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The hierarchical hub covering problem with an innovative allocation procedure covering radiuses

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Abstract. Hub location problems deal with locating hub facilities at one level of services or one type of facility, but some systems are performed by several types of facility. So, this paper attempts to study a single allocation hierarchical hub covering a facility location problem over a complete network linking at the first level, which consists of hub facilities known as central hubs. In addition, the study proposes a mixed integer programming formulation and finds the location of the hubs at the second level and central hubs at the first level, so that the non-hub and hub nodes are allocated to the opening hub and central hub nodes. Thus, the travel time between any origin destination pair is within a given time bound. The current study presents an innovative method for computing the values of radiuses in order to improve the computational time of the model and to test the performance of the mentioned heuristic method on the CAB data set and on the Turkish network. Helpful results were obtained, including: severe reduction in the time of solution, rational distribution of the centers for presenting results, equality (Justice) and appropriate accessibility consistent with different levels of servicing. A computational experience was applied to the Iranian hub airport location.

1. Introduction

The hub location problems deal with locating hub facilities and the assignment of demand nodes to them, and obtaining minimum total cost for network establishment. There are two types of allocation for links in hub networks: single assignment and multiple assignment. The former allocates demand nodes exactly to a hub and the latter connects non-hubs to one or more than one hub. The single assignment hierarchical hub location problem with two levels locates hub facilities at any level of hierarchy and allocates demand nodes to exactly one hub and one central hub, such that the travel time between any origin destination pair is within a given time bound. Frequently, the routing costs between hubs and central hubs and between central hubs are discounted at a certain rate, in order to take the benefits of economies of scale. Usually, in classical hub networks, hub facilities at the top level are connected to a complete network and at the second and third level links form a star network. These problems are applied to many fields, such as transportation (airline, cargo delivery), and telecommunication systems. Regarding the applied field of the present problem and related problems, the following cases could be mentioned:

a) Health care systems;
b) Solid waste management systems;
c) Production-distribution systems;
d) Education systems;
e) Emergency medical services (EMS);
f) Telecommunication network systems;

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g) Cargo delivery;

h) Security systems (for example: police offices).

Other relevant ones above have been examined by Sahina and Sural [1], which includes comprehensive research into hierarchical problems.

The hub location problem was presented by O'Kelly [2] with single allocation and minimizing total routing costs in a quadratic model formulation. This model has become the basis for all hub location problems. O'Kelly illustrated $\alpha \in [0,1]$ as the constant discount factor, in order to reflect economies of scale in hubbing links. Among several research fields in the hub location problem, the main one is the linearization of the quadratic model introduced by O'Kelly [2], which was investigated by Aykin [3], Campbell [4], Ernst and Krishnamoorthy [5], O'Kelly et al. [6] and Skorin-Kapov et al. [7].

Following the same trend of conducted research, Campbell [4] presented a new version of the hub location problem, entitled the hub covering problem. The hub covering problem finds the location of a hub and allocates demand nodes to opening hub facilities, so that the travel time between any origin destination pair is within a given time bound, and these problems have been presented with different objective functions. The hub covering problem presented by Campbell [4] created two new models for hub location problems (p-hub center & hub covering) with different objective functions. There are two types of hub covering problem: Hub set-covering that minimizes the cost of establishing the hubs for covering all demand nodes, and hub maximal covering that maximizes the non-hubs covered with a predetermined number of hubs. The mixed integer formulations for both problems are introduced by Campbell [8]. Kara and Tansel [9] studied the single allocation hub set-covering problem and demonstrated that it is NP-hard. They also presented a new linear model of their quadratic model. Wagner [10] proposed new formulations for hub covering problems in two situations of single and multiple assignment; these formulations need fewer numbers of variables and constraints compared to the Kara and Tansel [9] formulation. Ernst et al. [11] presented new formulations for the single and multiple assignment hub set covering problem via β as the covering radius. These formulations are better than Kara and Tansel's [9] because they require lower CPU processing time. Tan and Kara [12] focused on cargo delivery systems and suggested a model based on various Turkish cargo delivery companies. They considered the constraints, requirements, and criteria of the cargo delivery sector in the hub location problem. Calik et al. [13] studied the single assignment hub set-covering problem over incomplete hub networks and presented an integer programming formulation with a heuristic solution method, based on the tabu search. The findings of a study conducted on a major cargo company in Turkey indicated that the company has a three-level structure with two central hubs located in Ankara and Istanbul, which were joined with direct links; and each hub at the lower level is linked exactly to one central hub and the demand nodes are connected to a hub or a central hub [14]. Elmastas designed a cargo delivery company applied to airplanes and trucks in order to minimize the fixed charge costs [14]. Another type of Elmastas problem was presented with two differences by Yaman [15]. Yaman combined the hierarchical hub median problem with delivery time constraints and called it The Restricted Hierarchical Hub Median Problem with Single Assignment (SA-TH-HM) [15]. Reviewing Yaman's study, we understood that without covering constraints, the demands nodes and hubs do not sometimes assign to the nearest hub and the central hub, respectively. Since Yaman's model depicts a general presentation of delivery time constraints, on the one hand there is a high traffic demand among central hubs and, on the other hand the objective function tends to minimize routing costs, which make central hubs closer to each other. But, the objective model neglects the distance between non-hubs and hubs and hubs from central hubs.

In spite of a feasible solution, it is possible that some nodes are assigned to a more distant facility in a hierarchy in order to decrease total routing costs, but this increases travel time for that node. So, other constraints must be defined until the model enables covering nodes in the best hierarchy, and assignment with unsuitable distances be avoided. While there exists no previous study on a single allocation hierarchical hub covering location model with a given delivery time bound (SA-TH-HC), this paper presents a SA-TH-HC, and considers, simultaneously, hub location and covering location problems in hierarchical structures.

To the best of our knowledge, the amount of radii is defined as decision variables and brought to delivery time constraints, but in the paper, the value of radii is set fixed and obtained by heuristic methods. So, the demand and hub nodes are exactly assigned to one hub and one central hub, respectively, and are located in their cover radiuses. The traffic demand from an origin to a destination can encounter four or less hubs on its route. The non-hubs and hub nodes connect to exactly a hub and a central hub, respectively. The central hubs are connected to each other on a complete network, so there is a direct link from any top level facility to another.

Figure 1 depicts a transportation network with two levels of hub facility and presents the problem with the best resolution. This network has 35 nodes. Non-hubs, hubs and central hubs are seen in the

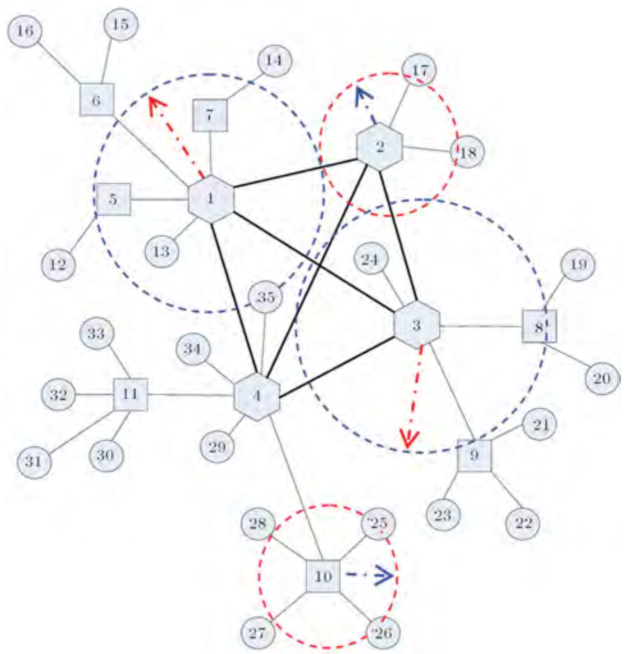

Figure 1. A three level network on 35 nodes with 11 hubs and 4 central hubs.

shapes of circles, squares and hexagons, respectively. The covering radiuses of the hubs are traced with incomplete circles. There are various routes for the model depicted in Figure 1; for example, a route that visits 4 hubs in which traffic demand flows from node 12 to node 21, and the flow paths including nodes 12 - 5 - 1 - 3 - 9 - 21. One path, which encounters 2 hubs, crosses from node 17 to node 35 and its flow paths are 17 - 2 - 4 - 35. There are many traffic paths in the problem, which can be observed in more detail in Figure 1.

This paper presents the hierarchical hub covering problem with different and fixed covering radii for all hub facilities at all hierarchy levels. It aims to find the location of hubs and central hubs, to allocate demand and hub nodes to the nearest located hub and central hub node in a unified covering situation, respectively. In addition, the paper guides the model for the better, seeking the solution area with predetermined radii. Radii are determined by an innovative method and they reduce CPU processing time. The main contributions of this paper are in proposing a new hub location problem and solving it in the best time by the heuristic identification of covering constraints. Computational results obtained for the CAB data, the Turkish network data and the Iranian Airport Data (IAD) on the proposed model show that this model is different from previous ones.

The rest of this paper is organized as follows: Section 2 presents a mixed integer programming formulation. Section 3 explains an innovative method for determining radii. Section 4 is dedicated to computational analysis, and investigating the performance of the model with the optimization solver, CPLEX 11.0, on the famous data sets of CAB, the Turkish network and IAD. The final section develops the outcome of the paper.

2. A mixed integer programming formulation for SA-TH-HC

In this section, the notations, parameters and decision variables for our model are defined and then the mathematical formulation for the SA-TH-HC problem is presented. It is assumed that I is the set of nodes, $H \subseteq I$ is a potential hub set and $C \subseteq H$ is a set of possible locations for the central hubs. The parameters of the model include: P_H, P_C, α_H, $\hat{\alpha}_H$, α_C, $\hat{\alpha}_C$, f_{ij}, C_{ij}, t_{ij}, d_{ij}, r_H, r_C. As the model has been designed for maximal covering, a constant number of hubs and central hubs must be established; we denote the number of hubs and central hubs by P_H and P_C, respectively. In the hub location problems, a discount rate of α is applied to established link costs (or travel time) between hubs and $\alpha \in [0, 1]$. The decreased coefficients of routing cost and travel time among hubs and central hubs are presented with α_H, $\hat{\alpha}_H$and those between central hubs are presented by α_C, $\hat{\alpha}_C$, respectively. It is assumed that these parameters take values based on $\alpha_H \geq \alpha_C$ and $\hat{\alpha}_H \geq \hat{\alpha}_C$ [15]. Parameter f_{ij} is the amount of traffic demand to be routed from node $i \in I$ to node $j \in I$, C_{ij} is the routing cost of unit flow demand between two nodes of i and j, t_{ij} is the travel time from node i to node j, and d_{ij} is the distance between node i and node j. r_H is the amount of covering radius for all hubs and r_C is the amount of covering radius for all central hubs. As mentioned in previous studies, f_{ii}, C_{ii}, t_{ii} and d_{ii} are equal to zero for all of $i \in I$. We symbolized the upper bound of the delivery time by β, which is similar to the literature; β is the maximum time by which any flow demand must reach its destination. The decision variables of the model are as follows:

- X_{ijl} — 1 if node i is assigned to a hub at node j and a central hub at node l; 0 otherwise.
- Y_{jl}^{i} — The flow originated by node $i \in I$ (as origin or destination) which crosses between the hub at node j and the central hub at node l.
- Z_{kl}^{i} — The total demand flow which crosses from the central hub at node k to the central hub at node l by node $i \in I$ as the origin or destination.
- $\hat{D}_I$ — The time by which all the demand flows from demand nodes and hubs assigned to central hub l arrive at l.

D_l The time by which all the demand flows cross from the origin to the destination central hub.

S_i The time at which all the traffic originating at node $i \in I$ is available at node i (in a telecommunications context, letting $S_i = 0$ for all $i \in I$ [15].

