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Network revenue management under specific choice models

F. Etebari[a,*], A. Aghaei[a] and A. Jalalimanesh[b]

a. *Department of Industrial Engineering, K.N. Toosi University of Technology, Tehran, P.O. Box 1999143344, Iran.*
b. *Department of Information Engineering, Iranian Research Institute for Information Science and Technology, 1090, Enqelab Ave., Tehran, P.O. Box 13185-1371, Iran.*

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Abstract. New challenges in the business environment, such as increased competition and the influence of the Internet on main distribution channels has led to fundamental changes in traditional revenue management models. Under these conditions, modeling individual decisions more accurately is becoming a key factor. Nearly all research studies about choice-based revenue management models use the well-known multinomial logit model. This model has one important restriction, that is, the independence of irrelevant alternatives; a property which states that the ratio of choice probabilities for two distinct alternatives is independent of the attributes of any other alternatives. In this paper, a nested logit model is proposed for removing this limitation and incorporating a correlation between alternatives in each nest. The new subproblem of column generation is introduced and a combination of heuristic and metaheuristic algorithms for solving this problem is provided. Interesting outcomes are obtained during analysis of the results of experimental computations, such as offer sets and iteration trends, with respect to the correlation measure inside each nest. Simulation results show that, although changing the choice model might lead to significant improvement in revenue under some conditions, during all scenarios, observing the correlation should not cause the choice model to change immediately.

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1. Introduction

Today's market environment can be explained with several simple words; increased competition pressure, the growing dominance of information technology such as the Internet, and the existence of rich sources about revealed preference data in a mutual environment. At the beginning of the previous decade, it was concluded that the assumption of independent demand in traditional models has serious limitations. Revenue management scientists preferred to use a new generation of choice based revenue management structures, including simple and well known choice models, such as the multinomial logit model, for capturing customer behavior and taking into account the effects of buy up, buy down and recapture. The independent demand model uses the assumption that the demand for each fare class is independent from firm availability controls. Then, traditional carriers felt the effects of these factors more seriously and tried to minimize their undesirable effects on their revenue.

All above mentioned factors have resulted in the need for better modeling of individual purchasing decisions. Application of a multinomial logit model to forecast ridership for a new transportation line in 1972, in San Francisco, provided a strong foundation and motivation for researchers to transit from modeling demands using aggregate data to modeling demands as a collection of individual choices. Nowadays, with the

*. *Corresponding author. Tel.: +98-21-84063482*
E-mail address: f_etebari@dena.kntu.ac.ir (F. Etebari)

growth in online shopping and booking, there is a new rich source of data to model customer preferences.

Cooper et al. (2006) show that if an airline wants to decide about the number of seats that should be reserved for sale at a high-fare, based on the sales history, while neglecting to account for the fact that the availability of low-fare tickets will reduce high-fare sales, then, high-fare sales will decrease, resulting in a lower future estimation of high-fare demand. This process is called the spiral-down effect and is observed only if historical data is used for demand forecasting. There are different reasons, such as those mentioned above, for using discrete choice models in order to forecast future demand.

The most popular choice model is the multinomial logit model (MNL). Despite its simplicity and efficiency, this model has some restrictions. The most important limitation of the MNL model is the assumption of the independent distribution of utility function errors across alternatives, which leads to the Independence of the Irrelevant Alternatives (IIA) effect. This property states that the ratio of choice probabilities is independent of the attributes of any other alternatives.

The property of IIA, which may not be a realistic assumption, means that a change or improvement in the utility of one alternative will draw shares proportionally from all other alternatives. For instance, in parallel flights, it is expected that a flight departing in the afternoon competes further with other afternoon departing flights.

Schön (2010) states that, recently, much attention has been given to (a) modeling how consumers choose among a set of multiple products, and (b) accommodating the realistic discrete choice model of consumer behavior in normative RM, while keeping problem complexity at a reasonable level, simultaneously. The nested logit model incorporates more realistic substitution patterns by relaxing the assumption of independent distribution of errors and grouping alternatives to the different nests.

The organization of this paper is as follows. In Section 2, a brief literature review of the related works is described. In Section 3, the Choice-based Deterministic Linear Programming (CDLP) approach and nested logit models are explained. At the beginning of section four, the column generation algorithm is described, and, subsequently, a new subproblem of this algorithm, based on the nested logit model, is proposed.

In the rest of this section, a new approach, composed of heuristic and genetic algorithms, is introduced for solving the problem. In the next section, a complete test problem is modeled and solved by with the aid of the proposed method, and the results are illustrated using a simulation of customer behavior with test problem data. The last section includes the discussion and conclusion, and finally, a number of interesting topics are presented for future research.

2. Literature review

Most traditional revenue management models are based on an independent demand assumption. A complete survey of traditional revenue management models can be found in Talluri and Van Ryzin [1]. Belobaba and Hopperstad [2] show the importance of considering customer choice decision behavior. They studied passenger purchase behavior using simulation for analyzing the sensitivity of airline time, date, path and price to passenger preferences. Anderson [3] and Algers and Baser [4] report the results of a project in the Scandinavian Airlines System (SAS) regarding estimation of recapture and buy up using stated and revealed preference data.

Zhang and Cooper [5] used the Markovian decision process for simultaneous seat-inventory control of a set of parallel flights from common origins to common destinations, considering customer choice among the flights. Their model assumes that the customer chooses within the same fare class among different flights but not between fare classes. They proposed heuristics and simulation-based techniques for solving this problem, and also applied a general choice model for considering customer behavior.

Van Ryzin and Vulcano [6] consider the network capacity control problem, where customers choose among various products offered by a firm. They model customer choice, assuming that each of them has an ordered list of preferences. They assume that the firm controls the availability of products using a virtual nesting control strategy.

Chen and Homem-de-Mello [7] consider network airline revenue management, when the customer choice model is based on the concept of preference orders. They proposed a new model using mathematical programming techniques to determine seat allocation.

Talluri and Van Ryzin [8] provide a complete characterization of an optimal policy under a general discrete choice model of customer behavior in a single leg revenue management model. They illustrate that an optimal policy is made up of a selected set of efficient offer sets, where these sets are a sequence of no dominated sets providing the highest positive exchange between expected capacity assumption and expected revenue.

Gallego et al. [9] provide a customer choice-based LP model for network revenue management. They suppose that the firm has the ability to provide customers alternative products to serve the same market demands with a flexible product offering. One limitation of their market demand model is that it does not allow any kind of segmentation.

Liu and Van Ryzin [10] use the analysis of the model provided by Gallego et al. [9] to extend the concept of efficient sets. They prove that when capacity and demand are scaled up proportionally, revenue obtained under choice-based deterministic linear programming converges to the optimal revenue under the exact formulation. They present a market segmentation model to describe choice behavior. The segments are defined by disjoint consideration sets of products, where a consideration set is a subset of the products provided by the firm, which customers consider options.

Bront et al. [11] extended the work of Liu and Van Ryzin [10] by allowing the customers to consider products which belong to overlapping segments. In this case, they proved that the column generation subproblem is Np-hard and proposed a greedy heuristic to solve it.

Kunnukal and Topaloglu [12] proposed a new deterministic linear program for the network revenue management problem with customer choice behavior. They generated bid prices that depended on the time left until the time of departure. The main drawback of their model is that the number of constraints in their model is significantly larger than linear programming formulation used by Liu and Van Ryzin [10].

Vulcano et al. [13] developed a maximum likelihood estimation algorithm in discrete choice models for airline revenue management. Their simulation results show 1%-5% average revenue improvements using choice-based revenue management.

