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Sensitivity analysis of vibration modes of rectangular cantilever beams immersed in fluid to surface stiffness variations

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Abstract. In this paper, the sensitivity of flexural and torsional vibration modes of a rectangular cantilever immersed in a fluid to surface stiffness variations has been analyzed and a closed-form expression is derived. To represent this sensitivity, we use analytical formulas for the vibrational resonant frequencies of a rectangular cantilever beam immersed in an inviscid fluid. The effect of the surface contact stiffness on both flexural and torsional sensitivities in a fluid is investigated and compared with cases in which the cantilever operates in air. The results show that in low surface stiffness, the first mode is the most sensitive. As sample surface stiffness is increased, higher resonant frequencies show a larger shift, compared with lower resonant frequencies. In addition, comparison between modal sensitivities in air and fluid shows that the resonance frequency shifts in air are greater than resonant frequency shifts in fluid.

1. Introduction

Dynamic analysis of cantilever beams immersed in fluids is necessary for numerous applications, including environmental sensing [1], micro/nano electromechanical system designing [2], and high resolution imaging [3]. In comparison with air or vacuum environments, cantilever dynamics in fluid remain much less understood and, thus, require more investigation [4].

By increasing cantilever size, the effect of viscosity on the dynamic response of the cantilever immersed in a fluid is decreased, and the fluid can be considered inviscid [5]. This assumption significantly simplifies analysis of cantilever dynamics in fluid. Based on this assumption, Chu [6] presented a simple expression for the flexural resonant frequency of a cantilever in fluid. The presented expression shows an acceptable agreement with experiments for macroscopic cantilevers, especially for fundamental and the next few modes [7].

Based on Chu's work [6], Elmer and Dreier [8] proposed an expression for flexural modes of arbitrary mode numbers. Based on the work carried out in [8] and [9], Eysden et al. [5] proposed an explicit analytical formula for both flexural and torsional resonant frequencies of a rectangular cantilever beam immersed in an inviscid fluid.

Jensen and Hegner recently undertook work to calculate the flexural resonance frequency in a fluid environment [10], and used the compressible fluid model of Van Eysden and Sader in [5] to calculate the flexural resonance frequencies of the micro cantilever.

The motion of the cantilever beam can be affected by the interactive stiffness of a cantilever with the sample surface [11].

Much research work has been carried out to study the vibration response of a cantilever beam to the interaction between the cantilever and the sample surface [12-19].

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As a pioneer work in this field, Turner and Wihen, in [12], have proposed analytical expressions to calculate the sensitivity of flexural and torsional micro cantilevers to surface stiffness variations for both rectangular and V-shaped cantilevers. However, all these studies are carried out in an air environment. In this paper, following the method presented in [12] for an air environment, we study the sensitivity of flexural and torsional vibration modes of a cantilever beam immersed in a fluid. We consider the inviscid fluid environment and use the analytical expression for the flexural and torsional resonance frequencies of the cantilever derived in [5]. Based on this assumption, we derive expressions for the vibration mode sensitivity of a cantilever beam immersed in fluid.

In Sections 2 and 3, analytical formulas for flexural and torsional vibrations are derived. Numerical results are presented in Section 4 and, finally, a conclusion is provided in Section 5.

2. Flexural Vibration

For the flexural modes of vibration, the governing equation for elastic deformation is:

$$EI\frac{\partial^4 y(x,t)}{\partial x^4}+\rho_c A\frac{\partial^2 y(x,t)}{\partial t^2}=F(x,t), \tag{1}$$

where $y(x,t)$ is the deflection function of the beam, E is the modulus of elasticity, I is the area moment of inertia, ρ_c is the volume density, and A is the uniform cross section area of the cantilever. $F(x,t)$ is the external applied force per unit length of the beam. The schematic of the cantilever beam is depicted in Figure 1.

