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Analyzing the price skimming strategy for new product pricing

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Abstract. This article presents a new model for pricing a new product considering a skimming pricing strategy in the presence of competition. We consider two periods for price setting, including skimming and an economy period. The problem is to decide on skimming as well as economy price, in order to maximize total profit. The derived model is a non-linear programming model and we have analyzed the structure and properties of an optimal solution to develop a solution method. Analytical results, as well as managerial insights, are presented by mathematical and numerical analyses.

1. Introduction and literature review

Pricing is the main step in marketing planning that generates revenue. Besides other factors, such as product quality and performance, brand image, distribution channels, and promotion plans, price plays a main role in encouraging customers to buy products. Companies have to consider and analyze many factors to set prices and, hence, developing pricing models to obtain managerial insight is of interest to marketing managers. A company can consider any of five major objectives for its pricing: survival, maximum current profit, maximum market share, maximum market skimming, or product-quality leadership [1].

Companies consider survival as their main objective when they are faced with overcapacity, intense competition, or changes in customer needs. Some companies set their prices to maximize current profit. They estimate demand and costs regarding alternative prices, and choose the price to maximize the profit. When the market is price sensitive and the company wishes to penetrate the market and increase its market share to maximize revenue over a long period, it will choose the maximum market share objective. The objective of price skimming involves setting a relatively high price for a short time where a new, innovative, or much-improved product is launched onto the market. The objective is to skim off consumers who are willing to pay more to have the product sooner. Prices are lowered later when demand from the early customers falls or competitors introduce the same product at a lower price. A company may decide to be the product-quality leader in the market. Price skimming is used by many companies, especially in the automobile, mobile phone, TV, laptop, and other luxury industries. For example, the Sony Company is a frequent practitioner of skimming pricing, where prices start high and are lowered over time [1]. Apple inc. introduced its new mobile phone, the "iphone", in June 2007, at a top price of $599 in the USA. Despite its high price, consumers across the country stood in a long line to buy the iphone on the first day of sales. Two months later, Apple cut the price from $599 to $399 [2].

In the skimming phase, setting a high price results in a high profit margin for the company, but it

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encourages competitors to introduce the same product at a lower price to penetrate the market. The success of skimming pricing depends on the following conditions:

1. A sufficient number of buyers who have a current high demand;
2. The unit cost of producing a small volume of the item is not so high;
3. The high initial price does not attract more competitors to the market;
4. The high price communicates the image of a superior product [3].

So, analyzing these parameters to decide on the optimal skimming price is very important to manage consumer demand and maximize revenue over skimming and economy phases.

Suppose a firm designs a new product and faces a monopolistic market at the entrance. The customers have different values for the product and there is a possibility of selling the product at a high price to the higher value customers who wish to have the product sooner. The firm decides to set its pricing policy to capture more profit from the market. The product is durable and each customer buys just one. This myopic behavior is assumed for customer behavior in purchasing. In myopic behavior, each customer is willing to pay (w.t.p), and when the price of the product is equal to or less than this w.t.p, he/she will buy the product. We also consider the finite population assumption, in which the customers who buy the product are removed from the customer population for future purchases. We consider two phases for pricing: the skimming phase and the economy phase. The length of the skimming phase depends on the volume of high value customers and the competitor's ability to entrance the market. The technological ability involved in producing the product, as well as production costs are main factors affecting the time of the competitors entrance into the market. However, the skimming price of the monopolist can encourage competitors to join the market faster. So, a high skimming price at the skimming phase increases profit, but also increases the penetration rate of competitors and decreases the market share at the economy phase, lowering revenue at this phase, which has a higher market volume. So, obtaining the optimal price at skimming and economy phases to maximize total profit as well as market shares is considered in this paper.

For the first time, Stokey [4] developed a model to consider the price discrimination policy when introducing a new product onto the market. It is assumed a monopolist and the customer's reservation price to buy the product is considered as a probability function. There is no competition and the monopolist wants to maximize the present value of profit over time. Zhang [5] developed a model to analyze price discrimination in multiple markets with constrained production capacity. This type of price discrimination is also called location pricing, where the problem charges different prices in different markets. Besanko and Winston [6] considered rational customers and analyzed the optimal skimming price. They assumed a monopolist seller with a product and a population of consumers. The seller does not know the reservation price of the consumer and if the consumer will decide to buy the product now or wait to buy it later at a lower price. So, a game is formed between seller and consumer. At each time, T, the seller sets a price, sees consumer behavior and decides on the price for the next time. Price discrimination is considered over time. The objective of the seller is to maximize its profit over time. Popescu and Wu [7] considered the reference price and analyzed the pricing strategy using dynamic pricing. The consumers at each time decide to buy the product based on their reference price, which is shaped by past prices. So, in the long run, the monopolist can decide to have a constant steady state price or a skimming price strategy. They investigated these situations using a dynamic programming method and showed the optimal policy.

