Transaction E: Industrial Engineering
Vol. 17, No. 2, pp. 143–149

Research Note

Determination of the Number of Kanbans and Batch Sizes in a JIT Supply Chain System

S.K. Chaharsooghi[1,*] **and A. Sajedinejad**[1]

Abstract. *Under stochastic demand in the multi-stage SCM in a JIT environment, the probability of generating a function of the stationary distributions of the backlogged demand was extended. The batch size of WIPs has a great impact on the packing, unpacking or transferring costs of a chain; it has been attempted to integrate the delivery batch size of each plant in a multi-stage serial chain with the production-ordering and supplier kanbans of the chain. An algorithm was developed to evaluate the optimal numbers of kanbans and batch sizes of each plant by minimizing the total cost of a chain. A numerical example is also provided to indicate the significance of adding the proposed assumptions, as well as demonstrating the approach adopted towards solving the problem.*

Keywords: *Supply chain; JIT; Stochastic demand; Batch size; Kanban.*

INTRODUCTION

Lean Production has become one of the most effective manufacturing approaches in many industrial countries.

Ford began critical phases in the evaluation of the evaluation of lean production in 1927, which was continued by Toyota in 1937. Today, a large number of books and articles are published to improve the different aspects of lean [1].

Naylor et al. made a comparison between the use of Lean and Agile paradigms in a supply chain, highlighting the similarities and differences, and also showed the role of pull strategies and decoupling points in SCM [2].

As an achievement in the benefits of a Lean system, JIT plays an important role in implementing the Lean successfully.

In 1999, Akturk and Erhun had a review of issues concerning JIT and kanban systems. They defined JIT as having the necessary amount of material where and when it is required. They described JIT as a pull system of coordination between stages of production. They also noted kanban as the major element of JIT philosophy and pull mechanisms [3].

Shah and Ward in 2007 showed that a pull system in the form of JIT production including kanban cards, which serves as a signal to start or stop production is one of the 10 most influential factors in implementing a lean system [1].

In 2007, Matsui analyzed the role of JIT production in Japanese companies and claimed that a JIT production system improves the competitive performance state of each company. He also implied that the JIT systems play a key role in operation management. He believed that JIT production strongly influences organizational behavior, technology development activities and manufacturing implementation strategies [4].

The present paper attempts to present a serial multi-stage supply chain system under stochastic demand with a kanban based philosophy. The researchers determined two numbers of kanban, each attached on a batch in the supply chain. In the following section, review literature concerning inventory decisions controlled by the JIT approach has been provided. It will be followed by a model, as well as an algorithm, developed to solve this problem, and finally the model has been illustrated using a numerical example.

LITERATURE REVIEW

There is so much research with the goal of optimizing inventory decisions under stochastic demand by means of minimizing the total cost of a chain using different

1. *Department of Industrial Engineering, Tarbiat Modares University, Tehran, P.O. Box 14155-4838, Iran.*
*. *Corresponding author. E-mail: skch@modares.ac.ir*

Received 7 February 2010; accepted 27 September 2010

methods like the model generated by Seliaman and Ahmad [5].

For the purpose of stochastic demand and a JIT system, Ohno et al. in 1995 presents the earliest non-deterministic demand for a single stage factory, using two kinds of kanban, one to withdraw the parts called the supplier kanban and another for triggering the production, called the production-ordering kanban. They proposed the model under stability condition assumptions. In the above-mentioned research, each kanban was attached to one product and no batch size was devised [6].

In 2005, Wang and Sarker classified the different systems of supply chains to single stage, multistage and assembly type supply chains, taking into consideration the batch size effect on the cost of the chain. They used only one kind of kanban on their model and assumed the order quantity in the system to be fixed [7].

In 2006, they improved their model and offered another solution method to the chain, but on the deterministic demand also [8].

The MA method for solving the probable shortcoming of the previous model was invented by Rabbani et al. in 2008 [9].

Hu et al. in 2008 represented a model in which the penalty expense for a shortage occurred downstream in a chain system, and passed to upstream stages. The cost was a linearly increasing function of the shortage of time, and they claimed that they had extracted the optimal policy for the SC members without the use of JIT principals in the model [10].