For analyzing the effects of applying mis-specified models, Amaruchkul and Sae-Lim [14] studied the static overbooking models. They assume that the decision model embedded in a commercial revenue management system is mis-specified. They explore the consequences of the modeling error and find that the performance of the model with mis-specification decreases as show-up probability decreases. Meissner and Strauss [15] propose a new heuristic for specifying bid prices in a choice-based network revenue management problem.

Derigs and Friederichs [16] propose a decision support system for maximizing the revenue generated in the area of waste and row material management. They use the dual variables of the linear program for setting bid prices. Schutze [17] applies price-based revenue management for hotel room pricing. Different pricing strategy clusters are proposed according to the hotel category.

Ben-Akiva and Lerman [18] analyzed different discrete choice models. Train [19] provided the most advanced elements of the estimation and usage of discrete choice models that require simulation. Garrow [20] provided a comprehensive overview of discrete choice models and application of these models to the airline industry.

Garrow et al. [9] completed the study of airline traveler no-show and standby behavior, based on passenger and directional outbound/inbound itinerary data. They focus on passenger behavior based on the estimation of a multinomial logit model and describe the benefits of using passenger data to improve forecasting accuracy and to support a broad range of managerial decisions.

3. Model

Consider a network with m resources (legs) providing n products. $N = \{1, 2, \cdots, n\}$ denotes the set of products and r_j is the associated revenue (fare) for product $j \in N$. We study capacity usage by defining vector $c = (c_1, c_2, \cdots, c_m)$, which denotes the initial capacities of resources (legs). Resource usage, according to the corresponding product, is presented by defining an incidence matrix, $A = [a_{ij}] \in B^{m \times n}$. The matrix entries are defined by $a_{ij} = 1$, if resource j is used by product j and $a_{ij} = 0$ otherwise. A_j, the jth column of A, denotes the incidence vector for product j, and notation $i \in A_j$ indicates that product j is using resource i. Note that one product can use more than one resource. Time has discrete periods and runs forward until a finite number, T; $t = 1, 2, \cdots, T$, and it is assumed that we have, at most, one arrival for each period of time, and that each customer can buy only a single product.

We divide customers into L different segments. A consideration set, $C_l \subset N$, $l = 1, 2, \cdots, L$ is used to describe each segment. Gallego et al. [9] consider a unique segment $C_1 = N$, Liu and Van Ryzin [10] represent non overlapping segments where $C_l \cap C_{l'} = \emptyset$, for all $l \neq l'$, and finally, Bront et al. [11] consider overlapping segments, where $C_l \cap C_{l'} \neq \emptyset$, for certain $l \neq l'$. We are going to analyze a model that has different nests in each segment. Alternatives that belong to the same nest, share common errors, whereas alternatives that are in different nests have independent errors. In this paper, we hypothesized the alternatives which are grouped share common, unobserved attributes. These unobserved attributes cannot be incorporated into the observed portion of the utility.

If we have one arrival, p_l represents the probability that an arriving customer belongs to segment l, with $\sum_{l=1}^{L} p_l = 1$. We consider a Poisson process of arriving streams of customers from segment l, with rate $\lambda_l = \lambda p_l$ and total arriving rate of $\lambda = \sum\limits_{l=1}^{L} \lambda_l$.

In each period of time t, the firm should decide about its offer set (i.e. a subset $S \subset N$ of products that the firm makes available for customers). If set S is offered, the deterministic quantity $P_j(S)$ indicates the probability of choosing product $j \in S$ and $P_j(S) = 0$, otherwise. Using the probability law, we have:

$$\sum_{j\in S} P_j(S) + P_0(S) = 1,$$

where $P_0(S)$ indicates the no-purchase probability.

3.1. Customer choice model

To model customer choice behavior, we can assume that each customer wants to maximize his utility, while his utility for alternatives is a random variable. The firm is offering a set of alternatives for customer n, where he/she has a consideration set, C_n with utility U_{in} for each alternative, $i \in C_n$. This utility can be decomposed into two deterministic (also called expected utility); denoted v_{in}, and a mean-zero random component ε_{in} without loss of generality. Hence, we have a utility function as follows:

$$U_{\text{in}} = v_{\text{in}} + \varepsilon_{\text{in}}. \tag{1}$$

In many cases, the representative component, v_{in}, is modeled as a linear combination of several attributes:

$$v_{\text{in}} = \beta^T x_{\text{in}}, \tag{2}$$

where β is an unknown vector of weights that should be computed from data and x_{in} is the vector of observable attributes for alternative i available to customer n at the time of purchase, such as time and date of departure, price, departure airport, airline brand, and so on.

Train [19] states that one of the best and most commonly used models for studying how customers make their choice is the multinomial logit (MNL) model. In this model it is assumed that the ε_{in} in the utility functions are independent and identically-distributed random variables with a Gumbel distribution. The probability whereby customer n chooses alternative $i \in C_n$ in a MNL model is given by:

$$P_n(i) = \frac{e^{\beta^T x_{\text{in}}}}{\sum_{j\in C_n} e^{\beta^T x_{jn}} + 1}. \tag{3}$$

Based on Garrow (2010), the Nested Logit (NL) model, which appeared just a few years after the MNL model, incorporates more realistic substitution patterns by relaxing the assumption that errors are independent. Within the airline industry, there are many applications in which the NL model can offer forecasting benefits over the MNL model. For each nest, the logsum parameter, μ_m, is a measure of the degree of correlation and substitution among alternatives in nest m. A higher value of μ_m implies less, and lower values imply more correlation among alternatives in the nest. In fact, higher correlation leads to greater competition effects among alternatives in the nest.

In the nested logit model, the probability that individual n selects alternative i is given by:

$$P_j(S) = \frac{e^{V_j/\mu_m}\left[\sum_{i\in A_m} e^{V_i/\mu_m}\right]^{\mu_m - 1}}{\sum_{l=1}^{M}\left[\sum_{i\in A_l} e^{V_i/\mu_l}\right]^{\mu_l}}, \quad 0 < \mu_m \leq 1. \tag{4}$$

A more intuitive expression for the NL choice probability can be derived as the product of a conditional and marginal probability.

$$P_j = P_{j|m} \times P_m = \frac{e^{V_j/\mu_m}}{\sum_{i\in A_m} e^{V_i/\mu_m}} \times \frac{e^{V_m + \mu_m \Gamma_m}}{\sum_{l=1}^{M} e^{V_l + \mu_l \Gamma_l}},$$

$$0 < \mu_m \leq 1, \tag{5}$$

$$\Gamma_m = \ln\left[\sum_{i\in A_m} e^{V_i/\mu_m}\right], \qquad 0 < \mu_m \leq 1. \tag{6}$$

The first component of the product is the probability of selecting alternative j among all i alternatives in nest m, conditional to the choice of m, and the second product is the probability of selecting nest m among all nests. Γ_m is often called the "log-sum term" because it is the log of a sum.