Su [8] developed a model of dynamic pricing with an endogenous inter-temporal demand. He assumed the finite inventory over a finite time horizon. The seller adjusts prices dynamically in order to maximize revenue, and customers arrive continually over the duration of the selling season and may buy the product at the current price, remaining in the market at a cost in order to purchase later or exit. Haji and Asadi [9] developed a fuzzy expert system for new product pricing. This fuzzy expert system includes practical rule bases to analyze the appropriate price of new products in a fuzzy environment. Dolgui and Proth [3] discussed pricing strategies and models. They discussed the benefits of the price skimming strategy for a company in the monopolistic market and recommended that high prices cannot be maintained for a long time. A good review of pricing models and their coordination with inventory decisions can be found in [10]. Berger and et al. [11] considered a problem in which the company is not satisfied with the current price and wishes to consider a new price for its items. The new price should be less than the current price and then it is similar to the situation of skimming pricing. In skimming pricing, the company starts with a high price and after skimming the high value customers, decreases the price to attract other lower valued customers.

In previous work, the authors considered a monopolist and a population of customers with different values, setting prices dynamically to maximize the monopolist's revenue. In this paper, we develop a new model considering price skimming and economy pricing in the presence of competitor effects and customer demand elasticity. The objective of the model is to maximize the total profit of the company in both skimming and economy phases. We consider customer behavior in defining market segments and develop a model to analyze the pricing strategy for a new product being introduced to the market. We consider two phase for pricing: skimming and economy phases. In the skimming phase, the company is a monopolist introducing a new product and, hence, there is an opportunity to apply the skimming strategy. The economy phase starts when all the high level customers buy the product or, at least, when a competitor enters the market with the same product at a lower price. We analyze the pricing strategy for both phases. We also introduce a market penetration function for the company in the economy phase considering skimming and economy prices as well as competitor considerations. We analyze the model to answer the following questions based on different market situations and customer behavior:

- Under what circumstances is applying the price skimming strategy recommended?
- What are the optimal prices in skimming and economy phases and the differences between them?
- Under what circumstances is it optimal to have a single price and ignore the price skimming strategy?
- How does the market situation and customer behavior affect application of the price skimming strategy?
- What is a reasonable estimate function for market penetration in the economy phase?

In this article, we try to analyze the structure of the problem and the optimal solution properties to derive a solution approach as well as managerial insights. A numerical study is also done to show the impact of parameters on optimal strategy.

2. Problem Formulation

Consider a market which can be segmented into two segments: A and B. Segment A contains the customers who are willing to purchase the product sooner at a higher price. Conversely, in segment B, the customers will purchase the product when the market price is lower than their reservation price. We set the skimming price for segment A and the economy price for segment B. The first time a new product is introduced onto the market, the skimming phase is considered and the company sets a higher price to skim segment A to achieve more profit. The length of the skimming phase is dependent on competitor's ability and the profit margin of the skimming price. The skimming phase will end when the demand falls or a competitor joins the market with a lower price. At this time, the economy phase is started and the company has to decrease its price based on skimming price, competitor price and customer elasticity.

We assume that it is possible to estimate the maximum volume of demand for each market segment, and the penetration rate of the company to capture demand depends on the price. The objective is to determine the best price for skimming and economy phases in order to maximize overall profit and market share. At first, two definitions that are considered to model the problem are presented in the following.

Definition 1. Maximum Reservation Price (MRP) is the price above which no customer will buy the product. In other words, it is the lowest price at which demand is equal to zero. Maximum Willing to Buy (MWB) is the lowest price at which all customers will buy the product [12].

Definition 2. A myopic customer is one who makes a purchase immediately, if the price is below his/her reservation price, without considering future prices. Conversely, a strategic (or rational) customer takes into account future estimated prices when making purchasing decisions [12].

In this article, we assume myopic behavior for customers. The parameters and variables needed to formulate the problem are defined as follows:

2.1. Notations

Parameters

FC :	The finished cost of the product;
MRP :	Maximum reservation price;
TV :	The maximum estimate of total market demand volume;
V :	The maximum estimate of market demand volume for the skimming phase;
PR^s :	The penetration rate function at the skimming phase;
PR^e :	The penetration rate function at the economy phase.

Variables

$P^s(P^e)$:	The skimming (economy) price for the product.

2.2. Analysis of penetration rate functions

The penetration rate at the skimming phase depends on the price of the product. If the skimming price is high, then the penetration rate is low. In a real situation, the relationship between penetration rate and price is non-linear, and the negative exponential function is more consistent and applied more than other functions in the literature. So, we apply the negative exponential function to model the penetration rate at the skimming and economy phases. We illustrate the penetration rate at the skimming phase as:

$$PR^s = e^{-\alpha\left(\frac{P^s - FC}{MRP - FC}\right)}. \tag{1}$$

It can be simplified by substituting the parameters as:

$$PR^s = e^{-\alpha a(P^s - FC)}, \tag{2}$$

where $a = \frac{1}{MRP - FC}$ and $\alpha \geq 1$. Parameter α is the shape parameter estimated by historical data from the market for previous products that present customer behavior.

