Introducing a new sliding manifold applied for control of uncertain nonlinear brushless DC and permanent magnet synchronous motors

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Abstract. In this paper, the problem of controlling chaotic uncertain brushless DC motor (BLDCM) and Permanent Magnet Synchronous Motor (PMSM), exposed to external disturbances, is considered. First, a new nonsingular terminal sliding surface is introduced, and its finite-time convergence to the zero equilibrium is proved. Then, it is assumed that the parameters of BLDCM and PMSM are fully unknown, and appropriate adaptive laws are derived to tackle the unknown parameters of the systems. Besides, the effects of the models uncertainties and external disturbances are also taken into account. Afterwards, based on the adaptive laws and robust finite-time control idea, a robust adaptive sliding mode controller is proposed to ensure the occurrence of the sliding motion in finite time. It is mathematically proved that the introduced nonsingular terminal sliding mode technique has finite-time convergence and stability in both reaching and sliding mode phases. Numerical simulations are presented to verify the efficiency of the proposed method and to validate the theoretical results of the paper.

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1. Introduction

Lately, chaos in electric motors has become one of the most attractive research topics due to its direct applications in industrial machinery, electrical locomotives and electrical submersibles thruster drives. In the late 1980s, since the chaos occurrences in electric motors have been concentrated on [1], a wide-ranging research has been done towards the detection of chaos and its control in various types of motors such as DC motors [2-4], step motors [5,6], synchronous reluctance motor drives [7], switched reluctance motor [8,9] and PMSMs [10-13].

A BLDC motor is a synchronous electric motor which is powered by direct-current electricity [14]. The main benefit of BLDCM is the removal of the physical contact between brushes and commutators. BLDCM has been extensively applied in direct-drive applications such as robotics, aerospace, etc. To date, there has been several works done in modeling and controlling of BLDCMs [15]. In [16], a new nonlinear model has been proposed for BLDCM which could demonstrate chaotic performance by some particular alternatives for parameters. Actually, it has shown that the unforced transformed model is equivalent to the well-known Lorenz system. Recently, some modern control methodologies have been proposed for control-
ling linear brushless permanent magnet DC motors such as nonlinear control [17], optimal control [18], variable structure control [19], PID control [20], adaptive control and backstepping approach [21]. On the other hand, these approaches are applied for a linear BLDCM that does not show chaotic and complex dynamics. Chaotic performance in BLDCM, which comes out mostly alternating ripples of torque and low-frequency oscillations of rotational speed of motor, can tremendously obliterate the stabilization and performance of the motor. Therefore, it is crucial to investigate the method of controlling and suppressing chaos in BLDCMs. In [22], piecewise quadratic state feedback controller has been proposed, but the equilibrium points of the uncontrolled chaotic BLDCM system did not apply as the control target. In [23], time-delay feedback and fuzzy control approaches have been used to stabilize the system around the origin, but a noticeable chattering phenomenon is observed. In [24], feedback linearization and sliding mode control techniques have been developed to control a chaotic BLDCM. Only stator current state equation has been perturbed by uncertainties. While in practical applications, the whole dynamics of the system is disturbed by uncertainties and external disturbances. Also, no control parameter selection procedure is used in the above mentioned methods. Recently, chaotic anticontrol and chaos synchronization of BLDCM systems have been studied [25-29]. In [30], a sliding mode controller has been designed, which has some practical advantages such as fast response, low sensitivity to external disturbances and robustness to the system uncertainties.

Furthermore, chaos in PMSM and its control are other areas of challenging research in the field of nonlinear control of electric motors. PMSM has been attracting more and more attention in high-accuracy and high performance electric drive systems in industrial motion control applications due to its advantages of superior power density, high-performance motion control with fast speed and better accuracy, large torque to inertia ratio and long life over other types of motors such as DC motors and induction motors [31]. But in industrial applications, there are many uncertainties such as system parameter uncertainty, external load disturbance, friction force, unmodeled uncertainty, etc., which always diminish the performance quality of the pre-design of the motor driving system. To cope with this problem, robust or adaptive or other control schemes dealing with parameter uncertainties and unknown external disturbances have extensively been studied up until now.

