Estimation of Hottest Spot Temperature in Power Transformer Windings with NDOF and DOF Cooling

M.A. Taghikhani1,* and A. Gholami1

Abstract. Power transformer outages have a considerable economic impact on the operation of an electrical network. One of the most important parameters governing a transformer’s life expectancy is the Hot-Spot Temperature (HST) value. The classical approach has been established to consider the hot-spot temperature as the sum of the ambient temperature, the top-oil temperature rise in the tank, and the hot-spot to top-oil (in tank) temperature gradient. In this paper, the heat conduction equation for temperature is solved. For numerical solution of the heat conduction equation, the finite element method is used. The selected model for simulation is a 32MVA transformer with Non-Directed Oil-Forced (NDOF) cooling and Directed Oil-Forced (DOF) cooling.

Keywords: Power transformer; Temperature distribution; Hot spot; Forced cooling; Heat equation; Finite element.

INTRODUCTION

A power transformer is a static piece of apparatus with two or more windings, which by electromagnetic induction transforms a system of alternating voltage and current into another system of voltage and current, usually of different values and at the same frequency for transmitting electrical power. Since large power transformers belong to the most valuable assets in electrical power networks, it is suitable to pay greater attention to these operating resources. An outage affects the stability of the network, and the associated financial penalties for the power utilities can be increased. In a power transformer operation, part of the electrical energy is converted into heat. Although this part is quite small, compared to total electric power transferred through a transformer, it causes a significant temperature rise in the transformer constructive parts, which represents the limiting criteria for possible power transfer through a transformer. That is why precise calculation of temperatures in critical points (top oil and the hottest solid insulation spot) is of practical interest. Thermal impact leads not only to long-term oil/paper-insulation degradation, it is also a limiting factor for transformer operation [1]. Therefore, knowledge of temperature, especially hot-spot temperature, is of great interest.

The basic criterion for transformer loading is the temperature of the hottest spot of the solid insulation (hot-spot). It must not exceed the prescribed value, in order to avoid insulation faults. In addition, long-term insulation ageing, depending on the hot-spot temperature diagram, should be less than the planned value. Hot-spot temperature depends on the load loss (i.e. on the current) diagrams and on the temperature of the external cooling medium. A hot-spot temperature calculation procedure is given in International Standards [2-4]. In [5,6], the algorithm for calculating the hot-spot temperature of a directly loaded transformer, using data obtained in a short circuit heating test, is given. These papers propose improvements in the modeling of thermal processes inside the transformer tank. Calculation methods have to be based on an energy balance equation. Some attempts at heat transfer theory result applications to heat transfer from winding to oil are exposed in [7]. The usage of the average heat transfer coefficient is
typical in a transformer designing process to calculate
the needed number (area) of cooling surfaces [8-13].

In this paper, a procedure for obtaining tem-
perature distribution in the transformer is proposed.
The procedure requires calculation of the heat transfer
coefficients. We use heat transfer coefficients in the
energy (thermal) equation. The model can be used
for temperature calculation on the arbitrary change of
current. For numerical simulation of the mentioned
equation, we use the finite element method. For this
reason, a code has been provided under MATLAB
software. This paper deals with a mathematical formul-
ation for the heat conduction equation and its solution,
using the finite element method for the steady state.
Discussion of the proposed work has been provided at
the end of the paper.

MATHEMATICAL FORMULATION FOR
HEAT CONDUCTION EQUATION

The structure of a transformer winding is complex and
does not conform to any known geometry in the strict
sense. Under general conditions, the transformer wind-
ings can be assumed cylindrical in formation, hence a
layer or a disc winding is a finite annular cylinder [10].
The thermal and physical properties of the system
would be equivalent to a composite system of insula-
tion and conductor. It has been assumed that heat is
generated throughout the body at a constant rate, and
cold in the vertical and horizontal ducts take away the
heat through the process of convection. However, in
an actual transformer winding, the conductor is the
only heat source. Later, in this section formulations
are given for calculating the different thermal and
physical properties of the system. It has been assumed
that temperature is independent of the space variable,
because the winding structure is symmetrical. The
temperature at any point on the periphery of the circle
for a specific value of \( r \) and \( z \) is deemed a constant (i.e.
presence of spacers has been ignored, thus reducing
three-dimensional problem to a two-dimensional) one
with \( r \) and \( z \) as space variables. Dielectric loss in
insulation is assumed small compared to copper losses
in the conductor. The surface of the disc or layer has
been assumed smooth.

