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Lattice Boltzmann simulation of natural convection in a nanofluid-filled inclined square cavity at presence of magnetic field

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Abstract. In this paper, the effect of a magnetic field on natural convection flow in a nanofluid-filled inclined square cavity has been analyzed by Lattice Boltzmann Method (LBM). The cavity is filled with water and nanoparticles of copper at the presence of a magnetic field. This study has been carried out for the pertinent parameters in the following ranges: the Rayleigh number of the base fluid, $\mathrm{Ra} = 10^3 - 10^5$, the volumetric fractions of nanoparticles between 0 and 6% and inclined angle (θ) of the cavity between $\theta = -60°$ and $60°$ with interval of $30°$. The Hartmann number varied from $\mathrm{Ha} = 0$ to 30, while the uniform magnetic field is considered horizontally. Results show that the heat transfer is decreased by the increment of Hartmann number for various Rayleigh numbers and the inclined angles. Magnetic field augments the effect of nanoparticles at high Rayleigh numbers. Negative inclined angles simultaneously decline heat transfer toward $\theta = 0°$ and the influence of nanoparticle.

1. Introduction

Flow in an enclosure driven by buoyancy force is a fundamental problem in fluid mechanics. This type of flow can be used as proof validation or in academic researches and various applications of engineering. Furthermore, the applications are listed by Davis [1], Ostrach [2], Catton [3], Bejan [4] and Patrick and David [5]. The analysis of flow and heat transfer in an inclination cavity was performed by Rasoul and Prinos [6]. They illustrated that the mean and local heat fluxes at the hot wall significantly depend on the inclination angle. They also obtained that the greatest dependence is observed at the inclination angles of $\theta > 90°$. Ozo et al. [7] investigated, numerically, two-dimensional natural circulation at the inclination angles of $0°$ to $180°$. Kuyper et al. [8] studied laminar and turbulent flows in an inclined cavity, numerically. The numerical predictions of the heat flux at the hot wall and the influence of the angle of inclination on the Nusselt number were presented into the investigation. The Nusselt number showed strong dependence on the orientation of the cavity and the power law dependence on the Rayleigh number of the flow. An appropriate method to accelerate shooting plasma into fusion devices or to produce high energy wind tunnels for the simulation of hypersonic flight is the usage of magnetic field. Moreover, the types of the problems arisen in electronic packages are microelectronic devices. Althoug application of magnetic field is not restricted to the applications and it was utilized for various industries such as crystal growth in liquids and cooling of nuclear reactors, various investigations on natural and forced convection in presence of magnetic field were implemented by researchers using different numerical methods. For instance, some of them,

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that have been published recently, can be named. Pirmohammadi and Ghassemi [9] studied the effect of magnetic field on convection heat transfer inside a tilted square enclosure. They found that for a given inclination angle (φ), as the value of Hartmann number (Ha) increases, the convection heat transfer reduces. Furthermore, they obtained that at $\text{Ra} = 10^4$, value of Nusselt number depends strongly upon the inclination angle for relatively small values of Hartmann number, and at $\text{Ra} = 10^5$, the Nusselt number increases from $\varphi = 0°$ to $45°$ and then decrease as φ increases. Kahveci and Öztuna [10] studied MHD natural convection flow and heat transfer in a laterally heated partitioned enclosure, using Polynomial Differential Quadrature (PDQ). They displayed that the magnetic field is more significant at high Rayleigh numbers and x-direction against y-direction, and also showed that the average heat transfer decreases with increment of partition distance from hot wall. Sathiyamoorthy and Chamkha [11] investigated the effect of magnetic field on natural convection flow in a liquid gallium filled square cavity for linearly heated side wall(s). They used penalty finite element method with bi-quadratic rectangular elements to solve the non-dimensional governing equations.

As these investigations showed, magnetic field causes heat transfer to decline. Therefore, a heat transfer improvement method is needful. The addition of nanoparticles to fluid can be a good solution for this problem. Fluids with nanoparticles suspended in them are called nanofluids. Nanoparticles enhance thermal conductivity of fluids with low volume fractions [12]. Therefore, many investigations were performed about nanofluid theoretically, experimentally and numerically. Kim et al. [13] studied the pool boiling characteristics of dilute dispersions of Al_2O_3, ZrO_2 and SiO_2 nanoparticles in water. It was found that a significant enhancement in critical heat flux can be obtained at the modest nanoparticle concentration. Khanafer et al. [14] numerically investigated the heat transfer enhancement in a two-dimensional enclosure, utilizing nanofluids for various pertinent parameters. They tested different models for nanofluid density, viscosity and thermal expansion coefficients. It was found that the suspended nanoparticles substantially increase the heat transfer rate at any given Grashof numbers. Wen and Ding [15] investigated the heat transfer enhancement, using water-TiO_2 nanofluid filled in a rectangular enclosure heated from below. They reported that the natural convection heat transfer rate increasingly decreased with the increase of particle concentration, particularly at low Rayleigh number. Oztop and Abu-Nada [16] investigated natural convection in partially heated rectangular enclosure with Cu/water nanofluid. They found that heater location influences the flow and temperature and the highest value of heat transfer enhancement occurs at low aspect ratios. Abu-Nada and Oztop [17] studied the effect of inclination angle on natural convection in a cavity with Cu/water nanofluid. They reported that the effect of nanofluid is more significant at low volume fraction as inclination of angle is a control parameter for nanofluids in the cavity. Recently, Ghasemi et al. [18] essayed magnetic field effect on natural convection in a nanofluid-filled square enclosure. They obtained that the heat transfer rate increases with the augmentation of the Rayleigh number, but it decreases with the increase in the Hartmann number. Furthermore, they exhibited that the enhancement of the solid volume fraction results in improvement or deterioration of the heat transfer performance, depending on the value of Hartmann and Rayleigh numbers.

The Lattice Boltzmann method has been applied extensively in recent years, as an effective and straightforward numerical method; it is utilized for simulation of complex flow problems with different boundary conditions. In comparison with the conventional CFD methods, the advantages of LBM include simple calculation procedure and efficient implementation for parallel computation, multiphase problems and complicated boundary conditions [19-25]. Kefayati [26] conducted a research into the effect of a magnetic field on natural convection in an open enclosure, which subjugated to water/copper nanofluid, using Lattice Boltzmann Method. It emerged that the heat transfer decreases by the increment of Hartmann number for various Rayleigh numbers and volume fractions. In addition, he mentioned that the magnetic field augments the effect of nanoparticles at Rayleigh number of $\text{Ra} = 10^6$ regularly.