This year, Kojima et al. deployed Ohno's model to a multistage supply chain system with an exact assumption of the original work, such as stochastic demand and the usage of two kinds of kanban. They did not note the role of batches in the total cost of the model either [11].

KANBAN SYSTEM

In the JIT systems, the cards used to give authorization of production are called Kanban Cards. Kanban cards are used to control production or transferring of parts or products between each supply chain or processing stages. In other words, kanbans are used as an authorization of production at each stage.

A station is only authorized to process parts if a kanban of those parts are available. Therefore, the number of kanbans of each stage limits the number of parts to be stored at that stage. When the processing is complete, the kanban moves with the part to the next station [12].

In a supply chain, kanbans control the inventory of each stage and also permit transportation.

As shown in Figure 1, a serial supply chain has n stages (each stage represents a firm), starts from plant 1 by an input of raw material from suppliers and ends with plant n with an output of finished products. Connections between each stage are made by withdrawing kanbans. Figure 1 demonstrates the role of supplier kanbans as a tool for withdrawing the needed parts at each stage of the chain.

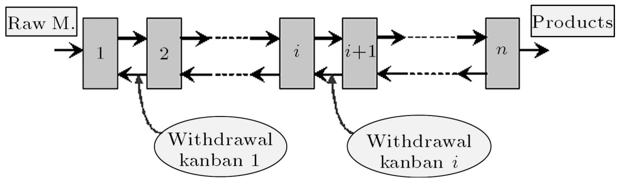

Figure 1. Withdrawal kanbans in a multi-stage supply chain [8].

In stage i, kanbans will attach to a container in batches and will go to plant $i+1$ (Figure 2). Parts are stored in storage before the plant $i+1$. When the first part of a batch is used in stage $i+1$, the kanban will release to a kanban post in order to be collected and transferred to the previous stage as a command for transporting batches from storage B_i, which is placed after plant i, to storage A_{i+1}, which, in turn, is placed before plant $i+1$. Kanbans may have such information as the time the part was produced, the parts number, the quantity of products needed to fill the container, etc. Withdrawal kanbans received in storage B_i will be replaced with another kind of kanban (Production Ordering Kanban) and will be transferred with batches to the next plant. Released production ordering kanbans will be collected in the production kanban post to permit the production and processing of the parts in firm i.

This mechanism is called a pull system, because the production authorizations are produced in plant n, and the order to produce the parts flows backward throughout the chain until it reaches the first plant. In fact, production is controlled or pulled by the quantity of demand occurred at the last stage, and demand information is shared throughout the chain by kanbans.

PRODUCTION MODEL

The JIT production system with two kinds of kanban (withdrawal and supplier), that was discussed by Ohno et al., has been improved and modified for the supply chain system in this paper. In order to universalize the model for a supply chain, some new variables have been defined. The chain begins at stage 1 (the supplier) to stage n (final customer). The demand during every period of time is stochastic, but the processing time of each plant is assumed deterministic. The following notation for modeling the system for each plant has been used:

