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Research Note

Estimation of Hottest Spot Temperature in Power Transformer Windings with NDOF and DOF Cooling

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Abstract. *Power transformer outages have a considerable economic impact on the operation of an electrical network. One of the most important parameters governing a transformer's life expectancy is the Hot-Spot Temperature (HST) value. The classical approach has been established to consider the hot-spot temperature as the sum of the ambient temperature, the top-oil temperature rise in the tank, and the hot-spot-to-top-oil (in tank) temperature gradient. In this paper, the heat conduction equation for temperature is solved. For numerical solution of the heat conduction equation, the finite element method is used. The selected model for simulation is a 32MVA transformer with Non-Directed Oil-Forced (NDOF) cooling and Directed Oil-Forced (DOF) cooling.*

Keywords: *Power transformer; Temperature distribution; Hot spot; Forced cooling; Heat equation; Finite element.*

INTRODUCTION

A power transformer is a static piece of apparatus with two or more windings, which by electromagnetic induction transforms a system of alternating voltage and current into another system of voltage and current, usually of different values and at the same frequency for transmitting electrical power. Since large power transformers belong to the most valuable assets in electrical power networks, it is suitable to pay greater attention to these operating resources. An outage affects the stability of the network, and the associated financial penalties for the power utilities can be increased. In a power transformer operation, part of the electrical energy is converted into heat. Although this part is quite small, compared to total electric power transferred through a transformer, it causes a significant temperature rise in the transformer constructive parts, which represents the limiting criteria for possible power transfer through a transformer. That is why precise calculation of temperatures in critical points (top oil and the hottest solid insulation spot) is of practical interest. Thermal impact leads not only to long-term oil/paper-insulation degradation, it is also a limiting factor for transformer operation [1]. Therefore, knowledge of temperature, especially hot-spot temperature, is of great interest.

The basic criterion for transformer loading is the temperature of the hottest spot of the solid insulation (hot-spot). It must not exceed the prescribed value, in order to avoid insulation faults. In addition, long-term insulation ageing, depending on the hot-spot temperature diagram, should be less than the planned value. Hot-spot temperature depends on the load loss (i.e. on the current) diagrams and on the temperature of the external cooling medium. A hot-spot temperature calculation procedure is given in International Standards [2-4]. In [5,6], the algorithm for calculating the hot-spot temperature of a directly loaded transformer, using data obtained in a short circuit heating test, is given. These papers propose improvements in the modeling of thermal processes inside the transformer tank. Calculation methods have to be based on an energy balance equation. Some attempts at heat transfer theory result applications to heat transfer from winding to oil are exposed in [7]. The usage of the average heat transfer coefficient is

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typical in a transformer designing process to calculate the needed number (area) of cooling surfaces [8-13].

In this paper, a procedure for obtaining temperature distribution in the transformer is proposed. The procedure requires calculation of the heat transfer coefficients. We use heat transfer coefficients in the energy (thermal) equation. The model can be used for temperature calculation on the arbitrary change of current. For numerical simulation of the mentioned equation, we use the finite element method. For this reason, a code has been provided under MATLAB software. This paper deals with a mathematical formulation for the heat conduction equation and its solution, using the finite element method for the steady state. Discussion of the proposed work has been provided at the end of the paper.

MATHEMATICAL FORMULATION FOR HEAT CONDUCTION EQUATION

The structure of a transformer winding is complex and does not conform to any known geometry in the strict sense. Under general conditions, the transformer windings can be assumed cylindrical in formation, hence a layer or a disc winding is a finite annular cylinder [10]. The thermal and physical properties of the system would be equivalent to a composite system of insulation and conductor. It has been assumed that heat is generated throughout the body at a constant rate, and oil in the vertical and horizontal ducts take away the heat through the process of convection. However, in an actual transformer winding, the conductor is the only heat source. Later, in this section formulations are given for calculating the different thermal and physical properties of the system. It has been assumed that temperature is independent of the space variable, because the winding structure is symmetrical. The temperature at any point on the periphery of the circle for a specific value of r and z is deemed a constant (i.e. presence of spacers has been ignored, thus reducing three-dimensional problem to a two-dimensional) one with r and z as space variables. Dielectric loss in insulation is assumed small compared to copper losses in the conductor. The surface of the disc or layer has been assumed smooth.