The corresponding boundary conditions are given by:

$$y(0,t)=0, \qquad \frac{\partial y(0,t)}{\partial x}=0,$$

$$\frac{\partial^2 y(L,t)}{\partial x^2}=0, \qquad EI\frac{\partial^3 y(L,t)}{\partial x^3}=K_n y(L,t), \tag{2}$$

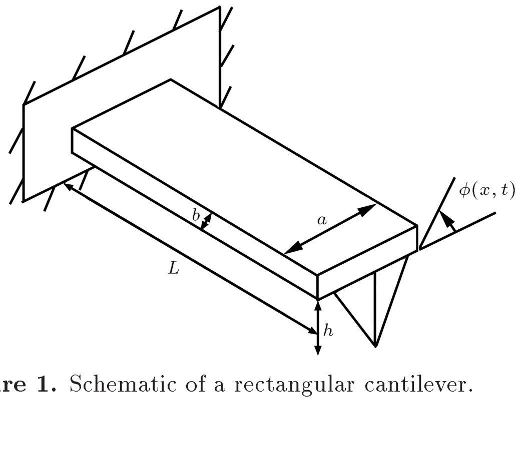

Figure 1. Schematic of a rectangular cantilever.

where K_n is normal contact stiffness. Following the analysis given in [5], the resonant frequencies in the fluid of a rectangular cantilever beam are given by:

$$\omega_{\text{fluid}}^{(n)}=\omega_{vac}^{(n)}\left[1+\frac{\pi\rho_f a}{4\rho_c b}\Gamma_f(k_n)\right]^{-1/2}, \tag{3}$$

where:

$$\omega_{vac}^{(n)}=\frac{\gamma_n^2}{L^2}\sqrt{\frac{EI}{\rho_c A}}, \tag{4}$$

where n is the mode order and γ_n is the nth positive root of the following characteristics equation:

$$C(\gamma_n,\beta_f)=\gamma_n^3(\cos\gamma_n\cosh\gamma_n+1)-\beta_f$$

$$(\sinh\gamma_n\cos\gamma_n-\sin\gamma_n\cosh\gamma_n)=0, \tag{5}$$

where $\beta_f=\frac{K_n}{EI/L^3}$, ρ_f is the density of the fluid, a is the cantilever width and b is the cantilever height. Also, the hydrodynamic function, $\Gamma_f(k_n)$, is given by [5]:

$$\Gamma_f(k_n)=\frac{1+0.74273k_n+0.14862k_n^2}{1+0.74273k_n+0.35004k_n^2+0.058364k_n^3}, \tag{6}$$

where $k_n=\gamma_n\frac{a}{L}$.

The flexural sensitivity of the cantilever can be calculated from the derivative of the flexural frequency with respect to the flexural surface stiffness, β_f.

For the flexural mode, the frequency is given by:

$$f_{\text{fluid},n}=f_{vac,n}\left[1+\frac{\pi\rho_f a}{4\rho_c b}\Gamma_f(k_n)\right]^{-1/2}, \tag{7}$$

where:

$$f_{vac,n}=\frac{\gamma_n^2}{2\pi L^2}\sqrt{\frac{EI}{\rho_c A}}. \tag{8}$$

So, based on Eqs. (7) and (8), we have:

$$\frac{\partial f_{\text{fluid},n}}{\partial\beta_f}=\frac{\partial f_{\text{fluid},n}}{\partial\gamma_n}\frac{\partial\gamma_n}{\partial\beta_f}. \tag{9}$$

From Eq. (7), we have:

$$\frac{\partial f_{\text{fluid},n}}{\partial\gamma_n}=\frac{\partial f_{vac,n}}{\partial\gamma_n}\left[1+\frac{\pi\rho_f a}{4\rho_c b}\Gamma_f(k_n)\right]^{-1/2}$$

$$-\frac{\pi\rho_f a}{8\rho_c b}\left[1+\frac{\pi\rho_f a}{4\rho_c b}\Gamma_f(k_n)\right]^{-3/2}$$

$$\times\frac{\partial\Gamma_f(k_n)}{\partial k_n}\frac{\partial k_n}{\partial\gamma_n}, \tag{10}$$

where: Eqs. (11)-(13) are shown in Box (I).

