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Introducing a new sliding manifold applied for control of uncertain nonlinear brushless DC and permanent magnet synchronous motors

B. Hashtarkhani[a] and M.P. Aghababa[b,*]

a. *Department of Control Engineering, Faculty of Electrical and Computer Engineering, University of Tabriz, Tabriz, P.O. Box 5166616471, Iran.*
b. *Department of Electrical Engineering, Urmia University of Technology, Urmia, P.O. Box 57155-419, Iran.*

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Abstract. In this paper, the problem of controlling chaotic uncertain brushless DC motor (BLDCM) and Permanent Magnet Synchronous Motor (PMSM), exposed to external disturbances, is considered. First, a new nonsingular terminal sliding surface is introduced, and its finite-time convergence to the zero equilibrium is proved. Then, it is assumed that the parameters of BLDCM and PMSM are fully unknown, and appropriate adaptive laws are derived to tackle the unknown parameters of the systems. Besides, the effects of models uncertainties and external disturbances are also taken into account. Afterwards, based on the adaptive laws and robust finite-time control idea, a robust adaptive sliding mode controller is proposed to ensure the occurrence of the sliding motion in finite time. It is mathematically proved that the introduced nonsingular terminal sliding mode technique has finite-time convergence and stability in both reaching and sliding mode phases. Numerical simulations are presented to verify the efficiency of the proposed method and to validate the theoretical results of the paper.

1. Introduction

Lately, chaos in electric motors has become one of the most attractive research topics due to its direct applications in industrial machinery, electrical locomotives and electrical submersibles thruster drives. In the late 1980s, since the chaos occurrences in electric motors have been concentrated on [1], a wide-ranging research has been done towards the detection of chaos and its control in various types of motors such as DC motors [2-4], step motors [5,6], synchronous reluctance motor drives [7], switched reluctance motor [8,9] and PMSMs [10-13].

A BLDC motor is a synchronous electric motor which is powered by direct-current electricity [14]. The main benefit of BLDCM is the removal of the physical contact between brushes and commutators. BLDCM has been extensively applied in direct-drive applications such as robotics, aerospace, etc. To date, there has been several works done in modeling and controlling of BLDCMs [15]. In [16], a new nonlinear model has been proposed for BLDCM which could demonstrate chaotic performance by some particular alternatives for parameters. Actually, it has shown that the unforced transformed model is equivalent to the well-known Lorenz system. Recently, some modern control methodologies have been proposed for control-

*. *Corresponding author. Tel.: +98 441 355 4180; Fax: +98 441 355 4184*
E-mail addresses: bijansagh@gmail.com (B.Hashtarkhani), m.p.aghababa@ee.uut.ac.ir (M.P. Aghababa)

ling linear brushless permanent magnet DC motors such as nonlinear control [17], optimal control [18], variable structure control [19], PID control [20], adaptive control and backstepping approach [21]. On the other hand, these approaches are applied for a linear BLDCM that does not show chaotic and complex dynamics. Chaotic performance in BLDCM, which comes out mostly alternating ripples of torque and low-frequency oscillations of rotational speed of motor, can tremendously obliterate the stabilization and performance of the motor. Therefore, it is crucial to investigate the method of controlling and suppressing chaos in BLDCMs. In [22], piecewise quadratic state feedback controller has been proposed, but the equilibrium points of the uncontrolled chaotic BLDCM system did not apply as the control target. In [23], time-delay feedback and fuzzy control approaches have been used to stabilize the system around the origin, but a noticeable chattering phenomenon is observed. In [24], feedback linearization and sliding mode control techniques have been developed to control a chaotic BLDCM. Only stator current state equation has been perturbed by uncertainties. While in practical applications, the whole dynamics of the system is disturbed by uncertainties and external disturbances. Also, no control parameter selection procedure is used in the above mentioned methods. Recently, chaotic anticontrol and chaos synchronization of BLDCM systems have been studied [25-29]. In [30], a sliding mode controller has been designed, which has some practical advantages such as fast response, low sensitivity to external disturbances and robustness to the system uncertainties.

Furthermore, chaos in PMSM and its control are other areas of challenging research in the field of nonlinear control of electric motors. PMSM has been attracting more and more attention in high-accuracy and high performance electric drive systems in industrial motion control applications due to its advantages of superior power density, high-performance motion control with fast speed and better accuracy, large torque to inertia ratio and long life over other types of motors such as DC motors and induction motors [31]. But in industrial applications, there are many uncertainties such as system parameter uncertainty, external load disturbance, friction force, unmodeled uncertainty, etc., which always diminish the performance quality of the pre-design of the motor driving system. To cope with this problem, robust or adaptive or other control schemes dealing with parameter uncertainties and unknown external disturbances have extensively been studied up until now.

In recent years, some intelligent control approaches [32-34] have a good robustness in spite of parameter variations and unknown external disturbances, since its design is independent of mathematical model of the plant. In [35], a detailed review study has been presented about the artificial intelligence-based control of PMSMs, and pointed out that the fuzzy logic and neural network control have been successfully applied for PMSM drive systems. Nonetheless, it is not an easy task to obtain an optimal set of fuzzy membership functions and rules, and real time implementation of these and suchlike methods is difficult due to their algorithm complexity.

A state-dependent Riccati equation-based controller has been studied in [36], which is also referred to nonlinear quadratic optimal control. The controller requires solving an algebraic Riccati equation [37,38].

A nonlinear speed control of interior PMSM has been proposed in [39], which has used Model Reference Adaptive Control (MRAC) for estimating state variables of nonlinear control systems during periods of time when the measurements of the related state variables are not available for feedback [40].

Backstepping control is a recursive design methodology for the feedback control of uncertain nonlinear systems, mostly for the systems with matched uncertainties [41,42]. An adaptive robust controller based on backstepping control method has been proposed for the speed control of PMSM [43]. The controller is robust against stator resistance, viscous friction uncertainties and load torque disturbance. However, this approach uses the feedback linearization, which cancels some useful nonlinearity out. Other adaptive nonlinear backstepping approaches that do not use the linearization theorems are proposed for the control of electromechanical systems [44-52].

In this paper, first a general model is presented for both BLDCM and PMSM. After introducing a novel nonsingular terminal sliding surface, its finite-time convergence is proved. Then, to tackle the unknown parameters, appropriate adaptive laws are proposed. Bounded unknown model uncertainties and external disturbances are considered in all state equations. Subsequently, based on the adaptive laws and robust finite-time control technique, a control law is designed to force the trajectories onto the sliding surface, and remain on it forever.

The rest of this paper is organized as follows: In Section 2, a brief description of the chaotic BLDCM and PMSM, and preliminary lemmas are given, and the control problem is formulated. In Section 3, a new nonsingular terminal sliding surface is introduced, and the proposed robust adaptive finite-time sliding mode controller is designed. In Section 4, some numerical simulations are given. Finally, this paper ends with some conclusions in Section 5.

2. System description, problem formulation and preliminary definitions and lemmas

In this section, first a brief description of nonlinear chaotic BLDCM and PMSM systems is given. Then, a general nonlinear model is represented for both systems. The problem of adaptive robust finite-time stabilization of the general nonlinear model with completely unknown parameters and uncertainties is formulated, and preliminary definitions and lemmas are presented.