Kefayati [27] studied the effects of aspect ratio in a long enclosure in the presence of magnetic field and nanofluid on natural convection by lattice Boltzmann method. It was found that the effect of nanoparticles rises for high Hartmann numbers when the aspect ratio increases. The rise in the magnetic field inclination improves heat transfer at aspect ratio of $A = 0.5$.

In the new investigation, Kefayati [28] analyzed the influence of sinusoidal temperature distribution on nanofluid and MHD flow simultaneously. The research demonstrated that at $\text{Ha} = 0$, the greatest effects of nanoparticles are obtained at $\theta = 3\pi/4$, 0 & $\pi/2$ for $\text{Ra} = 10^3$, 10^4 and 10^5, respectively.

The main aim of the present study is to identify the ability of Lattice Boltzmann Method (LBM) for solving nanofluid in the presence of a magnetic at an inclined cavity. In fact, the effects of the inclination on the flow field and isotherms as nanofluid and MHD flow are solved simultaneously. Moreover, the Cu-water nanofluid on laminar natural convection heat transfer at presence of magnetic field in inclined cavity by LBM was investigated. The results of LBM are validated with previous numerical investigations. Effects of all parameters (Rayleigh number, volume fraction, Hartmann number and inclination angle) on flow field and temperature distribution are also considered.

2. Problem statement

In this section, the proposed model is applied to simulate natural convection in a square cavity with side walls maintained at different temperatures. The left vertical wall is maintained at a high temperature, T_H, while the right vertical wall is kept at a low temperature, T_L. The horizontal walls are assumed to be insulated, nonconducting, and impermeable to mass transfer. The hot wall is inclined at an angle of θ with the horizontal axis. The cavity is filled with a mixture of water and solid copper. The nanofluid in the cavity is Newtonian, incompressible, and laminar. Thermo-physical properties of the nanofluid are assumed to be constant (Table 1). The density variation in the nanofluid is approximated by the standard Boussinesq model. The uniform magnetic field with a constant magnitude B_0 is applied in the X direction. It is assumed that the induced magnetic field produced by the motion of an electrically conducting fluid is negligible compared to the applied magnetic field. Further, it is assumed that the viscous dissipation and Joule heating are neglected (Figure 1).

Table 1. Thermophysical properties of water and copper.

Property	Water	Copper
μ (kg/ms)	8.9×10^{-4}	-
c_p (j/kg k)	4179	383
ρ (kg/m^3)	997.1	8954
β (k^{-1})	2.1×10^{-4}	1.67×10^{-5}
k (w/m k)	0.6	400

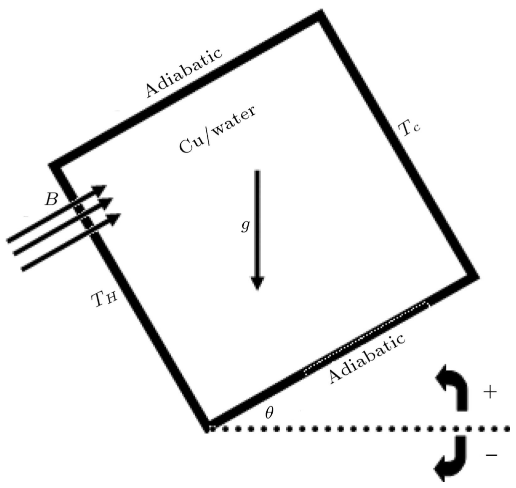


Figure 1. Geometry of the present study.

3. Governing equations

3.1. Lattice Boltzmann method

LBM method with standard two dimensional, nine velocities (D2Q9) for flow and temperature are used in this work; for completeness, only a brief discussion is given in the following paragraphs [19].

The Lattice Boltzmann equation with external forces can be written as:

For the flow field:

$$f_i(x + c_i\Delta t, t + \Delta t) - f_i(x,t) = -\frac{1}{\tau_v}[f_i(x,t) - f_i^{\mathrm{eq}}(x,t)] + \Delta t F. \quad (1)$$

For the temperature field:

$$g_i(x + c_i\Delta t, t + \Delta t) - g_i(x,t) = -\frac{1}{\tau_c}[g_i(x,t) - g_i^{\mathrm{eq}}(x,t)]. \quad (2)$$

D2Q9 model for flow and temperature are used in this work, so the weighting factors and the discrete particle velocity vectors are different for these two models and they are calculated as follows:

$$f_i^{\mathrm{eq}}(x,t) = \omega_i\rho\left[1 + \frac{c_i.u}{c_s^2} + \frac{1}{2}\frac{(c_i.u)^2}{c_s^4} - \frac{1}{2}\frac{u.u}{c_s^2}\right], \quad (3)$$

$$g_i^{\mathrm{eq}} = \omega_i T\left[1 + \frac{c_i.u}{c_s^2}\right]. \quad (4)$$

For D2Q9:

$$\omega_i = \begin{cases} 4/9 & i = 0 \\ 1/9 & i = 1-4 \\ 1/36 & i = 5-8 \end{cases} \quad (5)$$

The discrete velocities, c_i, for the D2Q9 are defined as follows:

$$c_i = \begin{cases} 0 & i = 0 \\ c\left(\cos\left[(i-1)\frac{\pi}{2}\right], \sin\left[(i-1)\frac{\pi}{2}\right]\right) & i = 1-4 \\ c\sqrt{2}\left(\cos\left[(i-5)\frac{\pi}{2} + \frac{\pi}{4}\right], \sin\left[(i-5)\frac{\pi}{2} + \frac{\pi}{4}\right]\right) & i = 5-8 \end{cases} \quad (6)$$

The kinematic viscosity (ϑ) and the thermal diffusivity (α) are then related to the relaxation times by:

$$\vartheta = \left[\tau_v - \frac{1}{2}\right] c_s^2\Delta t, \quad \text{and} \quad \alpha = \left[\tau_c - \frac{1}{2}\right] c_s^2\Delta t. \quad (7)$$

Also the external force appears for LBM as follows:

$$F_i = \omega_i.F.\frac{c_i}{c_s^2}. \tag{8}$$

Because of $c_s = c/\sqrt{3}$, Eq. (14) is written as:

$$F_i = 3.\omega_i.F. \tag{9}$$

Finally, macroscopic variable can be calculated in terms of these variables, with the following formula:

Flow density:

$$\rho(x,t) = \sum_i f_i(x,t). \tag{10}$$

Mommentum:

$$\rho u(x,t) = \sum_i f_i(x,t)c_i. \tag{11}$$

Temperature:

$$T = \sum_i g_i(x,t). \tag{12}$$

Viscosity is selected to insure that Mach number is within the limit of incompressible flow (Ma < 0.3) where Mach number is fixed at 0.1 in this investigation.