3.2. Dynamic and linear programming models

In the general case, as a firm cannot recognize the corresponding segment of an arrival in advance, we consider $P_j(S)$; the probability whereby the firm sells product j to an arriving customer as:

$$P_j(S) = \sum_{l=1}^{L} p_l P_{lj}(S). \tag{7}$$

In this equation, $P_{lj}(S)$ represents the probability of choosing product j by a customer who belongs to segment l. The expected revenue, by offering set $S \subset N$ for an arriving customer is given by:

$$R(S) = \sum_{j\in S} r_j P_j(S). \tag{8}$$

Given that we offer set S, let $P(S) = (P_1(S), \cdots, P_n(S))^T$ be the vector of purchase probabilities and A the incidence matrix of resources used by products. Then the vector of capacity consumption probabilities $Q(S)$ is given by:

$$Q(S) = A.P(S), \tag{9}$$

where $Q(S) = (Q_1(S), \cdots, Q_m(S))^T$, and $Q_i(S)$ indicates the probability of using a unit of capacity on leg i, $i = 1, 2, \cdots, m$. Based on Liu and Van Ryzin [10] this problem can be formulated as a dynamic program problem:

$$V_t(x) = \max_{S\subset N}\left\{\sum_{j\in S}\lambda P_j(S)(r_j + V_{t+1}(x-A_j)) + (\lambda P_0(S)+1-\lambda)V_{t+1}(x)\right\}$$

$$= \max_{S\subset N}\left\{\sum_{j\in S}\lambda P_j(S)(r_j - (V_{t+1}(x) - V_{t+1}(x-A_j)))\right\} + V_{t+1}(x), \tag{10}$$

and the boundary conditions are $V_t(0) = 0$; $t = 1, 2, \cdots, T$ and $V_{T+1}(x) = 0$; $\forall x \geq 0$. Since the state space is multi dimensional, this problem is intractable and is approximated with a linear programming.

The firm's decision consists of deciding which set of products should be offered at any period of time t, while it cannot distinguish each customer related segment in advance. However, as choice probabilities are time-homogeneous and demand is deterministic, it only matters how many times each set S is offered; knowing during exactly which period is not important. The variable $t(S)$ represents the number of periods during which set S is going to be offered. Another assumption is that we let variable $t(S)$ be continuous as well (i.e. the firm could offer a set S for a whole or a fraction of a period of time). The model's objective is to maximize firm revenue by deciding the number of periods of time for each set of products. Formulation of the CDLP problem will be:

$$V^{\mathrm{CDLP}} = \max\sum\lambda R(S)t(S).$$

S.t.

$$\sum\lambda Q(S)t(S) \leq C,$$

$$\sum t(S) \leq T,$$

$$t(S) \geq 0; \qquad \forall S\subset N. \tag{11}$$

Liu and Van Ryzin [10] prove that since demand and capacity are scaled up proportionately, the revenue obtained under the CDLP model is asymptotically optimal for the original stochastic network choice model.

4. Using column generation to solve the CDLP model

In the model (11), there are an exponential number of primal variables. This means that a problem with n products, has $2^n - 1$ possible non-empty subsets of products of set N. In spite of an enormous number of variables for practical real world problems which makes it impossible to enumerate all offer sets, there are at most $m+1$ constraints. Gallego et al. [9] represent the idea of using a column generation technique to solve real world practical problems.

The first step in applying a column generation algorithm starts by solving reduced linear programming; that is, just considering a limited number of columns (subsets) indicated by $N = \{S_1, S_2, \cdots, S_k\}$. This takes us to the reduced CDLP model as follows:

$$V^{\mathrm{CDLP}-R} = \max\sum\lambda R(S)t(S).$$

S.t.

$$\sum\lambda Q(S)t(S) \leq C,$$

$$\sum t(S) \leq T,$$

$$t(S) \geq 0; \qquad \forall S\subset N. \tag{12}$$

Let $\pi \in R^m$ be the dual prices for the first m-dimensional capacity constraints and $\sigma \in R$ be the dual price for the one-dimensional time constraint. Now for the next step in column generation, we construct a column generation subproblem to find the next column with the most positive reduced cost to add to our set collection, N, which is not included yet. This column is obtained by solving the following sub problem:

$$\max_{S\subset N}\left\{\lambda R(S) - \lambda\pi^T Q(S) - \sigma\right\}$$

$$= \max_{S\subset N}\left\{\lambda R(S) - \lambda\pi^T Q(S)\right\} - \sigma. \tag{13}$$

Afterwards, to explicit Eq. (13), a binary vector, $y \in B^n$, is defined as follows. Suppose a set, S, is offered now, then we denote:

$$y_j = \begin{cases} 1, & \text{if} \quad j\in S\\ 0, & \text{otherwise}\end{cases} \tag{14}$$

A nesting structure of products can be determined in two general different ways, according to a no-purchase alternative. The first approach is to divide products in such a way whereby the no-purchase alternative is placed in the one nest, and all firms' products are allocated in another nest. In this condition, problem (13) can be expressed as follows:

$$\max_{y\in\{0,1\}^n}\left\{\sum_{l=1}^{L}\lambda_l\right.$$

$$\max_{y\in\{0,1\}^n}\left\{\sum_{l=1}^{L}\lambda_l\frac{\sum_{j\in C_l}(r_j-A_j^T\pi)e^{V_{lj}/\mu_m}y_j\left[\sum_{i\in B_m}\left(e^{V_{li}/\mu_m}+\alpha_{im}^{1/\mu_m}e^{V_{l0}/\mu_m}\right)\right]^{\mu_m-1}}{\sum_{k=1}^{M}\left[\sum_{i\in B_k}\left(e^{V_{li}/\mu_k}y_i+\alpha_{ik}^{1/\mu_k}e^{V_{l0}/\mu_k}\right)\right]^{\mu_k}}\right\}-\sigma,\qquad 0<\mu\leq 1, \tag{16}$$

Box I.

$$\frac{\sum_{j\in C_l}(r_j-A_j^T\pi)e^{V_{lj}/\mu_m}y_j\left[\sum_{i\in B_m}e^{V_{li}/\mu_m}\right]^{\mu_m-1}}{\sum_{k=1}^{M}\left[\sum_{i\in B_k}e^{V_{li}/\mu_k}y_i\right]^{\mu_k}}\Bigg\}$$
$$-\sigma,\qquad 0<\mu\leq 1. \tag{15}$$

Another approach is to assume that the no-purchase alternative exists in all nests with the same share. With this definition, the column generation subproblem will be calculated in Eq. (16) as shown in Box I. Or, equivalently:

$$\max_{y\in\{0,1\}^n}\left\{\sum_{j=1}^{n}(r_j-A_j^T\pi)y_j\sum_{l=1}^{L}\right.$$
$$\left.\times\frac{\lambda_l\left[\sum_{i\in B_m}\left(e^{V_{li}/\mu_m}+\alpha_{im}^{1/\mu_m}e^{V_{l0}/\mu_m}\right)\right]^{\mu_m-1}}{\sum_{k=1}^{M}\left[\sum_{i\in B_k}\left(e^{V_{li}/\mu_k}y_i+\alpha_{ik}^{1/\mu_k}e^{V_{l0}/\mu_k}\right)\right]^{\mu_k}}\right\}$$
$$-\sigma,\qquad 0<\mu\leq 1, \tag{17}$$

where m is the nest that product j belongs to, and $V_{l0}>0$; $\forall l$ assumes that the denominator is greater than zero all the time. We assume that the no-purchase alternative belongs to all nests in each segment. We assign the allocation parameter, α_{jk} which reflects the extent to which alternative j is a member of nest k. This parameter must be nonnegative: $\alpha_{jk}\geq 0$; $\forall j,k$. Interpretation is facilitated by having the allocation parameters sum to one over nests for any alternative: $\sum_k \alpha_{jk}=1$; $\forall j$. If Problem (15) or (17) have a positive optimal value, then the optimal solution for the problem will be the next entering column to the reduced primal problem. Then we update the reduced CDLP (12) with the new column, and iterations are continued. Finally, if there is no solution for Problem (15) or (17) with a positive value, then the current solution for the reduced CDLP problem (12) is optimal.