2.1. Dynamics of brushless DC motor

Using time scaling and an affine state transformation, a non-dimensionalized model has been derived for BLDCM with smooth air gap [28]. It has been shown that the unforced transformed system exhibits chaotic behavior. The model is given by:

$$\dot{I}_q = -I_q - I_d\omega + \rho\omega + v_q,$$

$$\dot{I}_d = -\delta I_d + I_q\omega + v_d,$$

$$\dot{\omega} = \varepsilon I_q - \mu\omega + \vartheta I_q I_d - T_L, \tag{1}$$

where ρ, δ, ϑ, μ and ε are system parameters; I_q, I_d and ω are the transformed quadrature-axis for stator current, the transformed direct-axis for stator current, and the transformed angular velocity, respectively; v_q, v_d and T_L are transformed quadrature-axis for stator voltage, the transformed direct-axis for stator voltage, and the transformed external load torque (including friction), respectively.

2.2. Dynamics of permanent magnet synchronous motor

The mathematical normalized model of a conventional surface mounted PMSM can be given with standard assumptions in the $d-q$ frame [53,54] in the following:

$$\dot{I}_q = -I_q - I_d\omega + \lambda\omega + v_q,$$

$$\dot{I}_d = -I_d + I_q\omega + v_d,$$

$$\dot{\omega} = \gamma\left(I_q - \omega\right) - T_L, \tag{2}$$

where I_d and I_q are the $d-q$ axis currents, v_d and v_q are the $d-q$ axis voltages, ω is the rotor speed and T_L also represents the applied load torque disturbance; λ and γ are system parameters. It follows from the equations above that PMSM is highly nonlinear system owing to the cross coupling effect between the electrical current and speed state equations. In a practical way, measurement and calculation of the electrical parameters are available; however, it must be noted that they vary with operating conditions, primarily temperature and saturation effects. As for the mechanical parameters, it is not possible to practically measure or calculate the exact values of them. These parameters vary with operating conditions as well, primarily applied load torque disturbance. Even worse, the load torque disturbance is always unknown. In that respect, it is clear that industrial PMSM drive systems encounter unavoidable parameter variations and immeasurable disturbances.

2.3. General model

From a control engineering point of view, BLDCM system in Eq. (1) and PMSM system (2) with control inputs can be rewritten in the following general model form:

$$\dot{x}_1 = \alpha f_1(x) + \Delta f_1(x,t) + d_1(t) + u_1(t),$$

$$\dot{x}_2 = \beta f_2(x) + \Delta f_2(x,t) + d_2(t) + u_2(t),$$

$$\dot{x}_3 = \theta f_3(x) + \Delta f_3(x,t) + d_3(t) + u_3(t), \tag{3}$$

where x_1, x_2 and x_3, ($x = [x_1, x_2, x_3]^T$), denote I_q, I_d and ω, respectively; u_1, u_2 and u_3, ($u = [u_1, u_2, u_3]^T$), are control inputs to be designed later; $\Delta f_i(x,t)$ and $d_i(t)$ ($i = 1,2,3$), are unknown model uncertainties and external disturbances, respectively; $f_1(x) = [x_1.x_2x_3, x_3]^T$, $f_2(x) = [x_2.x_1x_3]^T$ and $f_3(x) = [x_1, x_3, x_1x_2]^T$ are nonlinear terms of equations, and $\alpha = [\alpha_1, \alpha_2, \alpha_3]$, $\beta = [\beta_1, \beta_2]$ and $\theta = [\theta_1, \theta_2, \theta_3]$ are the vector parameters of the system (3).

Assumption 1: We assume that system uncertainties are bounded [55] as follows:

$$|\Delta f_i(x,t)| \le a_i, \qquad i = 1,2,3, \tag{4}$$

where a_i, $i = 1,2,3$, are given positive constants.

Assumption 2: In general, it is assumed that external disturbances are norm-bounded in C^1, i.e.:

$$|d_i(t)| \le b_i, \qquad i = 1,2,3, \tag{5}$$

where b_i, $i = 1,2,3$ are known positive constants.

Assumption 3: The unknown vector parameters α, β and θ are norm-bounded, i.e.:

$$\|\alpha\| \le D_\alpha, \qquad \|\beta\| \le D_\beta, \qquad \|\theta\| \le D_\theta, \tag{6}$$

where D_α, D_β and D_θ are known positive constants.

Lemma 1. [56] Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \le -\nu V^\eta(t), \qquad \forall t \ge t_0, \qquad V(t_0) \ge 0, \tag{7}$$

where $\nu > 0$, $0 < \eta < 1$, are two constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0) - \nu(1-\eta)(t - t_0),$$

$$t_0 \le t \le t_1, \tag{8}$$

and $V(t) \equiv 0$, $\forall t \geq t_1$, with t_1 given by:

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{\nu(1-\eta)}. \tag{9}$$

Lemma 2. [57] Suppose $a_1, a_2, \cdots, a_n$ and $0 < q < 2$ are all real numbers, then the following inequality holds:

$$|a_1|^q + |a_2|^q + \cdots + |a_n|^q \geq (a_1^2 + a_2^2 + \cdots + a_n^2)^{\frac{q}{2}}. \tag{10}$$

3. Design of finite-time sliding mode controller

In this section, a new nonsingular terminal sliding mode controller is designed to stabilize the chaotic system in Eq. (3) with unknown uncertainties, external disturbances and unknown parameters in finite time. The design procedure of the proposed finite-time controller comprises two main steps:

(a) Selecting a nonsingular terminal sliding surface for the desired sliding motion;

(b) Designing a robust adaptive finite-time control law to guarantee the existence of the sliding motion in a given finite time.

Consequently, in this paper, a new nonsingular terminal sliding surface is proposed as:

$$s_i(t) = |x_i(t)| + \gamma_i \left(\int_0^t |x_i(t)|\, d\tau \right)^{p/q},$$

$$i = 1, 2, 3, \tag{11}$$

where $s_i(t) \in R$, $\gamma_i > 0$, p and q are positive odd integers and satisfy $p < q$.

Let $\omega_i(t) = \int_0^t |x_i(t)| d\tau$, it is obvious that $\omega_i(t) \geq 0$, $i = 1, 2, 3$. For the existence of the sliding mode, it is necessary and sufficient that $s(t) = 0$, $\dot{s}(t) = 0$ [58]. Therefore, the dynamics of the proposed nonsingular terminal sliding mode can be obtained as:

$$s_i(t) = \dot{\omega}_i(t) + \gamma_i(\omega_i(t))^{p/q} = 0,$$

$$i = 1, 2, 3, \tag{12}$$

$$\dot{\omega}_i(t) = -\gamma_i(\omega_i(t))^{p/q},$$

$$i = 1, 2, 3. \tag{13}$$

It is clear that as $\omega_i(t)$ can reach the equilibrium point in finite time T_1, system states $x_i(t)$, $(i = 1, 2, 3)$ can reach equilibrium point in finite time T_1, too.

Theorem 1. Consider the sliding mode dynamics in Eq. (13). This system is finite-time stable and its trajectories converge to the equilibrium point, in a finite time, $T_1 = \frac{V_1(0)^{1-\frac{p+q}{2q}}}{2\sigma(1-\frac{p+q}{2q})}$.