By fixing Rayleigh number, Prandtl number and Mach number, the viscosity and thermal diffusivity are calculated from definition of:

$$\vartheta = \sqrt{\frac{Ma^2 M^2 \mathrm{Pr} c^2}{\mathrm{Ra}}}, \tag{13}$$

where M is number of lattices in y-direction (parallel to gravitational acceleration). Rayleigh and Prandtl numbers are defined as $\mathrm{Ra} = \frac{\beta g_y H^3 \mathrm{Pr}(T_H - T_C)}{\vartheta^2}$, and $\mathrm{Pr} = \frac{\vartheta}{\alpha}$, respectively. In addition, speed of lattice is constant ($c = \frac{1}{\sqrt{3}}$). Finally, the values of relation times for flow and temperature can be found by the obtained viscosity and thermal diffusivity.

3.2. Lattice Boltzmann method for inclined cavity

It is assumed that the cavity is fixed, and just the force term is altered by the inclination of the angle. Thus, the whole previous conditions were stabled except the force term that was ameliorated by the following conditions:

X direction:

$$F_x = 3\omega_i(g_y \sin\theta)\beta\Delta T. \tag{14a}$$

Y direction:

$$F_y = 3\omega_i(g_y \cos\theta)\beta\Delta T. \tag{14b}$$

3.3. Lattice Boltzmann method in presence of magnetic field

The effect of magnetic field was shown only at the force term where it adds to the buoyancy force term. For natural convection driven flow, the force term is:

$$F_n = \rho g_y \beta \Delta T, \tag{15}$$

where g_y is the gravitational vector, ρ is the density, ΔT is the temperature difference between hot and cold boundaries and β is the thermal expansion coefficient.

But, for magnetic field at X-direction, the force term is:

$$F_B = -\rho(\mathrm{Ha}^2)\left(\frac{\nu}{m^2}\right)v. \tag{16}$$

The force added to the collision processes as:

$$F_i = 3\omega_i F c_i, \tag{17}$$

where $F = F_B + F_n$ and values of ω_i and c_i shown in Eqs. (5) and (6) are density and temperature distribution functions, respectively.

3.4. Lattice Boltzmann method for nanofluid

The dynamical similarity depends on two dimensionless parameters: the Prandtl number, Pr, and the Rayleigh number, Ra. It is assumed that nanofluid is similar to a pure fluid where nanofluid qualities are obtained by Eqs. (18)-(21); thereafter, they applied for the two parameters.

The thermo-physical properties of the nanofluid are assumed to be constant (Table 1) except for the density variation, which is approximated by the Boussinesq model.

The effect of density at reference temperature is given by [14]:

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s, \tag{18}$$

where the heat capacitance of the nanofluid and thermal expansion coefficient are calculated by [16]:

$$(\rho c_p)_{nf} = (1-\varphi)(\rho c_p)_f + \varphi(\rho c_p)_s, \tag{19}$$

$$(\rho\beta)_{nf} = (1-\varphi)(\rho\beta)_f + \varphi(\rho\beta)_s, \tag{20}$$

with φ being the volume fraction of the solid particles and subscripts f, nf and s stand for base fluid, nanofluid and solid, respectively. The viscosity of the nanofluid containing a dilute suspension of small rigid spherical particles is given by Brinkman model [17] as:

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}. \tag{21}$$

Nusselt number, Nu, is one of the most important dimensionless parameters in describing the convective heat transport. The local Nusselt number and the average value at the hot and cold walls are calculated as:

$$\mathrm{Nu}_y = -\frac{H}{\Delta T}\frac{\partial T}{\partial x}, \tag{22}$$

$$\mathrm{Nu}_{\mathrm{avg}} = \frac{1}{H}\int_0^H \mathrm{Nu}_y dy. \tag{23}$$

Because of considering different parameters effect exactly, two normalized average Nusselt numbers are defined. The first normalized average Nusselt number expresses the ratio of Nusselt number at any volume fraction of nanoparticles to that of pure water, that is:

$$\mathrm{Nu}_{\mathrm{avg}}(dv)(\varphi) = \frac{\mathrm{Nu}_{\mathrm{avg}}(\varphi)}{\mathrm{Nu}_{\mathrm{avg}}(\varphi=0)}. \tag{24}$$

The second normalized average Nusselt number exhibits the ratio of Nusselt number at any inclination angle to inclination of $\theta = 0°$, that is:

$$\mathrm{Nu}_{\mathrm{avg}}(da)(\theta) = \frac{\mathrm{Nu}_{\mathrm{avg}}(\theta)}{\mathrm{Nu}_{\mathrm{avg}}(\theta=0)}. \tag{25}$$

The convergence criterion is defined by the following expression:

$$\mathrm{Error} = \left|\frac{2(\mathrm{Nu}_{\mathrm{avg},H} - \mathrm{Nu}_{\mathrm{avg},c})}{\mathrm{Nu}_{\mathrm{avg},H} + \mathrm{Nu}_{\mathrm{avg},c}}\right| \le 10^{-5}. \tag{26}$$

4. Code validation and grid independence

This problem was investigated at different inclination angles from $\theta = -60°$ to $60°$, and various Rayleigh numbers of ($10^3 < \mathrm{Ra} < 10^5$), while Hartmann number changes between 0 and 30°, and is considered at X direction. Lattice Boltzmann method scheme is utilized for obtaining the numerical simulations in a cavity that is filled with nanofluid of water/Cu. An extensive mesh testing procedure is conducted to guarantee a grid independent solution. Five different mesh combinations were explored for the case of $\mathrm{Ra} = 10^5$, $\varphi = 0$ and $\mathrm{Ha} = 0$. The present code was tested for grid independence by calculating the local Nusselt number on the left wall. In harmony with this, it was confirmed that the grid size (51-51) ensures a grid independent solution as portrayed by Figure 2. To check the accuracy of the present results, the present code is validated against the published works in the literature on the inclined cavity in presence of magnetic field [9]. The results are compared in Figure 3. As shown in this figure, the streamlines and isotherms have appropriate agreement between both compared methods. Moreover, in this code, the method of Kefayati [22-28] for nanofluid and MHD flows is applied, where in the literature the ability of Lattice Boltzmann Method for simulating of nanofluids and MHD flow is demonstrated.