4.1. Complexity of the column generation subproblem

0-1 fractional programming problems (15) and (17) can be considered a special case of the sum of ratios problem with more firmly connected variables. Bront et al. [11] proved that the minimum vertex problem, which is known to be an Np-hard, can be reduced to the 0-1 fractional programming problem with the multinomial logit model. Then, this problem is an Np-hard problem. The multinomial logit model is a special case of the nested logit model in which if $\mu_m=1$ for all nests, the nested logit model will be equivalent to the MNL model. Then, Problems (15) and (17) are also Np-hard.

4.2. Solution approaches for the column generation subproblem

In this section, we study different solution approaches for the column generation subproblem starting by a heuristic method, followed with a metaheuristic.

4.2.1. Greedy heuristic

The fact that the column generation subproblem is an Np-hard optimization problem, guides us to use an alternative approach, which makes it possible to implement this algorithm in practical problems. Bront et al. [11] propose a greedy heuristic to their own problem with complexity $O(n^2L)$, based on the heuristic proposed by Prokopyev et al. [21] to overcome the complexity of the exact algorithm. We are going to propose a new heuristic inspired by this one.

Comparing local and global optimum results shows that, in most cases where a greedy heuristic stops at local optimum, same products exist in the global optimum with some extras. This fact leads us to insert a Boltzmann operator, after stopping the algorithm. This idea stems from the Simulated Annealing (SA) approach. This heuristic starts with an empty set, S, taking into account the maximum marginal contribution to the current solution, by adding progressively new products to the current set S. If the algorithm cannot find any column with a positive reduced cost and the new product does not improve the value of the new set, then the Boltzmann operator will be used and the new product will be inserted with a certain probability.

For applying this operator, we should set the number, say N, of iterations and the initial temperature, T_0, and the final temperature, $T_1 (T_0>T_1)$. We decrease temperature T after every iteration, usually by proportion α (cooling rate). So that, after N iterations, the temperature becomes $t_N=\alpha^N T_0$.

The algorithm is presented in the following steps

$$j_1^* = \arg\max_{j\in S_1'}\left\{\sum_{l=1}^{L}\frac{\sum_{j\in C_l\cap(S\cup\{j\})}(r_j-A_j^T\pi)e^{V_{lj}/\mu_m}\left[\sum_{i\in B_m\cap(S\cup j)}\left(e^{V_{li}/\mu_m}+\alpha_{im}^{1/\mu_m}e^{V_{l0}/\mu_m}\right)\right]^{\mu_m-1}}{\sum_{k=1}^{M}\left[\sum_{i\in B_k\cap(S\cup j)}\left(e^{V_{li}/\mu_k}y_i+\alpha_{ik}^{1/\mu_k}e^{V_{l0}/\mu_k}\right)\right]^{\mu_k}}\right\}.$$

Box II.

$$j^* = \arg\max_{j\in S_1'}\left\{\sum_{l=1}^{L}\lambda_l\frac{\sum_{j\in C_l\cap(S\cup\{j\})}(r_j-A_j^T\pi)e^{V_{lj}/\mu_m}\left[\sum_{i\in B_m\cap(S\cup j)}\left(e^{V_{li}/\mu_m}+\alpha_{im}^{1/\mu_m}e^{V_{l0}/\mu_m}\right)\right]^{\mu_m-1}}{\sum_{k=1}^{M}\left[\sum_{i\in B_k\cap(S\cup j)}\left(e^{V_{li}/\mu_k}y_i+\alpha_{ik}^{1/\mu_k}e^{V_{l0}/\mu_k}\right)\right]^{\mu_k}}\right\}.$$

Box III.

(this algorithm has been proposed for Relation (17) and can be adapted with Relation (15) easily):

- Step 1: For all product j, such that $r_j - A_j^T\pi \le 0$, set $y_j = 0$;
- Step 2: Let $S_1' \subset N$ be the set of products j with no assigned value for y_j;
- Step 3: Compute:

$$j_1^* = \arg\max_{j\in S_1'}\left\{\sum_{l=1}^{L}(r_j - A_j^T\pi)\frac{e^{V_{lj}/\mu_m}\left[\sum_{i\in B_m}\left(e^{V_{li}/\mu_m}+\alpha_{im}^{1/\mu_m}e^{V_{l0}/\mu_m}\right)\right]^{\mu_m-1}}{\sum_{k=1}^{M}\left[\sum_{i\in B_k}\left(e^{V_{li}/\mu_k}y_i+\alpha_{ik}^{1/\mu_k}e^{V_{l0}/\mu_k}\right)\right]^{\mu_k}}\right\}.$$

  Set $S_1 := \{j_1^*\}$, $S_1' := S_1' - \{j_1^*\}$;
- Step 4: **Repeat**:
  - Compute the equation given in Box II.
  - If Value$(S_1\cup\{j^*\}) >$ Value(S_1), then $S_1 := S_1\cup\{j^*\}$, and $S_1' := S_1 - \{j^*\}$.

  Until S_1 is not modified.
- Step 5: If

$$\sum_{l=1}^{L}(r_j - A_j^T\pi)\frac{e^{V_{lj}/\mu_m}\left[\sum_{i\in B_m}\left(e^{V_{li}/\mu_m}+\alpha_{im}^{1/\mu_m}e^{V_{l0}/\mu_m}\right)\right]^{\mu_m-1}}{\sum_{k=1}^{M}\left[\sum_{i\in B_k}\left(e^{V_{li}/\mu_k}y_i+\alpha_{ik}^{1/\mu_k}e^{V_{l0}/\mu_k}\right)\right]^{\mu_k}} - \sigma < 0,$$

  for all $j \in S_1$,
  - Set $y_j = 1$. For $j \notin S_1$, set $y_j = 0$ and stop;
  - Else $S_2 := S_1$ & $S_2' := S_1'$ & T_0 & T_1 & $\alpha N := 0$ and go to the step 6.
- Step 6: **Repeat**:
  - Compute the equation given in Box III.
  - Update $N := N + 1$ $t_N = \alpha^N\ T_0$ and generate random number r;
  - If Value$(S_2\cup\{j^*\}) <$ Value(S_2), then compute

$$\Pr ob = \exp\left[\frac{\text{Value}(S_2\cup\{j^*\}) - \text{Value}(S_2)}{t_N}\right].$$

  - If $\Pr ob > r$, then $S_2 := S_2\cup\{j^*\}$, and $S_2' := S_2' - \{j^*\}$;
  - Else $S_1 := S_2 \& S_1' := S_2'$ & go back to Step 4.

  Until $t_N \le T_1$ **or** $\Pr ob < r$.
- Step 7: Stop.

Thus, a new set may be accepted temporarily with some probability if it is inferior compared to the old set. It is easy to see that, initially, when the temperature is high, and the distance from the best set is low, there is a higher probability of accepting an inferior set. This exponential function is called a Boltzmann function, thus, the operation is called a Boltzmann-type operator.

4.2.2. Genetic algorithm

The genetic algorithm was developed by J. Holland in the 1970s to understand the adaptive processes of natural systems [22]. GAs are a very popular class of population-based metaheuristic. These algorithms start from an initial population of solutions. Then, they iteratively generate a new population and replace it with a current population. This replacement is based on selection methods.

One of the most suitable algorithms to tackle the column generation subproblem is the genetic algorithm. According to the nature of this problem, we can transform it to a binary unrestricted problem, this type of which, the GA algorithm successfully solves.

Firstly, the structure of chromosomes is a binary vector with a dimension equal to the number of products. A gene of the chromosome, denoted as $g(j)$, where $g(j) = 1$, means product number j is available in the set, and $g(j) = 0$, otherwise.