Consistent with the literature, if the variable $X_{jjl} = 1$ for each $j \in H$, node j is assigned to a central hub at node l, which means that node j is a hub node, and if $X_{lll} = 1$ for each $l \in C$, node l is a central hub node.

The objective function of our mathematical model minimizes the total routing costs of demand flow between non-hubs and their hubs, between the hubs and their central hubs and between central hubs together. According to the previously defined decision variables and parameters, the objective function is described in detail, as follows:

$$\min z_1 = \sum_{i\in I}\sum_{r\in I}(f_{ir} + f_{ri})\sum_{j\in H} C_{ij}\sum_{l\in C} x_{ijl} + \sum_{i\in I}\sum_{j\in H}\sum_{l\in C:l\neq j}\alpha_H C_{jl} y^i_{jl} + \sum_{i\in I}\sum_{k\in C}\sum_{l\in C:l\neq k}\alpha_C C_{kl} z^i_{kl}. \quad (1)$$

The objective function could be investigated in three parts: (a), (b) and (c):

(a) $\sum_{i\in I}\sum_{r\in I}(f_{ir} + f_{ri})\sum_{j\in H} C_{ij}\sum_{l\in C} x_{ijl}$;

(b) $\sum_{i\in I}\sum_{j\in H}\sum_{l\in C:l\neq j}\alpha_H C_{jl} y^i_{jl}$;

(c) $\sum_{i\in I}\sum_{k\in C}\sum_{l\in C:l\neq k}\alpha_C C_{kl} z^i_{kl}$.

According to part (a):
This formula calculates the routing cost of the established route between all demand nodes and their hub nodes, which is the result of forward flow (f_{ir}) and return flow (f_{ri}) at each demand node with other demand nodes.

According to part (b):
This term calculates the total routing cost caused by the interaction of demand node i with other nodes, which passes the connecting rout of hub j and central hub l. In this case, the hub and the central hub must be different in order to form a route between them.

According to part (c):
This term calculates the total routing cost caused by the interaction of demand node i with other nodes that passes the connecting rout of two central hubs, l and k.

We classified and explained the constraints of the mathematical model for SA-TH-HC as follows:
Single assignment hierarchical hub constraints:

$$\sum_{j\in H}\sum_{l\in C} x_{ijl} = 1 \qquad \forall i \in I, \quad (2)$$

$$x_{ijl} \leq x_{jjl} \qquad \forall i \in I, j \in H : j \neq i, l \in C, \quad (3)$$

$$\sum_{m\in H} x_{jml} \leq x_{lll} \qquad \forall j \in H, l \in C : l \neq j, \quad (4)$$

$$x_{ljl} = 0 \qquad \forall j \in H, l \in C : l \neq j, \quad (5)$$

$$x_{ijl} \in \{0,1\} \qquad \forall i \in I, j \in H, l \in C. \quad (6)$$

Since the model was designed with the single assignment attribute, all non-hub nodes and hub nodes should be allocated to exactly one hub facility and one central hub facility, respectively. Constraints (2), (4) and (6) guarantee the single assignment attribute for the model. Constraints (4) ensure that each hub cannot be linked to other nodes unless that node is a central hub node. Constraints (3) state that if a demand node is assigned to another node, then, that node should be a hub node. Constraints (5) are unnecessary; however, they do make the LP relaxation stronger.

Number of hub and central hub constraints:

$$\sum_{j\in H}\sum_{l\in C} x_{jjl} = p_H, \quad (7)$$

$$\sum_{l\in C} x_{lll} = p_C. \quad (8)$$

As the model was based on a P-hub median and maximal covering problems, we denoted the number of hubs and central hubs with Constraints (7) and (8), respectively:

Flow balance constraints:

$$y^i_{jl} \geq \sum_{r\in I:r\neq j}(f_{ir} + f_{ri})(x_{ijl} - x_{rjl}) \qquad \forall i \in I, \; j \in H, l \in C : l \neq j, \quad (9)$$

$$\sum_{k\in C:k\neq l} z^i_{lk} - \sum_{k\in C:k\neq l} z^i_{kl} = \sum_{r\in I} f_{ir}\sum_{j\in H}(x_{ijl} - x_{rjl}) \qquad \forall i \in I, l \in C, \quad (10)$$

$$y^i_{jl} \geq 0 \qquad \forall i \in I, j \in H, l \in C, \quad (11)$$

$$z^i_{kl} \geq 0 \qquad \forall i \in I, k \in C, l \in C : l \neq k. \quad (12)$$

The flow balance constraints are defined similar to Yaman's [15]. For each link, from a hub to its central hub, the amount of traffic demand will be computed by Constraints (9) and (11). Through these constraints, the amount of traffic crossing between a hub and its central hub is added as a lower bound for the y^i_{jl} variable; since the objective function is of a minimizing type, the least value of y^i_{jl} is estimated. According to flow network rules, the sum of input flow to a node is equal to the sum of output flow from the same node. This situation is ensured by flow balance Constraint (10) and Constraint (12), because the central hubs are end nodes of their hierarchy.

Covering constraints:

$$\sum_{j\in H}\sum_{l\in C:j\neq l} d_{ij}x_{ijl} \leq r_H \qquad \forall i\in I, \tag{13}$$

$$\sum_{l\in C} d_{il}x_{ill} \leq r_H \qquad \forall i\in I, \tag{14}$$

$$d_{jl}x_{jjl} \leq r_C \qquad \forall j\in H, \quad l\in C. \tag{15}$$

We classified the solution space and modified the assignment procedure of each hub and non-hub node to the nearest facility at their top level. We specified that each assignment between two nodes occurs when they are in a specific closeness to each other. The specific closeness criteria are equal to the possible minimal distance between two node and they are characterized by r_H and r_C for second and first level assignments, respectively. We solved the model by investigating few local optimum solutions and, therefore, obtained the global optimum solution in the least possible time.

In short, all assignments were conducted by two fixed amounts for r_H and r_C, which are computed by a a heuristic method and discussed in the next section. According to the above points, we designed Constraints (13), (14) and (15). Constraint (13) ensures that the assignment of demand nodes to hubs receives low services in a specific closeness criterion of r_H. On the other hand, the hierarchical model has a nested characteristic (in a nested hierarchy, a higher-level facility provides all the services provided by a lower level facility and at least one additional service; this is presented by Marianov and Serra [16]). So, it is possible that the non-hubs are assigned to a central hub to achieve low service. Then, Constraint (14) guarantees that assignment of demand nodes to central hubs receives low service in a specific closeness criterion of r_H. Also, Constraint (15) guarantees that hub nodes are assigned to central hubs and receive top service in a specific closeness criterion of r_C. The differences between the current model and other models in literature stem from covering Constraints (13), (14) and (15), which have the specific closeness criteria characterized by r_H and r_C for the second and first level assignments, respectively.

Time bound constraints:

$$\hat{D}_l \geq \sum_{j\in H}(S_i + t_{ij} + \hat{\alpha}_H t_{jl})x_{ijl} \qquad \forall i\in I, \quad l\in C, \tag{16}$$

$$D_l \geq \hat{D}_k + \hat{\alpha}_C t_{kl}x_{kkk} \qquad \forall l\in C, \quad k\in C, \tag{17}$$

$$D_l + \sum_{j\in H}(\hat{\alpha}_H t_{lj} + t_{ji})x_{ijl} \leq \beta \qquad \forall i\in I, \quad l\in C, \tag{18}$$

$$\hat{D}_l \geq 0 \qquad \forall l\in C. \tag{19}$$

We defined the time bound constraints by using the ideas developed in Ernst et al. [11] and chiefly Yaman [15]. For the central hub of l, let $\hat{D}_l$ be the time by which all the traffic from demand nodes and hubs assigned to central hub l arrives at l. Constraint (16) guarantee that the time by which all the traffic from demand nodes and hubs assigned to central hub l arrives at l is no earlier than the time the traffic reaches from i to $\hat{D}_l$. Constraint (17) calculates travel time from each origin to the destination central hub and then the result is inserted into its $\hat{D}_l$.

Constraint (18) ensures that the travel time of traffic at each origin arrives at the destination at time β or under. We presented a comprehensive output of the mathematical model with an example that is exhibited in Figures 2 and 3. The node set in Figure 2 is used as

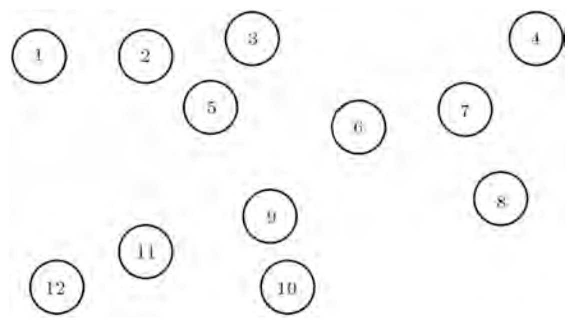

Figure 2. The node set I.

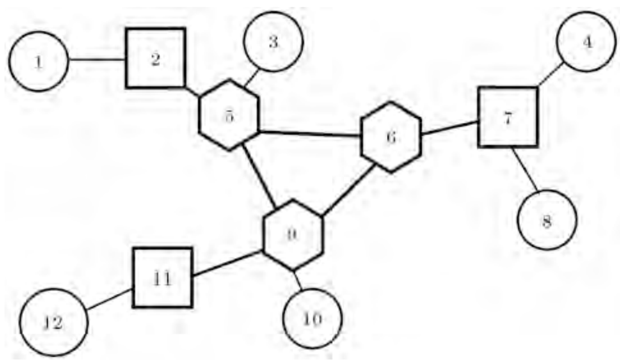

Figure 3. The resulting network of a solution.

Figure 4. The assignment state of m_1 and m_2 nodes where traffic of m_2 is greater than the traffic of m_1.

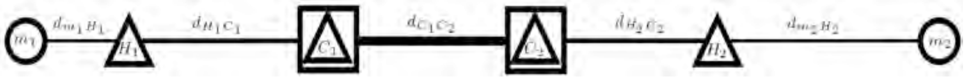

Figure 5. The assignment state of m_1 and m_2 nodes where traffic of m_1 is greater than traffic of m_2.

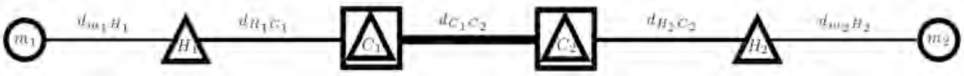

Figure 6. The assignment state of m_1 and m_2 nodes where traffic of m_2 equals traffic of m_1.

an input model and the resulting network of the model is presented in Figure 3. According to this solution, $x_{225} = x_{555} = x_{666} = x_{776} = x_{999} = x_{11119} = 1$, so, the nodes of 2, 5, 6, 7, 9 and 11 are chosen as hub nodes and also $x_{555} = x_{666} = x_{999} = 1$; therefore, nodes of 5, 6, and 9 are selected as central hub nodes. The below notations are the time bound and covering constraints for the route between node 1 and node 8. The time bound constraints include $(t_{12} + \alpha_H t_{25})x_{125} \leq \hat{D}_5$, $\hat{D}_5 + \alpha_C t_{56} x_{555} \leq D_6$ and $D_6 + (\alpha_H t_{67} + t_{78})x_{876} \leq \beta$. The covering constraints are $d_{12}x_{125} \leq r_H$, $d_{25}x_{225} \leq r_C$, $d_{76}x_{776} \leq r_C$ and $d_{87}x_{876} \leq r_H$. On the other hand, the notations of flow balance Constraint (9) for nodes $i = 1$, $j = 2$ and $l = 5$ are $(f_{12}+f_{21})(x_{125}-x_{225})+(f_{12}+f_{21})x_{125}+(f_{13}+f_{31})x_{125}+(f_{14}+f_{41})x_{125}+\cdots+(f_{112}+f_{121})x_{125} \leq y^1_{125}$. Constraint (10) is another flow balance, so we selected the nodes of $i = 1$ and $l = 5$ as an example. The constraint is equal to $(z^1_{56} + z^1_{95}) - (z^1_{65} + z^1_{95}) = (f_{12} + f_{13} + f_{14} + \cdots + f_{112})x_{125} - f_{12}x_{255} - f_{13}x_{355} - f_{15}x_{555}$. Also, this paper attempts to improve the hierarchical network structure which is investigated in Yaman [15]. Her output has some problems which lead to a one-way orientation, and create a special focus in one location. The present paper attempts to tackle the shortcomings of previous work using a transparent language, with perceived changes and a correct explanation of hierarchical structure. In addition to reduction in costs, which is investigated also in Yaman [15], we consider the balanced structure of the network, a reduction in the time of solution, an equal distribution of services, and promptness in providing services at all levels, by a change in the previous structure. Please consider the following logical reasoning;

Suppose points m_1 and m_2 as demand centers (or third level), H_1 and H_2 as second level service centers (or hub) and points C_1 and C_2 as first level service centers (or central hubs), which are shown by a circle, a triangle, and a square of the triangle, respectively. Also, f_{ij} is considered as traffic, d_{ij} the distance and t_{ij} the travel time between two locations, i and j, which is $d_{ij} = t_{ij}$ as the solution structure in Yaman [15] in the literature, and, according to the above three conditions, 1, 2 and 3 are plotted.