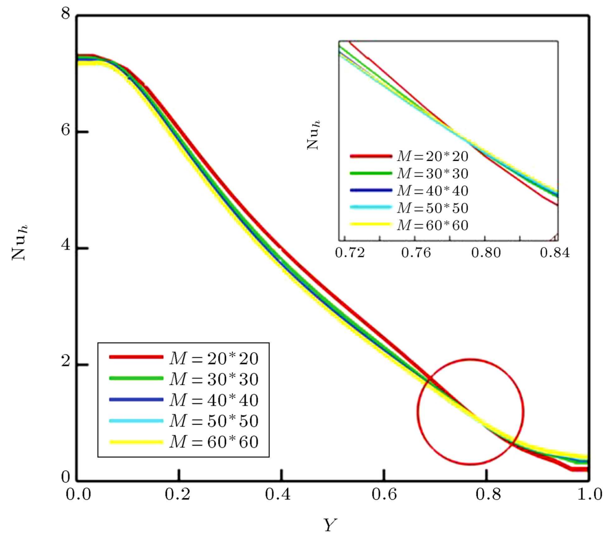


Figure 2. Local Nusselt number on the left wall for different uniform grids.

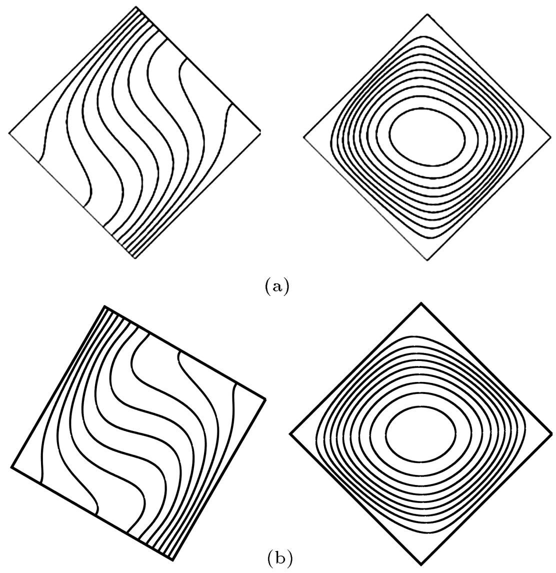


Figure 3. Comparison of the streamlines and isotherms between (a) numerical results by Pirmohammadi and Ghassemi [9] and (b) the present results.

5. Results and discussion

Figure 4 displays a comparison between pure fluid ($\varphi = 0$) and nanofluid ($\varphi = 0.06$) in the streamlines contours at various Rayleigh and Hartmann numbers for $\theta = 0°$. It is obvious that the core of streamline expands and its value increases as Rayleigh number grows. Moreover, augmentation of Hartmann number causes the streamlines to deform from horizontally circulation to vertically circulation in the cavity, and the value of the stream function declines. At various Hartmann and Rayleigh numbers, the streamline of nanofluid traverse more distance within the cavity in comparison with the pure fluid for an equal value of

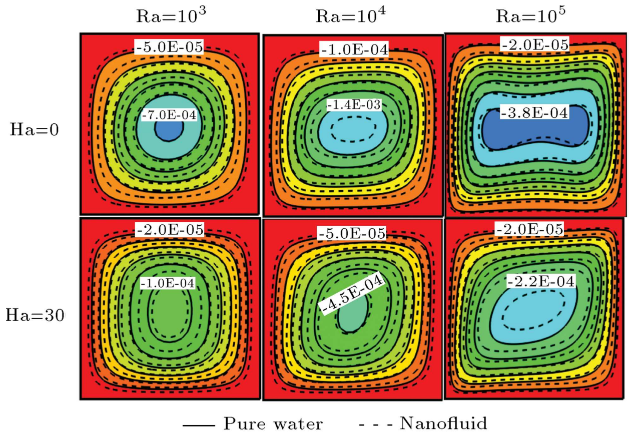


Figure 4. A comparison between pure fluid ($\varphi = 0$) and nanofluid ($\varphi = 0.06$), in the streamlines, at various Rayleigh and Hartmann numbers for $\theta = 0°$.

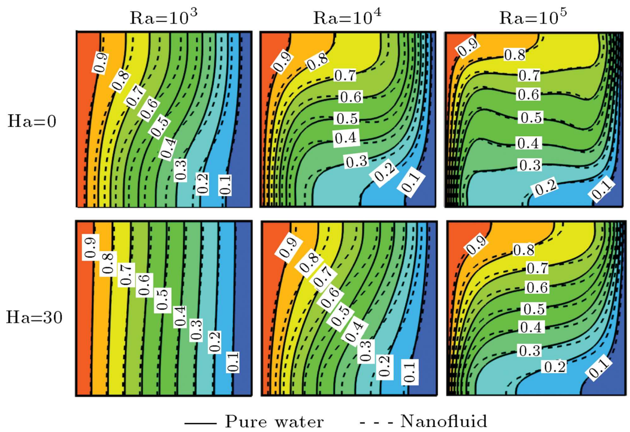


Figure 5. A comparison between isotherms contours for pure fluid ($\varphi = 0$) and nanofluid ($\varphi = 0.06$) at various Hartmann and Rayleigh numbers at $\theta = 0°$.

streamline. The phenomenon demonstrates that free convection is stronger at nanofluids.

Figure 5 illustrates the isotherms contours comparison for pure fluid ($\varphi = 0$) and nanofluid ($\varphi = 0.06$) at various Hartmann and Rayleigh numbers at $\theta = 0°$. Increment of Rayleigh number provokes the gradient of temperature on the hot wall augments and heat transfer increases. When Hartmann number enhances from Ha=0 to 30, it is obvious that the movement of the isotherm form the hot wall to the cold one reduces insofar as at $\mathrm{Ra} = 10^3$ and $\mathrm{Ha} = 30$, the isotherms behave similar to a pure conduction in the cavity. Figure 5 shows that the effect of nanoparticle at $\mathrm{Ra} = 10^3$ for $\mathrm{Ha} = 30$ against pure fluid is weaker, but it is diverse for $\mathrm{Ra} = 10^5$ where nanofluid makes convection process stronger in contrast with the fluid.