$$\frac{\partial f_{vac,n}}{\partial \gamma_n} = \frac{\gamma_n}{\pi L^2}\sqrt{\frac{EI}{\rho_c A}}, \tag{11}$$

$$\frac{\partial \Gamma_f(k_n)}{\partial k_n} = \frac{(0.74273+0.29724k_n)\left(1+0.74273k_n+0.35004k_n^2+0.058364k_n^3\right)-\left(1+0.74273k_n+0.14862k_n^2\right)\left(0.74273+0.70008k_n+0.175092k_n^2\right)}{(1+0.74273k_n+0.35004k_n^2+0.058364k_n^3)^2}, \tag{12}$$

$$\frac{\partial k_n}{\partial \gamma_n} = \frac{a}{L}. \tag{13}$$

Box (I).

On the other hand, $\partial\gamma_n/\partial\beta_f$ can be calculated from:

$$\frac{\partial \gamma_n}{\partial \beta_f} = -\frac{\partial C/\partial \beta_f}{\partial C/\partial \gamma_n}. \tag{14}$$

So, based on the characteristics Eq. (5), we have:

$$\frac{\partial \gamma_n}{\partial \beta_f} = (\cos\gamma_n \sinh\gamma_n - \sin\gamma_n \cosh\gamma_n) \times \left\{3\gamma_n^2\left(1+\cos\gamma_n\cosh\gamma_n\right)+2\beta_f\sin\gamma_n\sinh\gamma_n + \gamma_n^3\left(\cos\gamma_n\sinh\gamma_n - \sin\gamma_n\cosh\gamma_n\right)\right\}^{-1}. \tag{15}$$

Therefore, by substituting Eqs. (11)-(13) into Eq. (10), $\frac{\partial f_{\text{fluid},n}}{\partial \gamma_n}$ is obtained, and by substituting Eq. (15) and the obtained $\frac{\partial f_{\text{fluid},n}}{\partial \gamma_n}$ into Eq. (9), $\frac{\partial f_{\text{fluid},n}}{\partial \beta_f}$ is calculated.

Finally, the dimensionless form of flexural sensitivity can be calculated by:

$$\sigma_{f,n} = \frac{\partial f_{\text{fluid},n}/\partial\beta_f}{\frac{1}{2\pi L^2}\sqrt{\frac{EI}{\rho_c A}}}. \tag{16}$$

3. Torsional Vibration

The governing equation for the torsional oscillations is:

$$G\xi\frac{\partial^2\phi(x,t)}{\partial x^2} - \rho_c J\frac{\partial^2\phi(x,t)}{\partial t^2} = M(x,t), \tag{17}$$

where $\phi(x,t)$ is the angle of torsion of the cross-sectional area, shown in Figure 1, $M(x,t)$ is the applied torque per unit length of the beam, G is the shear modulus, J is the polar moment of inertia, and ξ is the torsional parameter for a rectangular beam, which is given in [20]:

$$\xi = \frac{1}{3}h^3 a\left(1 - 0.63\frac{h}{a}\right). \tag{18}$$

The corresponding boundary conditions are:

$$\phi(0,t) = 0,$$

$$G\xi\frac{\partial\phi(L,t)}{\partial x} = -K_t h^2\phi(L,t), \tag{19}$$

where K_t is lateral surface stiffness and h is tip height. Following the analysis given in [5], the torsional resonant frequencies in the fluid of a rectangular cantilever beam are given by:

$$\omega_{\text{fluid}}^{(n)} = \omega_{vac}^{(n)}\left[1 + \frac{3\pi\rho_f a}{2\rho_c b}\Gamma_t(k_n)\right]^{-1/2}, \tag{20}$$

where:

$$\omega_{vac}^{(n)} = \frac{\gamma_n}{L}\sqrt{\frac{G\xi}{\rho_c J}}m, \tag{21}$$

where n is the mode order and γ_n is the nth positive root of the following characteristics equation:

$$C(\gamma_n,\beta_t) = \gamma_n\cos\gamma_n + \beta_t\sin\gamma_n = 0, \tag{22}$$

where $\beta_t = K_t h^2 L/G\xi$. Also, the hydrodynamic function, $\Gamma_t(k_n)$, is given by [5]:

$$\Gamma_t(k_n) = \frac{1}{16}\left(\frac{1+0.37922k_n+0.072912k_n^2}{1+0.37922k_n+0.088056k_n^2+0.010737k_n^3}\right), \tag{23}$$

where $k_n = \gamma_n\frac{a}{L}$.