In these figures, for a simpler description and preventing the expression of complex conditions, a particular state which will cover another scenario is studied, i.e. $d_{c_1C_2}, d_{H_2C_2}$ and $d_{H_1}C_1$ are considered constant in all three states. Now consider Constraints (16) to (19) (i.e. time bound constraints). These constraints regard Figures 4, 5 and 6 as similar ones, because the time of providing services at all three states is equal, and they have the condition of being smaller than β, unless exchange degrees, m_1 and m_2, are different to other connections. In this case, three following states will occur, respectively:

1. If $\sum_{j\in m_2,H_1,H_2,C_1,C_2} f_{m_1j} < \sum_{j\in m_1,H_1,H_2,C_1,C_2} f_{m_2j}$, without considering the fair distribution of services, the model chooses Figure 4 in order to reduce routing costs, because the total cost of the objective function is obtained by summation of distance multiplied by the traffic.
2. If $\sum_{j\in m_2,H_1,H_2,C_1,C_2} f_{m_1j} > \sum_{j\in m_1,H_1,H_2,C_1,C_2} f_{m_2j}$ the model, as the above, chooses Figure 3 to reduce routing costs.
3. If $\sum_{j\in m_2,H_1,H_2,C_1,C_2} f_{m_1j} = \sum_{j\in m_1,H_1,H_2,C_1,C_2} f_{m_2j}$ the model will be apathetic in the choosing of figures, because routing costs in all three cases are the same.

As mentioned earlier, hierarchical problems, such as health and treatment or security forces, etc., have considered an equal distribution of services within the area of their goals and strategic values. Even the contemporary world provides various services in accordance with the fair distribution of services to draw attention and create a competitive advantage with other companies.

Thus, the model descriptions of Yaman [15] have a major weakness in the third case, which state we have controlled by expressing covering Constraints (13)-(15). We are not allowed to exceed authorized and justifiable limits, r_H, r_C, at the first and second servicing levels, and the demand nodes receive the service of different levels in a fair condition.

But, in cases 1 and 2, as mentioned in the computational results section, in return for an increase in Constraints (13)-(15), the total costs do not make many changes, and services are insignificant in return for creating fair distribution conditions.

According to the above, our model has considered the subject importance of fairness in providing services, which is significantly important in the area of servicing; so that today, in addition to the general time limit (i.e. β) for providing various services, the time for providing primary care services or low level services (here, the second level ones) has a certain individual limit.

Figures 7 and 8 show feasible solutions with a structure of classification created by our allocation pattern for the second and first service levels, respectively. A heuristic method for computing r_H and r_C is introduced in the next section.

3. Proper parameters determination

A global optimal solution of the NP-complete problems is challenging in realistic sized cases. Similarly, for the problem discussed in this paper, it is difficult to determine a global solution. It can be seen that the problem is solved in a better time with the time bound constraints. Also, there are other constraints in order to reach a global solution in the minimal amount of time. Hence, in this paper, we have provided an innovative method to determine the proper values of parameters r_H and r_C. So, we classify the total solution space, and modify the assignment procedure of each non-hub node and each hub node to the next facilities at their top levels. We specify that each assignment occurs between two nodes when they are close to each other. The specific closeness criterion is equal to the least distance between two nodes. The specific closeness criteria are characterized by r_H and r_C for second and first level assignments, respectively. So, in the process of solving the model, fewer solutions are surveyed and getting stuck at a local optimum solution is avoided. Therefore, the global solution is obtained more rapidly. We used a distance table to determine the radii. The columns of the distance table show the distances of a city to other cities. Initially, we obtained minimum distances in any column (without zero); they are the distance of cities close to each other. Also, we chose a maximum distance among the minimum distances of columns in the distance table for r_H. This distance is the best value for the covering radius of the second level, because if each distance is selected as being less, the model will be infeasible, and if a distance is selected further than that, the model will be farther from its ideal situation. Logically, the covering radiusof first level is bigger than that of $r_H (r_H \leq r_C)$, so, we will need a dominant covering radius that is not very large, nor very small.

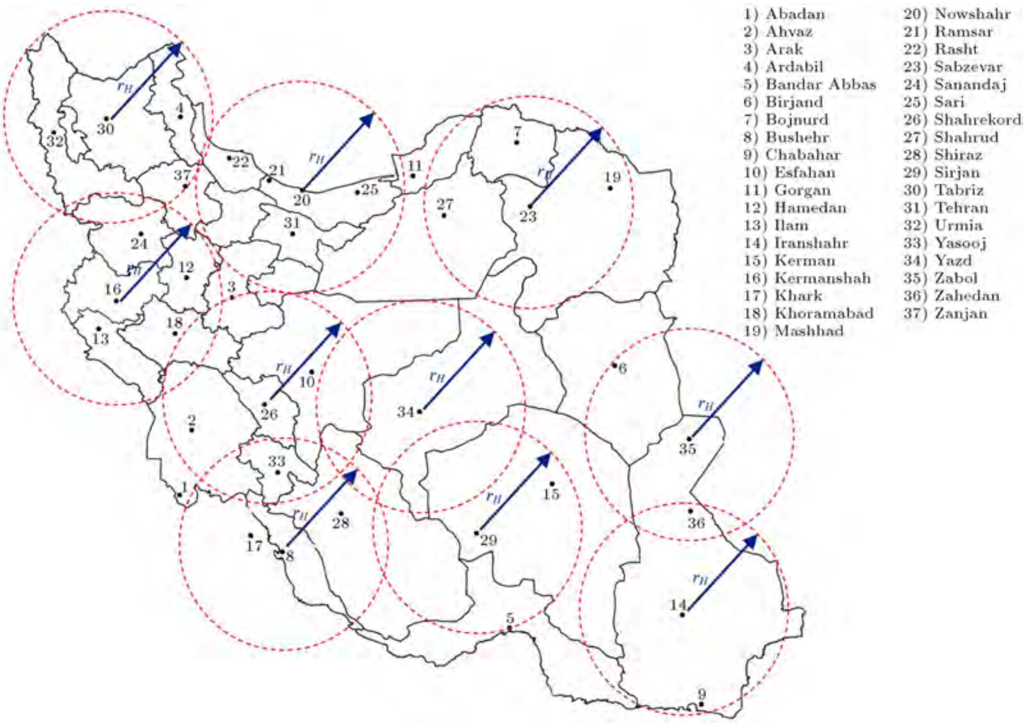

Figure 7. Pattern classification for the second level of services.

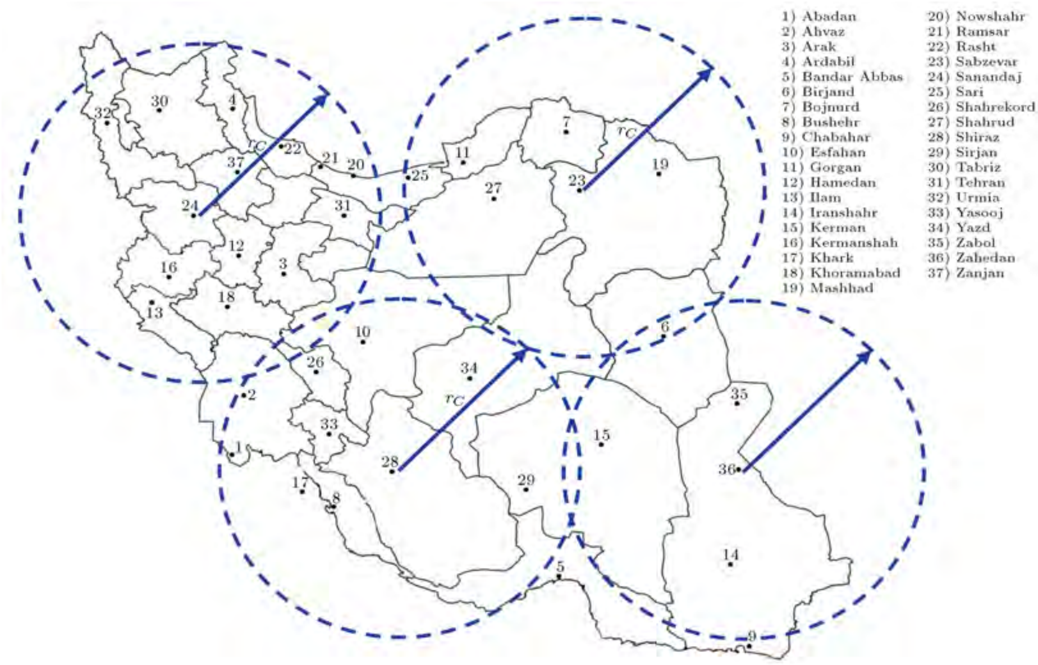

Figure 8. Pattern classification for the first level of services.

Table 1. The heuristic method for determining the covering radiuses on distance table.

Row	Descriptions
1	max{minimum numbers into the columns (regardless of zero) of distance table} $= r_H$
2	min{maximum numbers into columns of distance table} $= r_C$

Hence, we preferred a minimum distance among the maximum distances of columns in the distance table for r_C. The maximum distances of columns denote two cities the greatest distance from each other, and the minimum distance between them gives a dominant covering radius. However, we decreased and limited the space of the solution with a heuristic method. A procedure for determining radii is depicted in Table 1. We computed the r_H and r_C for the model. As a result, we take $r_H = 695$ and $r_C = 1506$ for the CAB data, $r_H = 377$ and $r_C = 918$ for the Turkish network data, and $r_H = 479$ and $r_C = 1390$ for the IAD data. The next section covers the computational analysis.

4. Computational Analysis

In the computational analysis, we applied the CAB data set, the Turkish network data set and the Iranian Aviation Dataset (IAD). The CAB data set is introduced by O'Kelly [2]. This data set contains flows and distances and is based on the airline passenger connections between 25 US cities in 1970 evaluated by the Civil Aeronautics Board (CAB) [1]. The global solutions are scaled through dividing by the total flow. We regarded all nodes as potential nodes for hubs and central hubs, i.e. $I = H = C$, similar to the one used in Yaman [15], in order to compare it with our model.