Proof. Consider the following positive definite function:

$$V_1(t) = \sum_{i=1}^{3} \omega_i^2. \tag{14}$$

Its derivative with respect to time is:

$$\dot{V}_1(t) = 2\sum_{i=1}^{3} \omega_i \dot{\omega}_i. \tag{15}$$

Replacing $\dot{\omega}_i$ from Eq. (13) into the above equation, it yields:

$$\dot{V}_1(t) = -2\sum_{i=1}^{3} \gamma_i(\omega_i)^{\frac{p+q}{q}}. \tag{16}$$

Since $1 < \frac{p+q}{q} < 2$, and assuming $\sigma = \min\{\gamma_i, (i = 1, 2, 3)\}$, using Lemma 2, we have:

$$\dot{V}_1(t) \leq -2\sigma \left(\sum_{i=1}^{3} \omega_i^2 \right)^{\frac{p+q}{2q}} = -2\sigma(V_1(t))^{\frac{p+q}{2q}}. \tag{17}$$

Since $0.5 < \frac{p+q}{2q} < 1$, using Lemma 1, one can conclude that the states, $x_i(t)$, $(i = 1, 2, 3)$ will converge to zero in the finite time, $T_1 = \frac{V_1(0)^{1-\frac{p+q}{2q}}}{2\sigma\left(1-\frac{p+q}{2q}\right)}$. Hence the proof is completed.

After establishing the suitable sliding manifold, the next step is to design a control law to force the state trajectories go on to the sliding surface within a finite time, and remain on it forever. Therefore, to ensure that the existence of the sliding motion (i.e. to ensure that the state trajectories $x_i(t)$, $(i = 1, 2, 3)$ converge to the sliding surface $s_i(t) = 0$, $(i = 1, 2, 3)$, the nonsingular sliding mode finite-time control laws are proposed as follows:

$$u_1(t) = -\hat{\alpha} f_1(x) - \gamma_1(p/q)x_1(t) \left(\int_0^t |x_1(t)|\, d\tau \right)^{(p/q)-1} - \left(D_D \left(\frac{s_1}{\|s\|^2} \right) + k_1 + a_1 + b_1 \right) sgn(x_1),$$

$$u_2(t) = -\hat{\beta} f_2(x) - \gamma_2(p/q)x_2(t) \left(\int_0^t |x_2(t)|\, d\tau \right)^{(p/q)-1} - \left(D_D \left(\frac{s_2}{\|s\|^2} \right) + k_2 + a_2 + b_2 \right) sgn(x_2),$$

$$u_3(t) = -\hat{\theta} f_3(x) - \gamma_3(p/q)x_3(t)\left(\int_0^t |x_3(t)|\, d\tau\right)^{(p/q)-1} - \left(D_D\left(\frac{s_3}{\|s\|^2}\right) + k_3 + a_3 + b_3\right) sgn(x_3), \quad (18)$$

where $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\theta}$ are estimations for α, β and θ, respectively; k_i, $(i = 1, 2, 3)$ are the switching gains and positive constants, and:

$$D_D = \mu\left(\|\hat{\alpha}\| + D_\alpha + \left\|\hat{\beta}\right\| + D_\beta + \left\|\hat{\theta}\right\| + D_\theta\right), \quad (19)$$

where μ is a positive constant.

We assume that $(\frac{s_i}{\|s\|^2}) = 0$, $(i = 1, 2, 3)$, if we have $\|s\| = 0$.

To tackle the unknown parameters, the appropriate adaptive laws are defined as:

$$\begin{aligned} &\dot{\hat{\alpha}}(t) = s_1 sgn(x_1)[f_1(x)]^T, \\ &\hat{\alpha}(0) = \hat{\alpha}_0, \\ &\dot{\hat{\beta}}(t) = s_2 sgn(x_2)[f_2(x)]^T, \\ &\hat{\beta}(0) = \hat{\beta}_0, \\ &\dot{\hat{\theta}}(t) = s_3 sgn(x_3)[f_3(x)]^T, \\ &\hat{\theta}(0) = \hat{\theta}_0, \end{aligned} \quad (20)$$

here, $\hat{\alpha}_0$, $\hat{\beta}_0$ and $\hat{\theta}_0$ are initial values of the adaptive vectors $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\theta}$, respectively.

The proposed adaptive control scheme in Eq. (18) with adaptive laws in Eq. (20) will guarantee the finite-timely occurrence of the sliding motion, which is proved in the following theorem.

Theorem 2. If the system in Eq. (3) is controlled with the control law in Eq. (18) and adaptive laws in Eq. (20), then the states of the system will move toward the sliding surface and will approach the sliding surface $s_i(t) = 0$, $(i = 1, 2, 3)$ in the finite time, $T_2 = \frac{\sqrt{V_2(0)}}{\sqrt{2}\zeta}$.

Proof. Choose a positive definite function in the form of:

$$V_2(t) = \frac{1}{2}\sum_{i=1}^{3} \|s_i\|^2 + \|\hat{\alpha} - \alpha\|^2 + \left\|\hat{\beta} - \beta\right\|^2 + \left\|\hat{\theta} - \theta\right\|^2, \quad (21)$$

where $\tilde{\alpha} = \hat{\alpha} - \alpha$, $\tilde{\beta} = \hat{\beta} - \beta$ and $\tilde{\theta} = \hat{\theta} - \theta$ are the parameters errors (It is clear that $\dot{\tilde{\alpha}} = \dot{\hat{\alpha}}$, $\dot{\tilde{\beta}} = \dot{\hat{\beta}}$ and $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$).

Taking the time derivative of $V_2(t)$, one has:

$$\dot{V}_2(t) = \sum\nolimits_{i=1}^{3} [s_i \dot{s}_i] + \tilde{\alpha}^T \dot{\hat{\alpha}} + \tilde{\beta}^T \dot{\hat{\beta}} + \tilde{\theta}^T \dot{\hat{\theta}}. \quad (22)$$

It is clear that:

$$\dot{V}_2(t) = \sum_{i=1}^{3}\left[s_i\left[\dot{x}_i(t) sgn(x_i(t)) + \gamma_i(p/q)\,|x_i(t)|\left(\int_0^t |x_i(t)|\, d\tau\right)^{(p/q)-1}\right]\right] + \tilde{\alpha}^T \dot{\hat{\alpha}} + \tilde{\beta}^T \dot{\hat{\beta}} + \tilde{\theta}^T \dot{\hat{\theta}}. \quad (23)$$

Inserting $\dot{x}_i(t)$, $(i = 1, 2, 3)$ from system in Eq. (3) and adaptive laws from Eq. (20) into Eq. (23), we have:

$$\begin{aligned} \dot{V}_2(t) = &\sum_{i=1}^{3}\left[s_i\left[\gamma_i(p/q)\,|x_i(t)|\left(\int_0^t |x_i(t)|\, d\tau\right)^{(p/q)-1}\right]\right] \\ &+ s_1[\alpha f_1(x) + \Delta f_1(x,t) + d_1(t) + u_1(t)] sgn(x_1(t)) \\ &+ s_2[\beta f_2(x) + \Delta f_2(x,t) + d_2(t) + u_2(t)] sgn(x_2(t)) \\ &+ s_3[\theta f_3(x) + \Delta f_3(x,t) + d_3(t) + u_3(t)] sgn(x_3(t)) \\ &+ (\hat{\alpha} - \alpha)^T s_1 sgn(x_1)[f_1(x)]^T \\ &+ (\hat{\beta} - \beta)^T s_2 sgn(x_2)[f_2(x)]^T \\ &+ (\hat{\theta} - \theta)^T s_3 sgn(x_3)[f_3(x)]^T, \end{aligned} \quad (24)$$

$$\begin{aligned} \dot{V}_2(t) = &\sum_{i=1}^{3}\left(s_i\left[\gamma_i(p/q)\,|x_i(t)|\left(\int_0^t |x_i(t)|\, d\tau\right)^{(p/q)-1}\right]\right] \\ &+ s_1[\alpha f_1(x) + \Delta f_1(x,t) + d_1(t) + u_1(t)] sgn(x_1(t)) \\ &+ s_2[\beta f_2(x) + \Delta f_2(x,t) + d_2(t) + u_2(t)] sgn(x_2(t)) \\ &+ s_3[\theta f_3(x) + \Delta f_3(x,t) + d_3(t) + u_3(t)] sgn(x_3(t)) \\ &+ \hat{\alpha}^T s_1 sgn(x_1)[f_1(x)]^T \\ &+ \hat{\beta}^T s_2 sgn(x_2)[f_2(x)]^T \\ &+ \hat{\theta}^T s_3 sgn(x_3)[f_3(x)]^T. \end{aligned} \quad (25)$$