The generalized system of a non-homogeneous
heat conduction equation with a non-homogeneous
boundary condition in a Cartesian coordinate system
is written, thus [10,14]:

\[
k \left( \frac{\partial T}{\partial x^2} + \frac{\partial T}{\partial y^2} \right) + Q = 0,
\]

in the region \( a < x < b \) and \( 0 < y < l \).

Equations 2 to 5 represent the general heat con-
duction equation with convection at all four boundary
surfaces.

At the inner cylindrical surface:

\[
-k_1 \frac{\partial T}{\partial x} + h_1 T = f_1(y).
\]
(2)

At the outer cylindrical surface:

\[
k_2 \frac{\partial T}{\partial x} + h_2 T = f_2(y).
\]
(3)

At the bottom flat surface:

\[-k_3 \frac{\partial T}{\partial y} + h_3 T = f_3(x).\]
(4)

At the top flat surface:

\[k_4 \frac{\partial T}{\partial y} + h_4 T = f_4(x).\]
(5)

In the above equations, temperature, \( T \), is a function
of the space variables, \( x \) and \( y \). The term \( Q \) is the heat
source function and has been modified here to take care
of the variation of resistivity of copper to temperature.
The heat source term, \( Q \), can be of the form:

\[Q = Q_0 \left[ 1 + \alpha_c (T - T_{amb}) \right].\]
(6)

\( \alpha_c \) is the temperature coefficient of electrical resistance
of copper wire. With this representation, function \( Q \)
becomes a temperature-dependent, distributed heat
source. Boundary functions \( f_1(y) \) and \( f_2(y) \) derived
from Newton’s law of cooling are of the following
equations:

\[f_1(y) = h_1(T_b + m_1y),\]
(7)

\[f_2(y) = h_2(T_b + m_2y).\]
(8)

The term \( T_b \) is the temperature at the bottom of the
disc or layer as applicable. Terms \( m_1 \) and \( m_2 \) are
the temperature gradient along the winding height (for
layer) or along the disc thickness for a disc. Similarly,
functions \( f_3 \) and \( f_4 \) representing temperature profiles
across bottom and top surfaces, have the same form
as shown in Equations 7 and 8, where the temperature
gradient term has been taken as zero:

\[f_3(x) = h_3 T_b,\]
(9)

\[f_4(x) = h_4 T_{top}.\]
(10)

Thermal conductivities are unequal in different direc-
tions. However, in an actual case, thermal conduc-
tivities in radial directions (\( k_1 \) and \( k_2 \)) are equal and
conductivities in the axial direction will be the same
(\( k_3 \) and \( k_4 \)). Thermal conductivity has been treated as
a vector quantity having components in both radial and
axial directions. The resultant thermal conductivity of the system can be estimated as:

$$K = \sqrt{k_x^2 + k_z^2},$$

(11)

where:

$$k_r = \left( \frac{\log \frac{R_2}{R_1}}{k_x} + \frac{\log \frac{R_3}{R_2}}{k_3} + \cdots + \frac{\log \frac{R_{n+1}}{R_n}}{k_n} \right),$$

$$k_z = \frac{k_{\text{Cu}} k_{\text{oil}} (t_{\text{Cu}} + t_{\text{in}})}{t_{\text{in}} k_{\text{Cu}} + t_{\text{Cu}} k_{\text{in}}}.$$