In recent years, some intelligent control approaches [32-34] have a good robustness in spite of parameter variations and unknown external disturbances, since its design is independent of mathematical model of the plant. In [35], a detailed review study has been presented about the artificial intelligence-based control of PMSMs, and pointed out that the fuzzy logic and neural network control have been successfully applied for PMSM drive systems. Nonetheless, it is not an easy task to obtain an optimal set of fuzzy membership functions and rules, and real time implementation of these and such-like methods is difficult due to their algorithm complexity.

A state-dependent Riccati equation-based controller has been studied in [36], which is also referred to nonlinear quadratic optimal control. The controller requires solving an algebraic Riccati equation [37,38]. A nonlinear speed control of interior permanent magnet has been proposed in [39], which has used Model Reference Adaptive Control (MRAC) for estimating state variables of nonlinear control systems during periods of time when the measurements of the related state variables are not available for feedback [40].

Backstepping control is a recursive design methodology for the feedback control of uncertain nonlinear systems, mostly for the systems with matched uncertainties [41,42]. An adaptive robust controller based on backstepping control method has been proposed for the speed control of PMSM [43]. The controller is robust against stator resistance, viscous friction uncertainties and load torque disturbance. However, this approach uses the feedback linearization, which cancels some useful nonlinearity out. Other adaptive nonlinear backstepping approaches that do not use the linearization theorems are proposed for the control of electromechanical systems [44-52].

In this paper, first a general model is presented for both BLDCM and PMSM. After introducing a novel nonsingular terminal sliding surface, its finite-time convergence is proved. Then, to tackle the unknown parameters, appropriate adaptive laws are proposed. Bounded unknown model uncertainties and external disturbances are considered in all state equations. Subsequently, based on the adaptive laws and robust finite-time control technique, a control law is designed to force the trajectories onto the sliding surface, and remain on it forever.

The rest of this paper is organized as follows: In Section 2, a brief description of the chaotic BLDCM and PMSM, and preliminary lemmas are given, and the control problem is formulated. In Section 3, a new nonsingular terminal sliding surface is introduced, and the proposed robust adaptive finite-time sliding mode controller is designed. In Section 4, some numerical simulations are given. Finally, this paper ends with some conclusions in Section 5.

2. System description, problem formulation and preliminary definitions and lemmas

In this section, first a brief description of nonlinear
chaotic BLDCM and PMSM systems is given. Then, a general nonlinear model is represented for both systems. The problem of adaptive robust finite-time stabilization of the general nonlinear model with completely unknown parameters and uncertainties is formulated, and preliminary definitions and lemmas are presented.

2.1. Dynamics of brushless DC motor
Using time scaling and an affine state transformation, a non-dimensionalized model has been derived for BLDCM with smooth air gap [28]. It has been shown that the unforced transformed system exhibits chaotic behavior. The model is given by:

\[\dot{I}_q = -I_q - I_d \omega + \rho \omega + v_q,\]
\[\dot{I}_d = -\delta I_d + I_q \omega + v_d,\]
\[\dot{\omega} = \varepsilon I_q - \mu \omega + \partial I_d - T_L,\]
where \(\rho, \delta, \varepsilon, \mu\), and \(\varepsilon\) are system parameters; \(I_q, I_d\), and \(\omega\) are the transformed quadrature-axis for stator current, the transformed direct-axis for stator current, and the transformed angular velocity, respectively; \(v_q, v_d\), and \(T_L\) are transformed quadrature-axis for stator voltage, the transformed direct-axis for stator voltage, and the transformed external load torque (including friction), respectively.

2.2. Dynamics of permanent magnet synchronous motor
The mathematical normalized model of a conventional surface mounted PMSM can be given with standard assumptions in the \(d-q\) frame [53, 54] in the following:

\[\dot{I}_q = -I_q - I_d \omega + \lambda \omega + v_q,\]
\[\dot{I}_d = -\gamma I_d + I_q \omega + v_d,\]
\[\dot{\omega} = \gamma (I_q - \omega) - T_L,\]
where \(\lambda, \gamma, \lambda, \text{ and } \gamma\) are system parameters. It follows from the equations above that PMSM is highly nonlinear system owing to the cross coupling effect between the electrical current and speed state equations. In a practical way, measurement and calculation of the electrical parameters are available; however, it must be noted that they vary with operating conditions, primarily temperature and saturation effects. As far as the mechanical parameters, it is not possible to practically measure or calculate the exact values of them. These parameters vary with operating conditions as well, primarily applied load torque disturbance. Even worse, the load torque disturbance is always unknown. In that respect, it is clear that industrial PMSM drive systems encounter unavoidable parameter variations and immeasurable disturbances.