The Turkish network data set contains the data of distances, travel times, flows and fixed link costs between 81 cities in Turkey and fixed hub costs for these cities. The flow data is offered by Çetiner, Spil and Süral, and the fixed link costs are suggested by Alumur, Kara and Karaşan. The distances, travel times and fixed hub costs are introduced by Tan and Kara and, finally, this data set is presented by Tan and Kara [12]. We applied Yaman's style in order to create a subset of the Turkish network data set, as we wanted to evaluate the effect of some parameters on the output of the model and investigate the procedure of computation times. To do this, we separated over 30 populated cities of the Turkish network data as the demand set I. There are the 21 populous cities in Turkey which belong to set I, from which we selected a set of 17 cities which had a cargo company. We chose them as a potential set of hubs (i.e., set of H) and determined a set of 10 cities with largest populations as a potential set of central hubs (i.e., set of C). Finally, we solved the model on all nodes in the Turkish network. The set of references were symbolized as I. We considered all 21 populous cities in Turkey as a potential set of hubs and located two central hubs in Ankara and Istanbul.

The IAD was presented by Karimi and Bashiri [17]. This data set includes the distances, costs, weights, capacities and fixed charge hub costs based on the hub airport location between 37 Iranian cities. We regarded all the nodes as the demand set I, and we separated the 24 populous cities of IAD as a potential set of hubs (i.e., set of H) similar to Yaman [15] on the Turkish network data set. There are 16 populous cities in set H that have more than 100 units of capacity, so, we selected them as a potential set of central hubs (i.e., set of C). We presented the computational experience of the IAD data in Section 4.5.

4.1. Effect of changes in some parameters on the total cost

In the first stage, we investigated the effects of time bound constraints and the number of central hubs on the routing costs. We solved the problem on the CAB data with $n = 25$ and $P_H = 5$, and the Turkish network data with $n = 30$ and $P_H = 7$, where $\alpha_H = 0.9$, $\alpha_C = 0.8$, $\hat{\alpha}_H = 0.9$, $\hat{\alpha}_C = \{0.9, 0.8\}$ and $P_C \in \{1, \cdots, P_H\}$. Similar to the application of each data set in Tan and Kara [12] (2007) and Yaman [15] (2009), we obtained the smallest β for the problem, so that it becomes feasible when $\alpha_H = \hat{\alpha}_H = 0.9$ and $\alpha_C = \hat{\alpha}_C = 0.8$. We calculated the average of the smallest amounts of β for various values of P_C. In order to have a strong time bound, we presented β amounts which are 120 and 240 times more than the average values. Therefore, we obtained $\beta \in \{2880, 3000, \infty\}$ for the CAB data and $\beta \in \{1740, 1860, \infty\}$ for the Turkish network data. For infeasible situations, we did not have any report. When great P_C was enlarged, the results were presented for both data sets, and the total cost had a decreasing trend in all cases. We assumed a basic state of cost with $\beta = r_H = r_C = \infty$ to see the procedure of total cost with more clarification. Then, for each state, we computed the percent of increasing cost with the well-known formula of:

$$\%\text{increasing cost} = \frac{\text{New cost} - \text{Basic cost}}{\text{Basic cost}} \times 100.$$

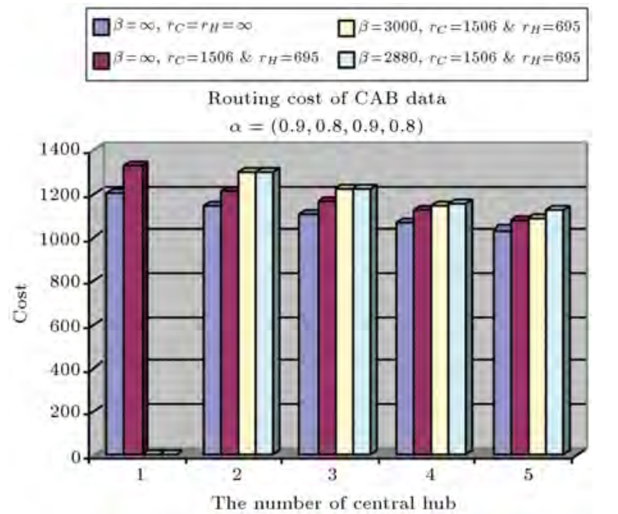

Figure 9. A chart of total routing cost for the CBA data with $n = 25$, $P_H = 5$ and $\hat{\alpha}_C = 0.8$.

The computation is presented in Table 2 and Figure 9 for the CAB data and Table 3 and Figure 10 for the Turkish network data.

For the CAB data, the total cost is increased on average by 6.27% for both $\hat{\alpha}_C = 0.8$ and $\hat{\alpha}_C = 0.9$, while covering radii are imposed on the model. The problem is infeasible while $P_C = 1$ and imposes time bound constraints on the model. Also, it is infeasible with two central hubs, while $\beta = 2880$ and $\hat{\alpha}_C = 0.9$. In most cases, when $P_C = 2$, the problem becomes feasible with the time bound constraints; however the total routing cost increases by 13.29% when it has a feasible solution. As revealed in Table 2, by decrease in value of β, the problem needs more central hubs to become a feasible solution. Generally, the total cost increases by the covering and the time bound

Table 2. Total routing cost and cost increasing percent for the CAB data with $n = 25$, $P_H = 5$ and $\beta = \{2880, 3000, \infty\}$.

$(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C)$	P_C	$(r_H, r_C) = (\infty, \infty)$	$(r_H, r_C) = (695, 1506)$		
		$\beta = \infty$	$\beta = \infty$	$\beta = 3000$	$\beta = 2880$
(0.9, 0.8, 0.9, 0.8)	1	1200.13	10.67	-	-
	2	1146.79	6.04	13.29	13.29
	3	1108.35	5.23	9.92	9.92
	4	1065.06	5.54	7.53	8.58
	5	1034.10	3.87	4.73	8.69
(0.9, 0.8, 0.9, 0.9)	1	1200.13	10.67	-	-
	2	1146.79	6.04	13.29	-
	3	1108.35	5.23	9.92	9.92
	4	1065.06	5.54	8.58	11.33
	5	1034.10	3.87	8.69	8.69

Table 3. Total routing cost and cost increasing percent for the Turkish network data with $n = 30$, $P_H = 7$ and $\beta = 1740, 1860$.

$(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C)$	P_C	$(r_H, r_C) = (\infty, \infty)$	$(r_H, r_C) = (377, 918)$		
		$\beta = \infty$	$\beta = \infty$	$\beta = 1860$	$\beta = 1740$
(0.9, 0.8, 0.9, 0.8)	1	29032861012	2.04	35.23	38.57
	2	28359343683	1.61	3.07	15.49
	3	27091874861	3.69	4.44	7.01
	4	26122191262	2.54	5.45	6.40
	5	25234752294	2.92	3.78	7.13
	6	2542462733	3.11	3.11	4.18
	7	23992650009	2.98	2.98	3.21
(0.9, 0.8, 0.9, 0.9)	1	29032861012	2.04	35.23	38.57
	2	28359343683	1.61	5.14	15.55
	3	27091874861	3.69	5.29	8.28
	4	26122191262	2.54	6.40	7.15
	5	25234752294	2.92	7.13	7.91
	6	24542462733	3.11	4.80	9.01
	7	23992650009	2.98	3.56	7.75

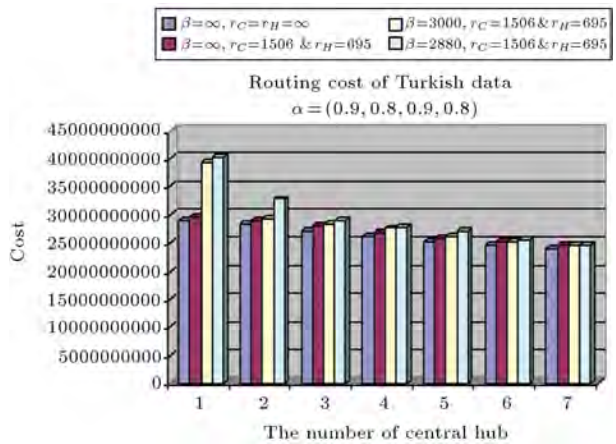

Figure 10. A chart of total routing cost for the Turkish data with $n = 30$, $P_H = 7$ and $\hat{\alpha}_C = 0.8$.

constraints, and the increased percentage of the total cost is equal to, on average, 3.69%, when $\hat{\alpha}_C = 0.8$, and 4.95% when $\hat{\alpha}_C = 0.9$ from $\beta = \infty$ to $\beta = 2880$. But, Figure 9 shows the growth rate of the total cost which is very negligible compared to the equal covering situation.

For the Turkish network data, the problem is feasible for all cases. Whenever the time bound constraint is imposed on the problem, the total cost increases on average by 5.59% when $\hat{\alpha}_C = 0.8$ and 6.95% when $\hat{\alpha}_C = 0.9$ from $\beta = \infty$ to $\beta = 1860$. The growth rate of the total cost increases with imposing the β, when P_C is equal to 1. Table 2 shows that while covering radii are imposed on the problem, the total cost increases, on average, 2.70% for all instances. Figure 10 displays the increasing procedure of the total cost, but this growth rate is very negligible compared to the equal covering situation.

Finally, we infer that, by imposing the covering radii and delivering the time bound, the total cost increases in both data sets. The distances between hubs and central hubs are in the range of r_C, and the range of distances among non-hubs and hubs is r_H by covering constraints. So, if two demand nodes and their hubs are assigned to different hubs and central hubs, the distances between origin-destination pairs include two r_H and one r_C. If the output of the model moves from a complete network to a star network by decreasing central hubs, the r_C does not come to the distances between each origin-destination pair.

4.2. *Effect of change in some parameters on the locations of central hub*

In this subsection, we report the locations of central hubs and hubs in the optimum solutions for the CAB data with $n = 25$, $P_H = 5$, $P_C = 2$, 3, 4, the Turkish network data with $n = 30$, $P_H = 7$, $P_C = 3$, 4, 5 and in both of them with different amounts of time bound (i.e., β), covering radii (i.e., r_H and r_C) and discount coefficients (i.e. α_H, α_C, $\hat{\alpha}_H$ and $\hat{\alpha}_C$).

These reports are presented in Tables 4 and 5 for CAB data and Turkish network data, respectively. Table 4 shows that when $\beta = \infty$ and $(r_H, r_C) = \infty$ cities 4 and 20 are selected as central hubs in 100% and 87% of cases, respectively. But when $\beta = \infty$ and $(r_H, r_c) = (695, 1506)$, these percentages are decreased by 27% and 34%. Cities 11 and 12 are selected as central hubs in all instances with $P_C = 2$, $\beta = 3000$ and $(r_H, r_C) = \infty$, cities 4 and 20 are selected as central

Table 4. The locations of central hubs and hubs for the CAB data.