With this chromosome structure, the mutation operator is a uniform function, whose rate is 0.05, whose crossover function is scattered and whose fraction rate is 0.8.

This approach first uses a greedy heuristic in order to identify an entering column to solve the CDLP by column generation. If this algorithm does not find an entering column, then, we use the metaheuristic GA algorithm. Experiments show that when there is still a column to enter the reduced problem, in most cases, the heuristic will find it.

The genetic algorithm for solving the column generation subproblem was implemented using the Matlab genetic toolbox.

5. Computational experiment

In this section, we consider a small network for a choice-based network revenue management model with nested segments in which customers choose their products based on a nested logit model.

Taking into account the computational results, we evaluate different solution approaches, based on the quality of the obtained solution and computational feasibility. According to the fact that the time complexity of the heuristic algorithm is $O(n^2L)$, we focus on analyzing the effect of the correlation of nests in the revenue. For the computational side, we analyze the numbers of iterations and their trend according to the correlation measure in nests.

Mont Carlo simulation has been used for simulating customer choice behavior. For analyzing the impact of the correlation between products in each nest, simulation has been done with 1000 streams of demand according to two distinct scenarios:

1. The assumption that customers choose products based on a nested logit model and firms use a standard logit model for determining the required offer sets.
2. The assumption that customers choose products based on a nested logit model and firms apply the nested logit model for determining the required offer sets.

For determining the offer sets in each period, we solve CDLP formulation and determine optimal offer sets and their related time periods to recommend them. These sets are offered, according to the lexicographic order of the indices of the LP variables. Since variable $t(S)$ could be fractional, we round them to the nearest integer.

Different network load factors are tested. To better evaluate the algorithms, we consider different capacities by multiplying a load factor, α, to the capacity of legs. Parameter α is to scale all the legs capacity, where $\alpha = 1$ corresponds to the original base case. The performance in choice behavior is analyzed by varying the no-purchase observed utility vector; $V_0 = (V_{10}, \cdots, V_{L0})$. We assume that in each segment, the no-purchase alternative belongs to all nests, with equal allocation parameters.

5.1. A small airline network

We evaluate different heuristic methods and product nesting effects with a small airline network. Consider a network with 4 airports and 7 flight legs. The capacities of the legs are $C = (100, 150, 150, 150, 150, 80, 80)$. The firm offers two high (H) and low (L) fares on each leg. Considering local and connecting itineraries, customers can choose between 22 available products defined by itineraries and fare class combinations. The problem consists of finding a policy which leads us to prepare a set of products at any period of time during the booking horizon to offer to the customers, while the revenue of the firm should be maximized. This airline network is illustrated in Figure 1, and Tables 1 and 2 describe available products in this network.

According to the customer prices and time sensitivities, their origin and ultimate destination, 10 overlapping segments and 20 nests (two nests in each segment) are defined in this example. This segmentation is described in Table 2.

The probability of customer arrival for the corresponding segment is given in the last column. Columns 3, 4 & 5 specify the nests, their consideration set and the observed utility for the indicated products, respectively.

Indeed, if the capacity of legs exceeds corresponding demand, the problem becomes much easier to solve and the firm could offer almost all of its products.

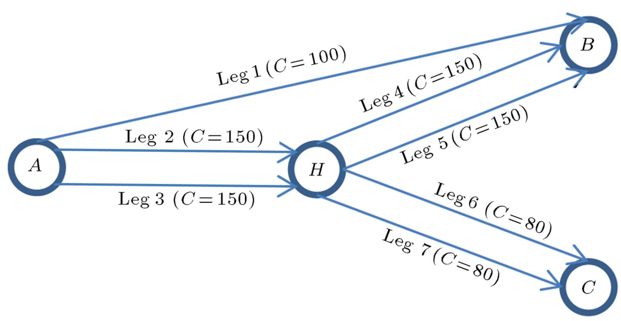

Figure 1. Small network airline.

Table 1. Product definition for small network problem.

Product	Legs	Class	Fare
1	1	H	1000
2	2	H	400
3	3	H	400
4	4	H	300
5	5	H	300
6	6	H	500
7	7	H	500
8	{2,4}	H	600
9	{3,5}	H	600
10	{2,6}	H	700
11	{3,7}	H	700
12	1	L	500
13	2	L	200
14	3	L	200
15	4	L	150
16	5	L	150
17	6	L	250
18	7	L	250
19	{2,4}	L	300
20	{3,5}	L	300
21	{2,6}	L	350
22	{3,7}	L	350

To better evaluate algorithms, we consider different capacities by multiplying scale factor α to the capacity of legs C. We use $\alpha = 0.6, 0.8, 1, 1.2$ & 1.4 to solve the problem and the booking horizon consists of 1000 periods of time.

Table 3 shows the results under the condition whereby the firm uses a nested logit model for determining offer sets, with respect to different correlations among alternatives in each nest, different load factors and no-purchase observed utility. For a no-purchase utility, we assume that a pair utility is repeated for all segments.

We assume that, correlations in different nests are equal. This assumption is considered for better analysis of results and can be relaxed easily.

Table 4 presents similar results when the firm uses a multinomial logit model for specifying offer sets, but customers choose products based on a nested logit model.

Table 5 represents a 95% confidence interval for the improvement percent when the firm switches to the nested logit model.

The first column in the result table is the case in which correlation is zero. We expect these models to have similar results in this condition, due to the fact that when correlation between each nest's alternatives became zero, the nested logit model restructures to a multinomial logit model. It can be observed that in all rows of the first column of results, the revenue gaps

Table 2. Customer segmentation in small network problem.

Segment	O-D	Nest	Con. Set	Observed utility	λ_1
1	A-B	1	{1, 8, 9}	(2.3000, 2.0800, 2.0800)	0.08
		2	{12, 19, 20}	(1.7900, 1.3900, 1.3900)	
2	A-B	1	{1, 8, 9}	(0.0001, 0.6900, 0.6900)	0.2
		2	{12, 19, 20}	(2.0800, 2.3000, 2.3000)	
3	A-H	1	{2, 3}	(2.3000, 2.3000)	0.05
		2	{13, 14}	(1.6100, 1.6100)	
4	A-H	1	{2, 3}	(0.6900, 0.6900)	0.2
		2	{13, 14}	(2.3000, 2.3000)	
5	H-B	1	{4, 5}	(2.3000, 2.3000)	0.1
		2	15, 16	(1.6100, 1.6100)	
6	H-B	1	{4, 5}	(0.6900.0.6900)	0.15
		2	{15, 16}	(2.3000, 2.0800)	
7	H-C	1	{6, 7}	(2.3000, 2.0800)	0.02
		2	{17, 18}	(1.6100, 1.6100)	
8	H-C	1	{6, 7}	(0.6900, 0.6900)	0.05
		2	{17, 18}	(2.3000.2.0800)	
9	A-C	1	{10, 11}	(2.3000.2.0800)	0.02
		2	{21, 22}	(1.6100, 1.6100)	
10	A-C	1	{10, 11}	(0.6900, 0.6900)	0.04
		2	21, 22	(2.3000, 2.3000)	

Table 3. Revenue results for simulation when firm offers products based on NL model.

