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Research Note

Stabilization of a vibrating non-classical micro-cantilever using electrostatic actuation

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Abstract. A closed-loop control methodology is investigated for stabilization of a vibrating non-classical micro-scale Euler-Bernoulli beam with nonlinear electrostatic actuation. The dimensionless form of governing nonlinear Partial Differential Equation (PDE) of the system is introduced. The Galerkin projection method is used to reduce the PDE of system to a set of nonlinear Ordinary Differential Equations (ODE). In non-classical micro-beams, the constitutive equations are obtained based on the non-classical continuum mechanics. In this work, proper control laws are constructed to stabilize the free vibration of non-classical micro-beams whose governing PDE is derived based on the modified strain gradient theory as one of the most inclusive non-classical continuum theories. Numerical simulations are provided to illustrate the effectiveness and performance of the designed control scheme. Also, the results have been compared with those obtained by the classical model of micro-beam.

1. Introduction

Many micro-cantilever beams based on MEMS instruments have drawn growing attention in modern technology, such as vibration and shock sensors, Atomic Force Microscopes (AFM), micro-switches, mass sensors, and chemical sensors [1-3]. One of the most recurrent actuating and sensing methods in MEMS systems is based on electrostatic force because of its high efficiency and simple structure and manufacturing. The combination of the electrostatic actuation and micro-beam structure has many applications in industrial and scientific fields like mass sensing systems, micro pressure sensors, micro flexible joints, micro rate gyros and ink injection printers [4-6].

Some papers were dedicated to derive the governing equation of motion of electrostatically actuated beams, while some others considered the vibration analysis of them [7-9]. Pull-in instability has also drawn much attention in the literature. Several works were also dedicated to predict pull-in voltage and its properties [9-11].

Since mechanical vibration can be a main source of damage and restricts the performance and resolution of micro-scale instruments, the necessity for existence of a high performance control system has emerged in recent decades. Vibration control of a clamped-free micro-beam made of stainless steel was studied by Cunningham et al., and the first two modes of vibration were actively suppressed [12]. In 1998, Wang considered a feedback control form to suppress mechanical vibration in a micro-cantilever beam by nonlinear electrostatic actuators via a switching controller [13]. Active vibration isolation of a stroke scanning probe microscope was accomplished by Yen et al. by using discrete sliding mode control to

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treat effectively the unavoidable ground vibration [14]. Zhang et al. used the Rayleigh–Ritz method to reduce the order of the dynamical model of micro-cantilever beams and proposed a rational linearizing feedback controller with a high gain observer to eliminate the unwanted deflection of the micro-cantilever beam system [15]. Many investigations considering feedback control problem of electrostatically actuated micro-systems have used lumped parameters assumption to simplify the governing PDEs of motion. Using this simplification, Vagia designed a switching PID control in 2008 [16]. In 2012, he suggested a sliding mode control to handle nonlinearity and uncertainty in the system parameters [17].

In various applications of MEMS-based micro-beams, the beam thickness is typically on the order of microns and sub-microns. In the recent decades, the size effect in micro-scale beams have been experimentally investigated in some metals and polymers such as those reported in [18-20]. These experiments signified that the classical strain-based mechanics theories cannot be used to describe the microstructure-dependent size effect. Hence, conventional continuum mechanics needs to be extended by using higher order continuum theories to interpret the size dependence phenomenon at small scale.

The effects of the strain gradients in linear elasticity were firstly investigated by Mindlin in 1964 [21]. As a common type of higher-order continuum theory, an improved version of the strain gradient theory was proposed by Fleck et al. in 1994 [18]. In 2003, a modified strain gradient elasticity theory as one of the most successful and inclusive higher-order continuum theories was elaborated by Lam et al. [22]. Recently, this theory has been broadly used to obtain the new governing equations and boundary conditions of micro-scale beams such as investigations illustrated in [23-25]. Also, a large number of publications and investigations in the field of micro-scale beams have been devoted to study the static and dynamic behaviors of strain gradient micro-beams [26-29]. It can be seen from the literature that, although several investigations and analyses have been initiated to discuss the static and dynamic behavior and vibration analysis of non-classical strain gradient micro-beams in recent years, vibration control of non-classical micro-beams, such as strain gradient ones, has been excluded absolutely.

The present work intends to investigate the problem of vibration suppression of a non-classical electrostatically actuated micro-cantilever using nonlinear control theory. For attaining this goal, first, the dimensionless form of the governing nonlinear partial differential equation of an electrostatically actuated Euler–Bernoulli micro-beam is determined based on the modified strain gradient theory introduced by Lam et al. and then Galerkin projection method is employed to reduce the order of the system. The first mode of the system is considered in the model of dynamics to design the controller and the first four modes are chosen to apply the proposed controller and check the performance of the closed loop system.