$(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C)$	β	(r_H, r_C)	P_C	Central hubs	Hubs
(1, 1, 1, 1)	∞	∞	2	4, 20	4, 8, 17, 20, 21
			3	4, 20, 21	4, 8, 17, 20, 21
			4	4, 7, 8, 20	4, 7, 8, 17, 20
	∞	(695, 1506)	2	11, 20	11, 12, 20, 23, 24
			3	1, 11, 20	1, 11, 12, 20, 23
			4	1, 8, 20, 21	1, 8, 20, 21, 22
	3000		3	19, 21, 23	19, 20, 21, 23, 24
			4	1, 19, 21, 23	1, 19, 20, 21, 23
(0.9, 0.9, 0.9, 0.9)	∞	∞	2	4, 20	4, 8, 17, 20, 21
			3	4, 20, 21	4, 12, 17, 20, 21
			4	1, 4, 11, 18	1, 4, 11, 12, 18
	∞	(695, 1506)	2	11, 20	11, 12, 20, 23, 24
			3	5, 7, 22	5, 7, 18, 22, 24
			4	4, 7, 20, 22	4, 7, 20, 22, 24
	3000		2	11, 12	11, 12, 20, 23, 24
			3	19, 21, 23	19, 20, 21, 23, 24
			4	1, 11, 12, 20	1, 11, 12, 20, 23
	2880		3	19, 21, 23	19, 20, 21, 23, 24
			4	5, 16, 19, 23	5, 16, 18, 19, 23
(0.8, 0.8, 0.8, 0.8)	∞	∞	2	4, 20	4, 12, 17, 20, 24
			3	4, 20, 21	4, 12, 17, 20, 21
			4	1, 4, 11, 18	1, 4, 11, 12, 18
	∞	(695, 1506)	2	5, 7	5, 7, 18, 22, 24
			3	5, 7, 22	5, 7, 18, 22, 24
			4	2, 4, 7, 22	2, 4, 7, 14, 22
	3000		2	11, 12	11, 12, 20, 23, 24
			3	1, 11, 20	1, 11, 12, 20, 23
			4	4, 8, 16, 18	4, 8, 16, 18, 22
	2880		2	19, 21	19, 20, 21, 23, 24
			3	5, 16, 19	5, 16, 18, 19, 23
			4	4, 16, 18, 19	4, 16, 18, 19, 23
(0.9, 0.8, 0.9, 0.8)	∞	∞	2	4, 20	4, 8, 17, 20, 21
			3	4, 12, 20	4, 12, 17, 20, 21
			4	4, 7, 12, 20	4, 7, 12, 17, 20
	∞	(695, 1506)	2	5, 22	1, 5, 7, 18, 22
			3	5, 7, 22	5, 7, 18, 22, 24
			4	2, 4, 7, 22	2, 4, 7, 14, 22
	3000		2	11, 12	11, 12, 20, 23, 24
			3	19, 21, 23	19, 20, 21, 23, 24
			4	1, 19, 21, 25	1, 19, 21, 23, 25
	2880		2	11, 12	11, 12, 20, 23, 24
			3	19, 21, 23	19, 20, 21, 23, 24
			4	1, 11, 12, 20	1, 11, 12, 20, 23

Table 4. Continued.

$(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C)$	β	(r_H, r_C)	P_C	Central hubs	Hubs
			2	4, 20	4, 8, 17, 20, 21
	∞	∞	3	4, 12, 20	4, 12, 17, 20, 21
			4	4, 7, 12, 20	4, 7, 12, 17, 20
			2	5, 22	1, 5, 7, 18, 22
	∞		3	5, 7, 22	5, 7, 18, 22, 24
(0.9, 0.8, 0.9, 0.9)			4	2, 4, 7, 22	2, 4, 7, 14, 22
	3000	(695, 1506)	2	11, 12	11, 12, 20, 23, 24
			3	19, 21, 23	19, 20, 21, 23, 24
			4	1, 11, 12, 20	1, 11, 12, 20, 23
	2880		3	19, 21, 23	19, 20, 21, 23, 24
			4	5, 16, 19, 23	5, 16, 18, 19, 23

hubs in 100% and 87% of cases, respectively. But, when $\beta = \infty$ and $(r_H, r_C) = (695, 1506)$. We observed that when the time bound does not exist, $P_C = 3$ and $(r_H, r_C) = (695, 1506)$, cities 5, 7 and 22 are selected as central hubs. The cities 19, 21 and 23 appear as central hubs in more instances when the time bound and the covering constraints are applied to the problem and when $P_C = 3$. Generally, the covering constraints cause many changes in the locations of central hubs and the change in parameters cannot set the exact procedure. We observed that by imposing covering constraints and simultaneously decreasing the procedure of values of discount coefficients, the locations of central hubs in the optimum solutions are significantly affected. For example, when $\beta = \infty$, and $P_C = 3$, cities 4, 20, and 21 are central hubs in a situation where all discount coefficients are 1 and $(r_H, r_C) = \infty$, and cities 5, 7, and 22 are central hubs in a situation where all discount coefficients are 0.8 and $(r_H, r_C) = (695, 1506)$. Not only do all central hubs transfer from one place to another, but also they are located far from cities 4, 20 and 21.

Table 5 shows that when $\alpha_C = \alpha_H$, $\beta = \infty$ and $(r_H, r_C) = \infty$, cities 6, 16 and 34 are chosen as central hubs in all cases, but these cities will be changed to cities 6, 34 and 35 in a situation when the covering constraints are considered in the problem. Also, city 55 is selected as the central hub in most cases where there exist a time bound and covering constraints. City 6 is chosen as the central hub in all situations except the state of $P_C = 3$, $\beta = 1860$ and $\alpha_C = \alpha_H = 1$. A memorable city is city 55 because it appears as a hub in all instances. Generally, we observed that imposing the time bound and covering constraints and further restricting the space of the solution with decreasing β lead to some changes in cities and a transfer of central hub locations to other cities.

Figure 11 shows the frequency percentage of selecting each potential city as a central hub. Considering this diagram, cities 6, 1 and 34 have been selected as central hubs in most items over total runs. Besides, city 7 appears as a top level facility in 2% of total instances.

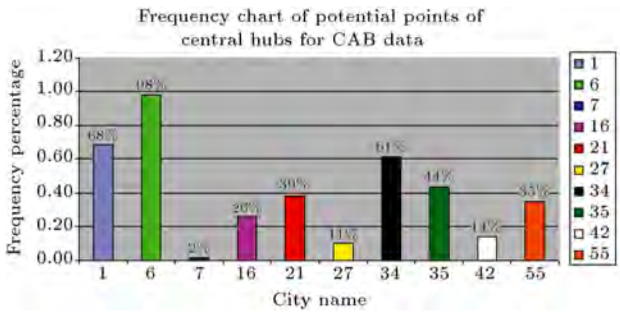

Figure 11. Frequency diagram of potential 10 cities of central hubs for Turkish data.

Also, in this data set, we noticed that covering restrictions are effective on locations of the top level facilities. For example, when $\beta = \infty$, $P_C = 3$ and $\alpha_H = \alpha_C = 1$, cities 6, 16 and 34 have been selected as central hubs for $(r_H, r_C) = \infty$, but the cities of 1, 6 and 21 have been selected as central hubs for $(r_H, r_C) = (377, 918)$, so, two facilities of three central hubs have been transferred. Cities 6 and 21 are selected as top level facilities in all instances by $\beta = 1740$. This shows that when the time bound restrictions are strict, these cities have the best covering. Imposing, simultaneously, the time bound and covering constraints, leads to convergence of the locations of central hubs.

4.3. *Effect of change in some parameters on computational time*

In this subsection, we studied the effect of the parameters of the problem on computation times. We solved our model using GAMS 21.7 and CPLEX 11.0.0 on a personal computer with a 2.10GHz Intel Core 2 Duo processor and 3GB of RAM operating under the system Windows 7 Ultimate.

Computational times have not been reported

Table 5. The locations of central hubs and hubs for the Turkish netword data.

$(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C)$	β	(r_H, r_C)	P_C	Central hubs	Hubs
			3	6, 16, 34	1, 6, 16, 34, 35, 38, 55
	∞	∞	4	6, 16, 34, 35	1, 6, 16, 34, 35, 38, 55
			5	6, 7, 16, 34, 35	6, 7, 16, 34, 35, 38, 55
			3	1, 6, 21	1, 3, 6, 21, 34, 38, 55
(1, 1, 1, 1)	∞		4	1, 6, 16, 34	1, 3, 6, 16, 21, 34, 55
			5	1, 6, 34, 35, 42	1, 6, 21, 34, 35, 42, 55
		(377, 918)	3	27, 42, 55	3, 21, 27, 34, 38, 42, 55
	1860		4	6, 27, 42, 55	3, 6, 21, 27, 34, 42, 55
			5	1, 6, 27, 42, 55	1, 3, 6, 21, 27, 42, 55
			3	6, 16, 34	1, 6, 16, 34, 35, 44, 55
	∞	∞	4	6, 16, 34, 35	1, 6, 16, 34, 35, 44, 55
			5	1, 6, 16, 34, 35	1, 6, 16, 27, 34, 35, 55
			3	6, 16, 34	1, 3, 6, 16, 21, 34, 55
	∞		4	1, 6, 34, 35	1, 6, 7, 21, 34, 35, 55
(0.9, 0.9, 0.9, 0.9)			5	1, 6, 34, 35, 42	1, 6, 21, 34, 35, 42, 55
			3	1, 6, 21	1, 3, 6, 21, 34, 55, 65
	1860	(377, 918)	4	1, 6, 21, 55	1, 3, 6, 21, 27, 34, 55
			5	1, 6, 21, 27, 55	1, 3, 6, 21, 27, 34, 55
			3	6, 21, 55	1, 3, 6, 21, 34, 55, 65
	1740		4	1, 6, 21, 55	1, 3, 6, 21, 34, 55, 65
			5	1, 6, 21, 34, 55	1, 3, 6, 21, 34, 55, 65
			3	6, 16, 34	1, 6, 16, 34, 35, 44, 55
	∞	∞	4	6, 16, 34, 35	1, 6, 16, 34, 35, 44, 55
			5	1, 6, 16, 34, 35	1, 6, 16, 27, 34, 35, 55
			3	6, 34, 35	1, 6, 7, 21, 34, 35, 55
	∞		4	1, 6, 34, 35	1, 6, 7, 21, 34, 35, 55
(0.8, 0.8, 0.8, 0.8)			5	1, 6, 34, 35, 42	1, 6, 21, 34, 35, 42, 55
			3	1, 6, 21	1, 3, 6, 21, 27, 34, 55
	1860	(377, 918)	4	1, 6, 27, 55	1, 3, 6, 21, 27, 34, 55
			5	1, 6, 34, 35, 55	1, 6, 7, 21, 34, 35, 55
			3	1, 6, 21	1, 3, 6, 21, 34, 55, 65
	1740		4	1, 6, 21, 55	1, 3, 6, 21, 27, 34, 55
			5	1, 6, 21, 27, 55	1, 3, 6, 21, 27, 34, 55
			3	6, 16, 34	1, 6, 16, 34, 35, 44, 55
	∞	∞	4	1, 6, 34, 35	1, 6, 7, 27, 34, 35, 55
			5	1, 6, 16, 34, 35	1, 6, 16, 27, 34, 35, 55
			3	6, 34, 35	1, 6, 7, 21, 34, 35, 55
	∞		4	1, 6, 34, 35	1, 6, 7, 21, 34, 35, 55
(0.9, 0.8, 0.9, 0.8)			5	1, 6, 34, 35, 42	1, 6, 21, 34, 35, 42, 55
			3	1, 6, 21	1, 3, 6, 21, 34, 38, 55
	1860	(377, 918)	4	6, 21, 34, 35	1, 6, 7, 21, 34, 35, 55
			5	1, 6, 34, 35, 55	1, 6, 7, 21, 34, 35, 55
			3	1, 6, 21	1, 3, 6, 21, 34, 55, 61
	1740		4	1, 6, 21, 55	1, 3, 6, 21, 27, 34, 55
			5	1, 6, 21, 34, 55	1, 3, 6, 21, 27, 34, 55

Table 5. Continued.

$(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C)$	β	(r_H, r_C)	P_C	Central hubs	Hubs
(0.9, 0.8, 0.9, 0.9)	∞	∞	3	6, 16, 34	1, 6, 34, 35, 44, 55
			4	1, 6, 34, 35	1, 6, 7, 27, 34, 35, 55
			5	1, 6, 16, 34, 35	1, 6, 16, 27, 34, 35, 55
	∞	(377, 918)	3	6, 34, 35	1, 6, 7, 21, 34, 35, 55
			4	1, 6, 34, 35	1, 6, 7, 21, 34, 35, 55
			5	1, 6, 34, 35, 42	1, 6, 21, 34, 35, 42, 55
	1860		3	1, 6, 21	1, 3, 6, 21, 34, 55, 65
			4	1, 6, 21, 55	1, 3, 6, 21, 27, 34, 55
			5	1, 6, 21, 34, 55	1, 3, 6, 21, 27, 34, 55
	1740		3	6, 21, 55	1, 3, 6, 21, 34, 55, 65
			4	1, 6, 21, 55	1, 3, 6, 21, 34, 55, 65
			5	1, 6, 21, 34, 55	1, 3, 6, 21, 34, 55, 65

Table 6. The CPU time for CAB data with $n = 25$ and $P_H = 5$.