Term $K$ represents the resultant thermal conductivity of the insulation and conductor system. Heat transfer coefficients, $h_1$ to $h_4$ (htc), are different across all four surfaces. To determine boundary functions, $f_1$ to $f_4$, it is necessary to calculate heat transfer coefficients across the four surfaces. Difficulty has been encountered in calculation of the heat transfer coefficient. It is reported elsewhere [10] that it depends on as many as 13 factors (e.g. winding size, type, duct dimensions, oil velocity, type of oil circulation, heat flux distribution, oil thermal properties etc.). In this work, corrections have been given for the temperature-dependence of the thermal and physical properties of oil, such as viscosity, specific heat, volumetric expansion and thermal conductivity. It was found that there is a negligible effect of specific heat, coefficient of volumetric expansion and conductivity in the present working range of loading (110-160°C). Following, are some of the heat transfer relations and relevant formulae in natural cooling (ON) mode. These formulae have been used to calculate the htc [14]. The local Nusselt number for laminar flow over vertical plates has been shown below:

$$\text{Nu} = 0.6 \text{Ra}^{0.2}_{h/f},$$

(12)

$$\text{Ra}_{h/f} = \text{Gr}_{h/f} P_r,$$

(13)

where $\text{Ra}_{h/f}$ and $\text{Gr}_{h/f}$ are the local Rayleigh and Grashof numbers based on heat flux ($q_w$) at characteristic dimension ($\delta$). $P_r$ is the Prandtl number of the transformer oil. The expression of the Rayleigh number based on constant heat flux is:

$$\text{Ra}_{h/f} = \frac{g \beta C_p \rho_0^2 q_w \delta^4}{\kappa_{\text{oil}} \mu_0^2},$$

(14)

The mean Nusselt number in this case can be computed as:

$$\text{Nu}_{\text{m}} = 1.5 [\text{Nu}]_{h/f}.$$  

(15)

However, correction to Equation 12 has to be given for cylindrical curvature. The correction factor in this case is of the following form ($30 < P_r < 50$):

$$f(\xi) \approx 1 + 0.12 \xi,$$

(16)

where:

$$\xi = \frac{2\sqrt{2}}{\text{Gr}^{0.25}} \times \frac{\delta}{r}.$$  

(17)

Here, $G_r$ is the Grashof number, based on temperature difference. The local heat transfer coefficient can be computed as:

$$h = \frac{\text{Nu} \cdot k_{\text{oil}}}{\delta}.$$  

(18)

The mean coefficient ($h_m$) can be calculated from the mean Nusselt number as in Equation 15. After knowing the $h_m$ for a particular surface, the temperature difference between the winding surface and oil can be found out dividing the $h_m$ by the heat flux through the surface. The mean Nusselt number of the top surface of annular cylindrical winding for the laminar and turbulent regimes will normally be of the form:

$$\text{Nu}_{\text{m}} = 0.54 \text{Ra}^{0.25}_{h/f} = 0.61 \text{Ra}^{0.2}_{h/f},$$

(19)

$$\text{Nu}_{\text{m}} = 0.15 \text{Ra}^{0.7}_{h/f} = 0.24 \text{Ra}^{0.7}_{h/f},$$

(20)

$$\delta = \frac{b - a}{2},$$

(21)

where $a$ and $b$ are the inner and outer radius of the annular disc or layer. The Nusselt number of the bottom surface of annular cylindrical winding for the laminar and turbulent regimes is in the following equation:

$$\text{Nu}_{\text{m}} = 0.27 \text{Ra}^{0.25}_{h/f} = 0.35 \text{Ra}^{0.2}_{h/f}.$$  

(22)

The axial oil temperature gradient in the presence of cooling by a constant heat flux can be found out by the following equation due to [14]:

$$\frac{\partial T}{\partial y} = 4.25 \times 10^{-2} \frac{q_w}{\kappa_{\text{oil}}} \left( \frac{l}{\pi D_m} \right)^{\frac{1}{2}} \text{Ra}^{\frac{1}{2}}_{h/f},$$