2. Dynamic modeling

Based on the modified strain gradient theory proposed by Lam et al. [22], for a linear Euler-Bernoulli clamped-free micro-beam with uniform cross-section A and length L, the governing PDE of motion and corresponding boundary conditions are derived with the aid of Hamilton's principle as follows [23,30]:

$$K_1 \frac{\partial^4 w}{\partial x^4} - K_2 \frac{\partial^6 w}{\partial x^6} + \rho A \frac{\partial^2 w}{\partial t^2} = 0, \tag{1}$$

$$\begin{cases} w(0,t) = \frac{\partial w(0,t)}{\partial x} = \frac{\partial^2 w(0,t)}{\partial x^2} = 0 \\ K_1 \frac{\partial^3 w(L,t)}{\partial x^3} - K_2 \frac{\partial^5 w(L,t)}{\partial x^5} = 0 \\ K_2 \frac{\partial^4 w(L,t)}{\partial x^4} - K_1 \frac{\partial^2 w(L,t)}{\partial x^2} = 0 \\ K_2 \frac{\partial^3 w(L,t)}{\partial x^3} = 0 \end{cases} \tag{2}$$

where x and t represent the independent spatial and time variables, respectively, ρ indicates the beam density and $w(x,t)$ denotes the lateral deflection. Furthermore,

$$K_1 = EI + \mu A \left(2l_0^2 + \frac{8}{15} l_1^2 + l_2^2 \right),$$

$$K_2 = \mu I \left(2l_0^2 + \frac{4}{5} l_1^2 \right), \tag{3}$$

where I is the area moment of inertia of the beam cross-section, E is the Young modulus and μ is the shear modulus. Moreover, l_0, l_1 and l_2 appeared in higher order stresses in the modified strain gradient theory [22], illustrate the additional independent material parameters.

Here, an electrostatically actuated non-classical strain gradient micro-beam formulation is attained using the Galerkin projection method. The beam is subjected to electrostatic force at its free end as stabilizing force. The electrostatic force is inherently unidirectional and one electrode is not able to exert force in both directions. So we need two opposite electrodes to produce both attracting and repelling forces. The suggested configuration is depicted in Figure 1.

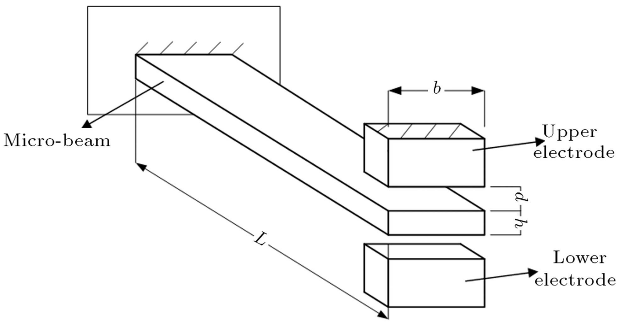

Figure 1. An electrostatically actuated micro-beam.

It must be noticed that it is not mandatory to place electrodes at the tip of the beam; they could be placed in every location along the length of the cantilever. The proper point for the location of electrodes can be obtained by controllability consideration which is beyond the scope of this study. For convenience, it is assumed that the electrodes are located at the tip.

The following equations show the magnitude of the electrostatic actuation force. When the electric potential is established between the electrode and the beam, a pulling force is founded and the beam is attracted toward the electrode.

$$F_{elec,l} = \frac{1}{2}\varepsilon b \frac{\psi(x)V_l^2(t)}{\left(d - w\left(x,t\right)\right)^2},$$

or:

$$F_{elec,u} = -\frac{1}{2}\varepsilon b \frac{\psi(x)V_u^2(t)}{\left(d + w\left(x,t\right)\right)^2}, \tag{4}$$

where d, b and ε are initial distances between electrode and the beam, overlapping width of the beam and electrode and vacuum permittivity, respectively. ψ is a spatial weighting function which is aimed to determine magnitude of the electrostatic force with respect to spatial independent variable. In the present work, a step function is used as a spatial weighting function in order to model the discontinuous geometry of electrodes. V is the applied voltage to the electrodes and w is the lateral displacement of the beam which is a function of temporal and spatial independent variables. "u" and "l" notations demonstrate the force that is generated due to upper and lower electrodes, respectively. To produce full control over the tip displacement of beams, each electrode has separate electrical circuit and different voltages are applied to each electrode. To avoid exerting opposite forces to the beam at once, each electrode is charged only if the other one is offline. It means that at each instant only one electrode is charged.