Figure 6 depicts the contour maps for the streamlines of various Hartmann numbers ($\mathrm{Ha} = 0, 30$), volume fractions ($\varphi = 0, 0.06$) and inclination angles at $\mathrm{Ra} = 10^5$. At $\theta = -60°$, the core of the cavity

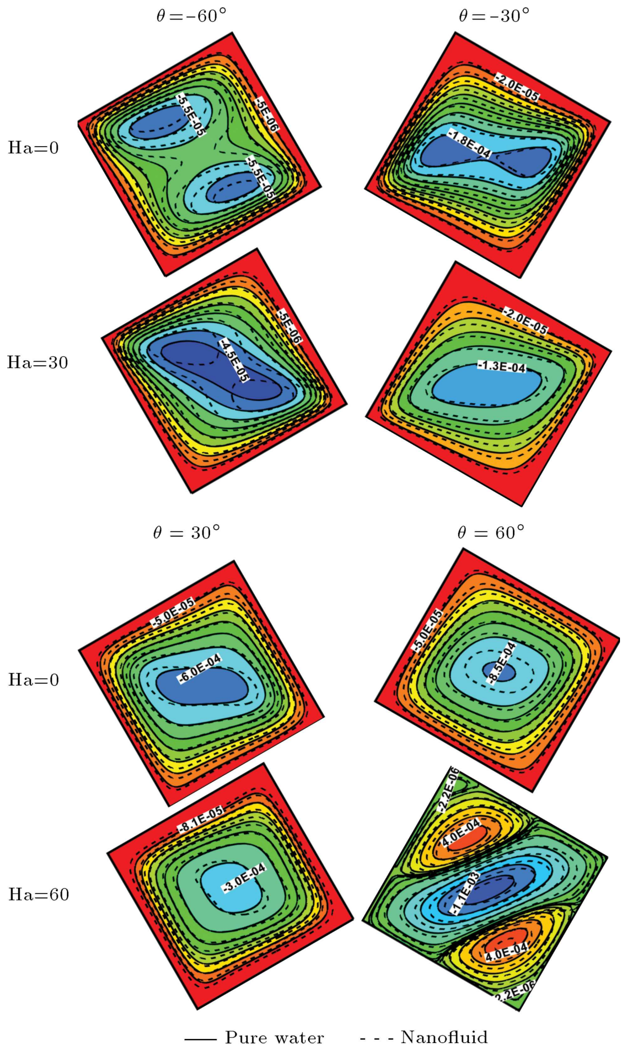


Figure 6. A comparison between streamlines for pure water and nanofluid with $\varphi = 0.06$ at various Hartman numbers and inclination angles for $\mathrm{Ra} = 10^5$.

divides into two vortexes elliptically. As Hartmann number increases by $\mathrm{Ha} = 30$ for $\theta = -60°$, the streamlines form regularly and two vortexes for the pure fluid vanishes. However, for nanofluid at the same value of stream function, two vortexes are created that proved the increment of convection with the presence of nanoparticle. At $\theta = -30°$ and $\mathrm{Ha} = 0$, it is clear that the value of stream functions declines against $\theta = 0°$; albeit, nanoparticles causes the core of the main circulation to divide into two vortexes. When Hartmann number grows to $\mathrm{Ha} = 30$, the fluid and the nanofluid form steady streamlines. Figure 6 shows that the inclination angle in positive direction augments the values of stream function. Furthermore, it is observed at $\theta = 60°$ that as Hartmann number enhances, the two counterclockwise circulations surround the main circulation that flows clockwise.

Figure 7 displays the isotherm contours' comparison for pure fluid ($\varphi = 0$) and nanofluid ($\varphi = 0.06$) at

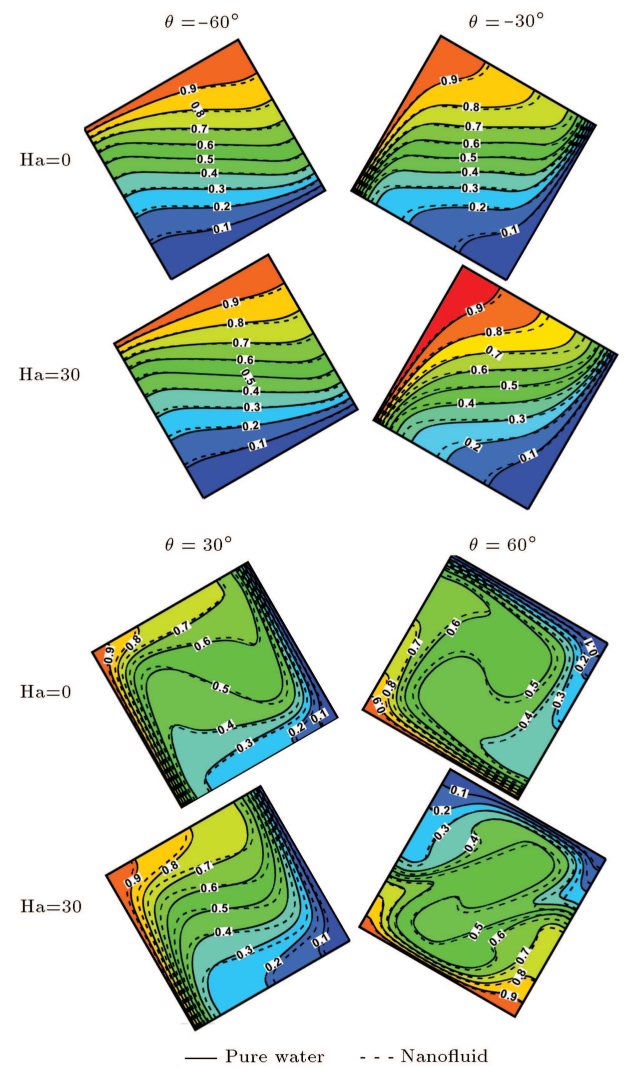


Figure 7. A comparison between isotherm contours for pure fluid ($\varphi = 0$) and nanofluid ($\varphi = 0.06$) at various inclination angle and two Hartmann numbers (Ha = 0, 30)