n: Plant number

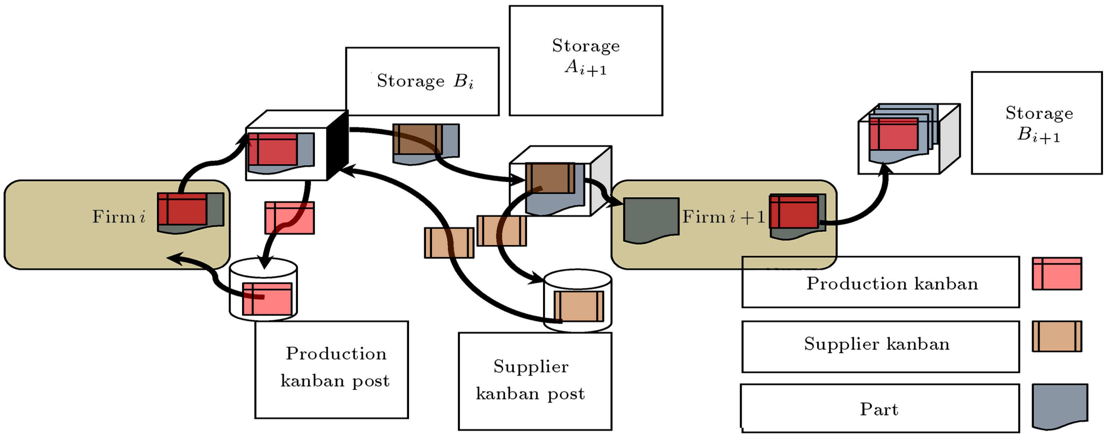

Figure 2. Kanbans operations between two firms.

i: Index of plant from 1 to n,

Al_i: Inventory cost for a part in one period in inventory A_i (placed before each plant),

Bl_i: Inventory cost for a part in one period in inventory B_i (placed after each plant),

Ab_i: Backlogged cost (if shortage occurs) for a part per period,

Cb: Backlogged cost per once,

Ao_i: Setup (ordering) cost for a part,

Aw_i: Withdrawal cost for a part,

L_i: Delivery lead time of each batch part at stage i,

As_i: Salvage cost and the end of each period,

$AE_i(j)$: Salvage cost and the end of each period elapsed j period after ordering,

Qw_i: Batch size number that plant i delivers,

Km_i: Number of production kanbans at plant i,

Kn_i: Number of supplier kanbans at plant i,

$Cow_i(Km, Kn)$: Fix cost per period for kanban space if the numbers of kanbans are Km, Kn,

$Ak_i(Qw_i, Qw_{i-1})$: Setup (packing and unpacking) cost placed if the input and output batch sizes are Qw_i, Qw_{i-1},

C_i: The maximum production quantity of plant i,

$D(k)$: Demand occurred in period k,

$B_i(k)$: The backlogged demand at the beginning of period k in plant i,

$I_i(k)$: Level of inventory at the beginning of period k (in batch),

$J_i(k)$: Number of production kanbans in kanban post at the beginning of stage k in plant i,

$P_i(k)$: Production at period k in plant i,

$Q_i(k)$: Quantity of batches delivered in period k by plant i,

Qw_i: Delivery batch size of plant i,

$X_i(k)$: Total backlogged demand at the beginning of period k in plant i,

$$X_i(k) = B_i(k) + J_i(k) * Qw_i. \quad (1)$$

M_i: Total number of parts that can be stored after plant i,

$$M_i = Qw_i * Km_i. \quad (2)$$

N_i: Total number of parts that can be stored before plant i,

$$N_i = Qw_{i-1} * Kn_i. \quad (3)$$

As shown in Figure 3, in period k, the number of

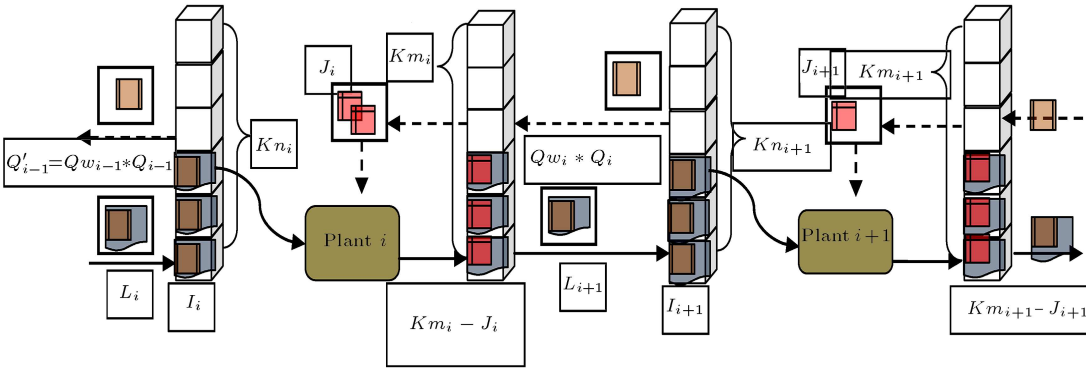

Figure 3. JIT production system in a chain.

supplier kanbans placed in the kanban post is:

$$Q'_{i-1} = Qw_{i-1} * Q_{i-1}.$$

So, the numbers of withdrawal kanbans can be determined as below:

$$Kn_i = I_i k + \frac{P_i(k-1)}{Qw_{i-1}} + B_{i-1}(k) + \sum_{m=k-L_i+1}^{k-1} Q_{i-1}(m), \tag{4}$$

and $Q_{i-1}(m)$ for m from 0 to L_{i+1} are given.