Using the modified strain gradient theory in well-known Euler-Bernoulli beam model, and the proposed electrostatic actuation terms, the following equation of motion is obtained.

$$K_1\frac{\partial^4 w(x,t)}{\partial x^4} - K_2\frac{\partial^6 w(x,t)}{\partial x^6} + \rho bh\frac{\partial^2 w(x,t)}{\partial t^2} = \frac{1}{2}\varepsilon b \left(\frac{\psi(x)V_l^2(t)}{\left(d - w\left(x,t\right)\right)^2} - \frac{\psi(x)V_u^2(t)}{\left(d + w\left(x,t\right)\right)^2}\right). \tag{5}$$

The first term in the right hand side of the equation represents the attracting force of the bottom electrode and the second term is corresponded to the upper electrode whose minus sign shows the opposite direction of the force [7].

In the above equation, h demonstrates the beam thickness. The dimensionless form of Eq. (5) can be obtained as below:

$$\hat{K}_1\frac{\partial^4 \hat{w}(\hat{x},\hat{t})}{\partial \hat{x}^4} - \hat{K}_2\frac{\partial^6 \hat{w}(\hat{x},\hat{t})}{\partial \hat{x}^6} + \frac{\partial^2 \hat{w}(\hat{x},\hat{t})}{\partial \hat{t}^2} = \frac{\psi(\hat{x})\,\hat{V}_l^2(\hat{t})}{\left(1 - \hat{w}(\hat{x},\hat{t})\right)^2} - \frac{\psi(\hat{x})\,\hat{V}_u^2(\hat{t})}{\left(1 + \hat{w}(\hat{x},\hat{t})\right)^2}, \tag{6}$$

where:

$$\hat{K}_1 = \frac{K_1}{EI}, \qquad \hat{K}_2 = \frac{K_2}{EIL^2}, \qquad \hat{w} = \frac{w}{d},$$

$$\hat{x} = \frac{x}{L}, \qquad \hat{t} = \frac{t}{L^2}\sqrt{\frac{EI}{\rho A}}, \qquad \hat{V} = \frac{V}{\sqrt{\frac{2d^3EI}{\varepsilon bL^4}}}. \tag{7}$$

The solution of Eq. (6) can be represented by a series of infinite terms. Using the decomposition of temporal and spatial parts of the preceding equation solution, the lateral displacement of the beam can be written in the following form:

$$\hat{w}(\hat{x},\hat{t}) = \sum_{i=1}^{\infty} u_i(\hat{t})\phi_i(\hat{x}), \tag{8}$$

where u_i is the temporal part of the ith mode of the solution and ϕ_i is the assumed mode shape of the ith mode. The functions ϕ_is must satisfy dimensionless form of six boundary conditions of the beam given in Eq. (2) and are preferably orthogonal to decouple thoroughly the linear part of Eq. (6) for different modes. To apply classical methods of nonlinear control theory for designing a stabilizing control, the PDE of motion should convert into a set of ODEs in which every equation of this set corresponds to one mode of the system. The Galerkin projection method is used to convert Eq. (6) into a desired set of ODEs. To use the Galerkin method, the actuation terms must be expanded using Taylor series as follows:

$$\hat{F}_{elec} = \frac{\psi\hat{V}_l^2}{\left(1 - \hat{w}\right)^2} - \frac{\psi\hat{V}_u^2}{\left(1 + \hat{w}\right)^2} =$$

$$\psi\hat{V}_l^2\left(1+2\hat{w}+3\hat{w}^2+4\hat{w}^3+...\right)$$
$$-\psi\hat{V}_u^2\left(1-2\hat{w}+3\hat{w}^2-4\hat{w}^3+...\right)=$$
$$\psi\hat{V}_l^2\sum_{i=1}^{\infty} i\hat{w}^{i-1}-\psi\hat{V}_u^2\sum_{i=1}^{\infty}(-1)^{i+1}\,i\hat{w}^{i-1}. \quad (9)$$

Substituting Eq. (8) and the result of Eq. (9) into Eq. (6) and using the Galerkin method lead to the next equation:

$$\int_0^1 \phi_j \begin{pmatrix} \hat{K}_1\frac{\partial^4}{\partial\hat{x}^4}\sum\limits_{i=1}^{\infty}u_i\phi_i-\hat{K}_2\frac{\partial^6}{\partial\hat{x}^6}\sum\limits_{i=1}^{\infty}u_i\phi_i \\ +\frac{\partial^2}{\partial\hat{t}^2}\sum\limits_{i=1}^{\infty}u_i\phi_i-\psi\hat{V}_l^2\sum\limits_{i=1}^{\infty}i\left(u_i\phi_i\right)^{i-1} \\ +\psi\hat{V}_u^2\sum\limits_{i=1}^{\infty}(-1)^{i+1}\,i\left(u_i\phi_i\right)^{i-1}\end{pmatrix} d\hat{x}=0. \quad (10)$$

