In the example of Section 4, there are 576 solutions satisfying $d = 200$ among 656,100 capacity combinations generated by complete enumeration. Here,let Z_v denote each solution satisfying $d = 200$ and $v = 1, 2, \cdots, 576$. System reliability is calculated by summing the probabilities of those solutions in which each probability is derived by each single solution. Hence, to derive system reliability by complete enumeration, the probability evaluation procedure should be executed 576 times by $\sum_{v=1}^{576} \Pr\{X = Z_v\}$. However, there is only one lower boundary vector generated from the proposed algorithms. In addition, system reliability is calculated using a single equation:

$$\begin{aligned} R_d &= \Pr\{X|X \geq Y\} = \Pr\{X|X \geq (y_1, y_2, \cdots, y_n)\} \\ &= \Pr\{x_1 \geq y_1\} \times \Pr\{x_2 \geq y_2\} \times \cdots \times \Pr\{x_n \geq y_n\}, \end{aligned}$$

which is executed only once.

Second, we address the issue of real numbers used in this work. Algorithms proposed in this paper are generalized in terms of real numbers, since we measure the input flow that each machine processes per unit time. That is, from the flow perspective, the amount of raw material/WIP/product is averaged by time and thus the value is a real number. Moreover, the machine success rate is always a real number and thus the output flow (WIP/products) processed by each machine is indeed a real number. However, the eventual exact amount of output product would be an integer, which can be obtained by [flow $\times$ time period] after a long-term time period. If the equations and constraints are modified by utilizing the $\lfloor x \rfloor$ or $\lceil x \rceil$ operations to solve integer cases (where $\lfloor x \rfloor$ is the largest integer, such that $\lfloor x \rfloor \leq x$ and $\lceil x \rceil$ is the smallest integer, such that $\lceil x \rceil \geq x$), the procedure in the algorithm would be distorted and very complicated to calculate. For example, given the input amount, I, and machine success rate, p, the output from the first machine would be $\lfloor Ip \rfloor$, the output from the second machine would be $\lfloor \lfloor IP \rfloor \rfloor \times p \rfloor$ and so on. Taking a numerical case for explanation, let input $I = 100$ and success rate $p = 0.98$ for each machine. Thus, the output of this SFMN is $O \lfloor \lfloor \lfloor \lfloor 100 \times 0.98 \rfloor \times 0.98 \rfloor \times 0.98 \rfloor = 92$. However, if we take the real number to calculate the output, the output should be $100 \times 0.98^4 = 92.237$. However, after 1000 units time period, the output of the integer case is underestimated to be 92000, while the real output is 92237. Moreover, this paper intends to determine the amount of input raw material in terms of success rate p before evaluating system reliability. That is, if there are four machines in the manufacturing network, the output products would be Ip^4, which it implies that the overall success rate is p^4. Given the demand, d, we can easily obtain that $I \geq d/p^4$ for satisfying the demand. If we consider the integer case, the output would be $\lfloor \lfloor \lfloor \lfloor Ip \rfloor \times p \rfloor \times p \times p \rfloor$ and there is no intuitive way to determine the amount of input raw material. The reworking actions would only complicate the algorithm further.

6. Conclusions

To assess the robustness of a manufacturing system from an industrial engineering perspective, this paper constructs a stochastic-flow network model considering reworking actions, herein named a Stochastic-Flow Manufacturing Network (SFMN). We evaluate the probability that the SFMN satisfies demand d, namely, system reliability. That is, system reliability is evaluated as a KPI to identify the demand satisfaction of the SFMN. Different from previous literature [3,8,11-21], the SFMN with reworking actions would violate the principle of flow conservation. The SFMN is decomposed into a general processing path (without reworking) and several reworking paths. In terms of these paths, we determine the capacity of each machine providing for eventually satisfying the output products amount, d. Two algorithms are proposed to generate the lower boundary vector for Model I (single reworking action) and Model II (two reworking actions), respectively. System reliability can be derived in terms of such a vector. The proposed models can also be easily extended to cases where each machine in the SFMN may possess a distinct success rate. That is, we can replace the same success rate, p, by a distinct success rate, p_i, of machine a_i. For instance, input I units of raw material, the products produced by the general processing path through machines $a_1, a_2, \cdots, a_n$ are $I\Pi_{t=1}^{n} p_t$. Based on the KPI, the production manager could conduct a sensitivity analysis to investigate the most important machine in an SFMN for improving the system more reliably. Results of the sensitivity analysis are also beneficial for adjusting production capacity as customer demand changes.

In future research, we plan to work on the development of an estimation method for SFMN reliability in cases of more than two reworking actions. This is because, if the reworking actions are more than two, the topology of the manufacturing system becomes rather complex. For instance, for three reworking actions, there are eight decomposed paths, including one general production path, three one-through reworking paths, three two-through reworking paths and one three-through reworking path. Theoretically, once all the decomposed paths are obtained, our present models and algorithms should be suitable for dealing with this scenario. However, it is challenging to develop a generic model in the case of more than two reworking actions. Hence, an estimation method for system reliability would be very useful and should be developed in the future.