	Small network								
	Correlation	**0**		**0.2**		**0.6**		**0.8**	
Scale factor	**No-purchase utility**	**Mean**	**%LF**	**Mean**	**%LF**	**Mean**	**%LF**	**Mean**	**%LF**
0.6	(0.00, 1.61)	207904.8	93.02054	207993.4	93.11279	206575.9	95.77636	203310.9	94.91705
	(1.61, 2.30)	193561.9	93.33527	193447.5	93.31357	185402.0	94.12519	175745.0	96.02054
	(2.30, 2.99)	164349.0	93.20271	162821.0	92.94729	155709.2	92.68023	148123.3	92.14690
0.8	(0.00, 1.61)	260799.2	89.15698	259923.4	89.13023	250344.1	92.08547	239459.5	92.82849
	(1.61, 2.30)	217825.6	92.25727	216111.5	92.13343	209488.6	92.24448	202329.2	93.25988
	(2.30, 2.99)	184764.5	88.60959	183928.0	88.22238	178145.3	86.55349	170175.0	85.69797
1.0	(0.00, 1.61)	276191.2	85.07023	276597.0	84.91302	267626.0	90.21372	258039.2	92.32186
	(1.61, 2.30)	230631.4	86.87116	230666.2	88.04605	228506.9	88.47000	224989.9	89.28651
	(2.30, 2.99)	190601.9	81.74186	189724.1	81.59651	184409.6	81.00651	175569.4	77.94977
1.2	(0.00, 1.61)	283230.4	75.27558	282294.0	75.06376	277296.8	86.46298	274703.4	88.85891
	(1.61, 2.30)	238092.5	74.27287	238249.8	74.27248	236356.0	74.16434	235514.1	76.76899
	(2.30, 2.99)	192334.8	70.91589	191800.6	70.80543	185848.2	68.68120	176161.1	65.30640
1.4	(0.00, 1.61)	286072.0	64.62724	285101.8	64.51362	283276.2	74.12907	285054.6	77.78189
	(1.61, 2.30)	238194.8	63.64817	238467.5	63.66196	238066.1	63.77824	236551.5	63.67342
	(2.30, 2.99)	192401.1	60.85963	191793.6	60.63505	185363.6	58.84302	176349.1	56.04385

Table 4. Revenue results for simulation when firm offers products based on MNL Model.

	Small network								
	Correlation	**0**		**0.2**		**0.6**		**0.8**	
Scale factor	**No-purchase utility**	**Mean**	**%LF**	**Mean**	**%LF**	**Mean**	**%LF**	**Mean**	**%LF**
0.6	(0.00, 1.61)	207392.6	92.84031	206979.4	92.58178	199437.8	89.53411	183400.2	82.27647
	(1.61, 2.30)	193320.5	93.27984	192556.2	92.99225	181320.9	87.55078	163316.4	78.17519
	(2.30, 2.99)	164512.8	93.31395	163366.6	92.99225	152727.3	87.60736	138054.6	79.38527
0.8	(0.00, 1.61)	260940.4	89.18866	260435.0	89.01802	248975.4	85.23895	227128.2	77.54157
	(1.61, 2.30)	216355.2	91.93227	216117.6	91.97471	209135.2	90.54942	198589.4	87.73721
	(2.30, 2.99)	184212.4	88.38488	183796.6	88.26017	177343.0	85.76638	167133.7	81.12558
1.0	(0.00, 1.61)	276746.8	85.19023	275401.1	84.92884	266761.8	83.36209	249905.2	79.41605
	(1.61, 2.30)	230924.5	86.97326	230723.8	86.94163	227675.1	86.73837	222632.8	86.26093
	(2.30, 2.99)	190760.8	81.75744	189653.5	81.43767	184303.0	79.62488	174843.1	75.98628
1.2	(0.00, 1.61)	283298.0	75.28566	282916.0	75.48411	276197.0	74.19012	264457.0	72.04981
	(1.61, 2.30)	237662.0	74.23760	237861.5	74.23876	236482.7	74.28973	232488.9	73.87132
	(2.30, 2.99)	192197.0	70.87403	191727.4	70.76202	185574.7	68.69554	176506.8	65.45543
1.4	(0.00, 1.61)	285996.8	64.69917	285703.2	64.60498	281134.2	64.27193	270573.4	62.90166
	(1.61, 2.30)	238691.1	63.67924	238940.1	63.68688	237893.7	63.67209	237212.1	63.79435
	(2.30, 2.99)	192351.6	60.79352	192221.1	60.80615	185570.0	58.84302	176927.0	56.26578

include zero, and then there is no significant difference between them.

General results show that when there is scarce capacity, the nested logit model outperforms the standard model and we have significant improvement in the revenue. However, it is obvious that if we have ample capacity or low correlation in the nests, it is recommended not to change the firm's choice model.

Figure 2 represents the improvement obtained in revenue in the case of switching to the nested logit

Table 5. Confidence interval for the improvement percent while firm switches to the NL model.

	Small network				
	Correlation	**0**	**0.2**	**0.6**	**0.8**
Scale factor	**No-purchase utility**	**C.I.**	**C.I.**	**C.I.**	**C.I.**
0.6	(0.00, 1.61)	(-0.017364, 0.675914)	(0.362213, 0.600817)	(3.388140, 3.935200)	(10.64630, 11.31920)
	(1.61, 2.30)	(-0.161641, 0.597681)	(0.177845, 0.937099)	(1.964460, 2.866910)	(7.305070, 8.382390)
	(2.30, 2.99)	(-0.394900, 0.400578)	(-0.637082, 0.171305)	(1.821490, 2.201360)	(7.154340, 7.619630)
0.8	(0.00, 1.61)	(-0.296974, 0.319497)	(-0.448020, 0.194134)	(0.302587, 1.013182)	(5.164140, 6.014120)
	(1.61, 2.30)	(-0.296502, 0.920363)	(-0.251740, 0.382133)	(-0.081722, 0.582707)	(1.608600, 2.371510)
	(2.30, 2.99)	(-0.135901, 0.790873)	(-0.209928, 0.513364)	(0.140065, 1.051149)	(1.501270, 2.442290)
1.0	(0.00, 1.61)	(-0.446172, 0.155106)	(0.319984, 0.569470)	(0.192665, 0.496207)	(3.112320, 3.461020)
	(1.61, 2.30)	(-0.387057, 0.273689)	(-0.272800, 0.372330)	(0.104934, 0.744230)	(0.817920, 1.424940)
	(2.30, 2.99)	(-0.393047, 0.482683)	(-0.258402, 0.533039)	(-0.244723, 0.595023)	(0.084582, 1.039203)
1.2	(0.00, 1.61)	(-0.283048, 0.380720)	(-0.484887, 0.200243)	(0.287579, 0.551087)	(3.740850, 4.056720)
	(1.61, 2.30)	(-0.108983, 0.645529)	(-0.120987, 0.623261)	(-0.323690, 0.388319)	(1.023920, 1.72100)
	(2.30, 2.99)	(-0.251667, 0.644250)	(-0.266928, 0.577070)	(-0.173553, 0.766554)	(-0.526092, 0.408106)
1.4	(0.00, 1.61)	(-0.232830, 0.427023)	(-0.471582, 0.194415)	(0.516539, 1.143274)	(5.111180, 5.707190)
	(1.61, 2.30)	(-0.500950, 0.310346)	(-0.492060, 0.302054)	(-0.231975, 0.566751)	(-0.569590, 0.216023)
	(2.30, 2.99)	(-0.295294, 0.636239)	(-0.548584, 0.373440)	(-0.438872, 0.488193)	(-0.651141, 0.323096)

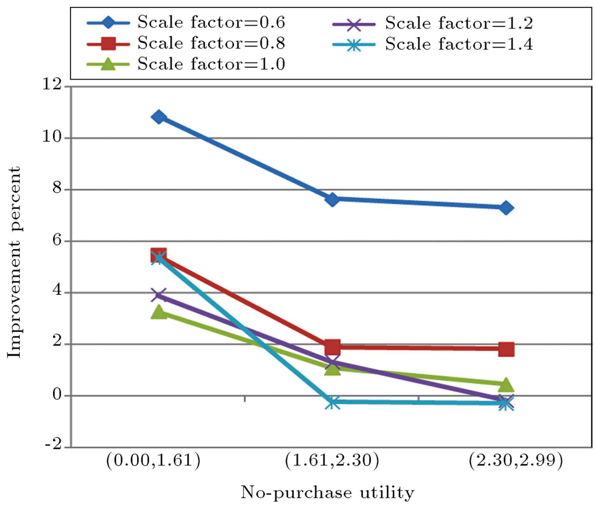

Figure 2. Improvement percent in revenue in the case of switching to the NL model.

model, with respect to the different scale factors and load factors when correlation is equal to 0.8.