The flexural sensitivity of cantilever can be calculated from the derivative of frequency, with respect to surface lateral stiffness, β_t.

For the flexural mode, the frequency is given by:

$$f_{\text{fluid},n} = f_{vac,n}\left[1 + \frac{3\pi\rho_f a}{2\rho_c b}\Gamma_t(k_n)\right]^{-1/2}, \tag{24}$$

where:

$$f_{vac,n} = \frac{\gamma_n}{2\pi L}\sqrt{\frac{G\xi}{\rho_c J}}. \tag{25}$$

So, based on Eqs. (24) and (25), we have:

$$\frac{\partial f_{\text{fluid},n}}{\partial \beta_t} = \frac{\partial f_{\text{fluid},n}}{\partial \gamma_n}\frac{\partial \gamma_n}{\partial \beta_t}. \tag{26}$$

From Eq. (25), we have:

$$\frac{\partial f_{\text{fluid},n}}{\partial \gamma_n} = \frac{\partial f_{vac,n}}{\partial \gamma_n}\left[1+\frac{3\pi\rho_f a}{2\rho_c b}\Gamma_t(k_n)\right]^{-1/2} - \frac{3\pi\rho_f a}{4\rho_c b}\left[1+\frac{3\pi\rho_f a}{2\rho_c b}\Gamma_t(k_n)\right]^{-3/2} \frac{\partial \Gamma_t(k_n)}{\partial k_n}\frac{\partial k_n}{\partial \gamma_n}, \tag{27}$$

where: Eqs. (28)-(30) are shown in Box (II).

On the other hand, $\frac{\partial \gamma_n}{\partial \beta_t}$ can be calculated from:

$$\frac{\partial \gamma_n}{\partial \beta_t} = -\frac{\partial C/\partial \beta_t}{\partial C/\partial \gamma_n}. \tag{31}$$

So, based on the characteristics of Eq. (22), we have:

$$\frac{\partial \gamma_n}{\partial \beta_t} = \frac{\sin\gamma_n}{\gamma_n \sin\gamma_n - (1+\beta_t)\cos\gamma_n}. \tag{32}$$

Therefore by substituting Eqs. (28)-(30) into Eq. (27), $\frac{\partial f_{\text{fluid}.n}}{\partial \gamma_n}$ is obtained, and by substituting Eq. (32) and the obtained $\frac{\partial f_{\text{fluid}.n}}{\partial \gamma_n}$ into Eq. (27), $\frac{\partial f_{\text{fluid}.n}}{\partial \beta_t}$ is calculated.

Also, the dimensionless form of torsional sensitivity can be obtained as:

$$\sigma_{t,n} = \frac{\partial f_{\text{fluid},n}/\partial \beta_t}{\frac{1}{2\pi L}\sqrt{\frac{G\xi}{\rho_c J}}}. \tag{33}$$

4. Results

In this section, the sensitivity of the vibrational modes of a rectangular cantilever immersed in fluid has been analyzed. The cantilever parameters are listed in Table 1.

Table 1. Parameters for a cantilever.