$(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C)$	P_C	$(r_H, r_C) = (\infty, \infty)$	$(r_H, r_C) = (695, 1506)$		
		$\beta = \infty$	$\beta = \infty$	$\beta = 3000$	$\beta = 2880$
(1, 1, 1, 1)	1	477.38	17.52	-	-
	2	19355.87	571.99	-	-
	3	12755.77	3005.96	158.50	-
	4	26657.54	2639.64	1141.48	-
	5	288.00	33.55	29.98	-
(0.9, 0.9, 0.9, 0.9)	1	685.46	18.85	-	-
	2	35826.54	868.42	254.59	-
	3	20098.73	2282.89	755.45	320.16
	4	70216.72	3788.04	1293.89	794.01
	5	253.02	90.86	19.83	17.52
(0.8, 0.8, 0.8, 0.8)	1	1642.01	21.50	-	-
	2	12126.83	698.31	3306.31	1568.32
	3	17357.99	2365.05	3301.53	4696.96
	4	34253.30	2950.95	4426.40	2360.16
	5	150.06	25.50	44.47	35.28
(0.9, 0.8, 0.9, 0.8)	1	635.74	16.11	-	-
	2	61561.77	621.98	1845.51	32.64
	3	51699.33	3026.62	3665.95	505.02
	4	19831.40	3022.81	2627.58	373.06
	5	141.30	24.42	46.18	21.25
(0.9, 0.8, 0.9, 0.9)	1	635.74	13.56	-	-
	2	61561.77	610.42	204.13	-
	3	51699.33	2812.99	618.65	247.27
	4	19831.40	2286.67	488.73	1005.40
	5	141.30	26.46	29.69	15.07

for infeasible cases. The computational study for CAB data was performed with $n = 25$, $P_H = 5$, $\beta = \{2880, 3000, \infty\}$ and different values of P_C and discount coefficients. The results for the CAB data are depicted in Table 6 and Figure 10. We reported the computational times in Yaman [15] for the basic state of CAB data (i.e., $\beta = \infty$ and $(r_H, r_C) = \infty$) in the first column of the left side (Table 6), in order for readers to observe the differences between the proposed model with that of previous studies. Table 6 shows that the different cases of the problem are solved, on average, 91.62% faster using covering constraints rather than non-covering ones, while the computational times of the problem are increased in all cases, except when

$P_C = 4$ and $\beta = 3000$. All cases with $P_H = P_C$ are the easiest.

Whereas the covering restrictions improve CPU times in all cases, imposing the time bound constraints do not improve the computation times; for example, when $\beta = 3000$ and all reduction factors are 0.8 for all P_C, the computational times get worse in all instances. When $\beta = 3000$, $\alpha_H = 0.9$ and $\alpha_C = 0.8$, the CPU time is improved only for $P_C = 4$, but it increases for other P_C. Also, the CPU times increase in $\alpha_H = \alpha_C = 0.8$ with $P_C = 2$ and 3 when the upper bound for the time bound is equal to 2880. The computational study for the Turkish network data was performed with $n = 30$, $P_H = 7$, $\beta = \{1740, 1860, \infty\}$ and different values of P_C and discount coefficients. The results are depicted in Table 7 and Figure 12. Table 7 shows that all the problems are solved, on average, 86.17% faster

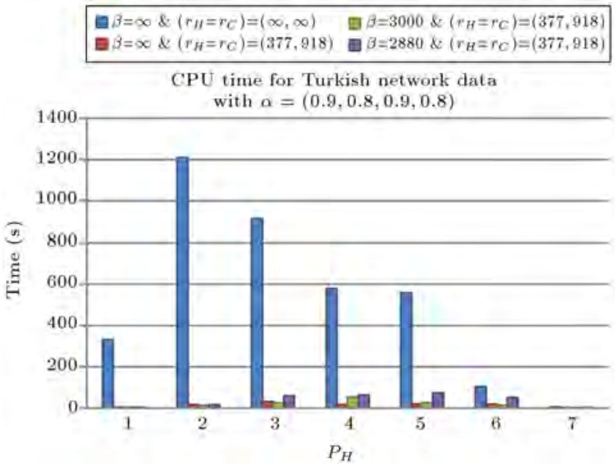

Figure 12. A chart of procedure of CPU time for the Turkish network data with $n = 30$ and $P_H = 7$.

Table 7. The CPU time for Turkish network data with $n = 30$, $P_H = 7$.

$(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C)$	P_C	$(r_H, r_C) = \infty$	$(r_H, r_C) = (377, 918)$		
		$\beta = \infty$	$\beta = \infty$	$\beta = 1860$	$\beta = 1740$
(1, 1, 1, 1)	1	317.53	5.15	-	-
	2	4044.978	23.60	6.41	-
	3	1593.769	47.67	13.33	-
	4	1129.881	62.36	16.73	-
	5	903.86	47.37	21.85	-
	6	518.87	34.92	13.79	-
	7	11.23	3.42	2.36	-
(0.9, 0.9, 0.9, 0.9)	1	175.51	4.53	4.54	4.78
	2	2214.94	35.29	12.49	6.75
	3	837.67	37.29	19.38	14.22
	4	700.64	43.39	26.34	16.16
	5	603.36	28.09	72.89	18.11
	6	325.08	35.25	23.21	13.88
	7	4.89	2.82	2.47	2.30
(0.8, 0.8, 0.8, 0.8)	1	165.52	3.39	5.46	6.07
	2	2520.53	36.75	15.83	27.55
	3	4128.00	46.04	42.94	66.57
	4	775.40	33.22	108.21	75.64
	5	447.10	18.80	24.24	124.43
	6	304.29	21.21	15.22	21.42
	7	4.06	3.16	2.66	2.47
(0.9, 0.8, 0.9, 0.8)	1	332.61	5.09	5.19	4.35
	2	1209.82	17.11	13.90	16.12
	3	914.50	30.82	25.76	60.48
	4	579.29	17.24	53.87	63.56
	5	559.03	21.10	26.77	73.09
	6	103.33	19.69	15.28	51.77
	7	5.57	2.42	2.60	2.22
(0.9, 0.8, 0.9, 0.9)	1	282.34	4.54	4.80	4.71
	2	1356.15	18.77	10.30	6.70
	3	936.87	35.58	22.88	14.38
	4	553.84	22.43	48.24	19.37
	5	360.47	24.48	43.27	18.13
	6	26.13	22.59	18.86	14.95
	7	5.69	3.72	2.24	2.19

with covering constraints rather than the state of no covering. The computation times of the problem are decreased in all instances. Also, in this data set, all cases with $P_H = P_C$ are the easiest. In this data set, when the time bound constraints are imposed on the model, CPU time gets worse in some cases. For example, when $\beta = 1860$, the computational times get worse in all instances (except when all reduction factors are 1) for $P_C = 5$. Also, when $\beta = 1740$, the CPU times increase in $P_C = 3$, 4, 5 and 6 for $\alpha_H = \alpha_C = 0.8$ and $\alpha_H = 0.9$, $\alpha_C = 0.8$.

CPU times are depicted in Figures 12 and 13 for Turkish network data and CAB data, respectively. The figures indicate that computational times have significantly decreased by our heuristic method, which is indicative of an excellent improvement. These improvements are so effective that they overshadow the poor increasing procedure of costs depicted in Figures 9 and 10. Figure 14 shows 30 selected nodes on the map of Turkey. The potential nodes of central hubs and hubs are characterized by squares and circles, respectively.

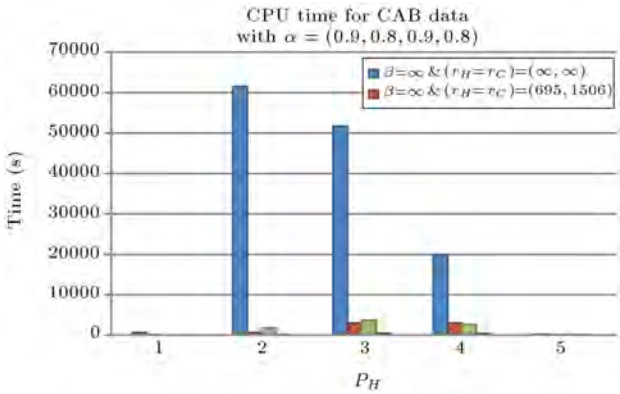

Figure 13. A chart of procedure of CPU time for the CAB data with $n = 25$ and $P_H = 5$.

Figure 14. The map for Turkish network data with $n = 30$, 17 and 10 number of potential nodes for hubs and central hubs, respectively. Adapted from Yaman (2009) [15].

4.4. *The problem with two central hubs located in Ankara and Istanbul*

We solved the problem with all nodes in the Turkish network data. We set $P_C = 2$ and located that in Ankara and Istanbul.

We computed r_C by the heuristic method, where $r_C = 1046$, but set $r_H = 477$, whose amount is 100 units higher than r_H for the set of 30 nodes, so, the problem needs fewer hubs for maximal covering. We took $\alpha_H = \hat{\alpha}_H = 0.9$, $\alpha_C = \hat{\alpha}_C = 0.8$, $\beta = \{2640, 2700, \infty\}$ and $P_H = 7$ similar to the work done by Yaman [15], as we wanted to have readers compare it with our work. These results are depicted in Table 8. We observed that the results are equal for each P_H

Table 8. The results for Turkish network with $n = 81$, $r_C = 1046$, $r_H = 477$ and two central hubs in Ankara and Istanbul.

β	P_H	Cost	Hubs	CPU time
∞	7	62778796527.80	1, 3, 6, 21, 25, 34, 55	7.293
	8	62306026267.80	1, 3, 6, 21, 25, 34, 44, 55	15.282
	9	62135009710.80	1, 3, 6, 16, 21, 25, 34, 44, 55	19.904
	10	62005078818.20	1, 3, 6, 16, 21, 25, 34, 42, 44, 55	22.483
	11	61881337592.50	1, 3, 6, 21, 25, 26, 34, 42, 44, 55, 81	16.069
	12	61777489330.20	1, 3, 6, 21, 25, 26, 34, 38, 42, 44, 55, 81	12.117
2700	7	62778796527.80	1, 3, 6, 21, 25, 34, 55	10.072
	8	62306026267.80	1, 3, 6, 21, 25, 34, 44, 55	18.781
	9	62135009710.80	1, 3, 6, 16, 21, 25, 34, 44, 55	25.905
	10	62005078818.20	1, 3, 6, 16, 21, 25, 34, 42, 44, 55	12.176
	11	61881337592.50	1, 3, 6, 21, 25, 26, 34, 42, 44, 55, 81	13.257
	12	61777489330.20	1, 3, 6, 21, 25, 26, 34, 38, 42, 44, 55, 81	12.978
2640	7	62778796527.80	1, 3, 6, 21, 25, 34, 55	9.890
	8	62306026267.80	1, 3, 6, 21, 25, 34, 44, 55	16.622
	9	62135009710.80	1, 3, 6, 16, 21, 25, 34, 44, 55	23.275
	10	62005078818.20	1, 3, 6, 16, 21, 25, 34, 42, 44, 55	17.181
	11	61881337592.50	1, 3, 6, 21, 25, 26, 34, 42, 44, 55, 81	16.188
	12	61777489330.20	1, 3, 6, 21, 25, 26, 34, 38, 42, 44, 55, 81	6.960

in different amounts of β. Table 8 shows that results have been converged by the covering radii. All CPU times occur within 26 seconds in all instances, which it is a very good result in the large scale problem. We solved the problem without covering restrictions for $P_H = 7$, whose CPU time amounted to 1609.409 seconds, total cost to 62103232960.50 and locations of hubs to 1, 3, 6, 25, 34, 44 and 55. First, we presented a map of 81 cities and the potential node sets for hubs and central hubs in Figure 15. Then, this result comes in Figure 16 for making a simpler comparison with the results by covering restrictions in Figure 17. It should be noted that hubs have been specified by black squares. Figures 16 and 17 indicate that the problem is solved in the best time and hubs are effectively dispersed by covering constraints.