(23)

where $l$ is the winding height, $D_m$ is the mean diameter of the annular disc or layer of windings. Determination of boundary conditions in forced convection (OF mode) also requires calculating htc. When the oil velocity is lower values, the mean Nusselt number, based on temperature difference, is expressed in the form of Equation 24 and the corresponding mean Nusselt number, based on constant heat flux, is expressed in the form of Equation 25:

$$\text{Nu}_{\text{m}} = 1.75 \left( G_z + 0.012 \left( G_z \text{Gr}^{0.7} \right)^{\frac{1}{2}} \times \left( \frac{\mu_k}{\mu_w} \right)^{0.14} \right).$$

(24)
\[ \text{Nu}_{m} = 1.63 \left[ 1.8C_0^{0.9} + 0.02 \left( G_z \text{Gr}_{zf}^{0.2} \right)^{1.10} \right]^{0.3} \]
\[ \times \left( \frac{\mu_s}{\mu_w} \right)^{0.125}, \quad (25) \]

\[ G_z = \text{Re Pr} \frac{D_t}{l}. \quad (26) \]

\( G_z \) is called the Graetz number. Terms \( \mu_s \) and \( \mu_w \) are the viscosity of the oil computed at the oil bulk mean temperature of oil and at the winding wall temperature, respectively. Re is the Reynolds number and Pr is the Prandtl number. The relative importance of natural and forced cooling is indicated by the factor \( f_s = \text{Gr}/\text{Re}^{0.2}. \) If \( f_s \approx 1 \), then both of the cooling modes have to be considered. At a lower value of this factor, natural cooling can be ignored. The oil viscosity is an important property, which depends on temperature. The formula to consider the oil temperature variation is given in the following equation [9,10]:
\[ \mu = \alpha \cdot \exp \left( \frac{\gamma}{T_m} \right), \quad (27) \]

where:
\[ \alpha = 0.0000013573 \text{ (kg.m}^{-1}\text{s}^{-1}), \]
\[ \gamma = 2797.3 \text{ (K)}. \]

The viscosity was calculated at the mean oil and wall temperature. Initially, the winding surface temperature is not known, so a starting guess for winding surface temperature has to be made. After calculating the value of \( h \), the temperature difference \( (T_w - T_{oil}) \) is to agree with the assumed value. To calculate the winding wall temperature at different surfaces, only the bottom oil temperature is necessary. In this paper, the bottom oil rise over the ambient temperature has been calculated as:
\[ \theta_u = \theta_f \cdot \left( \frac{I^2R + 1}{R + 1} \right)^n, \quad (28) \]

where \( \theta_u \) is the bottom oil temperature rise over the ambient temperature; \( \theta_f \) is the full load bottom oil temperature rise over the ambient temperature obtained from an off-line test; \( R \) is the ratio of load loss to no-load loss. The variable, \( I_v \), is the ratio of the specified load to rated load:
\[ I_v = \frac{I}{I_{\text{rated}}}. \quad (29) \]

The exponent \( n \) depends upon the cooling state. The loading guide recommends the use of \( n = 0.8 \) for natural convection and \( n = 0.9 - 1.0 \) for forced cooling.

Assumptions have been made in the calculation of the \( h \) in the ducts provided in the disc-type windings under Oil-Forced (OF) modes of heat transfer (DOF and ND OF). The cooling in OF mode is due to mixed mode (natural and forced) convection. While the oil-flow velocity in both vertical and horizontal ducts has been assumed equal, we are in the DOF mode. If the flow velocity in horizontal ducts is assumed negligible compared to the velocity in the vertical ducts, we are in ND OF mode. In case of the DOF mode, the \( h \) was assumed as a function of both heat flux through the surface and the oil-flow velocity. The mechanism of heat transfer in this mode of cooling is the same in both axial and radial directions of the disc. In case of the ND OF mode, the \( h \) in the vertical duct has been estimated by the same formula as for DOF. However, the convection of heat in the horizontal ducts has been assumed as a purely natural type.