Orthogonality of the mode shapes brings about the decoupling of the linear parts of the equation of motion, but because of the nonlinear terms which correspond to actuation forces, the ODE of each mode contains some temporal terms of the other modes. Integrating Eq. (10) leads to the following ODE:

$$\alpha u_j+\ddot{u}_j=\int_0^1\left(\phi_j\psi\hat{V}_l^2\sum_{i=1}^{\infty}i\left(u_i\phi_i\right)^{i-1}\right)d\hat{x}$$
$$-\int_o^1\left(\phi_j\psi\hat{V}_u^2\sum_{i=1}^{\infty}(-1)^{i+1}i(u_i\phi_i)^{i-1}\right)d\hat{x}, \quad (11)$$

where α is defined as:

$$\alpha=\int_0^1\left(\hat{K}_1\phi_j\frac{d^4}{d\hat{x}^4}\phi_j-\hat{K}_2\phi_j\frac{d^6}{d\hat{x}^6}\phi_j\right)d\hat{x}. \quad (12)$$

3. Control system

The objective of this section is deriving a feedback control law to stabilize undesired vibration of the micro-cantilever beam with taking into account the effect of strain gradient phenomenon. The design is based on the set of ODEs of motion which was derived in the previous section. The aim of the control system is to stabilize the advert vibrations of the micro-cantilever and restore it to its rest point. The first mode of the strain gradient micro-cantilever is considered in the model for controller design. This mode dominates the dynamic response of the beam, and stabilization of this mode would stabilize significantly the entire vibration of the beam. On the other hand, including the higher modes imposes need of more electrodes to actuate the beam. The actuation force is approximated by its first four terms of Taylor expansion. This approximation is used only for controller model, but in the plant model the right hand side of Eq. (11) is integrated directly over spatial domain, without approximation by Taylor expansion. It can be interpreted that the whole terms of the Taylor series are considered in the plant model. This method significantly improved accuracy of the numerical solution in the expense of increasing calculations. Simulations show that approximation by four terms has enough accuracy and increasing the terms would not improve the response of the system. The equation of motion for the first mode is depicted as follows:

$$\alpha u_1+\ddot{u}_1=\int_0^1\psi\phi_1\hat{V}_l^2$$
$$\left(1+2u_1\phi_1+3u_1^2\phi_1^2+4u_1^3\phi_1^3\right)d\hat{x}. \quad (13)$$

The controller will convert the characteristic equation of the closed loop system into the following form:

$$\ddot{u}_1+c_1\dot{u}_1+c_2u_1=0, \quad (14)$$

where c_1 and c_2 are positive coefficients that must be chosen in a way to ensure the stability of the system. To do so, a feedback linearization method is used to eliminate the nonlinear part of the equation and form Eq. (14). Doing some math the actuating voltage is obtained in the form of:

$$\hat{V}_l^2=\frac{-c_1\dot{u}_1-c_2u_1+\alpha u_1}{a_1+a_2u_1+a_3u_1^2+a_4u_1^3}, \quad (15)$$

where a_is are coefficients which are obtained by integrating the left part of Eq. (13) with respect to $\hat{x}$. They are defined as follows:

$$a_1=\int_0^1\psi\phi_1 d\hat{x}, \qquad a_2=\int_0^1 2\psi\phi_1^2 d\hat{x},$$
$$a_3=\int_0^1 3\psi\phi_1^3 d\hat{x}, \qquad a_4=\int_0^1 4\psi\phi_1^4 d\hat{x}. \quad (16)$$

Substituting Eq. (15) in Eq. (13) would convert Eq. (13) into Eq. (14). The controller can be tuned by choosing proper c_1 and c_2 coefficients. To generate force in the opposite side (upward force) one must use the upper electrode as well. From Eq. (15), it is obvious that the left side of the equation must be positive to have physical meaning; this arises from unidirectional nature of the electrostatic actuation. When the left side term becomes negative, it means that upward force is needed. The upper electrode can generate this upward force by applying a voltage that is specified by using the below model:

$$\alpha u_1+\ddot{u}_1=-\int_0^1\psi\hat{V}_u^2$$
$$\left(1-2u_1\phi_1+3u_1^2\phi_1^2-4u_1^3\phi_1^3\right)d\hat{x}. \quad (17)$$

To convert the above equation into asymptotic stable form of Eq. (14), the following voltage is derived:

$$\hat{V}_u^2 = -\frac{-c_1\dot{u}_1 - c_2 u_1 + \alpha u_1}{a_1 - a_2 u_1 + a_3 u_1^2 - a_4 u_1^3}, \qquad (18)$$

where a_is are defined in Eq. (16). A supervisory control would decide which control voltage is valid based on the sign of the left parts of Eqs. (15) and (18). The positive sign is the criterion of applying the voltage to electrode.