4.5. Computational research on the IAD data

We solved the problem on the IAD data with $n = 37$ and $P_H = 9$, and examined the effect of time bound

Figure 15. The map for Turkish network data with $n = 81$ and 21 number of potential nodes for hubs and two central hubs located in Ankara and Istanbul. Adapted from Yaman (2009) [15].

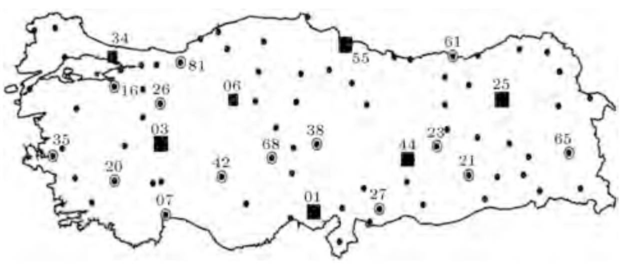

Figure 16. The hub network for $n = 81$, $\alpha_C = 0.8$, $\alpha_H = 0.9$, $P_H = 7$ and $\beta = \infty$.

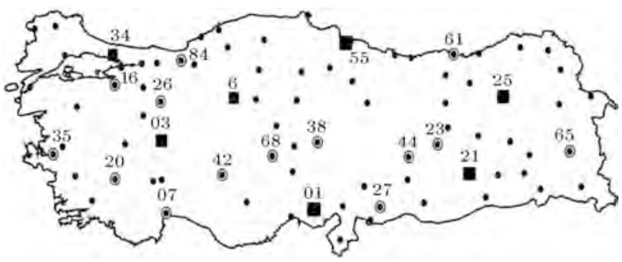

Figure 17. The hub network for $n = 81$, $\alpha_C = 0.8$, $\alpha_H = 0.9$, $P_H = 7$ and $\beta = \infty$, 2700, 2640.

constraints, the number of central hubs and covering constraints for various values of decreasing coefficients on the routing costs.

We defined the problem on the IAD data with $n = 37$ and $P_H = 9$, where α_H, α_C, $\hat{\alpha}_H$, $\hat{\alpha}_C = \{0.9, 0.8\}$ and $P_C \in \{1, \cdots, P_H\}$. Similar to the work by Tan and Kara (2007) and Yaman (2009), we calculated the amounts of β. Therefore, we defined $\beta \in \{2880, 3000, \infty\}$ for the IAD data. For an infeasible situation, we do not have any report. We reported locations of central hubs and hubs in the optimum solutions with $n = 37$, $P_H = 9$, $P_C = 1, 2, 3$ and 4 with different types of time bound (i.e., β), covering radii (i.e., r_H and r_C) and $\alpha_H = \alpha_C = \hat{\alpha}_H = \hat{\alpha}_C = \{0.8, 0.9\}$ (see Table 9).

Cities 10 and 31 are chosen as central hubs in 92% of cases, but city 31 has been changed to city 36 in 4 instances, when $P_C = 2$ and the covering constraints are considered in the problem. For example, when $P_C = 2$, $(r_H, r_C) = (479, 1390)$, $\beta = 3000$ and $(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C) = (0.9, 0.8, 0.9, 0.8)$. Also cities 2, 5, 10, 31 and 34 are selected as hub nodes for 100% of cases with the time bound and covering constraints and without them. City 19 is chosen as the hub node in all instances except when $P_C = 1$, $\beta = \infty$, $r_H = r_C = \infty$ and $\alpha_H = \hat{\alpha}_H = 0.9$. Memorable cities are 10 and 31 in Table 9, since they appear as central hub and hub nodes in more than 92% and 100% of instances, respectively. In general, we see that imposing the time bound and covering constraints and further restricting the space of solution with decreasing β lead to changes in cities and transfer of central hub locations to other cities. The computational times for infeasible cases have not been reported. The computational study for IAD data is conducted by $n = 37$, $P_H = 9$, $\beta = \{2880, 3000, \infty\}$ and different values of P_C and discount coefficients. The CPU time for IAD data are reported in Table 10. Figure 18 shows that, in different cases, the problem

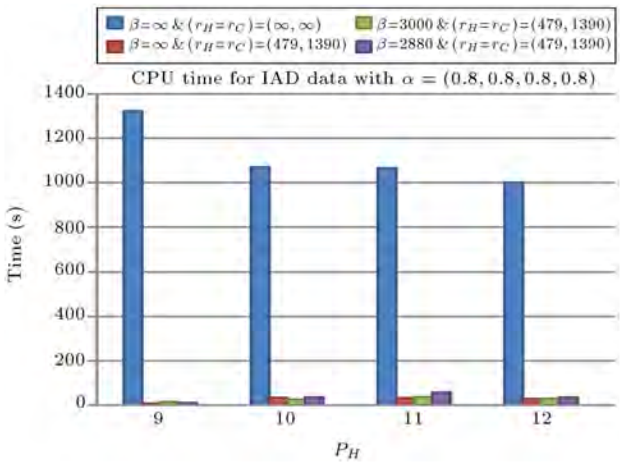

Figure 18. A chart of procedure of CPU time for the IAD data with $n = 37$, $P_H = 9$ and $\alpha = (0.8, 0.8, 0.8, 0.8)$.

Table 9. The locations of central hubs and hubs for the IAD data with $n = 37$ and $P_H = 9$.

$(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C)$	β	(r_H, r_C)	P_C	Central hubs	Hubs
(0.9, 0.9, 0.9, 0.9)	∞	∞	1	10	2, 5, 8, 10, 18, 26, 28, 31, 34
			2	10, 31	2, 5, 10, 16, 19, 30, 31, 34, 36
			3	10, 19, 31	2, 5, 10, 16, 19, 25, 30, 31, 34
			4	10, 16, 19, 31	2, 5, 10, 16, 19, 25, 30, 31, 34
	∞	(479, 1390)	1	10	2, 4, 5, 10, 19, 24, 31, 34, 36
			2	10, 31	2, 5, 10, 16, 19, 30, 31, 34, 36
			3	10, 19, 31	2, 5, 10, 16, 19, 30, 31, 34, 36
			4	10, 19, 31, 36	2, 5, 10, 16, 19, 30, 31, 34, 36
	3000		1	28	2, 5, 19, 22, 24, 28, 31, 34, 36
			2	10, 36	2, 4, 5, 10, 19, 24, 31, 34, 36
			3	10, 19, 31	2, 5, 10, 16, 19, 30, 31, 34, 36
			4	10, 19, 31, 36	2, 5, 10, 16, 19, 30, 31, 34, 36
	2880		1	-	-
			2	10, 36	2, 4, 5, 10, 19, 24, 31, 34, 36
			3	10, 31, 36	2, 5, 10, 16, 19, 30, 31, 34, 36
			4	10, 19, 31, 36	2, 5, 10, 16, 19, 30, 31, 34, 36
(0.8, 0.8, 0.8, 0.8)	∞	∞	1	10	2, 5, 10, 15, 18, 19, 28, 31, 34
			2	10, 31	2, 5, 10, 16, 19, 28, 30, 31, 34
			3	10, 19, 31	2, 5, 10, 16, 19, 28, 30, 31, 34
			4	10, 16, 19, 31	2, 5, 10, 16, 19, 28, 30, 31, 34
	∞	(479, 1390)	1	10	2, 4, 5, 10, 19, 24, 31, 34, 36
			2	10, 31	2, 5, 10, 16, 19, 30, 31, 34, 36
			3	10, 19, 31	2, 5, 10, 16, 19, 30, 31, 34, 36
			4	10, 19, 31, 36	2, 5, 10, 16, 19, 30, 31, 34, 36
	3000		1	10	2, 4, 5, 10, 19, 24, 31, 34, 36
			2	10, 31	2, 5, 10, 16, 19, 30, 31, 34, 36
			3	10, 19, 31	2, 5, 10, 16, 19, 30, 31, 34, 36
			4	10, 19, 31, 36	2, 5, 10, 16, 19, 30, 31, 34, 36
	2880		1	10	2, 4, 5, 10, 19, 24, 31, 34, 36
			2	10, 31	2, 5, 10, 16, 19, 30, 31, 34, 36
			3	10, 19, 31	2, 5, 10, 16, 19, 30, 31, 34, 36
			4	10, 19, 31, 36	2, 5, 10, 16, 19, 30, 31, 34, 36
(0.9, 0.8, 0.9, 0.8)	∞	∞	1	10	2, 5, 8, 10, 18, 26, 28, 31, 34
			2	10, 31	2, 5, 10, 16, 19, 25, 30, 31, 34
			3	10, 19, 31	2, 5, 10, 16, 19, 25, 30, 31, 34
			4	10, 16, 19, 31	2, 5, 10, 16, 19, 25, 30, 31, 34
	∞	(479, 1390)	1	10	2, 4, 5, 10, 19, 24, 31, 34, 36
			2	10, 31	2, 5, 10, 16, 19, 30, 31, 34, 36
			3	10, 19, 31	2, 5, 10, 16, 19, 30, 31, 34, 36
			4	10, 19, 31, 36	2, 5, 10, 16, 19, 30, 31, 34, 36
	3000		1	28	2, 5, 19, 22, 24, 28, 31, 34, 36
			2	10, 36	2, 4, 5, 10, 19, 24, 31, 34, 36
			3	10, 19, 31	2, 5, 10, 16, 19, 30, 31, 34, 36
			4	10, 19, 31, 36	2, 5, 10, 16, 19, 30, 31, 34, 36
	2880		-	-	
			2	10, 36	2, 4, 5, 10, 19, 24, 31, 34, 36
			3	10, 19, 31	2, 5, 10, 16, 19, 30, 31, 34, 36
			4	10, 19, 31, 36	2, 5, 10, 16, 19, 30, 31, 34, 36

Table 10. The CPU time for IAD data with $n = 37$ and $P_H = 9$.

$(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C)$	P_C	$(r_H, r_C) = \infty$	$(r_H, r_C) = (479, 1390)$		
		$\beta = \infty$	$\beta = \infty$	$\beta = 3000$	$\beta = 2880$
(0.9, 0.9, 0.9, 0.9)	1	1691.04	9.59	22.15	-
	2	2358.87	29.30	37.55	28.17
	3	1139.30	34.99	51.13	134.20
	4	1215.54	41.27	31.49	29.42
(0.8, 0.8, 0.8, 0.8)	1	1323.30	8.49	15.10	11.99
	2	1070.69	34.95	26.64	35.93
	3	1066.78	33.30	37.68	58.31
	4	1001.85	29.71	31.38	36.53
(0.9, 0.8, 0.9, 0.8)	1	1244.33	9.85	22.25	-
	2	1644.72	23.80	58.27	40.71
	3	1373.03	21.60	73.73	51.04
	4	1084.30	31.47	29.11	27.77

Figure 19. The map for IAD data with $n = 37$, 24 and 16 number of potential nodes for hubs and central hubs, respectively. Adapted from Karimi (2011).

is solved, on average, 97% faster (i.e., the CPU time decreases more than 97%) with covering constraints rather than with non-covering ones, so it is similar to Tables 6 and 7).