Replacing $u_i(t)$, $(i = 1, 2, 3)$ from Eq. (20) into the above equation and simplifying it, gives:

$$\dot{V}_2(t)=\sum_{i=1}^{3}\left[s_i\left[\gamma_i(p/q)\left|x_i(t)\right|\left(\int_0^t\left|x_i(t)\right|d\tau\right)^{(p/q)-1}\right]\right]$$
$$+s_1\Bigg[\alpha f_1(x)+\Delta f_1(x,t)+d_1(t)-\hat{\alpha}f_1(x)$$
$$-\gamma_1(p/q)x_1(t)\left(\int_0^t\left|x_1(t)\right|d\tau\right)^{(p/q)-1}$$
$$-\left(D_D\left(\frac{s_1}{\|s\|^2}\right)+k_1+a_1+b_1\right)sgn(x_1)\Bigg]sgn(x_1(t))$$
$$+s_2\Bigg[\beta f_2(x)+\Delta f_2(x,t)+d_2(t)+-\hat{\beta}f_2(x)$$
$$-\gamma_2(p/q)x_2(t)\left(\int_0^t\left|x_2(t)\right|d\tau\right)^{(p/q)-1}$$
$$-\left(D_D\left(\frac{s_2}{\|s\|^2}\right)+k_2+a_2+b_2\right)sgn(x_2)\Bigg]sgn(x_2(t))$$
$$+s_3\Bigg[\theta f_3(x)+\Delta f_3(x,t)+d_3(t)-\hat{\theta}f_3(x)$$
$$-\gamma_3(p/q)x_3(t)\left(\int_0^t\left|x_3(t)\right|d\tau\right)^{(p/q)-1}$$
$$-\left(D_D\left(\frac{s_3}{\|s\|^2}\right)+k_3+a_3+b_3\right)sgn(x_3)\Bigg]sgn(x_3(t))$$
$$+\hat{\alpha}^T s_1 sgn(x_1)[f_1(x)]^T+\hat{\beta}^T s_2 sgn(x_2)[f_2(x)]^T$$
$$+\hat{\theta}^T s_3 sgn(x_3)[f_3(x)]^T. \tag{26}$$

Using the fact $\sum_{i=1}^{3}s_i(\frac{s_i}{\|s\|^2})=1$, we have:

$$\dot{V}_2(t)=s_1[\Delta f_1(x,t)+d_1(t)$$
$$-(k_1+a_1+b_1)sgn(x_1)]sgn(x_1(t))$$
$$+s_2[\Delta f_2(x,t)+d_2(t)$$
$$-(k_2+a_2+b_2)sgn(x_2)]sgn(x_2(t))+s_3[\Delta f_3(x,t)$$
$$+d_3(t)(k_3+a_3+b_3)sgn(x_3)]sgn(x_3(t))-D_D. \tag{27}$$

It is obvious that:

$$\dot{V}_2(t)\le|s_1|\,[|\Delta f_1(x,t)|+|d_1(t)|$$
$$-(k_1+a_1+b_1)sgn(x_1)]sgn(x_1(t))$$
$$+|s_2|\,[|\Delta f_2(x,t)|+|d_2(t)|$$
$$-(k_2+a_2+b_2)sgn(x_2)]sgn(x_2(t))$$
$$+|s_3|\,[|\Delta f_3(x,t)|$$
$$+|d_3(t)|\,(k_3+a_3+b_3)sgn(x_3)]sgn(x_3(t))-D_D. \tag{28}$$

Using Assumptions 1 and 2 and Eq. (19), one can conclude that:

$$\dot{V}_2(t)\le-\sum_{i=1}^{3}k_i\left|s_i\right|-\mu(\|\hat{\alpha}\|+D_\alpha+\left\|\hat{\beta}\right\|+D_\beta$$
$$+\left\|\hat{\theta}\right\|+D_\theta). \tag{29}$$

Using Assumption 3 and since $\|\hat{\alpha}-\alpha\|\le\|\hat{\alpha}\|+\|\alpha\|\le\|\hat{\alpha}\|+D_\alpha$, $\|\hat{\beta}-\beta\|\le\|\hat{\beta}\|+\|\beta\|\le\|\hat{\beta}\|+D_\beta$ and $\|\hat{\theta}-\theta\|\le\|\hat{\theta}\|+\|\theta\|\le\|\hat{\theta}\|+D_\theta$, one can conclude that $-(\|\hat{\alpha}\|+D_\alpha)\le-\|\hat{\alpha}-\alpha\|$, $-(\|\hat{\beta}\|+D_\beta)\le-\|\hat{\beta}-\beta\|$ and $-(\|\hat{\theta}\|+D_\theta)\le-\|\hat{\theta}-\theta\|$; this yields:

$$\dot{V}_2(t)\le-\sum\nolimits_{i=1}^{3}k_i\left|s_i\right|$$
$$-\mu\left(\|\hat{\alpha}-\alpha\|+\left\|\hat{\beta}-\beta\right\|+\left\|\hat{\theta}-\theta\right\|\right). \tag{30}$$

According to Lemma 2, one has:

$$\dot{V}_2(t)\le$$
$$-\zeta\left[\sum_{i=1}^{3}|s_i|+\left(\|\hat{\alpha}-\alpha\|+\left\|\hat{\beta}-\beta\right\|+\left\|\hat{\theta}-\theta\right\|\right)\right]$$
$$\le-\sqrt{2}\zeta\left(\sum_{i=1}^{3}\frac{1}{2}\left\|s_i\right\|^2+\frac{1}{2}\left\|\hat{\alpha}-\alpha\right\|^2\right.$$
$$\left.+\frac{1}{2}\left\|\hat{\beta}-\beta\right\|^2+\frac{1}{2}\left\|\hat{\theta}-\theta\right\|^2\right)^{1/2}=-\sqrt{2}\zeta V_2(t)^{1/2}, \tag{31}$$

where $\zeta=\min\{k_i,\mu(i=1,2,3)\}$.

Therefore, from Lemma 1, the state trajectories $x_i(t)$, $(i=1,2,3)$ will converge to the sliding surface $s_i(t)=0$, $(i=1,2,3)$, in the finite time, $T_2=\frac{\sqrt{V_2(0)}}{\sqrt{2}\zeta}$. Hence the proof is completed.

Remark 1. According to the Theorems 1 and 2, the sliding mode control law in Eq. (18) with adaptive laws in Eq. (20) and the nonsingular terminal sliding surface in Eq. (13) can make the system in Eq. (3) states reach zero in the finite time, $T = T_1 + T_2$.