The penetration rate at the economy phase depends on the skimming price, competitor's price and economy price. We assume that the competitor will join the market with a lower price. So, the competitor will set the price lower than the skimming price and higher than the finished cost of the product. We assume the same production cost function for both the competitor and the company. We also assume that there is just one opportunity to set the price and the company cannot estimate the exact price of the competitor. A high skimming price increases the penetration rate of the competitor and the company will lose its market share in the economy phase. We apply this concept by defining coefficient, η. Deciding the best skimming price to gain more profit at the skimming phase, as well as more market share and profit at the economy phase, is the aim of this model. Customers who buy the product at the skimming phase are removed from the market and the penetration rate at the economy phase is calculated for remained market volume. When the skimming price is equal to MRP, the competitor will have a good opportunity to increase its penetration rate. However, we assume that it cannot capture the whole market because of the originality of the company brand, and we apply this concept in defining coefficient β. The penetration rate of the company at the economy phase is formulated as:

$$PR^e = e^{-\beta\left(\frac{P^s - FC}{MRP - FC}\right) - \eta\left(\frac{P^e - FC}{P^s - FC}\right)}. \tag{3}$$

It can be also simplified as:

$$PR^e = e^{-\beta a(P^s - FC) - \eta\left(\frac{P^e - FC}{P^s - FC}\right)}, \tag{4}$$

where $\beta \geq 0$ and $\eta \geq 1$ are the shape parameters for the economy phase and are estimated based on the market situation. The penetration function presented in Eq. (4) has the form of an exponential price response function. The penetration rate at the economy phase depends on the skimming price, economy price and the effect of competition. Parameters β and η are estimated by historical data, which include the effects of competition and customer behavior. The behavior of this function is reasonable and logical, as presented in the experimental results section. The more exact and real parameters, β and η, can be estimated using complete and updated historical data.

Therefore, by considering these penetration rate functions at skimming and economy phases, we can write the model of the problem, which appears in the next sub-section.

2.3. The model

The problem can be formulated as a Non-Linear Programming (NLP) model as follows:

$$\begin{aligned}\max Z =& V.PR^s(P^s - FC)\\ &+ (TV - V.PR^s)PR^e(P^e - FC),\end{aligned} \tag{5}$$

s.t.

$$P^s \leq MRP, \tag{6}$$

$$P^e \leq P^s, \quad P^s \geq 0, \quad P^e \geq 0. \tag{7}$$

The objective function (5), attempts to maximize company profit over skimming and economy phases. By substituting the penetration rate functions from Relations (2) and (4) in the objective function, we see that it is a non-linear function. All constraints are in the form of linear and constraint (6) states, in which the price cannot be larger than the maximum reservation price and the economy price cannot be larger than the skimming price (7).

3. Structural analysis

3.1. Optimal solution analysis

In this section, we will undertake some structural analyses on the model to find the properties of the optimal solution. By replacing the penetration functions, the objective function is transformed as follows:

$$\begin{aligned} z(P^s, P^e) =& Ve^{-\alpha a(P^s - FC)}(P^s - FC)\\ &+ \left(TV - Ve^{-\alpha a(P^s - FC)}\right)\\ &\times \left(e^{-\alpha a(P^s - FC)}\right)^{\frac{\beta}{\alpha}}\\ &\times \left(e^{-\frac{\eta(P^e - FC)}{P^s - FC}}\right)(P^e - FC).\end{aligned}$$

Theorem 1. The optimal economy price is derived based on the optimal skimming price by the following equation:

$$P^{e*} = \frac{\eta FC + P^s - FC}{\eta}. \tag{8}$$

Proof. By taking the first derivative condition of the objective function with respect to P^e, we have:

$$\frac{\partial Z}{\partial P^e} = -\frac{1}{P^s - FC}\left[\left(TV - Ve^{-\alpha a(P^s-FC)}\right) \times \left(e^{-\alpha a(P^s-FC)}\right)^{\frac{\beta}{\alpha}} \times \left(\eta e^{-\frac{\eta(P^e-FC)}{P^s-FC}}(P^e - FC)\right)\right] + (TV - Ve^{-\alpha a(P^s-FC)})(e^{-\alpha a(P^s-FC)})^{\frac{\beta}{\alpha}} \times \left(e^{-\frac{\eta(P^e-FC)}{P^s-FC}}\right) = \left(TV - Ve^{-\alpha a(P^s-FC)}\right) \times \left(e^{-\alpha a(P^s-FC)}\right)^{\frac{\beta}{\alpha}}\left(e^{-\frac{\eta(P^e-FC)}{P^s-FC}}\right) \times \left(1 - \frac{\eta(P^e - FC)}{P^s - FC}\right)\frac{\partial Z}{\partial P^e} = 0 \xrightarrow{\text{yields}} P^{e*} = \frac{\eta FC + P^s - FC}{\eta}.$$

The second derivative of Z, with respect to P^e, ensures that the maximum value of Z can reach P^{e*}, and hence, it is the optimal economy price.