Elastic modulus E (GPa)	170
Shear modulus G (GPa)	66.4
Density of the cantilever ρ_c (kg/m^3)	2330
Density of the Fluid ρ_f (kg/m^3)	1000
Length L (μm)	445
Width a (μm)	44
Thickness b (μm)	2.18
Tip length h (μm)	10

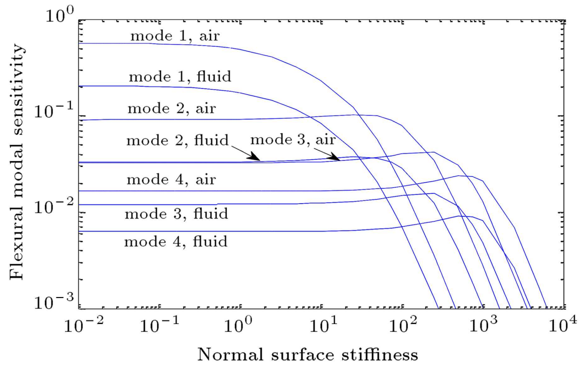

Figure 2. Comparison between the cantilever flexural modal sensitivity in air and fluid.

The flexural sensitivity of the first five modes for the rectangular cantilever in fluid is plotted in Figure 2. Also, for the purpose of comparison with an air environment, the numerical results of the air environment are added to this figure.

When the cantilever is more compliant than the sample surface, the first mode is most sensitive to surface stiffness. By increasing sample stiffness, the sensitivity of the first mode is decreased, and when β_f is about 25, the second mode is the most sensitive. Also, when β_f is in the range of 200, the third mode is the most sensitive. By increasing β_f, higher modes become more sensitive than lower ones. In other words, when β_f is small, the first resonance frequency has the largest shift among all resonance frequencies.

By increasing sample stiffness, higher resonance

$$\frac{\partial f_{vac.n}}{\partial \gamma_n} = \frac{1}{2\pi L}\sqrt{\frac{G\xi}{\rho_c J}}, \tag{28}$$

$$\frac{\partial \Gamma_t(k_n)}{\partial k_n} = \frac{(0.37922+0.145824k_n)\left(1+0.37922k_n+0.088056k_n^2+0.010737k_n^3\right)-\left(1+0.37922k_n+0.072912k_n^2\right)\left(0.37922+0.176112k_n+0.032211k_n^2\right)}{(1+0.37922k_n+0.088056k_n^2+0.010737k_n^3)^2}, \tag{29}$$

$$\frac{\partial k_n}{\partial \gamma_n} = \frac{a}{L}. \tag{30}$$

Box (II).

frequencies show more frequency shift in comparison with lower resonance frequencies. As shown in Figure 2, cantilever sensitivity in air is greater than cantilever sensitivity in fluid. In other words, the frequency shifts (changes) due to the variation of surface stiffness in fluid are less than those in the air. Also, comparing the sensitivity of mode 2 of fluid with mode 3 of air shows that for low values of surface stiffness, they are very close to each other, but when surface stiffness is increased, the sensitivity of mode 3 of air is greater than the sensitivity of the second mode of fluid. Also, for the any value of surface stiffness, the sensitivity of mode 4 in air is greater than the sensitivities of modes 3 and 4 in fluid. In other words, the frequency shift of the fourth resonance frequency in air is greater than the frequency shifts of the third and fourth resonance frequencies in fluid.

The torsional sensitivity of the first five modes for the rectangular cantilever in fluid is plotted in Figure 3.

As seen from Figure 3, torsional modal sensitivity shows similar behavior with the flexural modal sensitivity. For smaller lateral surface stiffness, the first mode is the most sensitive. For the stiffer sample, the sensitivity starts to decrease as β_t is increased; the shift in the resonance frequencies of higher torsional modes becomes larger than that of the lower modes.

Comparison shows that the torsional sensitivity in air is greater than in fluid. Also, the sensitivities of the third mode of the cantilever in air, is greater than the second mode sensitivity in fluid. We can conclude that, generally, the frequency shifts of the torsional resonance frequencies of a cantilever in air are significantly greater than those in fluid.