Remark 2. The proposed nonsingular terminal sliding mode in Eq. (11) is different from the previously reported terminal sliding mode control ($s = \dot{e} + \beta e^{p/q}$) and fast terminal sliding mode control ($s = \dot{e} + \alpha e + \beta e^{p/q}$) where α, $\beta > 0$, $p > q > 0$ are odd integers. Since the control inputs of the conventional terminal sliding mode control and fast terminal sliding mode control approaches contain the term $e^{\frac{q}{p}-1}$, one can see that for $e < 0$, the fractional power $\frac{q}{p} - 1$ may lead to the term $e^{\frac{q}{p}-1} \notin R$, which leads to $\dot{e} \notin R$. Our proposed nonsingular terminal sliding mode in Eq. (11) overcomes this singularity.

4. Numerical simulations

In this section, numerical simulations are presented to verify the efficiency and effectiveness of the proposed controllers. Numerical simulations are carried out using MATLAB software. The ode45 solver is used for solving differential equations.

4.1. BLDCM

The nonlinear equations of BLDCM are as follows:

$$\begin{cases} \dot{x}_1 = -x_1 - x_2x_3 + 7.961x_3 + \Delta f_1(x,t) \\ \qquad + d_1(t) + u_1(t) \\ \dot{x}_2 = -0.84x_2 + x_1x_3 + \Delta f_2(x,t) + d_2(t) \\ \qquad + u_2(t) \\ \dot{x}_3 = -3.708x_3 + x_2x_2 + \Delta f_3(x,t) + d_3(t) \\ \qquad + u_3(t) \end{cases} \tag{32}$$

where $\alpha = [\alpha_1, \alpha_2, \alpha_3] = [-1, -1, 7.961]$, $\beta = [\beta_1, \beta_2] = [-0.84, 1]$ and $\theta = [\theta_1, \theta_2, \theta_3] = [0, -3.708, 1]$ are the vector parameters of the system in Eq. (3).

The following uncertainties and external disturbances are added to the BLDCM system.

$$\Delta f_1(x,t) = 0.5\sin(x_1) - 0.1\cos(2t),$$

$$\Delta f_2(x,t) = 0.3\cos(3x_2) + 0.1\cos(5t),$$

$$\Delta f_3(x,t) = -0.4\sin(2x_3) + 0.2\sin(3t),$$

$$d_1(t) = 0.3\cos(5t),$$

$$d_2(t) = 0.2\sin(3t),$$

$$d_3(t) = -0.25\cos(2t). \tag{33}$$

The initial conditions of the BLDCM system in Eq. (32) is selected as $x_1(0) = 3$, $x_2(0) = 5$ and $x_3(0) = -1$. The initial values and the norm bounds of the adaptive vectors are $\hat{\alpha}_0 = 0$, $\hat{\beta}_0 = 0$, $\hat{\theta}_0 = 0$ and $D_\alpha = 33$, $D_\beta = 33$, $D_\theta = 33$, respectively. The controller parameters are $K = [K_1, K_2, K_3] = [0.1, 0.1, 0.1]$, $\gamma = [\gamma_1, \gamma_2, \gamma_3] = [10, 10, 2]$, $p = 3$, $q = 5$, $a = [a_1, a_2, a_3] = [0.5, 0.5, 0.5]$, $b = [b_1, b_2, b_3] = [0.5, 0.5, 0.5]$.

The state trajectories of the controlled uncertain chaotic BLDCM and the applied control inputs are depicted in Figures 1 and 2. It can be seen that the closed loop system is not chaotic anymore. The time responses of adaptive vector parameters $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\theta}$, are depicted in Figure 3. It is clear that the adaptive parameters converge to some constants. It can be seen that the control inputs are bounded and feasible in practice. Also, it is obvious that not only the system is stabilized around the origin, but also the closed loop system is robust against model uncertainties and external disturbances.

4.2. PMSM

PMSM is known to exhibit chaotic behavior for a certain range of system parameters. It was shown in [59] that PMSM system given in Eq. (34) behaves in a chaotic mode for the following system parameters and initial conditions.

$$\begin{cases} \dot{x}_1 = \underbrace{-x_1 - x_2x_3 + 20x_3}_{\alpha f_1(x)} + \Delta f_1(x,t) + d_1(t) + u_1(t) \\ \dot{x}_2 = \underbrace{-x_2 + x_1x_3}_{\beta f_2(x)} + \Delta f_2(x,t) + d_2(t) + u_2(t) \\ \dot{x}_3 = \underbrace{5.5(x_1 - x_3)}_{\theta f_3(x)} + \Delta f_3(x,t) + d_3(t) + u_3(t) \end{cases} \tag{34}$$

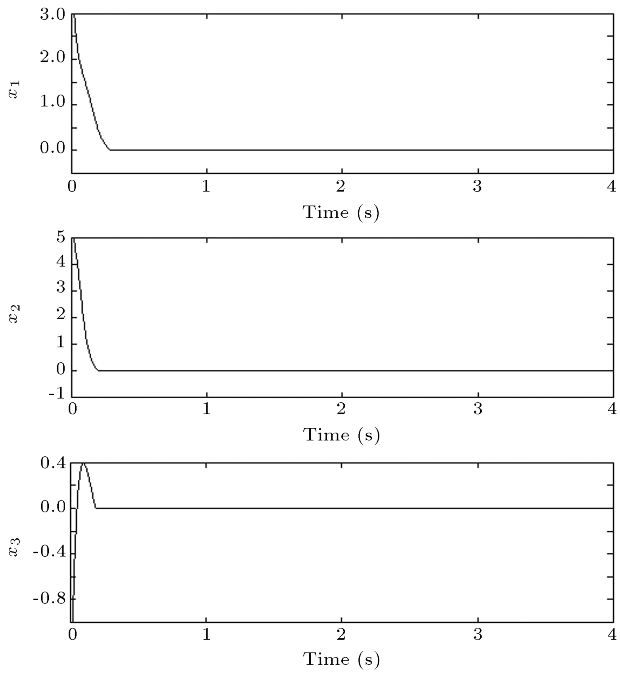

Figure 1. State trajectories of the controlled uncertain chaotic BLDCM.

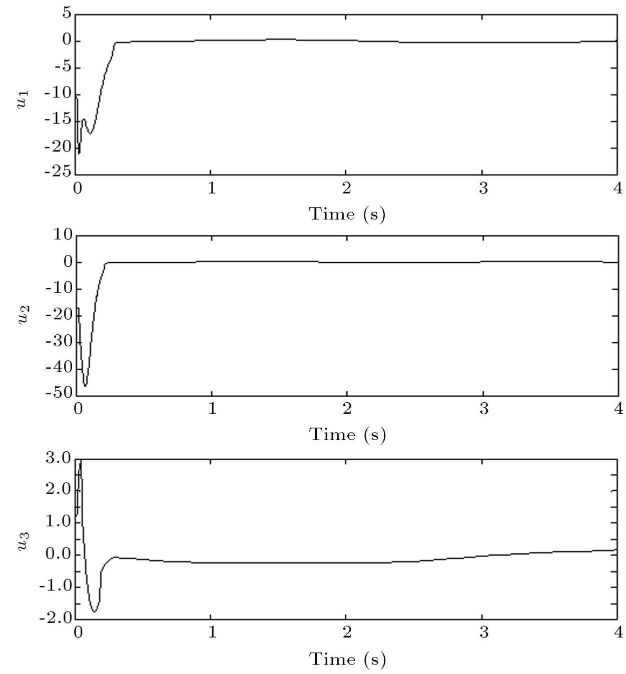

Figure 2. Control inputs applied for the uncertain chaotic BLDCM.