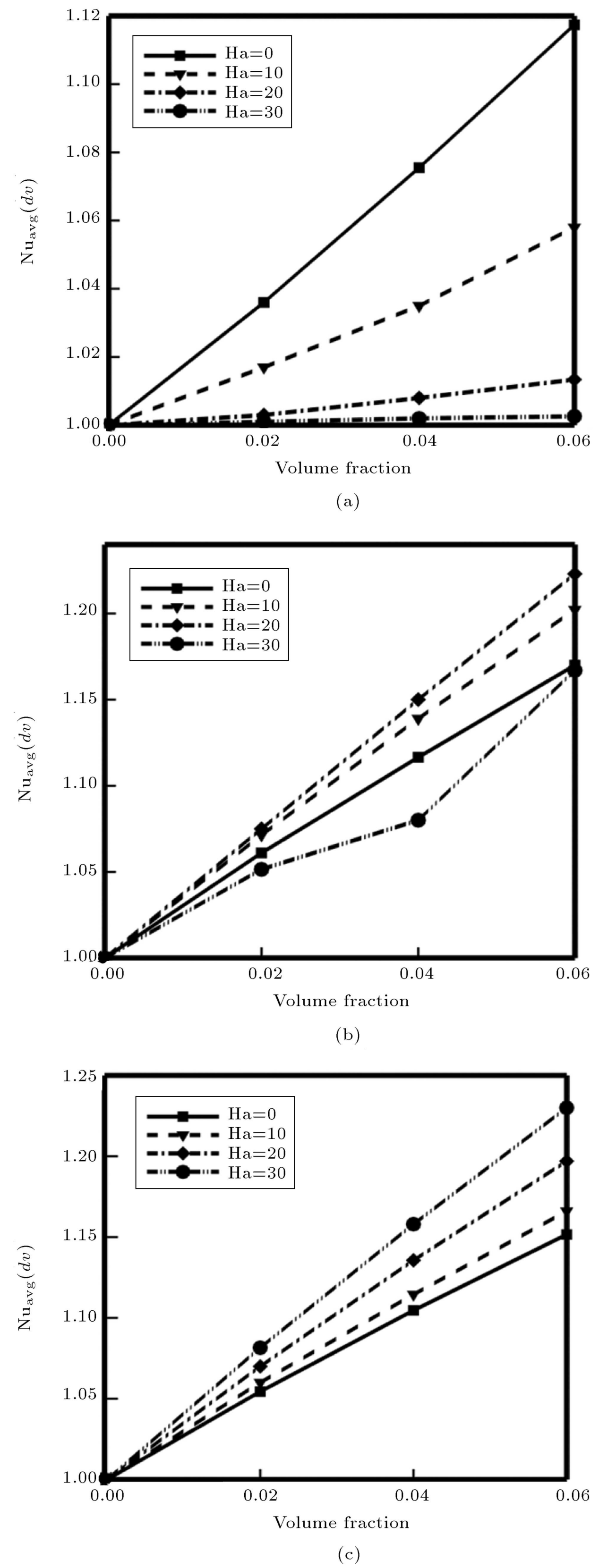


Figure 8. Non-dimensional average Nusselt number distributions on the hot wall for $\theta = 0°$ at various volume fractions, Hartmann and Rayleigh numbers: (a) $\mathrm{Ra} = 10^3$; (b) $\mathrm{Ra} = 10^4$; and (c) $\mathrm{Ra} = 10^5$.

various inclination angle and two Hartmann numbers (Ha = 0, 30) as $\mathrm{Ra} = 10^5$. It shows that the effect of Hartmann number is more significant at positive angles, as at $\theta = -60°$ and $-30°$, the isotherms have equal trend for various Hartmann numbers. At $\theta = 30°$ and $60°$, as Hartmann number increases in Ha = 30, the shape of the isotherm completely alters, and moreover the effect of nanoparticle on the isotherms ameliorates.

In Figure 8, non-dimensional average Nusselt number toward pure fluid is emerged for different volume fractions, Rayleigh and Hartmann numbers at $\theta = 0°$. As volume fraction grows, generally heat transfer for various pertinent parameters augments, but the pattern of the increment is non-uniform for different Rayleigh and Hartmann numbers. At $\mathrm{Ra} = 10^3$, the non-dimensional average Nusselt number reduces with the enhancement of Hartmann number, while for

$\mathrm{Ra} = 10^4$ an irregular trend is observed and the ratio is grown by Hartmann number until $\mathrm{Ha} = 20$, but suddenly a drop occurs at $\mathrm{Ha} = 30$. The manner of the ratio against the rise of Hartmann number at $\mathrm{Ra} = 10^4$ is completed when $\mathrm{Ra} = 10^5$, as it increases regularly with augmentation of Hartmann number.

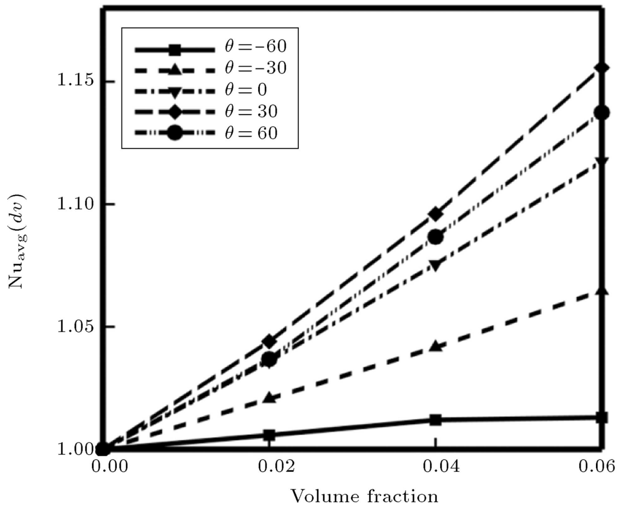
Figure 9 presents the non-dimensional average Nusselt number toward pure fluid for various inclination angles and volume fractions at three Rayleigh numbers ($\mathrm{Ra} = 10^3$, 10^4 and 10^5) and $\mathrm{Ha} = 0$. For the whole Rayleigh numbers, negative inclination angles of $\theta = -60°$ and $-30°$ have the least non-dimensional average Nusselt number value. At positive inclination angle, there exists an irregular manner for various Rayleigh numbers, although totally the ratio declines considerably from $\theta = 30°$ to $60°$.

Figure 10 exhibits the dimensionless average Nusselt number toward angle of $\theta = 0°$ for different Hartmann and Rayleigh numbers at various Rayleigh numbers, as $\varphi = 0.06$. This ratio has a linearly manner from $\theta = -60°$ to $0°$, and it enhances for various Hartmann numbers. However, at positive angles, the ratio changes erratically. High Hartmann number causes this ratio to jump noticeably at $\mathrm{Ra} = 10^4$ from $\theta = 30°$ to $60°$, while the value falls for low Hartmann number. In addition, at $\mathrm{Ra} = 10^3$, the variation confine decreases and its value is almost equal to 1 as Hartmann number augments.