**FINITE ELEMENT SOLUTION OF HEAT EQUATION FOR THREE-PHASE TRANSFORMER WINDINGS**

For finite element solution of the heat conduction equation, a software program using the MATLAB code has been provided. The model has been validated on a transformer of rating 32 MVA. The Cu-loss (at \( T_{\text{amb}} = 25^\circ \text{C} \)) in the winding per disc/layer of the above transformer has been tabulated in Table 1. The result of temperature distribution in the \( x, y \) plane from the LV layer 1 winding in the 1 p.u. load, with \( \text{Re} = 750 \), has been shown in Figure 1. It can be pointed out that, for LV layer 1 winding, the maximum temperature location is around 90 to 95% of the winding height from the bottom and at about 50% of the radial thickness of the layer.

The result of the temperature distribution from LV layer 2 winding has been shown in Figure 2. It can be pointed out that for LV layer 2 winding, the maximum temperature location is the same as LV.
Figure 2. Temperature distribution of LV layer 2 winding at \( T_{\text{amb}} = 25^\circ \text{C} \) with \( \text{Re} = 750 \).

Figure 3. Temperature distribution of HV disc winding in NDOF mode at \( T_{\text{amb}} = 25^\circ \text{C} \) with \( \text{Re} = 750 \).

layer 1 winding. Temperature distribution from a disc of HV winding has been shown in Figures 3 and 4. It may be observed that the maximum temperature occurs in the neighborhood of 30-35% of the axial and 50% of the radial thickness of the disc in NDOF mode and 50-55% of the axial and 50% of the radial thickness of the disc in DOF mode. Magnitudes of HST at different loading and OF modes are given in Table 2. In this work, different rates of forced oil circulation are taken into account. Figures 5 to 8 show variations of local HST with load and Reynolds number in different windings and different OF modes. Tables 2 and 3 show a comparison of the proposed method with the boundary value method, using the finite integral transform used in [10] and the experimental test in [9].

It is clear from Table 2 that the HST in the case of the NDOF mode is greater than that of the corresponding DOF mode and so is the average temperature rise. From Figures 5, 6 and 8, it is also borne out that an increase in the amount of cooling, by way of increased oil flow speed, would provide a reduced winding temperature, which is obvious. However, an increased oil flow has a lesser effect on HST in case of the NDOF mode rather than the DOF mode (Figures 7 and 8). From the results of this investigation, it is clear

<table>
<thead>
<tr>
<th>Transformer Rating (MVA)</th>
<th>Winding Type</th>
<th>Cu-Loss (W)</th>
<th>Dimensions (mm) a,b,l</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>LV layer 1</td>
<td>757</td>
<td>263, 287, 5, 1460</td>
<td>Per layer</td>
</tr>
<tr>
<td>32</td>
<td>LV layer 2</td>
<td>8536</td>
<td>301, 5, 326, 1460</td>
<td>Per layer</td>
</tr>
<tr>
<td>32</td>
<td>HV disc</td>
<td>570</td>
<td>371, 438, 21</td>
<td>Per disc</td>
</tr>
</tbody>
</table>

Table 2. HST(°C) magnitudes at \( T_{\text{amb}} = 25^\circ \text{C} \) with \( \text{Re} = 750 \).

<table>
<thead>
<tr>
<th>Load (p.u.)</th>
<th>HST (°C)-LV Layer 1</th>
<th>HST (°C)-LV Layer 2</th>
<th>HST (°C)-HV Disc NDOF</th>
<th>DOF</th>
<th>Global HST (Proposed)</th>
<th>HST-Ref. [10]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>95</td>
<td>94</td>
<td>87</td>
<td>67</td>
<td>95</td>
<td>92</td>
</tr>
<tr>
<td>0.9</td>
<td>108</td>
<td>107</td>
<td>99</td>
<td>74</td>
<td>108</td>
<td>104</td>
</tr>
<tr>
<td>1.0</td>
<td>123</td>
<td>122</td>
<td>112</td>
<td>82</td>
<td>123</td>
<td>117</td>
</tr>
<tr>
<td>1.1</td>
<td>140</td>
<td>138</td>
<td>126</td>
<td>91</td>
<td>140</td>
<td>134</td>
</tr>
<tr>
<td>1.2</td>
<td>158</td>
<td>156</td>
<td>141</td>
<td>101</td>
<td>158</td>
<td>152</td>
</tr>
<tr>
<td>1.3</td>
<td>177</td>
<td>175</td>
<td>157</td>
<td>112</td>
<td>177</td>
<td>183</td>
</tr>
<tr>
<td>1.4</td>
<td>190</td>
<td>197</td>
<td>174</td>
<td>124</td>
<td>199</td>
<td>200</td>
</tr>
<tr>
<td>1.5</td>
<td>222</td>
<td>220</td>
<td>192</td>
<td>137</td>
<td>222</td>
<td>220</td>
</tr>
</tbody>
</table>
Table 3. HST°C locations (in LV winding) at $T_{amb} = 25°C$.