Moreover, for the purpose of investigating the effect of a fluid environment on the sensitivity of a cantilever, other numerical analyses are carried out. As shown in Figures 4-6, for the first three modes of flexural vibration, and in Figures 7-9 for the first three modes of torsional vibration, if the density of the fluid is increased, the sensitivity of the cantilever

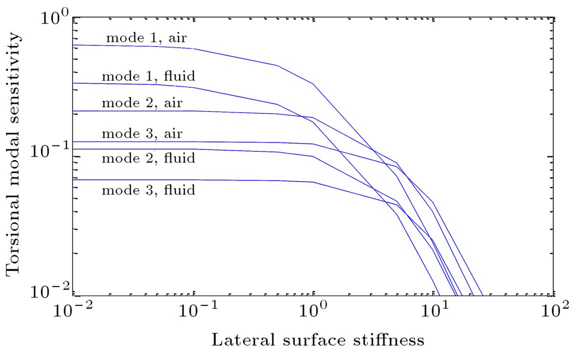

Figure 3. Comparison between the cantilever torsional modal sensitivity in air and fluid.

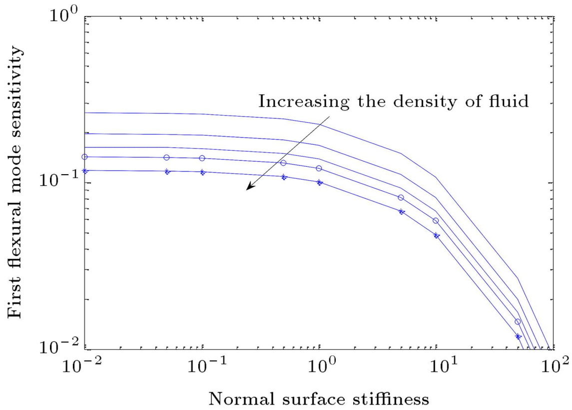

Figure 4. First flexural modal sensitivity, σ_f, as a function of normal contact stiffness for different fluids.

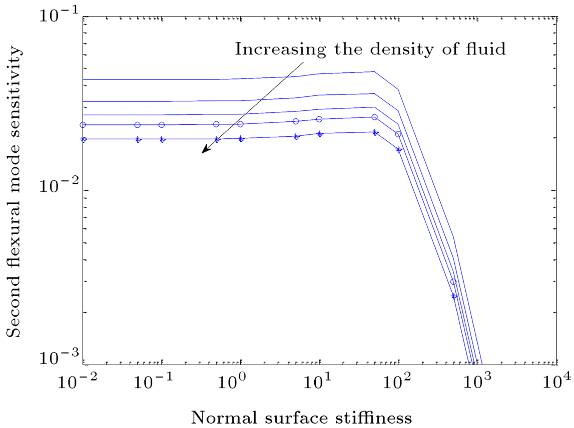

Figure 5. Second flexural modal sensitivity, σ_f, as a function of normal contact stiffness for different fluids.

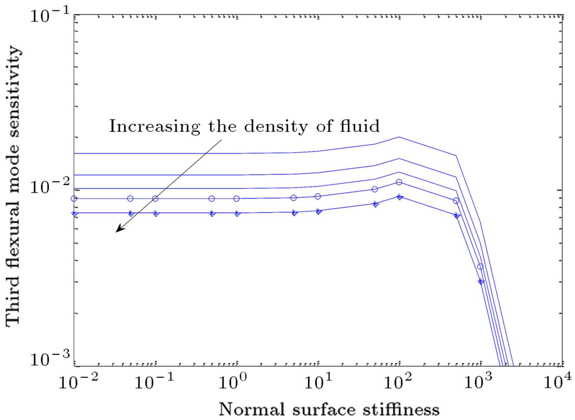

Figure 6. Third flexural modal sensitivity, σ_f, as a function of normal contact stiffness for different fluids.

is reduced, which means that for AFM imaging in a fluid environment, imaging in an environment with higher density leads to a decrease in image contrast, and, based on the stiffness of the sample, excitation of higher eigenmodes as the driving frequency is recommended.