$$\frac{\partial^2 Z}{\partial P^{e^2}} = \frac{1}{(P^s - FC)^2}\left[\left(TV - Ve^{-\alpha a(P^s-FC)}\right) \times \left(e^{-\alpha a(P^s-FC)}\right)^{\frac{\beta}{\alpha}}\left(\eta^2 e^{-\frac{\eta(P^e-FC)}{P^s-FC}}(P^e - FC)\right)\right] - \frac{1}{P^s - FC}\left[2\left(TV - Ve^{-\alpha a(P^s-FC)}\right) \times \left(e^{-\alpha a(P^s-FC)}\right)^{\frac{\beta}{\alpha}}\left(\eta e^{-\frac{\eta(P^e-FC)}{P^s-FC}}\right)\right]$$

$$= \frac{\left(TV - Ve^{-\alpha a(P^s-FC)}\right)\left(e^{-\alpha a(P^s-FC)}\right)^{\frac{\beta}{\alpha}}\left(\eta e^{-\frac{\eta(P^e-FC)}{P^s-FC}}\right)}{P^s - FC} \times \left(\frac{\eta(P^e - FC)}{P^s - FC} - 2\right)\left.\frac{\partial^2 Z)}{\partial P^{e^2}}\right|_{P^e = \frac{\eta FC + P^s - FC}{\eta}}$$

$$= \frac{\left(TV - Ve^{-\alpha a(P^s-FC)}\right)\left(e^{-\alpha a(P^s-FC)}\right)^{\frac{\beta}{\alpha}}\left(\eta e^{-1}\right)}{P^s - FC} \times (1-2) < 0.$$

□

Proposition 1. The economy price is increasing in the skimming price and the skimming strategy is reasonable for $\eta > 1$. Considering the constraint, $P^e \leq P^s$, if $\eta \leq 1$, the skimming and economy prices are equal, which means that obtaining a single price is optimal and the skimming strategy is not acceptable. Therefore, the economy price is always equal to or less than the skimming price, and, hence, constraint $P^e \leq P^s$ is surplus in the model and can be eliminated.

Observation 1. Based on proposition 1, the company can estimate parameter η using historical data regarding the price and demand of previous products, and decide to apply the skimming strategy according to parameter η. If $\eta \leq 1$, then the price skimming strategy is not reasonable. The historical data show the behavior of customers regarding different price values, and if the value of parameter η is equal to or less than 1, it means that customers prefer to buy the product in one and at a lower price.

Observation 2. The economy price is the average of the finished cost and skimming price in case of $\eta = 2$:

$$P^e = \frac{FC + P^s}{2}.$$

The model can be modified by replacing the equation of the optimal economy price in the objective function and transforming it into a function of single variable P^s. Therefore, the model becomes:

$$\max Z = (P^s - FC)\left[Ve^{-\alpha a(P^s-FC)} + \frac{e^{-1}(TVe^{-\beta a(P^s-FC)} - Ve^{-(\alpha+\beta)a(P^s-FC)}}{\eta}\right],$$

s.t.

$$P^s \leq \text{MRP}, \qquad P^s \geq 0.$$

By replacing $x = P^s - FC$, the objective function can be transformed as:

$$Z = x\left[Ve^{-\alpha a x} + \frac{e^{-1}(TVe^{-\beta a x} - Ve^{-(\alpha+\beta)a x}}{\eta}\right]. \tag{9}$$

In order to analyze the objective function and optimal solution, it can be simplified as:

$$Z = x\left[a_1 e^{-k_1 x} + a_2 e^{-k_2 x} - a_3 e^{-(k_1+k_2)x}\right]. \tag{10}$$

Parameters k_1 and k_2 are positive and:

$$k_1 = a\alpha, \qquad k_2 = a\beta.$$

Parameters a_1, a_2 and a_3 are as follows:

$$a_1 = V, \qquad a_2 = \frac{e^{-1}TV}{\eta}, \qquad a_3 = \frac{e^{-1}V}{\eta}.$$

Since $\eta \geq 1$ and $TV > V$, we have $a_1 > a_3$ and $a_2 > a_3$.

Now, by the first derivative condition in Eq. (10), the optimal value of x can be determined as:

$$x^* = \frac{a_1 e^{-k_1 x} + a_2 e^{-k_2 x} - a_3 e^{-(k_1+k_2)x}}{a_1 k_1 e^{-k_1 x} + a_2 k_2 e^{-k_2 x} - a_3(k_1+k_2)e^{-(k_1+k_2)x}}. \tag{11}$$

Let to consider the right hand side of Eq. (11) as $g(x)$. In the following we try to find a lower bound and an upper bound for x^*.

Theorem 2. The lower and upper bounds for the optimal value of x are:

$$\frac{1}{\max(k_1, k_2)} \leq g(x) \leq \max\left(\frac{1}{\min(k_1,k_2)}, \frac{a_1 + a_2 - a_3}{a_1k_1 + a_2k_2 - a_3(k_1+k_2)}\right). \tag{12}$$

Proof

a. Proof of the lower bound:

For the lower bound we have:

$$\max(k_1, k_2) \geq k_1,$$

$$\max(k_1, k_2) \geq k_2,$$

$$k_1 + k_2 \geq \max(k_1, k_2).$$

Therefore, we can write:

$$\max(k_1,k_2)\left[a_1 e^{-k_1 x} + a_2 e^{-k_2 x} - a_3 e^{-(k_1+k_2)x}\right] \geq a_1 k_1 e^{-k_1 x} + a_2 k_2 e^{-k_2 x} - a_3(k_1+k_2)e^{-(k_1+k_2)x} \xrightarrow{\text{the right hand side is positive}} g(x) \geq \frac{1}{\max(k_1,k_2)}.$$

b. Proof of the upper bound:

To prove the upper bound, we analyze the equation in all possible cases:

Case 1: $k_1 < k_2$ and $\frac{a_2}{a_3} > \frac{k_2}{k_2 - k_1}$;

Case 2: $k_1 > k_2$ and $\frac{a_1}{a_3} > \frac{k_1}{k_1 - k_2}$;

Case 3: $k_1 < k_2$ and $\frac{a_2}{a_3} < \frac{k_2}{k_2 - k_1}$;

Case 4: $k_1 > k_2$ and $\frac{a_1}{a_3} < \frac{k_2}{k_2 - k_1}$.