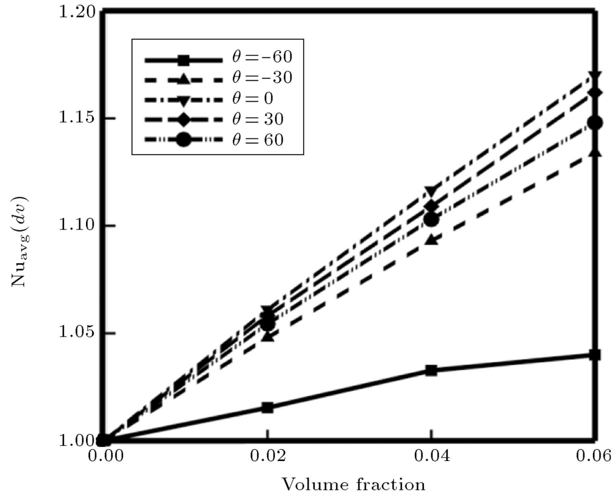
Figure 11 shows distribution of local Nusselt number on hot wall for various considered parameters at $\mathrm{Ra} = 10^5$. Local Nusselt number is examined for various Hartmann numbers in $\varphi = 0.06$ and $\theta = 0°$ in Figure 11(a). Figure 11(b) demonstrates the effect of inclination angles on local Nusselt numbers for $\varphi = 0.06$ and $\mathrm{Ha} = 0$. A noticeable note in local Nusselt number plot occurs for $\theta = 30°$ and $60°$ where they are placed under values of $\theta = 0°$ at $Y < 0.5$, but at $Y > 0.5$ acts oppositely. Figure 11(c) illustrates Nusselt number distributions on the hot wall at different volume fractions when $\mathrm{Ha} = 30$ and $\theta = 0°$. It demonstrates that the local Nusselt numbers along the hot wall at different volume fractions have equivalent trend. Nevertheless, $\mathrm{NU}_{\max}$ on the hot wall enhances with an increase in volume fraction.

6. Conclusions

In this paper, the effect of the magnetic field on nanofluid flow in an inclined cavity has been analyzed with lattice Boltzmann method. This study has been carried out for the pertinent parameters in the following ranges: the Rayleigh number of the base fluid, $\mathrm{Ra} = 10^3 - 10^5$, the Hartmann number of magnetic field between $\mathrm{Ha} = 0$ and 30, and the direction of the magnetic field being fixed at X direction, where inclination angles are conducted for $\theta = -60°$ to $60°$.



(a)



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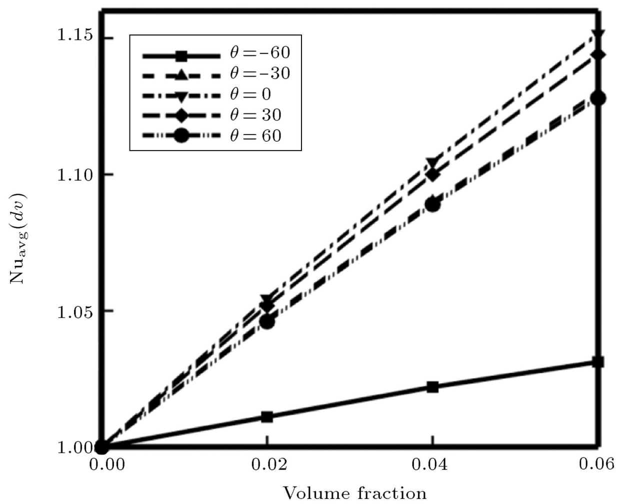

(c)

Figure 9. Non-dimensional average Nusselt number distributions on the hot wall for Ha=0 at various volume fractions, inclination angle and Rayleigh numbers: (a) $\mathrm{Ra} = 10^3$; (b) $\mathrm{Ra} = 10^4$; and (c) $\mathrm{Ra} = 10^5$.

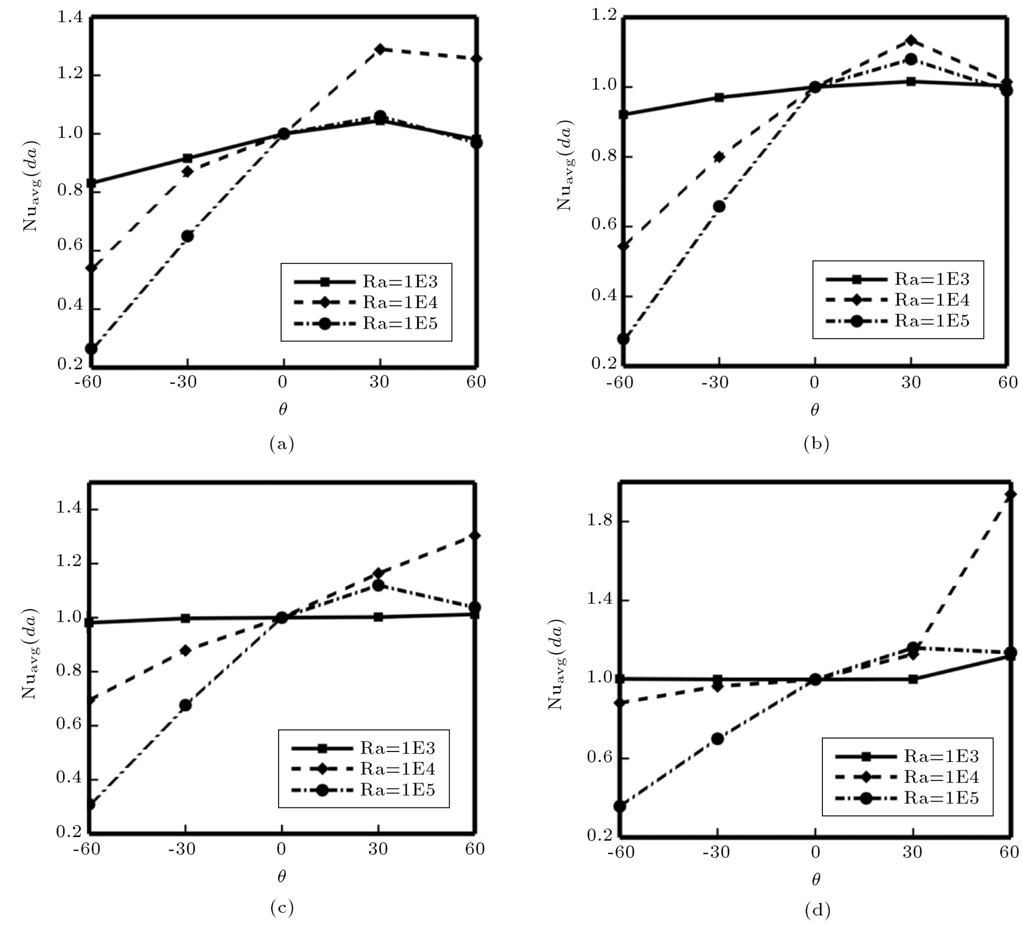

Figure 10. Non-dimensional average Nusselt number distributions on the hot wall for $\varphi = 0.06$ at various inclination angle, Hartmann and Rayleigh numbers: (a) Ha = 0; (b) Ha = 10; (c) Ha = 20; and (d) Ha = 30.