4. Simulation results

In the previous section, a controller was designed to stabilize the vibration of a strain gradient micro-cantilever. In this part of the present work, numerical simulations are used to validate the proposed feedback control system. The parameters of the micro-cantilever which is modeled as plant of the system are shown in Table 1. To investigate the effect of unmodeled dynamics on controller performance, the plant is modeled by first four modes of the beam. So, the robustness of the proposed system will be tested by assuming higher modes and unmodeled fast dynamics of the system in simulations. In this research, it is assumed that all necessary states of the system are available via a proper estimation system, but the observation system is not included in present simulations. In future investigation, the authors will consider a state estimation system by designing a proper nonlinear observer. The observabillity of similar nonlinear system was proved in pervious works [31-32].

In the first simulation, the vibration of the beam due to a constant voltage is investigated. The input voltage is applied as a step function. The simulation is carried out for both classical and strain gradient Euler-Bernoulli micro-beam models. Figure 2 shows the result of the simulation in this case.

Comparison between the simulation results of these two models shows significant differences in the frequency and amplitude of the time response. The time response of the tip displacement of classical Euler-Bernoulli beam model to the step input exhibits lower frequency which is in contrast to fast response of the strain gradient model. Also, the tip displacement amplitude of the classical model is higher than that of the second one. This matter can be explained by considering the difference between stiffness of the two models. Taking the effect of strain gradient into account increases the stiffness of the beam. Much thinner the beam is, this added stiffness is larger. In Figure 3, the result of the same simulation is carried out but the thickness of the beam is taken to be about 5 times greater than the first simulation. It is obvious that the difference in response of the two models is far less than the first simulation in which the thickness of the beam was lesser. But the difference in amplitude of the response is again considerable. In the second simulation, the performance of the proposed stabilizing control system is studied. An initial displacement is applied at the tip of the beam and the controller would stabilize the resulted vibration. In the simulation, the parameters of Table 1 are used.

In Figure 4, the result of the simulation for the

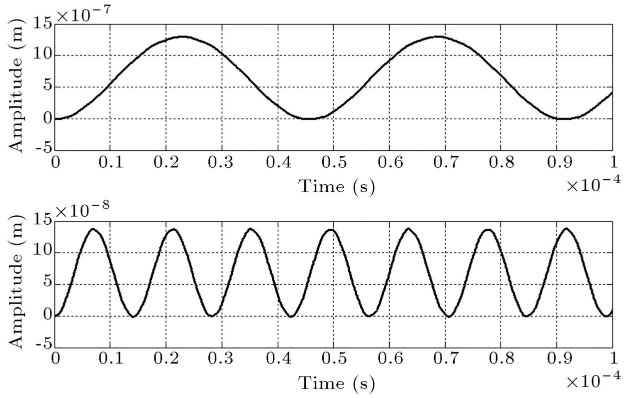

Figure 2. Tip displacement response of the beam to a step input voltage for classical (upper) and non-classical (lower) beam models. The thickness of the micro-beam is 17 μm.

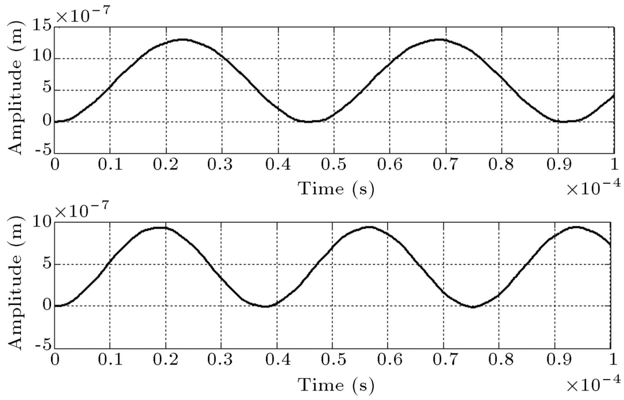

Figure 3. Tip displacement response of the beam to a step input voltage for classical (upper) and non-classical (lower) beam models. The thickness of the beam is 80 μm.

Table 1. Parameters of the non-classical micro-cantilever [33].