According to parameters k_1 and k_2, these four cases cover all possible cases. According to Eq. (12), the upper bound is the maximum of two terms. Now we should show, in each case, which term is considered as the upper bound. In the following we prove that in both Cases 1 and 2, the upper bound is $\frac{1}{\min(k_1,k_2)}$, and in Cases 3 and 4, the upper bound is $\frac{a_1+a_2-a_3}{a_1k_1+a_2k_2-a_3(k_1+k_2)}$.

Case 1: Assume $k_1 < k_2$ and $\frac{a_2}{a_3} > \frac{k_2}{k_2-k_1}$, we have:

$$\frac{a_2}{a_3} > \frac{k_2}{k_2 - k_1} \Rightarrow a_3 k_2 < a_2(k_2 - k_1)$$

$$\Rightarrow a_3(k_1 + k_2) - a_3 k_1 < a_2 k_2 - a_2 k_1$$

$$\xrightarrow{\text{by adding } a_1k_1 \text{ to both sides of equations}} a_3(k_1+k_2) - a_3k_1 + a_1k_1 < a_2k_2 - a_2k_1 + a_1k_1 \Rightarrow a_1k_1 + a_2k_1 - a_3k_1 < a_1k_1 + a_2k_1 - a_3(k_1+k_2)$$

$$\Rightarrow \frac{a_1 + a_2 - a_3}{a_1k_1 + a_2k_2 - a_3(k_1+k_2)} < \frac{1}{k_1} = \frac{1}{\min(k_1,k_2)}.$$

It means that:

$$\max\left(\frac{1}{\min(k_1,k_2)}, \frac{a_1 + a_2 - a_3}{a_1k_1 + a_2k_2 - a_3(k_1+k_2)}\right) = \frac{1}{\min(k_1,k_2)}. \tag{13}$$

Case 2: $k_1 > k_2$ and $\frac{a_1}{a_3} > \frac{k_1}{k_1-k_2}$: with the same approach as Case 1, it is proved that:

$$\max\left(\frac{1}{\min(k_1,k_2)}, \frac{a_1 + a_2 - a_3}{a_1k_1 + a_2k_2 - a_3(k_1+k_2)}\right) = \frac{1}{\min(k_1,k_2)}.$$

Therefore, for the other two cases (Cases 3 and 4) the maximum of two terms $\frac{1}{\min(k_1,k_2)}$ and $\frac{a_1+a_2-a_3}{a_1k_1+a_2k_2-a_3(k_1+k_2)}$ is $\frac{a_1+a_2-a_3}{a_1k_1+a_2k_2-a_3(k_1+k_2)}$. In other words, for two Cases 3 and 4, we have:

$$\max\left(\frac{1}{\min(k_1,k_2)}, \frac{a_1 + a_2 - a_3}{a_1k_1 + a_2k_2 - a_3(k_1+k_2)}\right) = \frac{a_1 + a_2 - a_3}{a_1k_1 + a_2k_2 - a_3(k_1+k_2)}.$$

Now, we will prove the upper bound in Cases 1 and 2 and, after that, the upper bound of Cases 3 and 4 is proved. Suppose:

$$\max\left(\frac{1}{\min(k_1,k_2)},\frac{a_1+a_2-a_3}{a_1k_1+a_2k_2-a_3(k_1+k_2)}\right)=\frac{1}{\min(k_1,k_2)}.$$

Without loss of generality, assume $k_1 < k_2$. So, condition $\frac{a_2}{a_3} > \frac{k_2}{k_2-k_1}$ is needed to have Case 1. Therefore, we have:

$$a_2(k_2-k_1) > a_3k_2 \xrightarrow{e^{k_1x}>0} a_2(k_2-k_1)e^{k_1x} > a_3k_2$$
$$\Rightarrow a_2(k_2-k_1)e^{-k_2x} > a_3k_2e^{-(k_1+k_2)x}$$
$$\Rightarrow a_2k_2e^{-k_2x} - a_3k_2e^{-(k_1+k_2)x} > a_2k_1e^{-k_2x}$$
$$\Rightarrow a_2k_2e^{-k_2x} - a_3k_2e^{-(k_1+k_2)x} + a_1k_1e^{-k_1x}$$
$$> a_2k_1e^{-k_2x} + a_1k_1e^{-k_1x} \Rightarrow a_1k_1e^{-k_1x}$$
$$+ a_2k_2e^{-k_2x} - a_3((k_1+k_2)-k_1)e^{-(k_1+k_2)x}$$
$$> a_1k_1e^{-k_1x} + a_2k_1e^{-k_2x} \Rightarrow a_1k_1e^{-k_1x}$$
$$+ a_2k_2e^{-k_2x} - a_3(k_1+k_2)e^{-(k_1+k_2)x}$$
$$> a_1k_1e^{-k_1x} + a_2k_1e^{-k_2x} - a_3k_1e^{-(k_1+k_2)x}$$
$$\Rightarrow \frac{a_1e^{-k_1x}+a_2e^{-k_2x}-a_3e^{-(k_1+k_2)x}}{a_1k_1e^{-k_1x}+a_2k_2e^{-k_2x}-a_3(k_1+k_2)e^{-(k_1+k_2)x}}$$
$$< \frac{1}{k_1} \Rightarrow g(x) < \frac{1}{\min(k_1,k_2)}.$$

With the same approach for Case 2 ($k_1 > k_2$ and $\frac{a_1}{a_3} > \frac{k_1}{k_1-k_2}$), it is proved that the upper bound is $\frac{1}{\min(k_1,k_2)}$.