2.3. General model
From a control engineering point of view, BLDCM system in Eq. (1) and PMSM system (2) with control inputs can be rewritten in the following general model form:

\[\dot{x}_1 = \alpha f_1(x) + \Delta f_1(x, t) + d_1(t) + u_1(t),\]
\[\dot{x}_2 = \beta f_2(x) + \Delta f_2(x, t) + d_2(t) + u_2(t),\]
\[\dot{x}_3 = \theta f_3(x) + \Delta f_3(x, t) + d_3(t) + u_3(t),\]
where \(x_1, x_2, x_3, (x = [x_1, x_2, x_3]^T)\), denote \(I_q, I_d\), and \(\omega\), respectively; \(u_1, u_2\), and \(u_3\), \((u = [u_1, u_2, u_3]^T)\), are control inputs to be designed later; \(\Delta f_i(x, t)\) and \(d_i(t)\) (\(i = 1, 2, 3\)) are unknown model uncertainties and external disturbances, respectively; \(f_1(x) = [x_1, x_2, x_3]^T, f_2(x) = [x_2, x_1, x_3]^T\) and \(f_3(x) = [x_1, x_3, x_1 x_3]^T\) are nonlinear terms of equations, and \(\alpha = [\alpha_1, \alpha_2, \alpha_3], \beta = [\beta_1, \beta_2]\) and \(\theta = [\theta_1, \theta_2, \theta_3]\) are the vector parameters of the system (3).

Assumption 1: We assume that system uncertainties are bounded [55] as follows:

\[|\Delta f_i(x, t)| \leq a_i, \quad i = 1, 2, 3,\]
where \(a_i, i = 1, 2, 3\), are given positive constants.

Assumption 2: In general, it is assumed that external disturbances are norm-bounded in \(C^1\), i.e.:

\[|d_i(t)| \leq b_i, \quad i = 1, 2, 3,\]
where \(b_i, i = 1, 2, 3\) are known positive constants.

Assumption 3: The unknown vector parameters \(\alpha, \beta\) and \(\theta\) are norm-bounded, i.e.:

\[|\alpha| \leq D_\alpha, \quad |\beta| \leq D_\beta, \quad |\theta| \leq D_\theta,\]
where \(D_\alpha, D_\beta\), and \(D_\theta\) are known positive constants.

Lemma 1. [56] Assume that a continuous, positive-definite function \(V(t)\) satisfies the following differential inequality:

\[\dot{V}(t) \leq -\nu V(t), \quad \forall t \geq t_0, \quad V(t_0) \geq 0,\]
where \(\nu > 0, 0 < \eta < 1\), are two constants. Then, for any given \(t_0\), \(V(t)\) satisfies the following inequality:

\[V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - \nu (1 - \eta)(t - t_0),\]
\[t_0 \leq t \leq t_1,\]
and $V(t) \equiv 0$, \( \forall t \geq t_1 \), with $t_1$ given by:

$$ t_1 = t_0 + \frac{V^{1-q}(t_0)}{\nu(1-q)} \quad (9) $$

**Lemma 2.** [57] Suppose $a_1, a_2, \ldots, a_n$ and $0 < q < 2$ are all real numbers, then the following inequality holds:

$$ |a_1|^q + |a_2|^q + \cdots + |a_n|^q \geq (a_1^q + a_2^q + \cdots + a_n^q)^{\frac{1}{q}}. \quad (10) $$

3. Design of finite-time sliding mode controller

In this section, a new nonsingular terminal sliding mode controller is designed to stabilize the chaotic system in Eq. (3) with unknown uncertainties, external disturbances and unknown parameters in finite time.

The design procedure of the proposed finite-time controller comprises two main steps:

(a) Selecting a nonsingular terminal sliding surface for the desired sliding motion;

(b) Designing a robust adaptive finite-time control law to guarantee the existence of the sliding motion in a given finite time.

Consequently, in this paper, a new nonsingular terminal sliding surface is proposed as:

$$ s_i(t) = |x_i(t)| + \gamma_i \left( \int_0^t |x_i(t)| \, dt \right)^{\frac{p}{q}}, \quad i = 1, 2, 3, \quad (11) $$

where $s_i(t) \in R$, $\gamma_i > 0$, $p$ and $q$ are positive odd integers and satisfy $p < q$.