Length	Modulus of elasticity	Width	Thickness	Density	l_0	l_1	l_2	Initial gap
340 μm	1.44 GPa	34 μm	17 μm	1000 kg/m^3	17.6 μm	17.6 μm	17.6 μm	3 μm

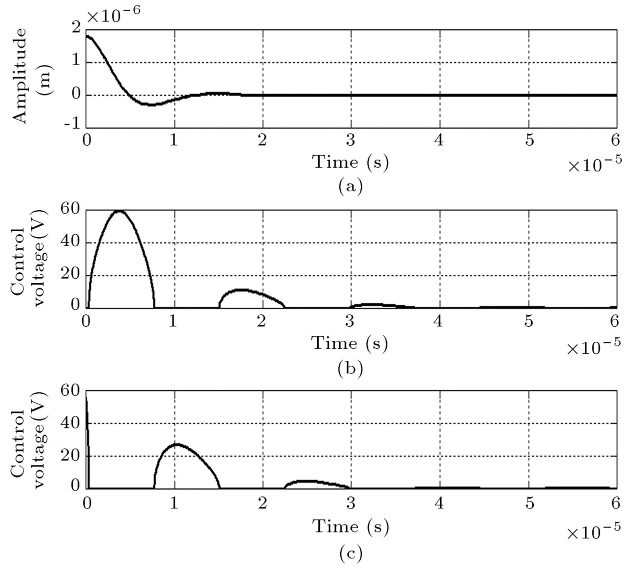

Figure 4. (a) Tip displacement of the beam, (b) lower electrode control voltage, and (c) upper electrode control voltage of the stabilized micro-cantilever with non-classical beam model.

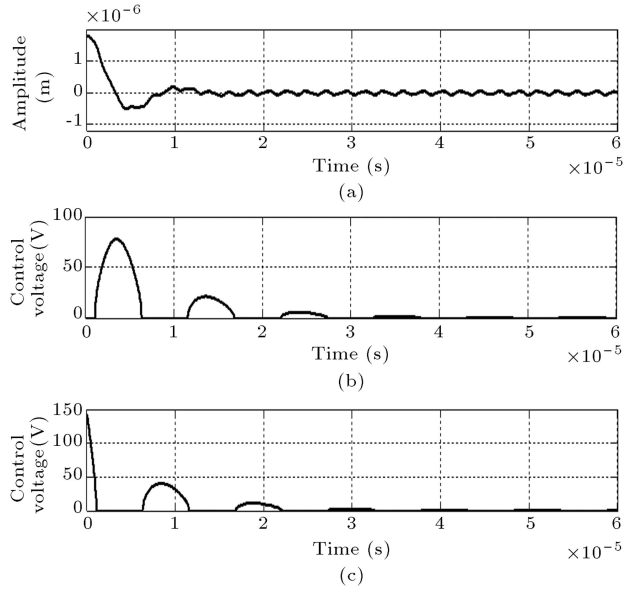

Figure 5. (a) Tip displacement of the beam, (b) lower electrode control voltage, and (c) upper electrode control voltage of the stabilized micro-cantilever with classical beam model.

beam with stabilizing feedback control laws is depicted. The result shows smooth stabilizing of the beam. In the next simulation, the effect of neglecting strain gradient term in the model, which is used to design the controller, is studied. Classical theory of the Euler-Bernoulli beam is used for controller design, but the plant is simulated as a strain gradient included beam. In Figure 5, the result of simulation is illustrated.

The result demonstrates that the response of the

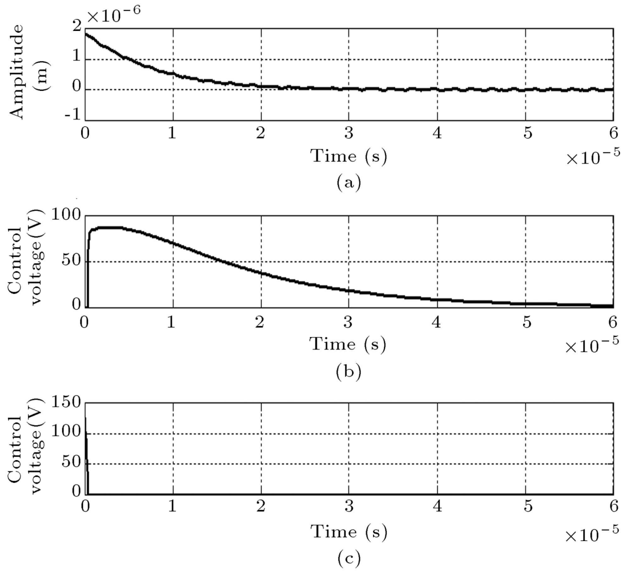

Figure 6. (a) Tip displacement, (b) lower electrode control voltage, and (c) upper electrode control voltage of the stabilized micro-cantilever with linear PID controller.

beam exhibits increasing in the controller signal which is significantly higher than the case where the strain gradient is included in the controller model. The mismatch between the actual plant and the controller model leads to the poor controller performance. The maximum control effort in the latter case is about two times greater than the first case. In addition, the higher order dynamics effects which are neglected in the controller model can be observed in the form of small oscillations in the time response of the system.