For Cases 3 and 4, we know that:

$$\max\left(\frac{1}{\min(k_1,k_2)},\frac{a_1+a_2-a_3}{a_1k_1+a_2k_2-a_3(k_1+k_2)}\right)=\frac{a_1+a_2-a_3}{a_1k_1+a_2k_2-a_3(k_1+k_2)}.$$

Therefore, in the following, we prove the upper bound in these cases.

Case 3: $k_1 < k_2$ and $\frac{a_2}{a_3} < \frac{k_2}{k_2-k_1}$. Now we should prove that:

$$g(x) \leq \frac{a_1+a_2-a_3}{a_1k_1+a_2k_2-a_3(k_1+k_2)}. \tag{14}$$

Since $a_1 > a_3$ and $a_2 > a_3$, Eq. (14) can be written as:

$$\left(a_1e^{-k_1x}+a_2e^{-k_2x}-a_3e^{-(k_1+k_2)x}\right)(a_1k_1$$
$$+a_2k_2-a_3(k_1+k_2)) - \left(a_1k_1e^{-k_1x}+a_2k_2e^{-k_2x}\right.$$
$$\left.- a_3(k_1+k_2)e^{-(k_1+k_2)x}\right)(a_1+a_2-a_3) < 0.$$

By considering the left hand side of the above relation as $Q(x)$, we should prove that $Q(x) < 0$.

To do this, $Q(x)$ is simplified as follows:

$$Q(x) = a_3(k_1a_2+a_1k_2)e^{-(k_1+k_2)x}$$
$$+ \left[(k_2-k_1)a_2 - a_3k_2\right]a_1e^{-k_1x}$$
$$+ a_2\ \left[(k_1-k_2)a_1 - a_3k_1\right]e^{-k_2x}$$
$$\xrightarrow{\text{Considering } \frac{a_2}{a_3} > \frac{k_2}{k_2-k_1} \text{ and } k_1<k_2} a_1[a_2(k_2-k_1)$$
$$- a_3k_2]e^{-k_1x} < a_1[a_2(k_2-k_1)-a_3k_2]e^{-k_2x}$$
$$\Rightarrow Q(x) \leq a_3(k_1a_2+a_1k_2)e^{-(k_1+k_2)x}$$
$$+ a_1[a_2(k_2-k_1)-a_3k_2]e^{-k_2x} + a_2[(k_1-k_2)a_1$$
$$- a_3k_1]e^{-k_2x} \Rightarrow Q(x) \leq e^{-(k_1+k_2)x}$$
$$\times\left[a_3(k_1a_2+a_1k_2)e^{-k_1x}(a_3a_1k_2+a_2a_3k_1)\right]$$
$$\Rightarrow Q(x) \leq e^{-(k_1+k_2)x}\left[a_3(k_1a_2+a_1k_2)(1-e^{-k_1x})\right]$$
$$\xrightarrow{\text{since } k_1>0} Q(x) < 0. \tag{15}$$

Therefore, we have:

$$g(x) \leq \frac{a_1+a_2-a_3}{a_1k_1+a_2k_2-a_3(k_1+k_2)}.$$

Case 4: $k_1 > k_2$ and $\frac{a_1}{a_3} < \frac{k_2}{k_2-k_1}$. With the same approach as Case 3, it is proved that:

$$g(x) \leq \frac{a_1+a_2-a_3}{a_1k_1+a_2k_2-a_3(k_1+k_2)}.$$

Now, we can conclude that:

$$\frac{1}{\max(k_1,k_2)} \leq g(x) \leq \max$$
$$\times\left(\frac{1}{\min(k_1,k_2)},\frac{a_1+a_2-a_3)}{a_1k_1+a_2k_2-a_3(k_1+k_2)}\right).$$

□

Proposition 2. By replacing parameters k_1 and k_2, the lower bound is:

$$x^* \geq \frac{1}{\max(k_1, k_2)} = \frac{1}{a \max(\alpha, \beta)} = \frac{\text{MRP} - FC}{\max(\alpha, \beta)}. \tag{16}$$

Now, we can conclude that if both α and β are equal to or less than one, then $x^* \geq \text{MRP} - FC$. Therefore, the problem has no feasible solution.