It is obvious that by decreasing initial capacity and no-purchase utility, the nested logit model outperforms the standard logit model.

As expected, with increasing correlation between nest products, in most cases, the gap between two models becomes greater, especially when there is capacity shortage. Then, it is recommended to use a nested logit model under these conditions.

Table 6 represents the number of iterations in the column generation method, according to the different correlation measures inside the nests. It shows that changing the correlation measure between each nest's products will alter the buy up and no-purchase probability. The algorithm tries to balance these effects and this leads to an increasing or decreasing number of iterations. It can be observed that, generally, when there is ample capacity in the high scale factor and high no-purchase utility, increasing correlation will lead to simplifying offer set structures. In order to explain the causes of this effect, consider a case in which the scale factor is 1, and the no-purchase observed utility is (2.3,2.99). If we solve this problem, the optimum solution will be degenerated with three offer sets. Results are summarized in Table 7.

These results should be analyzed based on the fact that products 19 and 20 are low fare products and belong to the same nest. Figure 3 shows the trend of different sets offering time periods according to the product correlations of the nest.

When the product correlation of the nests is zero, removing products 19 and 20 will lead to an increase in other product choice probabilities, with the same rate. But, increasing correlation will result in the choice probability of product 12 (low fare product) to be increased more than other high fare products, and this leads to reductions in buy up probability

Table 6. Number of iterations in the column generation algorithm.

	Correlation	0	0.2	0.6	0.8
Scale factor		**Iterations**	**Iterations**	**Iterations**	**Iterations**
0.6	(0.00, 1.61)	18	19	28	31
	(1.61, 2.30)	25	33	28	37
	(2.30, 2.99)	22	22	23	20
0.8	(0.00, 1.61)	17	17	22	26
	(1.61, 2.30)	21	20	26	23
	(2.30, 2.99)	13	12	13	13
1.0	(0.00, 1.61)	13	10	21	22
	(1.61, 2.30)	14	16	13	18
	(2.30, 2.99)	10	10	9	2
1.2	(0.00, 1.61)	8	7	11	15
	(1.61, 2.30)	3	2	2	6
	(2.30, 2.99)	2	2	2	2
1.4	(0.00, 1.61)	5	7	4	10
	(1.61, 2.30)	2	2	2	2
	(2.30, 2.99)	2	2	2	2

Table 7. Offer sets and their optimal offering period.

		Correlation			
		0	**0.2**	**0.6**	**0.8**
Offer set 1	(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22)	572	604	947	1000
Offer set 2	(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22)	224	208	39	0
Offer set 3	(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22)	204	288	14	0

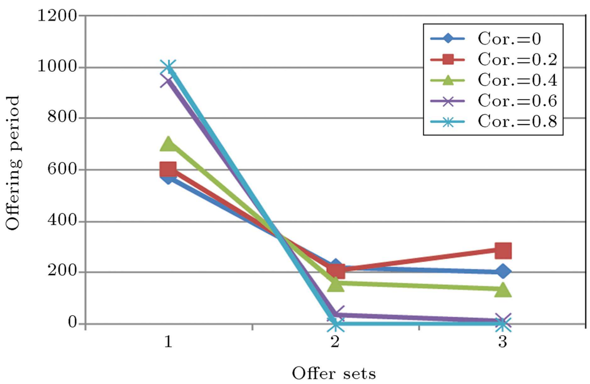

Figure 3. Optimal offering period of different sets.

and increases departure with no-purchase probability. Then, as seen in Figure 3, increasing correlation will result in a decrease in the offering time period of sets 2 and 3. But, when we decrease capacity, buy up will become more important than departure with no-purchase, and then we will not observe the mentioned behavior.

Computational results show that in 88% of cases, a heuristic without the Boltzmann operator and genetic

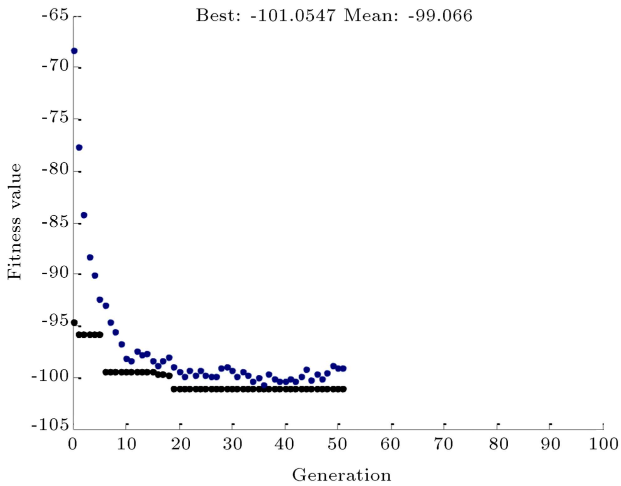

Figure 4. Path of the best and mean of population in different generations.

algorithm manages to find the entering column and, then, the heuristic works well itself.

Figure 4 represents the path of the best and mean of populations in different generations in a genetic algorithm with parameters $\alpha = 0.6$. The no-purchase utility is (1.61,2.30) and the nest correlation is 0.8.

5.2. Railroad network

A specific railroad network in Europe is used by Hosseinalifam [23] for a test problem, a part of which we will consider with five cities and four legs. There are two high (H) and low (L) fare classes on each leg. Figure 5 illustrates this railroad network.

In this problem, there are 10 trains with a capacity of 100 passengers going from Paris to Amsterdam. Each train stops in Brussels, Rotterdam, Schiptol and Amsterdam. Thus, there are 10 markets shown in Table 8. Two fare classes and 10 markets produce a total of 60 products.

Customers are divided into 20 different segments based on their sensitivity to prices, their origin and ultimate destination and 40 nests. Table 9 shows each segment's definition according to our assumptions. We assume that the booking horizon includes 2000 time periods and each segment includes two nests. The experiments are done for three scale factors, including 0.5, 1 and 1.5 for time periods.

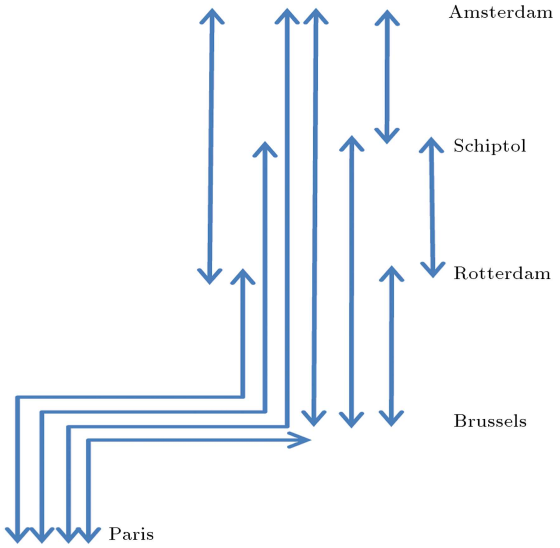

Figure 5. Railroad network.