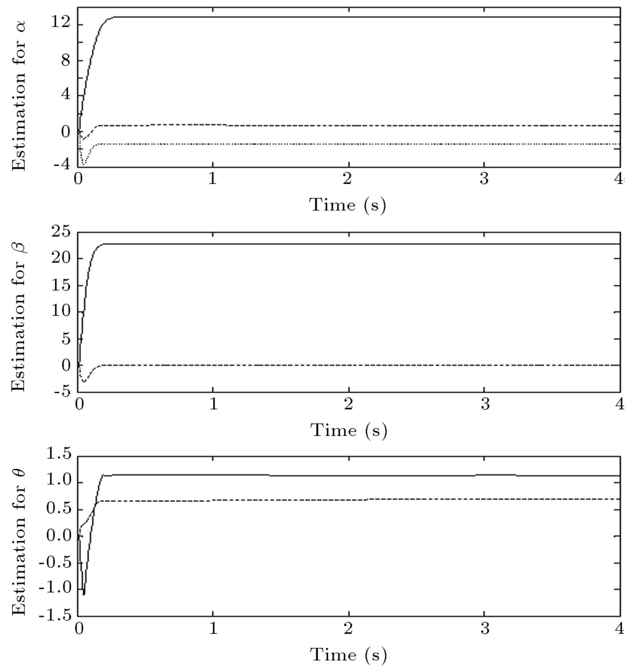

Figure 3. Time responses of adaptive vector parameters $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\theta}$ for the uncertain chaotic BLDCM.

where $\alpha = [\alpha_1, \alpha_2, \alpha_3] = [-1, -1, 20]$, $\beta = [\beta_1, \beta_2] = [-1, 1]$ and $\theta = [\theta_1, \theta_2, \theta_3] = [5.5, 5.5, 0]$ are the vector parameters of the system in Eq. (3).

The following uncertainties and external disturbances are imposed to PMSM system.

$$\Delta f_1(x, t) = 0.2\cos(3x_1) + 0.15\cos(t),$$

$$\Delta f_2(x, t) = 0.2\sin(x_2) - 0.5\cos(5t),$$

$$\Delta f_3(x, t) = 0.25\sin(2x_3) + 0.4\sin(4t),$$

$$d_1(t) = -0.2\cos(5t),$$

$$d_2(t) = 0.1\sin(3t),$$

$$d_3(t) = -0.25\sin(t). \tag{35}$$

The initial conditions of the PMSM system are selected as $x_1(0) = 1$, $x_2(0) = -1$, and $x_3(0) = 2$. The initial values and norm bounds of the adaptive vectors are $\hat{\alpha}_0 = 0$, $\hat{\beta}_0 = 0$, $\hat{\theta}_0 = 0$ and $D_\alpha = 40$, $D_\beta = 50$, $D_\theta = 40$, respectively. The controller parameters are $K = [K_1, K_2, K_3] = [0.2, 0.1, 0.3]$, $\gamma = [\gamma_1, \gamma_2, \gamma_3] = [6, 7, 4]$, $p = 3$, $q = 5$, $a = [a_1, a_2, a_3] = [0.7, 0.7, 0.7]$, $b = [b_1, b_2, b_3] = [0.5, 0.5, 0.5]$.

Figure 4 illustrates PMSM system states. It is obvious that the controller works well for stabilization of the system states around the zero equilibrium even when the whole system with unknown parameters is perturbed by unknown uncertainties and external disturbances. Control inputs are depicted in Figure 5, respectively. One can see that the control signals are practical. The time responses of adaptive vector parameters $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\theta}$, are depicted in Figure 6, respectively. One can see that the adaptive parameters are bounded.

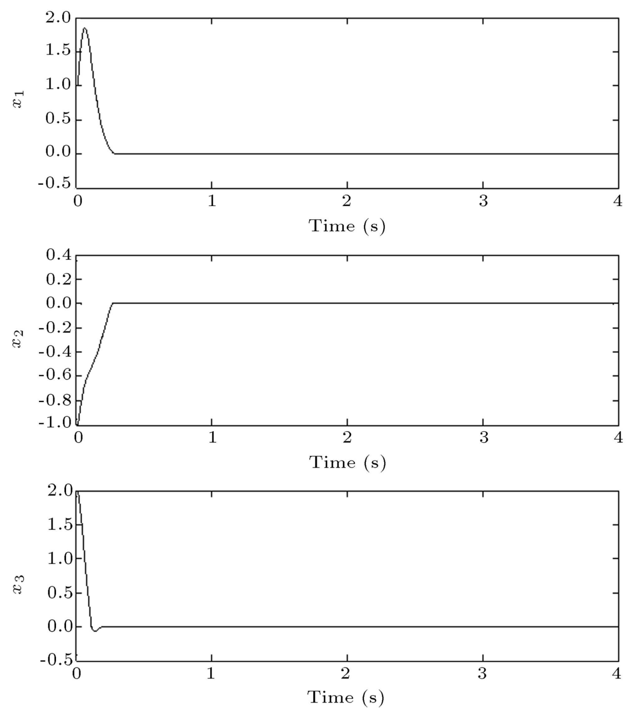

Figure 4. State trajectories of the controlled uncertain chaotic PMSM.

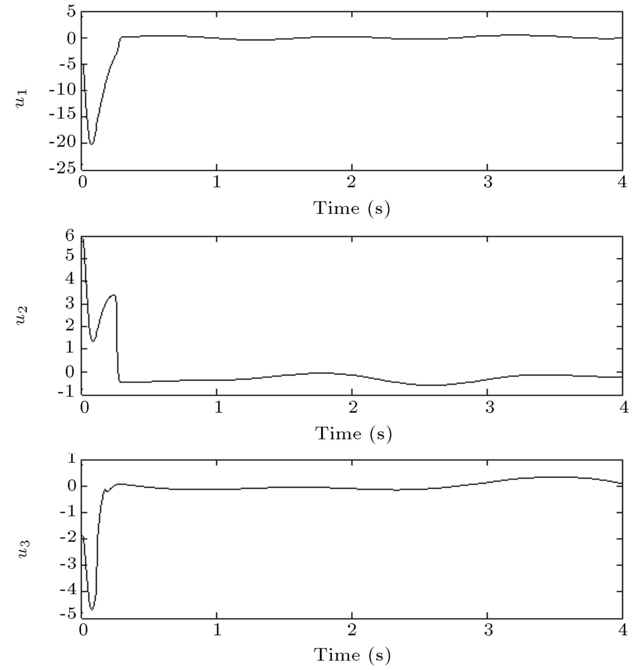

Figure 5. Control inputs applied for the uncertain chaotic PMSM.

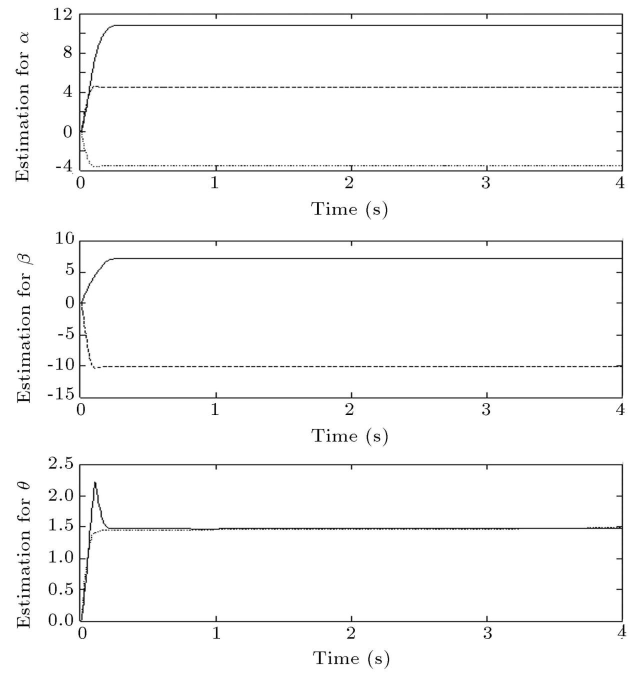

Figure 6. Time responses of adaptive vector parameters $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\theta}$ for the uncertain chaotic PMSM.