The last simulation is dedicated for a comparison between the proposed nonlinear control scheme and a linear controller. A PID controller is used and its performance is indicated in Figure 6. As shown in this figure, the linear controller can stabilize the vibration of the beam using very large control effort in comparison to the proposed nonlinear controller. Also, the effect of the higher order dynamics in the system response is stronger than the case of using nonlinear controller.

5. Conclusion

In this research, the problem of vibration suppression of a clamped-free strain gradient Euler-Bernoulli micro-beam is studied. The nonlinear electrostatic actuation is considered to achieve the control objective. Nonlinear PDE of the motion of a strain gradient micro-beam with electrostatic actuation is obtained in dimensionless form. State space representation of the system is instituted by employing the Galerkin method. Then, for the obtained ODE model, a feedback control law is designed to stabilize the undesired vibration of the micro-cantilever beam with taking into account the effect of strain gradient phenomenon.

Finally, computer simulation is implemented to observe the effectiveness of the proposed control technique. Numerical results obtained by considering the non-classical model are compared with those obtained by the classical one. Numerical simulations show significant differences between the results of non-classical and classical models. These differences decrease and diminish by increasing the micro-beam thickness. In this work, the effect of observer errors in estimating the needed states for the controller is ignored, and it is assumed that the whole states are available for feedback. In the future work, this important factor will be studied and considered in the system performance.

References

1. de Boer, M.P., Luck, D.L., Ashurst, W.R., Maboudian, R., Corwin, A.D., Walraven, J.A. and Redmond, J.M. "High-performance surface-micromachined inchworm actuator", *J. Microelectromech. Syst.*, **13**(1), pp. 63-74 (2004).
2. Lun, F.Y., Zhang, P., Gao, F.B. and Jia, H.G. "Design and fabrication of micro-optomechanical vibration sensor", *Microfab. Technol.*, **120**(1), pp. 61-64 (2006).
3. Batra, R.C., Porfiri, M. and Spinello, D. "Vibrations of narrow microbeams predeformed by an electric field", *J. Sound Vib.*, **309**(3-5), pp. 600-612 (2008).
4. Ekinci, K.L., Yang, Y.T. and Roukes, M.L. "Ultimate limits to inertial mass sensing based upon nanoelectromechanical systems", *J. Appl. Phys.*, **95**, pp. 2682-2687 (2004).
5. Yasuda, T., Shimoyama, I. and Miura, H. "Electrostatically driven micro elastic joints", IEEE Computer Society, *International Conference on Intelligent Robots and Systems*, pp. 242-245 (1995).
6. Chu, L.L., Que, L. and Gianchandani, Y.B. "Measurements of material properties using differential capacitive strain sensors", *J. Microelectromech. Syst.*, **11**(5), pp. 489-498 (2002).
7. Abdel-Rahman, E.M., Younis, M.I. and Nayfeh, A.H. "Characterization of the mechanical behavior of an electrically actuated microbeam", *J. Micromech. Microeng.*, **12**, pp. 759-765 (2002).
8. Brusa, E., De Bona, F., Gugliotta, A. and Somà, A. "Modeling and prediction of the dynamic behaviour of microbeams under electrostatic load", *Analog. Integr. Circ. S.*, **40**(2), pp. 155-164 (2004).
9. Mojahedi, M., Moghimi Zand, M. and Ahmadian, M.T. "Static pull-in analysis of electrostatically actuated microbeams using homotopy perturbation method", *Appl. Math. Model.,* **34**(4), pp. 1032-1041 (2011).
10. Rezazadeh, G., Tahmasebi, A. and Ziaei-rad, S. "Nonlinear electrostatic behavior for two elastic parallel fixed-fixed and cantilever microbeams", *Mechatronics.*, **19**(6), pp. 840-846 (2009).
11. Lakrad, F. and Belhaq, M. "Suppression of pull-in instability in MEMS using a high-frequency actuation", *Commun. Nonlinear Sci. Numer. Simul.*, **15**(11), pp. 3640-3646 (2010).
12. Cunningham, M.J., Jenkins, D.F.L., Clegg, W.W. and Bakush, M.M. "Active vibration control and actuation of a small cantilever for applications in scanning probe instruments", *Sens. Actuators. A.*, **50**(1), pp. 147-150 (1995).
13. Wang, P.K.C. "Feedback control of vibrations in a micromachined cantilever beam with electrostatic actuators", *J. Sound Vib.*, **213**(3), pp. 537-550 (1998).
14. Yen, J.Y., Lan, K.J. and Kramar, J.A. "Active vibration isolation of a large stroke scanning probe microscope by using discrete sliding mode control", *Sens. Actuators. A.*, **121**(1), pp. 243-250 (2005).
15. Zhang, W., Meng, G. and Li, H. "Adaptive vibration control of micro-cantilever beam with piezoelectric actuator in MEMS", *Int. J. Adv. Manuf. Technol.*, **28**(3), pp. 321-327 (2006).
16. Vagia, M., Nikolakopoulos, G. and Tzes, A. "Design of a robust PID-control switching scheme for an electrostatic micro-actuator", *Control Eng. Pract.*, **16**(11), pp. 1321-1328 (2008).
17. Vagia, M. "A frequency independent approximation and a sliding mode control scheme for a system of a micro-cantilever beam", *ISA. Trans.,* **51**(2), pp. 325-332 (2012).
18. Fleck, N.A., Muller, G.M., Ashby, M.F. and Hutchinson, J.W. "Strain gradient plasticity: Theory and experiment", *Acta. Metall. Mater.*, **42**(2), pp. 475-487 (1994).
19. Ma, Q. and Clarke, D.R. "Size dependent hardness of silver single crystals", *J. Mater. Res.*, **10**(04), pp. 853-863 (1995).
20. Stolken, J.S. and Evans, A.G. "A microbend test method for measuring the plasticity length scale", *Acta Mater.*, **46**(14), pp. 5109-5115 (1998).
21. Mindlin, R.D. "Micro-structure in linear elasticity", *Archs Ration. Mech. Analysis*, **16**, pp. 51-78 (1964).
22. Lam, D.C.C., Yang, F., Chong, A.C.M., Wang, J. and Tong, P. "Experiments and theory in strain gradient elasticity", *J. Mech. Phys. Solids.*, **51**(8), pp. 1477-1508 (2003).
23. Kong, S., Zhou, S., Nie, Z. and Wang, K. "Static and dynamic analysis of micro beams based on strain gradient elasticity theory", *Int. J. Eng. Sci.,* **47**(4), pp. 487-498 (2009).
24. Ma, H.M., Gao, X.L. and Reddy, J.N. "A microstructure-dependent Timoshenko beam model based on a modified couple stress theory", *J. Mech. Phys. Solids*, **56**(12), pp. 3379-3391 (2008).
25. Xia, W., Wang, L. and Yin, L. "Nonlinear non-classical microscale beams: Static bending, postbuckling and free vibration", *Int. J. Eng. Sci.*, **48**(12), pp. 2044-2053 (2010).