Let $\omega_i(t) = \int_0^t |x_i(t)| \, dt$, it is obvious that $\omega_i(t) \geq 0$, $i = 1, 2, 3$. For the existence of the sliding mode, it is necessary and sufficient that $s(t) = 0$, $\dot{s}(t) = 0$ [58]. Therefore, the dynamics of the proposed nonsingular terminal sliding mode can be obtained as:

$$ s_i(t) = \dot{\omega}_i(t) + \gamma_i(\omega_i(t))^{\frac{p}{q}} = 0, \quad i = 1, 2, 3, \quad (12) $$

$$ \dot{\omega}_i(t) = - \gamma_i(\omega_i(t))^{\frac{p}{q}}, \quad i = 1, 2, 3. \quad (13) $$

It is clear that as $\omega_i(t)$ can reach the equilibrium point in finite time $T_1$, system states $x_i(t)$, $(i = 1, 2, 3)$ can reach equilibrium point in finite time $T_1$, too.

**Theorem 1.** Consider the sliding mode dynamics in Eq. (13). This system is finite-time stable and its trajectories converge to the equilibrium point, in a finite time, $T_1 = \frac{V_1(0)^{1-q}}{2q(1-q)}$.

**Proof.** Consider the following positive definite function:

$$ V_1(t) = \sum_{i=1}^3 \omega_i^2. \quad (14) $$

Its derivative with respect to time is:

$$ \dot{V}_1(t) = 2 \sum_{i=1}^3 \omega_i \dot{\omega}_i. \quad (15) $$

Replacing $\omega_i$ from Eq. (13) into the above equation, it yields:

$$ \dot{V}_1(t) = -2 \sum_{i=1}^3 \gamma_i(\omega_i(t))^{\frac{p}{q}}. \quad (16) $$

Since $1 < \frac{p+2}{2q} < 2$, and assuming $\sigma = \min \{ \gamma_i, (i = 1, 2, 3) \}$, using Lemma 2, we have:

$$ \dot{V}_1(t) \leq -2\sigma \left( \sum_{i=1}^3 \omega_i^2 \right)^{\frac{p}{q}+\frac{q}{p}} = -2\sigma(V_1(t))^{\frac{p+2}{2q}}. \quad (17) $$

Since $0.5 < \frac{p+2}{2q} < 1$, using Lemma 1, one can conclude that the states, $x_i(t)$, $(i = 1, 2, 3)$ will converge to zero in the finite time, $T_1 = \frac{V_1(0)^{1-q}}{2q(1-q)}$. Hence the proof is completed.

After establishing the suitable sliding manifold, the next step is to design a control law to force the state trajectories go on to the sliding surface within a finite time, and remain on it forever. Therefore, to ensure that the existence of the sliding motion (i.e. to ensure that the state trajectories $x_i(t)$, $(i = 1, 2, 3)$ converge to the sliding surface $s_i(t) = 0$, $(i = 1, 2, 3)$, the nonsingular sliding mode finite-time control laws are proposed as follows:

$$ u_1(t) = - \alpha f_1(x) $$

$$ \quad - \gamma_1(p/q)x_1(t) \left( \int_0^t |x_1(t)| \, dt \right)^{(p/q)-1} $$

$$ \quad - \left( D_D \left( \frac{s_1}{\|s\|^2} \right) + k_1 + a_1 + b_1 \right) \text{sgn}(x_1). $$

$$ u_2(t) = - \beta f_2(x) $$

$$ \quad - \gamma_2(p/q)x_2(t) \left( \int_0^t |x_2(t)| \, dt \right)^{(p/q)-1} $$

$$ \quad - \left( D_D \left( \frac{s_2}{\|s\|^2} \right) + k_2 + a_2 + b_2 \right) \text{sgn}(x_2). $$
Taking the time derivative of $V_2(t)$, one has:

\[
\dot{V}_2(t) = \sum_{i=1}^{3} s_i \dot{x}_i(t) + \alpha^T \dot{\alpha} + \beta^T \dot{\beta} + \theta^T \dot{\theta}.
\]  

(22)

It is clear that:

\[
\dot{V}_2(t) = \sum_{i=1}^{3} \left[ s_i \left[ x_i(t) \right] \right] 
+ \gamma_i (p/q) |x_i(t)| \left( \int_{0}^{t} |x_i(t)| \, dt \right)^{(p/q) - 1} \]

\[ + \alpha^T \dot{\alpha} + \beta^T \dot{\beta} + \theta^T \dot{\theta}. \]  