Observation 3. Based on Proposition 2, and assumption $\alpha \geq 1$, if $\alpha = 1$ and $\beta < 1$, then the skimming price is equal to MRP, which means that the skimming strategy is not reasonable. The company should set a single price in both periods equal to:

$$P^e = \frac{\eta FC + \text{MRP} - FC}{\eta}. \tag{17}$$

Now, we are going to show the uniqueness of the optimal skimming price. Recalling Eq. (11), it can be transformed as Eq. (16). If we show that Eq. (16) has just one solution, we can develop a procedure to obtain the optimal solution.

$$\begin{aligned} L(x) =& a_1 e^{-k_1 x}(1 - k_1 x) + a_2 e^{-k_2 x}(1 - k_2 x) \\ & - a_3 e^{-(k_1+k_2)x}(1 - (k_1 + k_2)x) = 0. \end{aligned} \tag{18}$$

To show the uniqueness of the solution for Eq. (18), we solved 560 sample, whose parameter summaries are shown in Table 1. In all sample problems, there was just one solution for Eq. (18). The behavior of Eq. (18) with respect to x is shown in Figure 1.

Based on this observation we can propose a solution algorithm to solve the model which is presented in the next sub-section.

3.2. Solution algorithm

Step 1. Determine the maximum feasible distance between the lower and upper bound of x (δ) and compute the lower and upper bound of x (LB and UB)

Table 1. The summary of example problems parameters.

FC	TV	V	MRP	α	β	γ
			15	1	1	1
			20	1.5	1.5	1.5
			50	2	3	2
10	1660	744	100	3	5	2.5
				5		3
						5
						10
Number of values			4	5	4	7

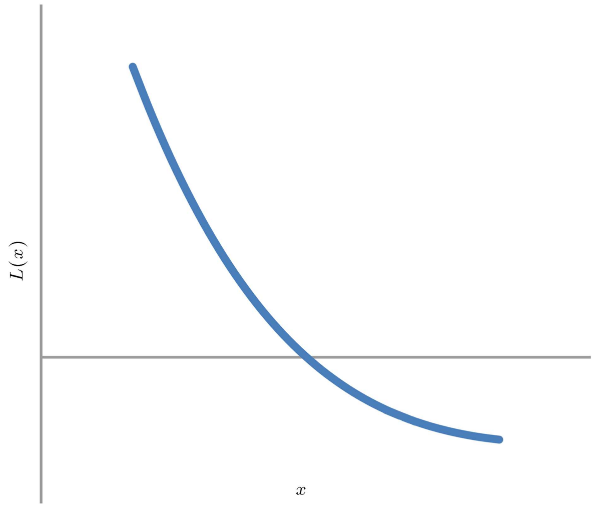

Figure 1. The behavior of Eq. (18) in x.

as:

$$\text{LB} = \frac{1}{\max(k_1, k_2)},$$

$$\text{UB} = \max\left(\frac{1}{\min(k_1, k_2)}, \frac{a_1 + a_2 - a_3}{a_1 k_1 + a_2 k_2 - a_3(k_1 + k_2)}\right).$$

Step 2. Let $x = \frac{\text{LB}+\text{UB}}{2}$.

Step 3. Compute the equation:

$$\begin{aligned} L(x) =& a_1 e^{-k_1 x}(1 - k_1 x) + a_2 e^{-k_2 x}(1 - k_2 x) \\ & - a_3 e^{-(k_1+k_2)x}(1 - (k_1 + k_2)x). \end{aligned}$$

Step 4.

If $L(x) = 0$ then set $x^* = x$ and go to Step 6;

If $L(x) > 0$ then set $\text{LB} = x$ and go to Step 5;

If $L(x) < 0$ then set $\text{UB} = x$ and go to Step 5.

Step 5: If $\text{UB} - \text{LB} \leq \delta$, then set $x^* = \frac{\text{LB}+\text{UB}}{2}$ and go to Step 6, else go to Step 2.

Step 6. The optimal solution is:

$$P^{s*} = x^* + FC,$$

$$P^{e*} = \frac{\eta FC + P^{s*} - FC}{\eta}.$$

4. Experimental results

In this section, we present the results observed by numerical study based on solving 560 sample problems. Table 2 shows a sample of these problems with their solutions.

Table 2. A sample of example problems.

FC	TV	V	MRP	α	β	η	P^e	P^s	Profit
1	1660	744	1.5	1	5	10	1.04976	1.49758	137.025
1	1660	744	2	1	1.5	3	1.32171	1.96513	311.836
1	1660	744	5	1	3	2.5	2.49292	4.73229	1137.95
1	1660	744	10	1	3	1.5	6.33635	9.00453	2631.55
1	1660	744	2	1.5	1.5	2.5	1.27797	1.69492	232.705
1	1660	744	10	1.5	1.5	3	3.0733	7.21989	2018.79
1	1660	744	10	3	1	1.5	4.82188	6.73282	1783.55
1	1550	842	10	5	5	10	1.18231	2.8231	587.819
1	1550	842	5	5	3	2	1.49208	1.98415	354.997
1	1550	842	5	5	0.5	10	1.78503	8.85028	168.143
1	1550	842	5	5	0.5	1.5	6.32132	8.98198	1119.05
1	1550	842	10	2	0.5	2.5	3.97514	8.43784	2204.82
1	1550	842	10	1.5	5	10	1.5934	6.93399	1868.41
1	1550	842	5	1.5	3	1	3.36684	3.36684	997.944
10	1660	744	15	1	3	2.5	11.8661	14.6654	1422.43
10	1660	744	20	1	1.5	1.5	16.2696	19.4043	3502.64
10	1660	744	50	1	1.5	3	22.8684	48.6053	12473.5
10	1660	744	100	1	1.5	5	27.5961	97.9803	26688.5
10	1660	744	50	1.5	0.25	5	17.786	48.9299	10067.1
10	1660	744	50	2	0.5	2	31.2643	49.5285	10996.9