The inventory level of a plant at each time period is defined as the difference between the inventory level of the last period and the received batches from the previous plant and the amount of batches produced in the plant. Therefore:

$$I_i(k+1) = I_i(k) + Q_{i-1}(k - L_{i+1}) - \frac{P_i(k)}{Qw_{i-1}}. \tag{5}$$

As the JIT system stands, the plant should produce equal to the production-ordering kanbans existing in the post. Also the plant cannot produce more than its capacity or more than the parts available in the store before the plant. So:

$$P_i(k) = \min(I_i(k) * Qw_{i-1}, J_i(k) * Qw_i, C_i). \tag{6}$$

Because the demand in period k is known at the beginning of period k, the backlogged demand at the beginning of period $k+1$ is:

$$B_i(k+1) = \max(0, B_i(k) + J_i(k) * Qw_i + P_{i+1}(k-1) - P_i(k) - Qw_i * Km_i). \tag{7}$$

The number of released production-ordering kanbans placed in the kanban post at the beginning of period $k+1$ is the minimum between total production-ordering kanbans (B_i capacity) and total backlogged demand;

$$J_i(k+1) = \min\left(K_m, \frac{B_i(k)}{Qw_i} + J_i(k) + \frac{P_{i+1}(k-1)}{Qw_i} - \frac{P_i(k)}{Qw_i}\right). \tag{8}$$

The number of delivered quantity of goods in period k equals the minimum between the sum of the demand of the next plant at the previous time period and the backlogged demand, and the sum of the production quantity and level of the inventory:

$$Q'_i = Qw_i * Q_i(k) = \min(P_{i+1}(k-1) + B_i(k), P_i(k) + \max(0, Qw_i * Km_i - X_i(k)), \tag{9}$$

and the total backlog at each period is a function of the backlog in the previous period and the production of the present and next plant:

$$X_i(k+1) = X_i(k) + P_{i+1}(k) - P_i(k). \tag{10}$$

Here, each plant has the state space consisting of total backlogged demand, inventory level and production quantities in period k. We are faced with a Markov chain for each plant in which $I_i(k)$, $X_i(k)$ and $Q'_i(k)$ could be obtained [11].

COST OPTIMIZATION ALGORITHM OF NUMBER OF KANBANS AND BATCH SIZES

With no problem in the generality of the model, we assume that at each stage the batch size equals one:

$$Qw_i = 1 \quad \text{for } i \text{ from 1 to } n.$$

Then, Equation 2 becomes:

$$M_i = Km_i,$$

and Equation 3 becomes:

$N_i = Kn_i$.

Also, the state space of a plant in period k ($I_i(k)$, $X_i(k)$, $Q_i(k)$) is defined as follows:

$$I_i(k+1) = I_i(k) + Q_{i-1}(k - L_i + 1) - P_i(k)$$
$$= I_i'(k+1),$$
$$X_i(k+1) = X_i(k) + P_{i+1}(k) - P_i(k),$$
$$Q_i(k) = \min(P_{i+1}(k-1) + B_i(k),$$
$$P_i(k) + \max(0, Qw_i * Km_i - X_i(k)),$$

and:

$$J_i(k+1) = \min\left(K_m, \frac{B_i(k)}{Qw_i} + J_i(k) + \frac{P_{i+1}(k-1)}{Qw_i} - \frac{P_i(k)}{Qw_i}\right) = J_i'(k+1).$$

Kojima et al. [11] showed that limiting probabilities:

$$\Pr\{I_i(\infty) = i_i, X_i(\infty) = x_i, Q'_{i-1}(\infty) = q_{i-1},$$
$$1 \le i \le n, 1 - L_i \le m \le -1\}$$
$$= \lim_{k\to\infty} \Pr\{I_i(\infty) = i_i, X_i(\infty) = x_i,$$
$$Q'_{i-1}(\infty) = q_{i-1},$$
$$1 \le i \le n, k - L_i \le m \le k-1\},$$

with integers i_i, x_i, q_{i-1} for $1 \le i \le n$, $k - L_i \le m \le k-1$ under stability conditions exist.