<table>
<thead>
<tr>
<th>Load (p.u.)</th>
<th>Proposed (Radial Thickness%-Height%)</th>
<th>Ref. [9] (Radial Thickness%-Height%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>50 / 95</td>
<td>44 / 94</td>
</tr>
<tr>
<td>0.9</td>
<td>50 / 94.8</td>
<td>45 / 95</td>
</tr>
<tr>
<td>1.0</td>
<td>50 / 94.6</td>
<td>45 / 94</td>
</tr>
<tr>
<td>1.1</td>
<td>50 / 94.5</td>
<td>45 / 93</td>
</tr>
<tr>
<td>1.2</td>
<td>50 / 94.5</td>
<td>45 / 93</td>
</tr>
<tr>
<td>1.3</td>
<td>50 / 94.3</td>
<td>49 / 91</td>
</tr>
<tr>
<td>1.4</td>
<td>50 / 94.2</td>
<td>49 / 91</td>
</tr>
<tr>
<td>1.5</td>
<td>50 / 94</td>
<td>52 / 90</td>
</tr>
</tbody>
</table>

Figure 5. HST (local) distribution of LV layer 1 winding at $T_{amb} = 25°C$.

Figure 6. HST (local) distribution of LV layer 2 winding at $T_{amb} = 25°C$.

Figure 7. HST (local) distribution of HV disc winding in NDOF mode at $T_{amb} = 25°C$.

Figure 8. HST (local) distribution of HV disc winding in DOF mode at $T_{amb} = 25°C$.

that DOF cooling is more effective than the NDOF cooling mode as expected.

CONCLUSIONS

In this paper, an attempt has been made to suggest a method to improve the accuracy of temperature prediction of the hottest spot in a power transformer by solving the heat transfer partial differential equation (PDE), numerically. The purely numerical approach followed in this paper for evaluating the hot spot and its location seems to correspond reasonably well with results of calculations and actual tests and on site measurements [9,10]. The authors wish to point out that the IEEE loading guide and other similar documents offer relations for calculation of the HST, based on per-unit load. The formulations tend to ignore the possibilities of two transformers that are rated identical but have a different winding structure and a varying heat loss/unit volume. The method suggested by the authors gives due representation for this omission and, hence, is believed to give more accurate estimates. The thermal model presented here can predict the hot spot location with a reasonable degree of accuracy. The authors are currently working on a 3D model of a transformer for the estimation of temperature at different points of the transformer, knowing the load conditions. A two-dimensional model is suitable for
estimation of the temperature at transformer windings and is not useful for the core; because in a two-
dimensional case, we use the assumption that the core is symmetrical in all directions, which is not a correct assumption. If we want to estimate temperature distribution at the core of the power transformer, we must use a three-dimensional model.