(23)

Inserting $\dot{x}_i(t), (i = 1, 2, 3)$ from system in Eq. (3) and adaptive laws from Eq. (20) into Eq. (23), we have:

\[
\dot{V}_2(t) = \sum_{i=1}^{3} \left[ s_i \left[ x_i(t) \right] \right] 
+ s_i [\alpha f_1(x) + \Delta f_1(x, t) + d_1(t) + u_1(t)] \text{sgn}(x_1(t)) 
\]

\[ + s_2 [\beta f_2(x) + \Delta f_2(x, t) + d_2(t) + u_2(t)] \text{sgn}(x_2(t)) \]

\[ + s_3 [\theta f_3(x) + \Delta f_3(x, t) + d_3(t) + u_3(t)] \text{sgn}(x_3(t)) \]

\[ + (\alpha - \dot{\alpha})^T s_1 \text{sgn}(x_1(t)) \]

\[ + (\beta - \dot{\beta})^T s_2 \text{sgn}(x_2(t)) \]

\[ + (\theta - \dot{\theta})^T s_3 \text{sgn}(x_3(t)) \]

\[ + (\dot{\alpha} - \dot{\beta} - \dot{\theta})^T s_1 \text{sgn}(x_1(t)) \]

\[ + (\dot{\beta} - \dot{\beta})^T s_2 \text{sgn}(x_2(t)) \]

\[ + (\dot{\theta} - \dot{\theta})^T s_3 \text{sgn}(x_3(t)) \]

\[ + \alpha^T \dot{\alpha} + \beta^T \dot{\beta} + \theta^T \dot{\theta}. \]  

(24)

Replacing $u_i(t), (i = 1, 2, 3)$ from Eq. (20) into the
above equation and simplifying it, gives:

\[ V_2(t) = \sum_{i=1}^{3} \left[ s_i \left[ \gamma_i(p/q) |x_i(t)| \left( \int_0^t |x_i(t)| d\tau \right)^{(p/q)-1} \right] \right] + s_1 \left[ \alpha f_1(x) + \Delta f_1(x, t) + d_1(t) - \hat{\alpha} f_1(x) \right] - \gamma_1(p/q)x_1(t) \left( \int_0^t |x_1(t)| d\tau \right)^{(p/q)-1} \]

\[ - \left( D_D \left( \frac{s_1}{||x||^2} \right) + k_1 + a_1 + b_1 \right) s_i \left[ \sigma_1 \left[ \gamma_i(p/q) |x_i(t)| \left( \int_0^t |x_i(t)| d\tau \right)^{(p/q)-1} \right] \right] \]

\[ + s_2 \left[ \beta f_2(x) + \Delta f_2(x, t) + d_2(t) + \hat{\beta} f_2(x) \right] - \gamma_2(p/q)x_2(t) \left( \int_0^t |x_2(t)| d\tau \right)^{(p/q)-1} \]

\[ - \left( D_D \left( \frac{s_2}{||x||^2} \right) + k_2 + a_2 + b_2 \right) s_i \left[ \sigma_2 \left[ \gamma_i(p/q) |x_i(t)| \left( \int_0^t |x_i(t)| d\tau \right)^{(p/q)-1} \right] \right] \]

\[ + s_3 \left[ \theta f_3(x) + \Delta f_3(x, t) + d_3(t) - \hat{\theta} f_3(x) \right] - \gamma_3(p/q)x_3(t) \left( \int_0^t |x_3(t)| d\tau \right)^{(p/q)-1} \]

\[ - \left( D_D \left( \frac{s_3}{||x||^2} \right) + k_3 + a_3 + b_3 \right) s_i \left[ \sigma_3 \left[ \gamma_i(p/q) |x_i(t)| \left( \int_0^t |x_i(t)| d\tau \right)^{(p/q)-1} \right] \right] \]

\[ + \hat{\alpha}^T s_1 \left[ \sigma_1 \left[ \gamma_1(p/q) |x_1(t)| \left( \int_0^t |x_1(t)| d\tau \right)^{(p/q)-1} \right] \right] \left[ \gamma_1(p/q) |x_1(t)| \left( \int_0^t |x_1(t)| d\tau \right)^{(p/q)-1} \right] \]