So for each plant, we have $E(X_i(\infty))$, $E(J_i'(\infty))$, $E(B_i(\infty))$ and $\Pr\{B_i(\infty) > 0\}$ [6].

For each plant, if Qw_i be a fixed number, we have the following relation:

$$J_i(k+1) = \min\left(K_m, \frac{B_i(k)}{Qw_i} + J_i(k) + \frac{P_{i+1}(k-1)}{Qw_i} - \frac{P_i(k)}{Qw_{i-1}}\right)$$
$$= \min\left(\frac{M}{Qw_i}, \frac{B_i(k)}{Qw_i} + J_i(k) + \frac{P_{i+1}(k-1)}{Qw_i} - \frac{P_i(k)}{Qw_i}\right). \quad (11)$$

Equation 11 shows that:

$$J_i(k) = \frac{J_i'(k)}{Qw_i}. \quad (12)$$

Then, until $E\{J_i'(\infty)\}$ exists, we have, from the previous equation, that $E(J_i(\infty))$ exists and equals:

$$E(J_i(\infty)) = \frac{E\{J_i'(\infty)\}}{Qw_i}, \quad (13)$$

and, according to Equation 9, until $E\{Q_i'(\infty)\}$ exists and Qw_{i-1} is a fixed number, $E\{Q_i(\infty)\}$ exists and equals:

$$E\{Q_i(\infty)\} = \frac{E\{Q_i'(\infty)\}}{Qw_i}.$$

According to the costs, considering the batch size setup cost, the total cost of each plant of the chain can be obtained from the following equation:

$$A_i(K_m, K_n) = Al_i(Kn_i * Qw_{i-1} - 1/2)D$$
$$+ Bl_i(Km_i * Qw_i - E(J_i(\infty) * Qw_i)$$
$$+ Ab_i * E(B_i(\infty)) + (Ao_i + Aw_i)D$$
$$+ Cb * \Pr\{B_\infty > 0\}$$
$$+ As_i * (Kn_i * Qw_{i-1})D + C(M_i, N_i)$$
$$+ Ak_i(Qw_{i-1}, Qw_i). \quad (14)$$

Algorithm 1 has been devised to determine the optimal values of number of kanbans and numbers of input batch sizes as well as output batch sizes of each plant regarding to minimal cost of the chain.

The output of the algorithm reveals the values of numbers of production-ordering and supplier kanbans, and the input and output batch sizes of each plant, regarding the minimum cost for the total chain.

NUMERICAL EXAMPLE

Algorithm 1 was applied to a chain with three plants with a JIT ruled mechanism. The average demand rate is $D = 7$ with Poisson distribution; each plant performs a deterministic process time and it is assumed that the lead time is $L = 4$ for each plant. The maximum capacity for all plants is 10. The supplier delivers the parts to the chain (to the first plant) in a batch size equal to 6. The case has some restrictions for batch sizes as follows:

1. The supplier delivers the parts in batch size 6.
2. There is equality between the output batch size of plants 1 and 2, because of using the same containers for carrying the parts.
3. The supply chain output (output of plant 3) is a product with no batch pack permitted.