**NOMENCLATURE**

- $l$: thickness of the disc or height of layer winding (m)
- $n$: temperature rise exponent to bottom oil
- $C_p$: specific heat of transformer oil (J · kg$^{-1}$ · K$^{-1}$)
- $k_{oil}$: thermal conductivity of transformer oil (W · m$^{-1}$ · K$^{-1}$)
- $a$: inner radius of the annular disc or layer (m)
- $b$: outer radius of the annular disc or layer (m)
- $D_m$: mean diameter of annular disc or layer of windings (m)
- $f_r$: relative index of free-to-forced convection
- $G_r$: Graetz number
- $Gr_{hf}$: Grashof number based on constant heat flux
- $Gr$: Grashof number based on temperature difference
- $k_1$: thermal conductivity at inner cylinder surface (W · m$^{-1}$ · K$^{-1}$)
- $k_2$: thermal conductivity at outer cylinder surface (W · m$^{-1}$ · K$^{-1}$)
- $k_3$: thermal conductivity at bottom cylinder surface (W · m$^{-1}$ · K$^{-1}$)
- $k_4$: thermal conductivity at top cylinder surface (W · m$^{-1}$ · K$^{-1}$)
- $h_1$: heat transfer coefficient at inner surface (W · m$^{-2}$ · K$^{-1}$)
- $h_2$: heat transfer coefficient at outer surface (W · m$^{-2}$ · K$^{-1}$)
- $h_3$: heat transfer coefficient at bottom surface (W · m$^{-2}$ · K$^{-1}$)
- $h_4$: heat transfer coefficient at top surface (W · m$^{-2}$ · K$^{-1}$)
- $f_1$: boundary function at inner cylinder surface
- $f_2$: boundary function at outer cylinder surface
- $f_3$: boundary function at bottom cylinder surface
- $f_4$: boundary function at top cylinder surface
- $k_{cu}$: thermal conductivity of copper (W · m$^{-1}$ · K$^{-1}$)
- $k_{in}$: thermal conductivity of insulation (W · m$^{-1}$ · K$^{-1}$)
- $m$: oil temperature gradient along winding height (K · m$^{-1}$)
- $Ntu$: mean Nusselt number
- $Nu$: local Nusselt number
- $q_w$: heat flux in (W · m$^{-2}$)
- $r_i$: radius of insulation and conductor layers (m), $i = 1, 2, \ldots$
- $Ra_{hf}$: Rayleigh number based on constant heat flux
- $Ra$: Rayleigh number based on temperature difference
- $T_{amb}$: ambient temperature (K)
- $T_b$: temperature at the bottom of the winding (K)
- $T_{top}$: temperature at the top of the winding (K)
- $T_m$: temperature average of oil and winding surface (K)
- $t_{cu}$: thickness of conductor (m)
- $t_{in}$: thickness of insulation (m)
- $g$: acceleration due to gravity (m · s$^{-2}$)
- $R$: loss ratio = load loss/no load loss
- $Re$: Reynolds number
- $Pr$: Prandtl number
- $Q$: volumetric heat source function (W · m$^{-3}$)
- $Q_0$: volumetric heat source at ambient temperature (W · m$^{-3}$)
- $k_r$: thermal conductivity in radial direction ($k_r = k_1 = k_2$)
- $k_z$: thermal conductivity in axial direction ($k_z = k_3 = k_4$)
- $K$: referred to as equivalent thermal conductivity
- $LV$: low voltage
- $HV$: high voltage
- $p.u.$: per unit
- $DOF$: directed oil-forced cooling
- $NDOF$: non-directed oil-forced cooling

**Greek Symbols**

- $\beta$: coefficient of volumetric expansion of oil (K$^{-1}$)
$\theta_u$ ultimate bottom oil temperature rise over ambient temperature at rated load (K)

$\theta_{fl}$ ultimate bottom oil temperature rise over ambient temperature at related load (K)

$\delta$ characteristic dimension (m)

$\rho$ density of transformer oil (kg.m$^{-3}$)

$f(\xi)$ correction factor for cylindrical curvature

$\alpha_c$ temperature coefficient of electrical resistance (K$^{-1}$)

$\mu$ viscosity of oil (kg.m$^{-1}$.s$^{-1}$)

$\mu_b$ viscosity of oil at oil bulk mean temperature (kg.m$^{-1}$.s$^{-1}$)

$\mu_w$ viscosity of oil at winding wall temperature (kg.m$^{-1}$.s$^{-1}$)

**Subscripts**

$b$ bulk property

$m$ mean value

$w$ wall value

**REFERENCES**