5. Conclusions

In this paper, the nonsingular terminal sliding mode controller is represented to stabilize chaotic nonlinear BLDCMs and PMSMs with unknown parameters. It is assumed that all states of BLDCM and PMSM are perturbed by unknown model uncertainties and external disturbances. A novel nonsingular terminal sliding mode manifold was proposed and its finite-time convergence was proved analytically. Suitable adaptive laws were designed to approach the unknown parameters. Based on the adaptive laws and robust finite-time control method, a robust adaptive finite-time sliding mode controller was introduced. The proposed technique had finite-time convergence and stability in both the reaching and the sliding mode phases. Numerical simulations show that the proposed controllers work well for finite-time stabilization of BLDCM and PMSM systems with unknown parameters, in the presence of both unknown model uncertainties and external disturbances.

References

1. Kuroe, Y. and Hayashi, S. "Analysis of bifurcation in power electronic induction motor drive system", *Proceedings of the IEEE Power Electronics Specialists Conference*, **2**, pp. 923-930 (1989).

2. Chen, J.H., Chau, K.T., Siu, S.M. and Chan, C.C. "Experimental stabilization of chaos in a voltage-mode DC drive system", *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, **47**(7), pp. 1093-1095 (2000).

3. Wang, Z. and Chau, K.T. "Anti-control of chaos of a permanent magnet DC motor system for vibratory compactors", *Chaos, Solitons & Fractals*, **36**(3), pp. 694-708 (2008).

4. Tang, T., Yang, M., Li, H. and Shen, D. "A new discovery and analysis on chaos and bifurcation in DC motor drive system with full-bridge converter", *First IEEE Conference on Industrial Electronics and Applications*, pp. 1-6 (2006).

5. Robert, B., Alin, F. and Goeldel, C. "Aperiodic and chaotic dynamics in hybrid step motor-new experimental results", *Proceedings of the IEEE International Symposium on Industrial Electronics*, Pusan, Korea, **3**, pp. 2136-2141 (2001).

6. Reiss, J., Alin, F., Sandler, M. and Robert, B. "A detailed analysis of the nonlinear dynamics of the electric step motor", *Proceedings of the IEEE International Conference on Industrial Technology*, Bangkok, Thailand, **2**, pp. 1078-1083 (2002).

7. Gao, Y. and Chau, K.T. "Hopf bifurcation and chaos in synchronous reluctance motor drives", *IEEE Trans Energy Conver*, **19**(2), pp. 296-302 (2004).

8. Chau, K.T., Chen, J.H., Chan, C.C. and Jiang, Q. "Subharmonics and chaos in switched reluctance motor drives", *IEEE Trans. Energy Conversion*, **17**(1), pp. 73-78 (2002).

9. Chau, K.T. and Chen, J.H. "Modeling, analysis, and experimentation of chaos in a switched reluctance drive system", *IEEE Trans. on Circuit and Systems-I: Fundamental Theory and Application*, **50**(5), pp. 712-716 (2003).

10. Li, Z., Park, J., Zhang, B. and Chen, G. "Bifurcations and chaos in a permanent-magnet synchronous motor", *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, **49**(3), pp. 383-387 (2002).

11. Jing, Z., Yu, C. and Chen, G. "Complex dynamics in a permanent-magnet synchronous motor model", *Chaos, Solitons & Fractals*, **22**(4), pp. 831-848 (2004).

12. Wei, D.Q., Luo, X.S., Wang, B.H. and Fang, J.Q. "Robust adaptive dynamic surface control of chaos in permanent magnet synchronous motor", *Physics Letters A*, **363**, pp. 71-77 (2007).

13. Zribi, M., Oteafy, A. and Smaoui, N. "Controlling chaos in the permanent magnet synchronous motor", *Chaos, Solitons and Fractals*, **41**(3), pp. 266-1276 (2009).

14. Zhang, C., *Brushless DC Motor Theory and Application*, 2nd. Edn., Machine Press, Beijing, China (1999).

15. Hemati, N. and Leu, M.C. "A complete model characterization of brushless dc motors", *IEEE Trans. Industry Applications*, **28**(1), pp. 172-180 (1992).

16. Hemati, N. "Strange attractors in brushless dc motors", *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, **41**(1), pp. 40-45 (1994).

17. Hemati, N., Thorp, J.S. and Leu, M.C. "Robust nonlinear control of brushless dc motors for direct-drive robotic applications", *IEEE Trans. Industrial Electronics*, **37**, pp. 460-468 (1990).

18. Pelczewski, P.M. and Kunz, U.H. "The optimal control of a constrained drive system with brushless dc motor", *IEEE Trans. Industrial Electronics*, **37**(5), pp. 342-348 (1990).

19. Lin, F.J., Shyu, K.K. and Lin, Y.S. "Variable structure adaptive control for PM synchronous servo motor drive", *IEE Proc. IEE B: Electric Power Applications*, **146**(2), pp. 173-185 (1999).

20. Lin, C.L. and Jan, H.Y. "Evolutionarily multiobjective PID control for linear brushless DC motor", *Proc. IEEE Int. Conf. Industrial Electronics Society*, **3**, pp. 39-45 (2002).

21. Cerruto, E., Consoli, A., Raciti, A. and Testa, A. "A robust adaptive controller for PM motor drives in robotic applications", *IEEE Trans. Power Electronics*, **10**(1), pp. 62-71 (1995).

22. Lin, H., Yan, W., Wang, J., Yao, Y. and Gao, B. "Robust nonlinear speed control for a brushless DC motor using model reference adaptive backstepping approach", *Proceedings of the IEEE International Conference on Mechatronics and Automation*, Changchun, China, pp. 335-340 (2009).

23. Ren, H.P. and Chen, G. "Control chaos in brushless dc motor via piecewise quadratic state feedback", *Advances in Intelligent Computing, Lecture Notes in Computer Science*, **3645**, pp. 149-158 (2005).

24. Dadras, S., Motallebzadeh, F., Motallebzadeh, F., Ozgoli, S. and Momeni, H.R. "Control of chaotic uncertain brushless DC motors", *IEEE International Conference on Control and Automation Christchurch*, New Zealand, pp. 2143-2148 (2009).

25. Ge, Z.M., Cheng, J.W. and Chen, Y.S. "Chaos anticontrol and synchronization of three time scales brushless DC motor system", *Chaos, Solitons and Fractals*, **22**(5), pp. 1165-1182 (2004).

26. Ge, Z.M. and Chang, C.M. "Chaos synchronization and parameters identification of single time scale brushless DC motors", *Chaos, Solitons and Fractals*, **20**(4), pp. 883-903 (2004).