Set initial numbers: $A_{min} = \infty$, $M_{\max} =$ appropriate M.
For $i = 1$ to n
For $M = [D]$ to $M_{\max}$
For $M = [D]$ to $M_{\max}$
For $\forall\ Qw_i$, $\{1 \leq Qw_i \leq M\}$, $\left\{\left[\frac{M}{Qw_i}\right] = \frac{M}{Qw_i}\right\}$, {case constructions for Qw_{i-1}}
For $\forall\ Qw_{i-1}$, $\{1 \leq Qw_{i-1} \leq N\}$, $\left\{\left[\frac{N}{Qw_{i-1}}\right] = \frac{N}{Qw_{i-1}}\right\}$, {case constructions for Qw_i}
Calculate $E(J'(\infty))$, $B(\infty)$, $\Pr\{B_\infty > 0\}$ from Ohno et al. equations [6] and $E(J(\infty))$ from Equation 13
Calculate $\sum_{i=1}^{N} A_i(K_m, K_n)$ from Equation 14
If $A_i(K_m, K_n) < A_{\min}$, then $A_{\min} = A_i(K_m, K_n)$, $Km_i = \frac{M}{Qw_i}$, $Kn_i = \frac{N}{Qw_{i-1}}$ for i from 1 to n)
Next M
Next N
Next Qw_i
Next Qw_{i-1}
Next i
Return $A_{\min}$, Km_i, Kn_i, Qw_i, for i from 1 to n

Algorithm 1. An approach to determine the value of each objective.

So:

$$Qw_0 = 6, \qquad Qw_1 = Qw_2, \qquad Qw_3 = 1.$$

D_k for $k = 1, 2, \cdots$ has the distribution of:

$$\Pr\{D_k = i\} = \frac{e^D * D^i}{i!}, \qquad i = 0, 1, \cdots$$

The cost parameters are valued below:

$$Al_1 = Al_2 = 1, \qquad Al_3 = 1.5,$$

$$Ao_1 + Aw_1 = 10, \qquad Ao_2 + Aw_2 = 10,$$

$$Ao_3 + Aw_3 = 5, \quad Bl_1 = Bl_2 = 5, \quad Bl_3 = 10,$$

$$Cb_1 = Cb_2 = 100, \qquad Cb_3 = 110,$$

$$Ab_1 = Ab_2 = Ab_3 = C(M, N) = 0,$$

$$Ak_1(Qm, Qn) = \frac{N}{2 * Qn} + \frac{M}{Qm},$$

$$Ak_2(Qm, Qn) = \frac{N}{3 * Qn} + 2 * \frac{M}{Qm},$$

$$Ak_3(Qm, Qn) = e^{\frac{20 * Qn}{N}}.$$

The minimum occurs at $M_1 = M_2 = M_3 = 9$ and $N_1 = N_2 = N_3 = 36$ with the batch size matrix equal to (6, 3, 3, 1). In Figure 4, some other values for $M_i = 9$ and equal N_i for $i = 1, 2, 3$ are illustrated.

As seen in Figure 4, the minimum cost for the chain with the batch sizes equal to:

$$Qw_0 = 6, \qquad Qw_1 = Qw_2 = 3, \qquad Qw_3 = 1,$$

for each plant could be determined as $A_{\min} = 262.8$; other variables are calculated as below:

$$Kn_1 = \frac{N_1}{Qw_0} = \frac{36}{6} = 6,$$

$$Km_1 = \frac{M_1}{Qw_1} = \frac{9}{3} = 3,$$

$$Kn_2 = 12, \qquad Km_2 = 3,$$

$$Kn_3 = 12, \qquad Km_3 = 9.$$

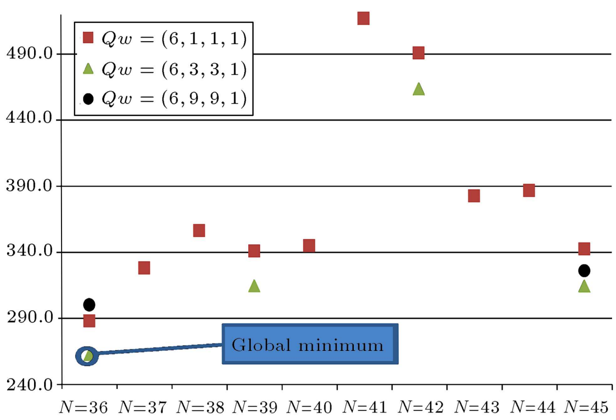

Figure 4. Behavior of A for $M = 9$.