\[ + \hat{\beta}^T s_2 \left[ \sigma_2 \left[ \gamma_2(p/q) |x_2(t)| \left( \int_0^t |x_2(t)| d\tau \right)^{(p/q)-1} \right] \right] \left[ \gamma_i(p/q) |x_i(t)| \left( \int_0^t |x_i(t)| d\tau \right)^{(p/q)-1} \right] \]

\[ + \hat{\theta}^T s_3 \left[ \sigma_3 \left[ \gamma_3(p/q) |x_3(t)| \left( \int_0^t |x_3(t)| d\tau \right)^{(p/q)-1} \right] \right] \left[ \gamma_i(p/q) |x_i(t)| \left( \int_0^t |x_i(t)| d\tau \right)^{(p/q)-1} \right] \]

Using the fact \( \sum_{i=1}^{3} s_i (\frac{s_i}{||x||^2}) = 1 \), we have:

\[ \dot{V}_2(t) = s_1 \left[ \Delta f_1(x, t) + d_1(t) \right] - (k_1 + a_1 + b_1) s_1 \left[ \sigma_1 \left[ \gamma_1(p/q) |x_1(t)| \left( \int_0^t |x_1(t)| d\tau \right)^{(p/q)-1} \right] \right] \]

\[ + s_2 \left[ \Delta f_2(x, t) + d_2(t) \right] - (k_2 + a_2 + b_2) s_2 \left[ \sigma_2 \left[ \gamma_2(p/q) |x_2(t)| \left( \int_0^t |x_2(t)| d\tau \right)^{(p/q)-1} \right] \right] \]

\[ + s_3 \left[ \Delta f_3(x, t) + d_3(t) \right] - (k_3 + a_3 + b_3) s_3 \left[ \sigma_3 \left[ \gamma_3(p/q) |x_3(t)| \left( \int_0^t |x_3(t)| d\tau \right)^{(p/q)-1} \right] \right] - D_D \]

It is obvious that:

\[ \dot{V}_2(t) \leq \left[ s_i \left[ ||\Delta f_i(x, t)|| + |d_i(t)| \right] \right] - (k_1 + a_1 + b_1) s_1 \left[ \sigma_1 \left[ \gamma_1(p/q) |x_1(t)| \left( \int_0^t |x_1(t)| d\tau \right)^{(p/q)-1} \right] \right] \]

\[ + s_2 \left[ \Delta f_2(x, t) + d_2(t) \right] - (k_2 + a_2 + b_2) s_2 \left[ \sigma_2 \left[ \gamma_2(p/q) |x_2(t)| \left( \int_0^t |x_2(t)| d\tau \right)^{(p/q)-1} \right] \right] \]

\[ + s_3 \left[ \Delta f_3(x, t) + d_3(t) \right] - (k_3 + a_3 + b_3) s_3 \left[ \sigma_3 \left[ \gamma_3(p/q) |x_3(t)| \left( \int_0^t |x_3(t)| d\tau \right)^{(p/q)-1} \right] \right] - D_D \]

(28)

Using Assumptions 1 and 2 and Eq. (19), one can conclude that:

\[ \dot{V}_2(t) \leq - \sum_{i=1}^{3} k_i |s_i| - \mu (\|\ddot{\delta}\| + D_a + \|\dddot{\delta}\| + D_B + \|\dddot{\theta}\| + D_B) \]

(29)

Using Assumption 3 and since \( ||\dot{\delta} - \alpha|| \leq ||\dot{\delta}|| + ||\alpha|| \leq ||\dot{\delta}|| + D_a \), \( ||\beta - \beta|| \leq ||\beta|| + ||\beta|| \leq ||\beta|| + D_B \), and \( ||\theta - \theta|| \leq ||\theta|| + ||\theta|| \leq ||\theta|| + D_B \), one can conclude that:

\[ - (||\dot{\delta}|| + D_a) \leq - ||\dot{\delta} - \alpha|| \leq - ||\dot{\delta}|| + D_B \] and \( - (||\theta|| + D_B) \leq - ||\theta - \theta|| \leq ||\theta|| + D_B \), this yields:

\[ \dot{V}_2(t) \leq - \sum_{i=1}^{3} k_i |s_i| - \mu (||\dot{\delta} - \alpha|| + ||\dot{\beta} - \beta|| + ||\ddot{\theta} - \theta||) \]

(30)