27. Ge, Z.M. and Cheng, J.W. "Chaos synchronization and parameter identification of three time scales brushless DC motor system", *Chaos, Solitons and Fractals*, **24**(2), pp. 597-616 (2005).

28. Ge, Z.M., Changa, C.M. and Chen, Y.S. "Anti-control of chaos of single time scale brushless DC motors and chaos synchronization of different order systems", *Chaos, Solitons and Fractals*, **27**(5), pp. 1298-315 (2006).

29. Ge, Z.M. and Lin, G.H. "The complete, lag and anticipated synchronization of a BLDCM chaotic system", *Chaos, Solitons and Fractals*, **34**(3), pp. 740-764 (2007).

30. Utkin, V., *Sliding Modes in Control and Optimization*, Springer Verlag, Berlin (1992).

31. Leonhard, W., *Control of Electrical Drives*, Springer Verlag, Berlin (1985).

32. Hu, H., Hu, Q., Lu, Z. and Xu, D. "Optimal PID controller design in PMSM servo system via particle swarm optimization", *31st Annual Conference of IEEE Industrial Electronics Society*, pp. 6-10 (2005).

33. Jan, R.M., Tseng, C.S. and Liu, R.J. "Robust PID control design for permanent magnet synchronous motor: a genetic approach", *Elect. Power Syst. Res.*, **78**, pp. 1161-1168 (2008).

34. Demirtas, M. "Off-line tuning of a PI speed controller for a permanent magnet brushless DC motor using DSP", *Energy Convers. Manag.*, **52**, pp. 264-273 (2011).

35. Gupta, R., Rajesh, K. and Bansal, A. "Artificial intelligence applications in permanent magnet brushless DC motor drives", *Artif. Intell. Rev.*, **33**, pp. 175-186 (2010).

36. Yucelen, T., Medagam P.V. and Pourboghrat, F. "Nonlinear quadratic optimal control for cascaded multilevel static compensators", *39th North American Power Symposium*, pp. 523-527 (2007).

37. Banks, H.T., Lewis, B.M. and Tran, H.T. "Nonlinear feedback controllers and compensators: A state-dependent Riccati equation approach", *Comput. Optim. Appl.*, **37**, pp. 177-218 (2007).

38. Medagam, P.V., Yucelen, and Pourboghrat, T.F. "Adaptive SDRE based nonlinear sensorless speed control for PMSM drives", *American Control Conference*, pp. 3866-3871 (2009).

39. Kim, K.H. "An MRAC-based nonlinear speed control of an interior PM synchronous motor with improved maximum torque operation", *Int. J. Electron.*, **90**, pp. 481-494 (2003).

40. Maiti, S., Chakraborty, C. and Sengupta, S. "Simulation studies on model reference adaptive controller based speed estimation technique for the vector controlled permanent magnet synchronous motor drive", *Simul. Model. Pract. Theory*, **17**(4), pp. 585-596 (2009).

41. Kokotovic, P.V. "The joy of feedback: nonlinear and adaptive", *IEEE Contr. Syst. Mag.*, **12**, pp. 7-17 (1992).

42. Krstic, M., Kanellakopoulos, I. and Kokotovic, P.V., *Nonlinear and Adaptive Control Design*, Wiley, New York (1995).

43. Zhou, J. and Wang, Y. "Real-time nonlinear adaptive backstepping speed control for a PM synchronous motor", *Contr. Eng. Pract.*, **13**, pp. 1259-1269 (2005).

44. Fung, R.F., Chiang, C.L. and Wu, G.C. "System identification of a pick-and-place mechanism driven by a permanent magnet synchronous motor", *Appl. Math. Model.*, **34**, pp. 2323-2335 (2010).

45. Jianhui, H., Yongxiang, X. and Jibin, Z. "Design and implementation of adaptive backstepping speed control for permanent magnet synchronous motor", *The Sixth World Congress on Intelligent Control and Automation*, pp. 2011-2015 (2006).

46. Rebouh, S., Kaddouri, A., Abdessemed, R. and Haddoun, A. "Nonlinear controller design for a permanent magnet synchronous motor", *IEEE International Electric Machines and Drives Conference*, pp. 776-780 (2007).

47. Shi, H. "A novel scheme for the design of backstepping control for a class of nonlinear systems", *Appl. Math. Model.*, **35**, pp. 1893-1903 (2011).

48. Kim, S.K. "Speed and current regulation for uncertain PMSM using adaptive state feedback and backstepping control", *IEEE International Symposium Industrial Electronics*, pp. 1275-1280 (2009).

49. Xie, Q., Han, Z. and Kang, H. "Adaptive backstepping control for hybrid excitation synchronous machine with uncertain parameters", *Exp. Syst. Appl.*, **37**, pp. 7280-7284 (2010).

50. Magri, A.E., Giri, F., Abouloifa, A. and Chaoui, F.Z. "Robust control of synchronous motor through AC/DC/AC converters", *Contr. Eng. Pract.*, **18**, pp. 540-553 (2010).

51. Yu, J., Chen, B., Yu, H. and Gao, J. "Adaptive fuzzy tracking control for the chaotic permanent magnet synchronous motor drive system via backstepping", *Nonlinear Anal. Real World Appl.*, **12**, pp. 671-681 (2010).

52. Karabacak, M. and Eskikurt, H.I. "Speed and current regulation of a permanent magnet synchronous motor via nonlinear and adaptive backstepping control", *Math. Comput. Model.*, **53**, pp. 2015-2030 (2011).

53. Li, Z., Park, J., Zhang, B. and Chen, G. "Bifurcations and chaos in a permanent-magnet synchronous motor", *IEEE Trans Circuits Syst-I*, **49**(3), pp. 383-7 (2002).

54. Pillay, P. and Krishnan, R. "Modeling of permanent magnet motor drives", *IEEE Trans. Indust. Electron.*, **35**, pp. 537-541 (1988).

55. Curran, P.F. and Chua, L.O. "Absolute stability theory and the synchronization problem", *Int. J. Bifurcat. Chaos*, **7**, pp. 1357-1382 (1997).

56. Wang, H., Han, Z., Xie, Q. and Zhang, W. "Finite-time chaos control via nonsingular terminal sliding mode control", *Commun. Nonlinear Sci. Numer. Simulat*, **14**, pp. 2728-2733 (2009).

57. Aghababa, M.P., Khanmohammadi, S. and Alizadeh, G. "Finite-time synchronization of two different chaotic systems with unknown parameters via sliding mode technique", *Applied Mathematical Modelling*, **35**, pp. 3080-3091 (2011).

58. Slotine, J. and Li, W., *Applied Nonlinear Control*, Prentice-Hall, Englewood Cliffs, New Jersey (1991).

59. Ren, H. and Liu, D. "Nonlinear feedback control of chaos in permanent magnet synchronous motor", *IEEE Trans Circuits Syst-II*, **53**(1), pp. 45-50 (2006).

Biographies

Bijan Hashtarkhani received his MS degrees in Control Engineering from the University of Tabriz in 2012. His research interests include nonlinear control, power systems control, robust control and adaptive control.

Mohammad Pourmahmood Aghababa received his BS degree in Biomedical Engineering from Isfahan University in 2005. He received his MS and PhD degrees both in Control Engineering from the University of Tabriz in 2007 and 2011, respectively. He is currently an Assistant Professor in the Department of Electrical Engineering of the Urmia University of Technology, Urmia, Iran. Dr. Aghababa has published over 50 ISI journal papers, and is an editorial member of 4 international journals. His research interests include nonlinear control, fractional calculus and artificial intelligence.