This investigation was performed for various mentioned parameters, and some conclusions are summarized as follows:

a) A good agreement with previous numerical investigations demonstrates that lattice Boltzmann Method is an appropriate method for different applicable problems.

b) Heat transfer declines with increment of Hartmann number for various Rayleigh numbers and inclination angles.

c) The effect of nanoparticle is different for various Rayleigh numbers in the presence of the magnetic field. At $\mathrm{Ra} = 10^3$, this effect is reduced with increment of Hartmann number but at $\mathrm{Ra} = 10^5$, the magnetic field augments the effect of nanoparticle.

d) Heat transfer decreases generally when inclination angle moves downward.

e) Negative angles ($\theta = -60°$ and $-30°$) cause the effect of nanoparticles to fall; nanoparticles behave erratically at positive angles for different Rayleigh numbers.

Nomenclature

B	Magnetic field
c	Lattice speed
c_i	Discrete particle speeds
c_p	Specific heat at constant pressure
F	External forces
f	Density distribution functions
f^{eq}	Equilibrium density distribution functions
g	Internal energy distribution functions
g^{eq}	Equilibrium internal energy distribution functions
g_y	Gravity
Gr	Grashof number $\left(\mathrm{Gr} = \frac{\beta g_y H^3 (T_H - T_C)}{\nu^2}\right)$
Ha	Hartmann number $\mathrm{Ha}^2 = \frac{B^2 L^2 \sigma_e}{\mu}$
M	Lattice number
Ma	Mach number
Nu	Nusselt number

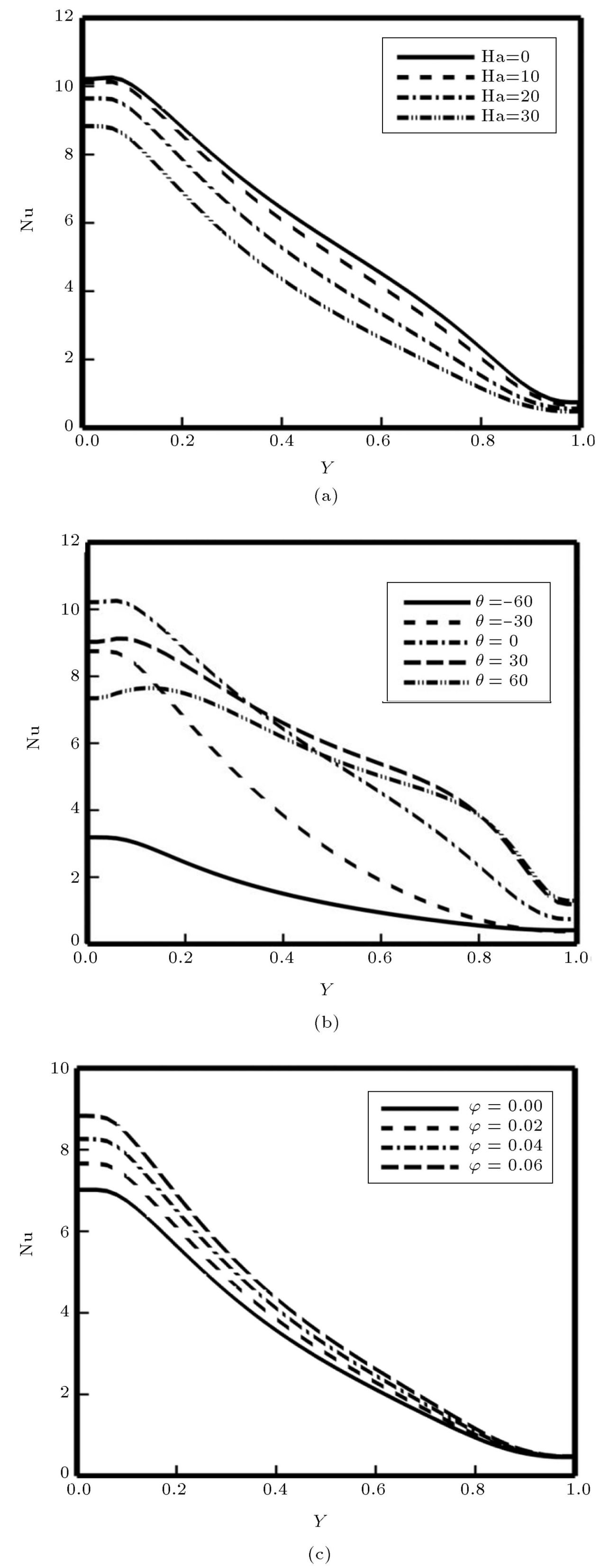

Figure 11. Local Nusselt number distributions on the left wall at various parameters: (a) $\varphi = 0.06$, $\theta = 0°$ and $\mathrm{Ra} = 10^5$; (b) $\varphi = 0.06$, $\mathrm{Ha} = 0$ and $\mathrm{Ra} = 10^5$; and (c) $\mathrm{Ha} = 30$, $\varepsilon = 0°$ and $\mathrm{Ra} = 10^5$.

Pr	Prandtl number
R	Constant of the gases
Ra	Rayleigh number $\left(\mathrm{Ra} = \frac{\beta g_y H^3 (T_H - T_C)}{\nu \alpha}\right)$
T	Temperature
x, y	Cartesian coordinates
u	Magnitude velocity

Greek letters

σ	Surface tension
ω_i	Weighted factor indirection i
β	Thermal expansion coefficient
τ_c	Relaxation time for temperature
τ_v	Relaxation time for flow
ν	Kinematic viscosity
Δx	Lattice spacing
Δt	Time increment
α	Thermal diffusivity
φ	Volume fraction
μ	Dynamic viscosity
ψ	Stream function value
θ	Inclination angle

Subscripts

avg	Average
C	Cold
H	Hot
f	Fluid
nf	Nanofluid
s	Solid

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Biography

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