Table 8. Products definition in railroad network.

O-D	Low fare	High fare
PAR-BRU	200	400
PAR-RTA	300	500
PAR-SCH	350	525
PAR-AMA	350	525
BRU-RTA	150	250
BRU-SCH	175	275
BRU-AMA	200	300
RTA-SCH	50	100
RTA-AMA	175	300
SCH-AMA	50	100

Table 10 represents a 95% confidence interval for the improvement percent, while the firm changes the choice model from a multinomial to a nested logit model.

The first column in this table is the case in which correlations in all nests are zero. As we expect, all confidence intervals in this condition include zero and both model outputs are the same.

Following the previous network, scarce capacity leads to an increase in the importance of choosing the correct choice model for offering the most suitable products for customers, and ample capacity decreases this sensitivity.

6. Conclusion

This article focuses on the effects of specific choice models on network revenue management. Most research focusing on choice-based revenue management, usually applies a multinomial logit choice model. In spite of the simplicity of this model, it has some serious limitations, such as the independence of irrelevant alternatives. We explained this restriction and compared it with the multinomial logit choice model in this article. In order to overcome this restriction, we introduce the nested logit model, the most well known model after multinomial logit. One of the challenges faced by scientists in revenue management is incorporation of more realistic choice models in traditional models without significantly increasing the complexity of the problem. One of the most applicable models of choice-based revenue management is choice-based deterministic linear programming. Considering the exponential number of variables in this problem, a column generation technique is used for solving it. The subproblem of this algorithm is sensitive to the specific choice model used in the original problem. We changed the choice model and introduced the new subproblem structure. By referring to previous studies, we showed that the new subproblem is Np-hard and a heuristic algorithm is required for solving it in real conditions. A combination of heuristic and metaheuristic algorithms is proposed for solving the new subproblem. The heuristic algorithm assures that we can reach a reasonable point and the metaheuristic would improve it if the previous algorithm stopped at a local optimum point. The simulation study was done under two different conditions. We assume that the real choice model customers use for specifying a product is nested logit, and we analyze the effects of using the multinomial logit model by the firm to determine the offer sets. The results show that when there is scarce capacity, specifying an accurate choice model is very important and can improve organization revenue.

Table 9. Customer segmentation in railroad network problem.

Segment	O-D	Nest	Consideration set	Preference vector	No-purchase utility	Arrival rate
1	PAR-BRU	1	1, 2, 3	10, 55, 25	8	0.08
		2	4, 5, 6	20, 4, 3	8	0.08
2	PAR-BRU	1	1, 2, 3	8, 38, 18	60	0.02
		2	4, 5, 6	60, 10, 7	60	0.02
3	PAR-RTA	1	7, 8, 9	15, 30, 20	2	0.08
		2	10, 11, 12	8, 2, 1	2	0.08
4	PAR-RTA	1	7, 8, 9	10, 25, 8	45	0.02
		2	10, 11, 12	25, 10, 4	45	0.02
5	PAR-SCH	1	13, 14, 15	25, 25, 20	10	0.08
		2	16, 17, 18	2, 2, 2	10	0.08
6	PAR-SCH	1	13, 14, 15	10, 12, 15	30	0.02
		2	16, 17, 18	21, 3, 3	30	0.02
7	PAR-AMA	1	19, 20, 21	20, 20, 2	4	0.08
		2	22, 23, 24	3, 4, 3	4	0.08
8	PAR-AMA	1	19, 20, 21	8, 5, 2	35	0.02
		2	22, 23, 24	20, 3, 3	35	0.02
9	BRU-RTA	1	25, 26, 27	10, 60, 50	15	0.08
		2	28, 29, 30	4, 3, 2	15	0.08
10	BRU-RTA	1	25, 26, 27	4, 25, 20	70	0.02
		2	28, 29, 30	45, 4, 6	70	0.02
11	BRU-SCH	1	31, 32, 33	5, 25, 10	5	0.08
		2	34, 35, 36	4, 3, 3	5	0.08
12	BRU-SCH	1	31, 32, 33	2, 14, 3	40	0.02
		2	34, 35, 36	7, 6, 4	40	0.02
13	BRU-AMA	1	37, 38, 39	30, 24, 4	6	0.08
		2	40, 41, 42	2, 2, 2	6	0.08
14	BRU-AMA	1	37, 38, 39	25, 12, 2	10	0.02
		2	40, 41, 42	6, 5, 4	10	0.02
15	RTA-SCH	1	43, 44, 45	10, 25, 20	4	0.08
		2	46, 47, 48	4, 3, 2	4	0.08
16	RTA-SCH	1	43, 44, 45	3, 13, 12	30	0.02
		2	46, 47, 48	36, 3, 2	30	0.02
17	PAR-AMA	1	49, 50, 51	20, 40, 10	5	0.08
		2	52, 53, 54	2, 1, 2	5	0.08
18	PAR-AMA	1	49, 50, 51	10, 15, 5	40	0.02
		2	52, 53, 54	25, 25, 3	40	0.02
19	SCA-AMA	1	55, 56, 57	30, 32, 20	5	0.08
		2	58, 59, 60	4, 3, 2	5	0.08
20	SCA-AMA	1	55, 56, 57	20, 24, 15	60	0.02
		2	58, 59, 60	20, 4, 4	60	0.02

Table 10. Confidence interval for the improvement percent while firm switches to the NL model.

	Correlation				
	0	**0.2**	**0.4**	**0.6**	**0.8**
Time periods	**C.I.**	**C.I.**	**C.I.**	**C.I.**	**C.I.**
1000	(-0.672,0.924)	(-0.309,1.097)	(-0.969,0.356)	(-1.044,0.590)	(0.840,2.147)
2000	(-0.678,0.241)	(-0.009,0.984)	(0.499,1.406)	(2.556,3.495)	(6.353,7.584)
3000	(-0.542,0.386)	(-0.344,0.550)	(0.258,1.112)	(3.096,4.110)	(8.299,9.500)

Improvement percent confidence intervals indicate that even if statistical tests demonstrate a correlation between unobserved parts of utility functions, it is not beneficial under all conditions to change the choice model in the optimization module immediately and increase the complexity of calculations. We studied the relationship between correlation measure and firm revenue under different conditions, and analyzed the number of iterations, with respect to the correlation measure in different nests. We also showed that changing the correlation will lead to a change in buy up and no-purchase probability. The algorithm tries to balance these effects, which causes a change in the number of iterations of the column generation algorithm.

For future work, the nested logit model could be applied to other choice-based revenue management algorithms. Further work can be done to prove, analytically, the improvement in changing specific choice models. Finally, research can be planned to incorporate other realistic choice models and analyze their effects in the choice-based revenue management models.

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Biographies

Farhad Etebari is PhD candidate in the Industrial Engineering Department of K.N. Toosi University of Technology. His research interests include optimization models, choice-based revenue management, transportation planning, AI algorithms and logistic systems. He has several publications in these fields.

Abdollah Aghaei is a professor in the Industrial Engineering Department of K.N. Toosi University of Technology. His research interests include modeling and computer simulation, knowledge management and technology, project management and e-marketing. Professor Aaghaie has authored several books and technical publications in archival journals.

Ammar Jalalimanesh is a faculty member of Iranian Research Institute for Information Science and Technology. He is PhD student in the Industrial Engineering and Management System Department of Amirkabir University of Technology. He holds a master degree in Industrial Engineering from K.N. Toosi University of Technology. His research interests include AI algorithms and modeling of complex systems.