By solving the 560 sample problems, we analyzed the sensitivity of parameters on the optimal solution. We considered the distance between the economy price and the skimming price as a criterion to analyze the effects of each parameter on this criteria. The distance between economy and skimming prices gives an insight for management regarding the importance of the skimming strategy. The more distance between economy and skimming prices, the more interest in applying the price skimming strategy. Our observations are as follows:

1. The distance between skimming and economy prices ($P^s - P^e$) is increasing in η. Figure 2, presents the relation between skimming and economy price in η. Skimming and economy prices are the same in $\eta = 0$. By increasing η, the economy price decreases to finished cost, and its distance from skimming price increases.

2. For parameters α and β, we observed that $P^s - P^e$ is decreasing in both parameters α and β. Figures 3 and 4 show the behavior of skimming as well as economy prices with respect to α and β. The decrease in skimming price is sharper than the economy price in both α and β. The most distance between skimming and economy prices is where α and β are equal to one, and by increasing these parameters, the distance between skimming and economy prices decreases.

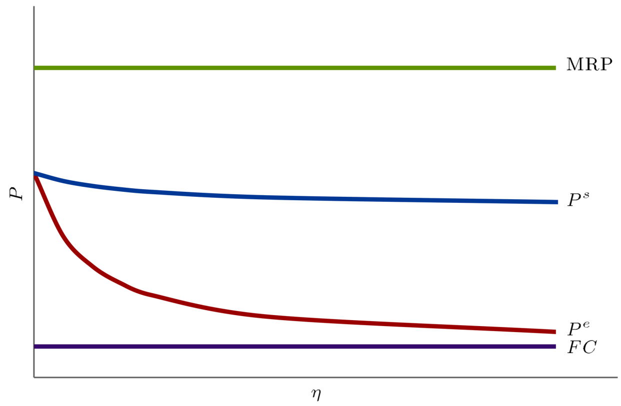

Figure 2. The behavior of skimming and economy price in η.

5. Conclusion and future researches

In this article, a new pricing model was developed considering the skimming pricing strategy for introducing a new product onto the market. We considered two periods for price setting: skimming and economy periods. In the skimming period, the company faces a monopolistic market, but in the economy period, it may compete with at least one competitor. We formulate the effect of competition in the economy phase by introducing a penetration function. The penetration rates at skimming and economy phases were formulated by an exponential function and the effect of competition is considered as the loss of market share in the economy phase penetration rate. The

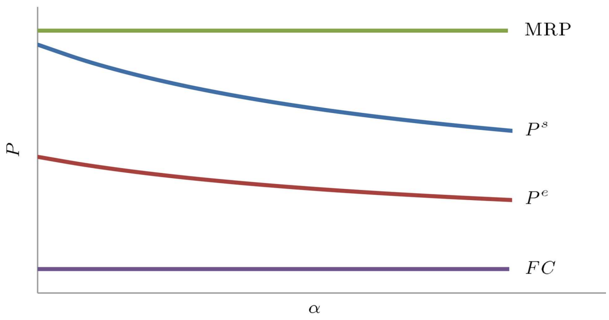

Figure 3. The behavior of skimming and economy price in α.

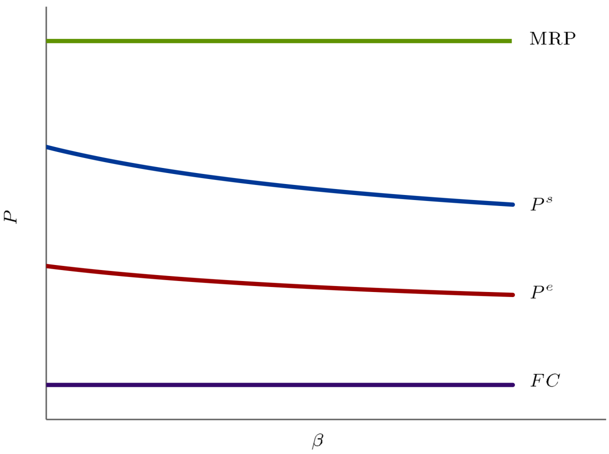

Figure 4. The behavior of skimming and economy price in β.

derived model is a non-linear programming model. The structural analysis presents valuable results concerning optimal solution properties. We also analyzed feasible and effective ranges of parameters for applying the price skimming strategy. The optimal economy price is calculated considering the skimming price. An algorithm was developed to solve the model based on the lower and upper bounds derived in structural analysis. Many sample problems were solved and some managerial insights were presented by numerical analysis. This is the first attempt to formulate the skimming pricing strategy considering competition in estimating the parameters and formulating the penetration rate functions. As an extension, this problem can be analyzed by game theory to realize the competition in a dynamic environment, considering the reaction of competitors and companies. Other functions to formulate penetration rates and other solution methods can be of interest for future research.

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