Figure 19 shows the map of 37 nodes on the map of Iran. This map was also presented in Karimi [17]. We denoted the potential nodes of central hubs and hubs by squares and circles, respectively.

Whereas the covering restrictions decrease the CPU times, imposing time bound constraints increases the computational times in all instances; for example, when $\beta = 2880$, all reduction factors are 0.9 and $P_C = 3$, the computation times get worse and are equal to 134.20s. But, when β changes to infinity, the CPU time decreases one hundred units and is equal to 34.99s.

The computational times procedure with the best resolution for IAD data is depicted in Figure 18. These improvements are so effective that they overshadow the poor increasing procedure of costs. The total costs for IAD data are reported in Table 11.

Table 11. The total cost for IAD data with $n = 37$ and $P_H = 9$.

$(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C)$	P_C	$(r_H, r_C) = \infty$	$(r_H, r_C) = (479, 1390)$		
		$\beta = \infty$	$\beta = \infty$	$\beta = 3000$	$\beta = 2880$
(0.9, 0.9, 0.9, 0.9)	1	143.21	159.66	217.65	-
	2	139.21	150.84	159.66	159.66
	3	136.60	147.19	150.60	154.61
	4	135.32	143.44	143.44	143.44
(0.8, 0.8, 0.8, 0.8)	1	135.43	147.43	147.43	147.43
	2	130.33	138.97	138.97	138.97
	3	128.02	135.71	135.71	135.71
	4	126.67	132.38	132.38	132.38
(0.9, 0.8, 0.9, 0.8)	1	143.21	159.66	217.65	-
	2	136.39	148.03	157.27	157.27
	3	132.24	142.18	144.74	144.74
	4	130.33	136.46	136.46	136.46

Table 12. The results for IAD data with $n = 37$ and two central hubs in Tehran and Esfahan.

β	P_H	(r_H, r_C)	Cost	Hubs	CPU time
3000	9	(∞, ∞)	133.419	2, 5, 10, 12, 23, 30, 31, 33, 34	6.155
	10		132.950	2, 5, 10, 12, 22, 23, 30, 31, 33, 34	7.770
	11		132.635	2, 5 10, 12, 22, 23, 25, 30, 31, 33, 34	10.416
	12		132.369	2, 5, 10, 12, 14, 22, 23, 25, 30, 31, 33, 34	12.600
	13		132.144	2, 5, 10, 12, 14, 17, 22, 23, 25, 30, 31, 33, 34	12.759
	14		131.928	2, 5, 10, 12, 14, 17, 22, 23, 25, 27, 30, 31, 33, 34	14.369
	15		131.719	2, 3, 5, 10, 12, 14, 17, 22, 23, 25, 27, 30, 31, 33, 34	14.591
	16		131.552	2, 3, 5, 10, 12, 14, 17, 22, 23, 25, 27, 30, 31, 33, 34, 37	15.612
3000	9	(479, 1390)	142.728	10, 12, 14, 17, 23, 29, 30, 31, 35	1.560
	10		138.530	2, 10, 12, 14, 23, 29, 30, 31, 33, 35	1.638
	11		136.053	2, 5 10, 12, 14, 23, 30, 31, 33, 34, 35	1.580
	12		135.084	2, 5, 10, 12, 14, 22, 23, 30, 31, 33, 34, 35	1.649
	13		134.607	2, 5, 10, 12, 14, 17, 22, 23, 30, 31, 33, 34, 35	1.865
	14		134.277	2, 5, 6, 10, 12, 14, 17, 22, 23, 30, 31, 33, 34, 35	1.828
	15		133.961	2, 5, 6, 10, 12, 14, 17, 22, 23, 25, 30, 31, 33, 34, 35	2.099
	16		133.746	2, 5, 6, 10, 12, 14, 17, 22, 23, 25, 27, 30, 31, 33, 34, 35	2.621

4.6. The problem with two central hubs located in Tehran and Esfahan for IAD data

We explained the problem with all nodes in IAD data, where $P_C = 2$, so that the central hubs are located only in Tehran and Esfahan, because memorable cities are 10 and 31 in Table 9, and appear as central hub and hub nodes in more than 92% instances. On the other hands, Esfahan and Tehran are the Islamic capital of the world and Iran's capital, respectively. We considered all nodes as a potential set of hubs, therefore, we defined $\alpha_H = \hat{\alpha}_H = 0.9$, $\alpha_C = \hat{\alpha}_C = 0.8$, $\beta = 3000$, $r_H = \{479, \infty\}$, $r_H = \{1390, \infty\}$ and $P_H = 9$ to 16 similar to work done by Yaman [15], as we wanted to have readers compare it with our work. These results are depicted in Table 12. We displayed the procedure of CPU time with a better resolution in Figure 15.

We observed that the results are equal for each P_H in different amounts of β. The outcome of the problem is depicted on Iran's map for IAD data, and two central hub nodes are fixed in cities 10 and 31,

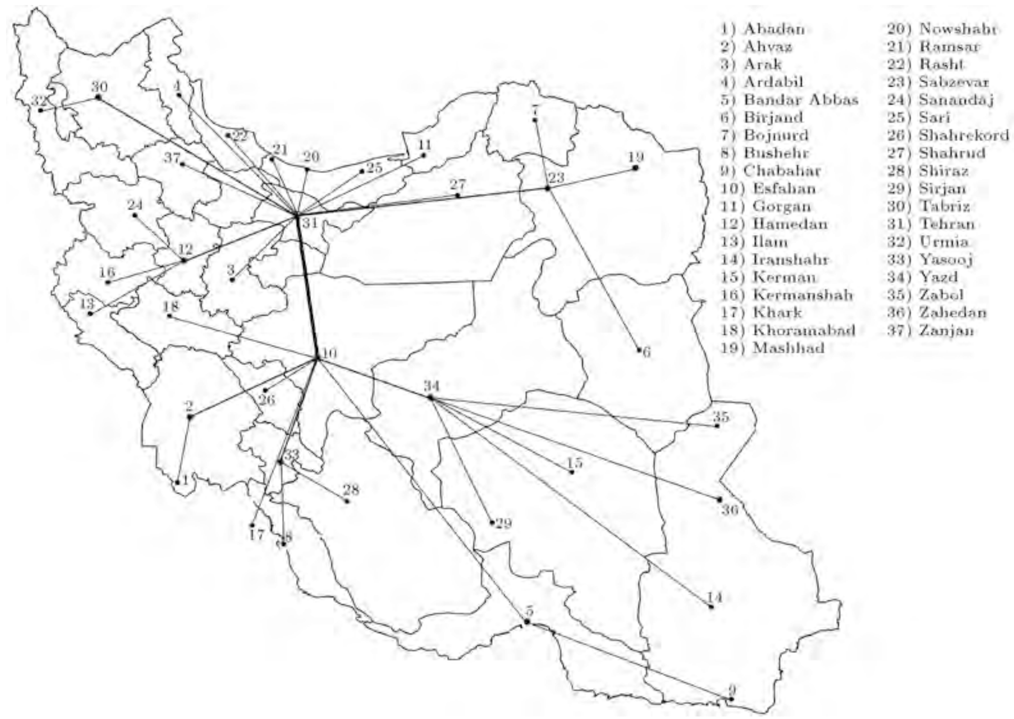

Figure 20. The hierarchical hub network for IAD data with $P_H = 9$, $r_H = r_C = \infty$ and $\beta = 3000$.

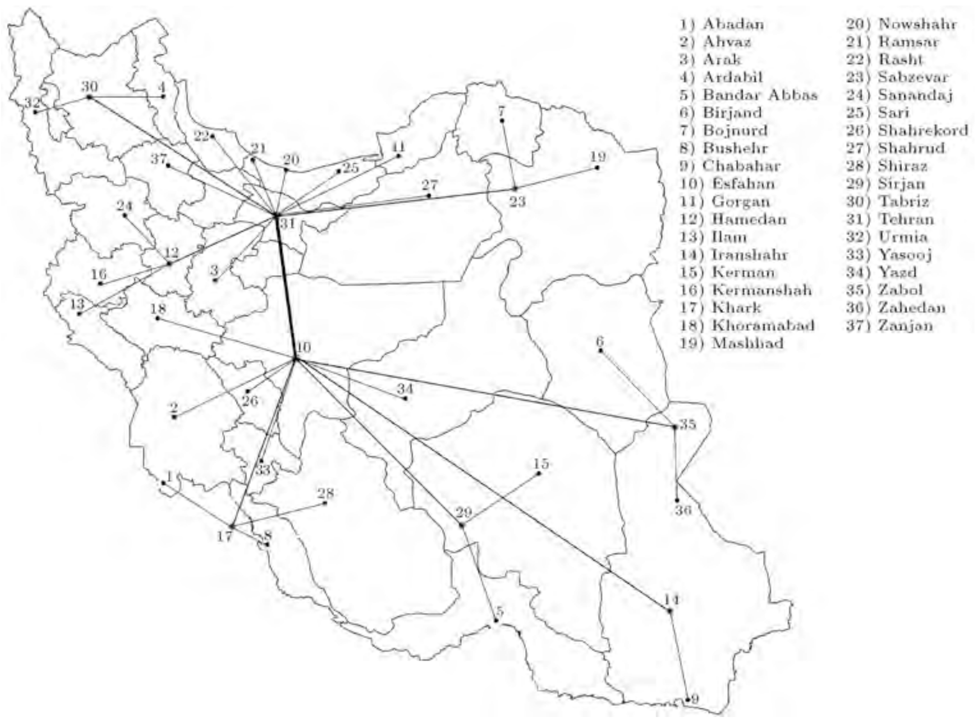

Figure 21. The hierarchical hub network for IAD data with $P_H = 9$, $r_H = 479$, $r_C = 1390$ and $\beta = 3000$.

where $n = 37$ (i.e. set of H equals set of I), $P_H = 9$ and $\beta = 3000$ in Figures 20 and 21 with $r_H = r_C = \infty$, and $r_H = 479$, $r_C = 1390$, respectively. According to the above conditions, Figure 22 has been presented for the CPU times, which represents the interval run time in the two situations.

5. Conclusion

In this paper, we studied a single assignment hierarchical hub covering problem over a complete network at the first level, and star networks in the second and third levels. We presented a mixed integer programming

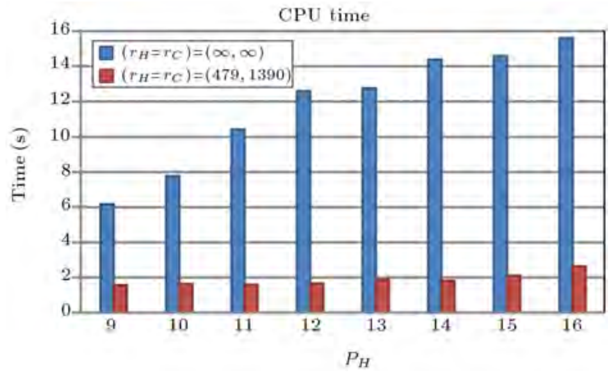

Figure 22. The CPU time of IAD data with $n = 37$, $\beta = 3000$ and two central hubs in Tehran and Esfahan for $r_H = \{479, \infty\}$ and $r_H = \{1390, \infty\}$.

formulation of the problem and, in order to rapidly solve the model on data sets, a heuristic method for calculating amounts of covering radii was proposed. The proposed model and innovative method are introduced in the literature for the first time. Our model has also considered the subject importance of fairness in providing services, which is significantly important in the area of servicing; so that, today, in addition to the general time limit (i.e. β) for providing various services, the time for providing primary care services or low level services has a certain individual limit. We tested the model on the CAB, the Turkish network and the IAD data sets, and compared the productivity of the model with that of the basic model on the data sets and understood that it produces efficient solutions with less CPU time. In addition, we did not notice considerable effects on the total routing cost.

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