26. Vatankhah, R., Kahrobaiyan, M.H., Alasty, A. and Ahmadian, M.T. "Nonlinear forced vibration of strain gradient microbeams", *Appl. Math. Model.*, **37**, pp. 8363-8382 (2013).
27. Kahrobaiyan, M.H., Asghari, M., Rahaeifard, M. and Ahmadian, M.T. "Investigation of the size-dependent dynamic characteristics of atomic force microscope microcantilevers based on the modified couple stress theory", *Int. J. Eng. Sci.*, **48**(12), pp. 1985-1994 (2010).
28. Zhao, J., Zhou, S., Wang, B. and Wang, X. "Nonlinear microbeam model based on strain gradient theory", *Appl. Math. Modell.*, **36**, pp. 2674-2686 (2012).
29. Ansari, R., Gholami, R. and Sahmani, S. "Free vibration analysis of size-dependent functionally graded microbeams based on the strain gradient Timoshenko beam theory", *Compos. Struct.*, **94**(1), pp. 221-228 (2011).
30. Akgoz, B. and Civarlek, O. "Comment on "Static and dynamic analysis of micro beams based on strain gradient elasticity theory" by S. Kong, S. Zhou, Z. Nie, and K. Wang, (International Journal of Engineering Science, 47, 487-498, 2009)", *Int. J. Eng. Sci.*, **55**, pp. 279-281 (2012).
31. Shirazi, M.J., Salarieh, H., Alasty, A. and Shabani, R. "Tip tracking control of a micro-cantilever Timoshenko beam via piezoelectric actuator", *J. Vib. Control.*, Article in Press, D.O.I.: 10.1177/1077546312447837.
32. Karami, F. and Salarieh, H. "Positioning control in micro-beams via electrostatic actuators and incomplete state feedback", MSc Thesis, Department of Mechanical Engineering, Sharif University of Technology, Tehran, Iran (2012).
33. Vatankhah, R., Najafi, A., Salarieh, H. and Alasty, A. "Boundary stabilization of non-classical micro-scale beams", *Appl. Math. Model.*, **37**, pp. 8709-8724 (2013).

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