According to Lemma 2, one has:

\[ \dot{V}_2(t) \]

\[ - \zeta \left[ \sum_{i=1}^{3} |s_i| + \left( ||\ddot{\delta} - \alpha|| + ||\ddot{\beta} - \beta|| + ||\ddot{\theta} - \theta|| \right) \right] \leq - \sqrt{2} \zeta \left( \sum_{i=1}^{3} \frac{1}{2} ||s_i||^2 + \frac{1}{2} ||\dot{\delta} - \alpha||^2 \right) \]

\[ + \frac{1}{2} \left( ||\dot{\beta} - \beta||^2 + ||\dot{\theta} - \theta||^2 \right)^{1/2} = - \sqrt{2} \zeta V_2(t)^{1/2} \]

(31)

where \( \zeta = \min \{k_i, \mu (i = 1, 2, 3) \} \).

Therefore, from Lemma 1, the state trajectories \( \{x_i(t), i = 1, 2, 3\} \) will converge to the sliding surface \( s_i(t) = 0, (i = 1, 2, 3) \) in the finite time, \( T_2 = \frac{\sqrt{V_2(0)}}{\sqrt{2} \zeta} \). Hence the proof is completed.
Remark 1. According to the Theorems 1 and 2, the sliding mode control law in Eq. (18) with adaptive laws in Eq. (20) and the nonsingular terminal sliding surface in Eq. (13) can make the system in Eq. (3) states reach zero in the finite time, $T = T_1 + T_2$.

Remark 2. The proposed nonsingular terminal sliding mode in Eq. (11) is different from the previously reported terminal sliding mode control ($s = \dot{e} + \beta e^{p/\gamma}$) and fast terminal sliding mode control ($s = \dot{e} + \alpha e + \beta e^{p/\gamma}$) where $\alpha, \beta > 0$, $p > q > 0$ are odd integers. Since the control inputs of the conventional terminal sliding mode control and fast terminal sliding mode control approaches contain the term $e^{p/\gamma}$, one can see that for $e < 0$, the fractional power $\frac{p}{\gamma} - 1$ may lead to the term $e^{\frac{p}{\gamma}-1} \notin R$, which leads to $\dot{e} \notin R$. Our proposed nonsingular terminal sliding mode in Eq. (11) overcomes this singularity.

4. Numerical simulations

In this section, numerical simulations are presented to verify the efficiency and effectiveness of the proposed controllers. Numerical simulations are carried out using MATLAB software. The ode45 solver is used for solving differential equations.

4.1. BLDCM

The nonlinear equations of BLDCM are as follows:

$$
\begin{align*}
\dot{x}_1 &= -x_1 - x_2x_3 + 7.961x_2 + \Delta f_1(x, t) + d_1(t) + u_1(t) \\
\dot{x}_2 &= -0.84x_2 + x_1x_3 + \Delta f_2(x, t) + d_2(t) + u_2(t) \\
\dot{x}_3 &= 3.708x_3 + 2x_2x_3 + \Delta f_3(x, t) + d_3(t) + u_3(t)
\end{align*}
$$

(32)

where $\alpha = [\alpha_1, \alpha_2, \alpha_3] = [-1, -1, 7.961]$, $\beta = [\beta_1, \beta_2, \beta_3] = [-0.84, 1]$ and $\theta = [\theta_1, \theta_2, \theta_3] = [0, -3.708, 1]$ are the vector parameters of the system in Eq. (3).

The following uncertainties and external disturbances are added to the BLDCM system:

$$
\begin{align*}
\Delta f_1(x, t) &= 0.5 \sin(x_1) - 0.1 \cos(2t), \\
\Delta f_2(x, t) &= 0.3 \cos(3x_2) + 0.1 \cos(5t), \\
\Delta f_3(x, t) &= -0.4 \sin(2x_3) + 0.2 \sin(3t), \\
d_1(t) &= 0.3 \cos(5t), \\
d_2(t) &= 0.2 \sin(3t), \\
d_3(t) &= -0.25 \cos(2t).
\end{align*}
$$

The initial values and the norm bounds of the adaptive vectors are $\hat{\alpha}_0 = 0$, $\hat{\beta}_0 = 0$, $\theta_0 = 0$ and $D_\alpha = 33$, $D_\beta = 33$, $D_\theta = 33$, respectively. The controller parameters are $K = [K_1, K_2, K_3] = [0.1, 0.1, 0.1]$, $\gamma = [\gamma_1, \gamma_2, \gamma_3] = [10, 10, 2]$, $p = 3$, $q = 5$, $a = [a_1, a_2, a_3] = [0.5, 0.5, 0.5]$, $b = [b_1, b_2, b_3] = [0.5, 0.5, 0.5]$.