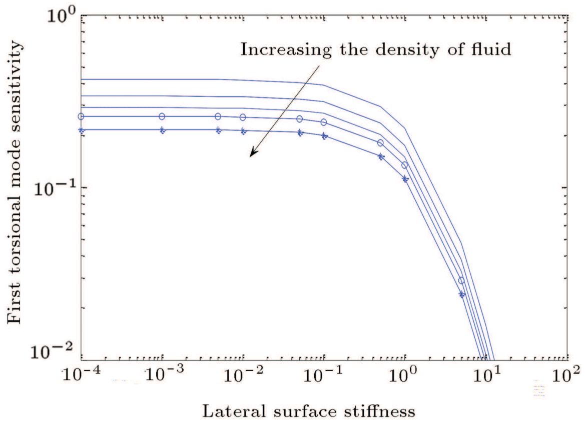

Figure 7. First torsional modal sensitivity, σ_t as a function of lateral contact stiffness for different fluids.

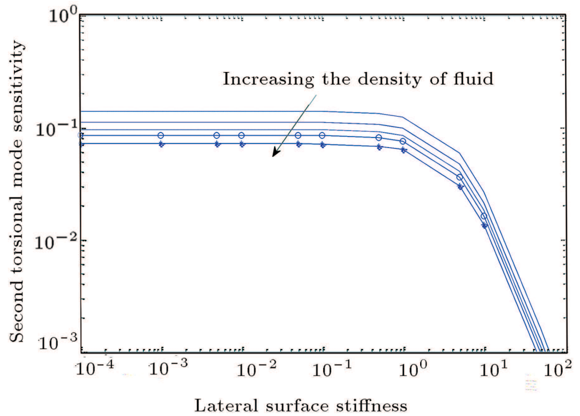

Figure 8. Second torsional modal sensitivity, σ_t, as a function of lateral contact stiffness for different fluids.

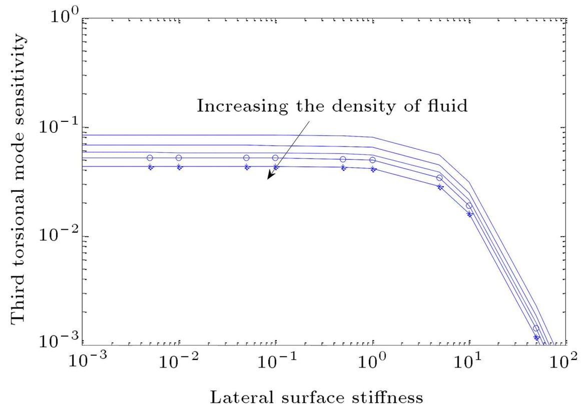

Figure 9. Third torsional modal sensitivity, σ_t, as a function of lateral contact stiffness for different fluids.

5. Conclusion

In this paper, we have employed an analytical expression for the flexural and torsional resonance frequency of a rectangular cantilever beam immersed in fluid. We derived analytical formulas for the flexural and torsional sensitivity of a cantilever immersed in an inviscid fluid to surface stiffness variations. It was concluded that for the lower value of normal and lateral stiffness, the sensitivity of the first mode is dominant, but, as stiffness is increased, the sensitivity of the first mode is decreased and the higher vibration modes become more sensitive. Comparison between the vibrational mode sensitivity of air and fluid shows that the modal sensitivity in air is greater than that in fluid, i.e the frequency shifts in the resonance frequencies of the cantilever to the variation of surface stiffness in the air are greater than those in fluid. Thus, if we use the cantilever for imaging, the image contrast in air is better than that in fluid for a similar mode and similar cantilever.

Nomenclature

$y(x,t)$	Deflection function of the beam
E	Modulus of elasticity
I	Area moment of inertia
ρ_c	Volume density
A	Uniform cross section area of the cantilever
$F(x,t)$	External applied force per unit length of the beam
K_n	Normal contact stiffness
ρ_f	Density of fluid
a	Cantilever width
b	Cantilever height
β_f	Flexural surface stiffness
$\phi(x,t)$	Angle of torsion of the cross-sectional area
$M(x,t)$	Applied torque per unit length of the beam
G	Shear modulus
J	Polar moment of inertia
ξ	Torsional parameter for a rectangular beam
K_t	Lateral surface stiffness
h	Tip height
β_t	Surface lateral stiffness
$\Gamma(k_n)$	Hydrodynamic function

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