CONCLUSION

In this paper, the researchers dealt with a multi-stage supply chain that performs under the philosophy of a JIT production environment. Supply Chain Management (SCM) in a JIT environment with two kinds of kanban (production-ordering and supplier) under stochastic demand has been considered in the present paper. Furthermore, deterministic processing time with withdrawal batch size constrain for each plant is taken into consideration. As long as the demand rate becomes stochastic, one of the very influential logistic parameters in the total chain cost is the size of batches delivered to or produced by a plant. The batch size and two kinds of kanban were integrated with the chain and play a significant role in determining the cost of SC more feasibly. The algorithm was devised to find the optimal cost of a chain, regarding constrains that the batches brought. All in all, a numerical example is exhibited to illustrate the effect of batches on supply chain cost and to find the optimal state of performance.

REFERENCES

1. Shah, R. and Ward, P.T. "Defining and developing measures of lean production", *Journal of Operations Management*, **25**, pp. 785-805 (2007).
2. Naylor, J.B., Naim, M.M. and Berry, D. "Leagility: Integrating the lean and agile manufacturing paradigms in the total supply chain", *Int. J. Production Economics*, **62**, pp. 107-118 (1999).
3. Akturk, M.S. and Erhun, F. "An overview of design and operational issues of kanban systems", *Int. J. Prod. Res.*, **37**(17), pp. 3859-3881 (1999).
4. Matsui, Y. "An empirical analysis of just-in-time production in Japanese manufacturing companies", *Int. J. Production Economics*, **108**, pp. 153-164 (2007).
5. Seliaman, M.E. and Ahmad, A.R. "Optimizing inventory decisions in a multi-stage supply chain under stochastic demands", *Applied Mathematics and Computation*, **206**, pp. 538-542 (2008).
6. Ohno, K., Nakashima, K. and Kojima, M. "Optimal numbers of two kinds of kanbans in a JIT production system", *International Journal of Production Research*, **33**(5), pp. 1387-1401 (1995).
7. Wang, S. and Sarker, B.R. "An assembly-type supply chain system controlled by kanbans under a just-in-time delivery policy", *European Journal of Operational Research*, **162**, pp. 153-172 (2005).
8. Wang, S. and Sarker, B.R. "Optimal models for a multi-stage supply chain system controlled by kanban under just-in-time philosophy", *European Journal of Operational Research*, **172**, pp. 179-200 (2006).
9. Rabbani, M., Layegh, J. and Ebrahim, R.M. "Determination of number of kanbans in a supply chain system via Memetic algorithm", *Advances in Engineering Software*, **40**(6), pp. 431-437 (June 2009).
10. Hu, W., Kim, S. and Banerjee, A. "An inventory model with partial backordering and unit backorder cost linearly increasing with the waiting time", *European J. of Operational Res.*, **197**(2), pp. 581-587 (Sept. 2009).
11. Kojima, M., Nakashima, K. and Ohno, K. "Performance evaluation of SCM in JIT environment", *Int. J. Production Economics*, **115**, pp. 439-443 (2008).
12. Sharma, S. and Agrawal, N. "Selection of a pull production control policy under different demand situations for a manufacturing system by AHP-algorithm", *Computers & Operations Research*, pp. 1622-1632 (2009).

BIOGRAPHIES

S. Kamal Chaharsooghi is Associate Professor of Industrial Engineering at the Industrial Engineering Department, Faculty of Engineering, Tarbiat Modares University, in Tehran, Iran. His research interests include: Manufacturing Systems, Supply Chain Management, Information Systems, Strategic Management, International Marketing Strategy and Systems Theory. His work has appeared in Decision Support Systems, the European Journal of Operational Research, the International Journal of Production Economics, the International Journal of Advanced Manufacturing Technology, Applied Mathematics and Computation, and Scientia Iranica. He obtained his Ph.D. from Hull University, England.

Arman Sajedinejad is a Ph.D. candidate of Industrial Engineering at the Industrial Engineering Department, Faculty of Engineering, Tarbiat Modares University, in Tehran, Iran. His research background includes: Supply Chain Management, JIT Production Systems, Simulation and also Performance Evaluation of Organizations.