The state trajectories of the controlled uncertain chaotic BLDCM and the applied control inputs are depicted in Figures 1 and 2. It can be seen that the closed loop system is not chaotic anymore. The time responses of adaptive vector parameters $\hat{\alpha}$, $\hat{\beta}$ and $\theta$ are depicted in Figure 3. It is clear that the adaptive parameters converge to some constants. It can be seen that the control inputs are bounded and feasible in practice. Also, it is obvious that not only the system is stabilized around the origin, but also the closed loop system is robust against model uncertainties and external disturbances.

4.2. PMSM

PMSM is known to exhibit chaotic behavior for a certain range of system parameters. It was shown in [59] that PMSM system given in Eq. (34) behaves in a chaotic mode for the following system parameters and initial conditions.

$$
\begin{align*}
\dot{x}_1 &= -x_1 - x_2x_3 + 20x_3 + \Delta f_1(x, t) + d_1(t) + u_1(t) \\
\dot{x}_2 &= -x_2 - x_1x_3 + \Delta f_2(x, t) + d_2(t) + u_2(t) \\
\dot{x}_3 &= 5.5(x_1 - x_3) + \Delta f_3(x, t) + d_3(t) + u_3(t)
\end{align*}
$$

(34)

Figure 1. State trajectories of the controlled uncertain chaotic BLDCM.
\[ \Delta f_2(x, t) = 0.2 \sin(x_2) - 0.5 \cos(5t), \]
\[ \Delta f_3(x, t) = 0.25 \sin(2x_3) + 0.4 \sin(4t), \]
\[ d_1(t) = -0.2 \cos(5t), \]
\[ d_2(t) = 0.1 \sin(3t), \]
\[ d_3(t) = -0.25 \sin(t). \]  

The initial conditions of the PMSM system are selected as \( x_1(0) = 1, x_2(0) = -1, \) and \( x_3(0) = 2. \) The initial values and norm bounds of the adaptive vectors are \( \hat{\alpha}_0 = 0, \hat{\beta}_0 = 0, \hat{\theta}_0 = 0 \) and \( D_\alpha = 40, D_\beta = 50, \)
\( D_\theta = 40, \) respectively. The controller parameters are \( K = [K_1, K_2, K_3] = [0.2, 0.1, 0.3], \gamma = [\gamma_1, \gamma_2, \gamma_3] = [6, 7, 4], p = 3, q = 5, \) \( a = [a_1, a_2, a_3] = [0.7, 0.7, 0.7], b = [b_1, b_2, b_3] = [0.5, 0.5, 0.5]. \)

Figure 4 illustrates PMSM system states. It is obvious that the controller works well for stabilization of the system states around the zero equilibrium even when the whole system with unknown parameters is perturbed by unknown uncertainties and external disturbances. Control inputs are depicted in Figure 5, respectively. One can see that the control signals are practical. The time responses of adaptive vector parameters \( \hat{\alpha}, \hat{\beta} \) and \( \hat{\theta}, \) are depicted in Figure 6, respectively. One can see that the adaptive parameters are bounded.
sliding mode manifold was proposed and its finite-time convergence was proved analytically. Suitable adaptive laws were designed to approach the unknown parameters. Based on the adaptive laws and robust finite-time control method, a robust adaptive finite-time sliding mode controller was introduced. The proposed technique had finite-time convergence and stability in both the reaching and the sliding mode phases. Numerical simulations show that the proposed controllers work well for finite-time stabilization of BLDCM and PMSM systems with unknown parameters, in the presence of both unknown model uncertainties and external disturbances.

References


5. Conclusions

In this paper, the nonsingular terminal sliding mode controller is represented to stabilize chaotic nonlinear BLDCMs and PMSMs with unknown parameters. It is assumed that all states of BLDCM and PMSM are perturbed by unknown model uncertainties and external disturbances. A novel nonsingular terminal

Figure 5. Control inputs applied for the uncertain chaotic PMSM.

Figure 6. Time responses of adaptive vector parameters $\dot{\alpha}$, $\dot{\beta}$ and $\dot{\theta}$ for the uncertain chaotic PMSM.


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