Nomenclature

n	Number of machines
a_i	ith machine where $i = 1, 2, \cdots, n$
p	Success rate of each machine
q	Defective rate of each machine, where $q = 1 - p$
M_i	Maximal capacity of machine a_i, $i = 1, 2, \cdots, n$
l_i	Loading of machine a_i, $i = 1, 2, \cdots, n$
L	$(l_1, l_2, \cdots, l_n)$: loading vector
x_i	Current capacity of a_i, $i = 1, 2, \cdots, n$
X	$(x_1, x_2, \cdots, x_n)$: the capacity vector
c_i	Number of possible capacities of a_i, $i = 1, 2, \cdots, n$
x_{ij}	jth possible capacity of a_i, $j = 1, 2, \cdots, c_i$. Thus, x_i takes possible values $0 = x_{i1} < x_{i2} < \cdots < x_{ic} = M_i$, $i = 1, 2, \cdots, n$
d	Demand
I	Amount of input raw material
O	Amount of output product
a_i'	ith machine doing the reworking action
$P^{(G)}$	General processing path (without reworking action)
$p^{R\|r_1,r_2-k_1}$	Reworking path with one reworking action, where the output defective WIP of the r_1th machine are reworked starting from the previous k_1 machine, i.e., from the $(r_1 - k_1)$th machine
$f_i^{(G)}$	Input flow for $a_i \in P^{(G)}$
$f_i^{(R\|r_1,r_1-k_1)}$	Input flow for $a_i \in p^{(R\|r_1,r_1-k_1)}$
$V(X)$	Maximum output under X
$p^{(R\|(r_1,r_1-k_1),(r_2,r_2-k_2))}$	Reworking path with two reworking actions, where the output defective WIP of the r_1th and the r_2th are reworked starting from the $(r_1 - k_1)$th and the $(r_2 - k_2)$th machines, respectively
$f_i^{(R\|(r_1,r_1-k_1),(r_2,r_2-k_2))}$	Input flow for $a_i \in P^{(R\|(r_1,r_1-k_1),(r_2,r_2-k_2))}$

Acronyms

KPI	Key Performance Indicator
MP	Minimal Path
PCB	Printed Circuit Board
SFMN	Stochastic-Flow Manufacturing Network
WIP	Work-In-Process

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Appendix A

(i) Products produced from regular manufacturing process are generally affected by the number of machines. That is, the number of output product/WIP processed from the ith machine is Ip^i. Since there are n machines in total, it can be easily concluded that Ip^n units of product would be produced from a regular manufacturing process by mathematical induction.

(ii) According to (i), WIP output from the r_1th machine should be Ip^{r_1}, and $Ip^{r_1-1}q$ are defective WIP. Since defective WIP output from the r_1th machine can be reworked, starting from the previous k_1 machine, it implies that defective WIP still has to be processed by machines $a_{r_1-k_1}, a_{r_1-k_1+1}, \cdots, a_n$. That is, the number of follow-up machines equals $[n-(r_1-k_1)+1]$. Then, the output products produced from the reworking process are $Ip^{r_1-1}q \times p^{n-(r_1-k_1)+1} = Ip^{n+k_1}q$.

Appendix B

Suppose that Y is not a lower boundary vector for d, then there exists a lower boundary vector Z for d, such that $Z < Y$, because Y fulfils d. Without loss of generality, we set $Z = (z_1, z_2, \cdots, z_i, \cdots, z_n)$ and there exists at least one $z_i < y_i$. The situation $z_i < y_i$ implies that $z_i < f_i^{(G)} + f_i^{(R|r_1,r_1-k_1)}$, which cannot provide sufficient capacity for the input units of WIP, and which contradicts Z being the lower boundary vector for d (note that $l_i = f_i^{(G)} + f_i^{(R|r_1,r_1-k_1)}$ and that y_i is the minimal capacity satisfying l_i). Thus, we conclude that Y generated from the algorithm is the lower boundary vector for d.

Appendix C

WIP output from the r_1th machine should be Ip^{r_1} and $Ip^{r_1-1}q$ are defective WIP. Since defective WIP output from the r_1th machine is reworked, starting from the previous k_1 machine, it implies that defective WIP has to be processed by machines $a_{r_1-k_1}, a_{r_1-k_1}, \cdots, a_{r_2}$ until the next machine, a_{r_2}, whose output can be reworked. That is, the number of follow-up machines equals $[r_2-(r_1-k_1)+1]$. Then, the output WIP produced from would be $Ip^{r_2+k_1}q$, and $Ip^{r_2+k_1-1}q^2$ are defective WIP, which still have to be processed by machines $a_{r_2-k_2}, a_{r_2-k_2+1}, \cdots, a_n$. That is, the number of follow-up machines equals $[n-(r_2-k_2)+1]$. Then, we can obtain that the output product produced from the reworking path with two reworking actions is $Ip^{r_2+k_1-1}q^2 \times p^{n-(r_2-k_2)+1} = Ip^{n+k_1+k_2}q^2$.

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