The generalized system of a non-homogeneous heat conduction equation with a non-homogeneous boundary condition in a Cartesian coordinate system is written, thus [10,14]:

$$k.\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+Q=0, \tag{1}$$

in the region $a < x < b$ and $0 < y < l$.

Equations 2 to 5 represent the general heat conduction equation with convection at all four boundary surfaces.

At the inner cylindrical surface:

$$-k_1\frac{\partial T}{\partial x}+h_1T=f_1(y). \tag{2}$$

At the outer cylindrical surface:

$$k_2\frac{\partial T}{\partial x}+h_2T=f_2(y). \tag{3}$$

At the bottom flat surface:

$$-k_3\frac{\partial T}{\partial y}+h_3T=f_3(x). \tag{4}$$

At the top flat surface:

$$k_4\frac{\partial T}{\partial y}+h_4T=f_4(x). \tag{5}$$

In the above equations, temperature, T, is a function of the space variables, x and y. The term Q is the heat source function and has been modified here to take care of the variation of resistivity of copper to temperature. The heat source term, Q, can be of the form:

$$Q=Q_0.[1+\alpha_c.(T-T_{\text{amb}})]. \tag{6}$$

α_c is the temperature coefficient of electrical resistance of copper wire. With this representation, function Q becomes a temperature-dependent, distributed heat source. Boundary functions $f_1(y)$ and $f_2(y)$ derived from Newton's law of cooling are of the following equations:

$$f_1(y)=h_1.(T_b+m_1.y), \tag{7}$$

$$f_2(y)=h_2.(T_b+m_2.y). \tag{8}$$

The term T_b is the temperature at the bottom of the disc or layer as applicable. Terms m_1 and m_2 are the temperature gradient along the winding height (for layer) or along the disc thickness for a disc. Similarly, functions f_3 and f_4 representing temperature profiles across bottom and top surfaces, have the same form as shown in Equations 7 and 8, where the temperature gradient term has been taken as zero:

$$f_3(x)=h_3.T_b, \tag{9}$$

$$f_4(x)=h_4.T_{\text{top}}. \tag{10}$$

Thermal conductivities are unequal in different directions. However, in an actual case, thermal conductivities in radial directions (k_1 and k_2) are equal and conductivities in the axial direction will be the same (k_3 and k_4). Thermal conductivity has been treated as a vector quantity having components in both radial and axial directions. The resultant thermal conductivity of the system can be estimated as:

$$K = \sqrt{k_r^2 + k_z^2}, \tag{11}$$

where:

$$k_r = \frac{\log \frac{r_n}{r_1}}{\left(\frac{\log \frac{r_2}{r_1}}{k_1} + \frac{\log \frac{r_3}{r_2}}{k_2} + \cdots + \frac{\log \frac{r_n}{r_{n-1}}}{k_n}\right)},$$

$$k_z = \frac{k_{\text{cu}}.k_{\text{in}}.(t_{\text{cu}} + t_{\text{in}})}{t_{\text{in}}.k_{\text{cu}} + t_{\text{cu}}.k_{\text{in}}}.$$

Term K represents the resultant thermal conductivity of the insulation and conductor system. Heat transfer coefficients, h_1 to h_4 (htc), are different across all four surfaces. To determine boundary functions, f_1 to f_4, it is necessary to calculate heat transfer coefficients across the four surfaces. Difficulty has been encountered in calculation of the heat transfer coefficient. It is reported elsewhere [10] that it depends on as many as 13 factors (e.g. winding size, type, duct dimensions, oil velocity, type of oil circulation, heat flux distribution, oil thermal properties etc.). In this work, corrections have been given for the temperature-dependence of the thermal and physical properties of oil, such as viscosity, specific heat, volumetric expansion and thermal conductivity. It was found that there is a negligible effect of specific heat, coefficient of volumetric expansion and conductivity in the present working range of loading (110-160°C). Following, are some of the heat transfer relations and relevant formulae in natural cooling (ON) mode. These formulae have been used to calculate the htc [14]. The local Nusselt number for laminar flow over vertical plates has been shown below:

$$\text{Nu} = 0.6\ \text{Ra}_{hf}^{0.2}, \tag{12}$$

$$\text{Ra}_{hf} = \text{Gr}_{hf} P_r, \tag{13}$$

where Ra_{hf} and Gr_{hf} are the local Rayleigh and Grashof numbers based on heat flux (q_w) at characteristic dimension (δ). P_r is the Prandtl number of the transformer oil. The expression of the Rayleigh number based on constant heat flux is:

$$\text{Ra}_{hf} = \frac{g.\beta.C_p.\rho^2.q_w.\delta^4}{k_{\text{oil}}^2.\mu}. \tag{14}$$

The mean Nusselt number in this case can be computed as:

$$\text{Nu}_m = 1.5[\text{Nu}]_{\delta=l}. \tag{15}$$

However, correction to Equation 12 has to be given for cylindrical curvature. The correction factor in this case is of the following form ($30 < P_r < 50$):

$$f(\xi) \approx 1 + 0.12\xi, \tag{16}$$

where:

$$\xi = \frac{2\sqrt{2}}{\text{Gr}^{0.25}} \times \frac{\delta}{r}. \tag{17}$$

Here, G_r is the Grashof number, based on temperature difference. The local heat transfer coefficient can be computed as:

$$h = \frac{\text{Nu}.k_{\text{oil}}}{\delta}. \tag{18}$$

The mean coefficient (h_m) can be calculated from the mean Nusselt number as in Equation 15. After knowing the h_m for a particular surface, the temperature difference between the winding surface and oil can be found out dividing the h_m by the heat flux through the surface. The mean Nusselt number of the top surface of annular cylindrical winding for the laminar and turbulent regimes will normally be of the form:

$$\text{Nu}_m = 0.54\ \text{Ra}^{0.25} = 0.61\ \text{Ra}_{hf}^{0.2}, \tag{19}$$

$$\text{Nu}_m = 0.15\ \text{Ra}^{\frac{1}{3}} = 0.24\ \text{Ra}_{hf}^{\frac{1}{4}}, \tag{20}$$

$$\delta = \frac{b-a}{2}, \tag{21}$$

where a and b are the inner and outer radius of the annular disc or layer. The Nusselt number of the bottom surface of annular cylindrical winding for the laminar and turbulent regimes is in the following equation:

$$\text{Nu}_m = 0.27\ \text{Ra}^{0.25} = 0.35\ \text{Ra}_{hf}^{0.2}. \tag{22}$$

The axial oil temperature gradient in the presence of cooling by a constant heat flux can be found out by the following equation due to [14]:

$$\frac{\partial T}{\partial y} = 4.25 \times 10^{-2}\, \frac{q_w}{k_{\text{oil}}}.\left(\frac{l}{\pi.D_m}\right)^{\frac{4}{9}} \text{Ra}_{hf}^{\frac{-1}{9}}, \tag{23}$$

where l is the winding height, D_m is the mean diameter of the annular disc or layer of windings. Determination of boundary conditions in forced convection (OF mode) also requires calculating htc. When the oil velocity is lower values, the mean Nusselt number, based on temperature difference, is expressed in the form of Equation 24 and the corresponding mean Nusselt number, based on constant heat flux, is expressed in the form of Equation 25:

$$\text{Nu}_m = 1.75\left[G_z + 0.012\left(G_z \text{Gr}^{\frac{1}{3}}\right)^{\frac{4}{3}}\right]^{\frac{1}{3}} \times \left(\frac{\mu_b}{\mu_w}\right)^{0.14}, \tag{24}$$

$$\mathrm{Nu}_m = 1.63 \left[1.8 G_z^{0.9} + 0.02 \left(G_z \mathrm{Gr}_{hf}^{\frac{1}{3}} \right)^{1.16} \right]^{\frac{1}{3}} \times \left(\frac{\mu_b}{\mu_w} \right)^{0.125}, \tag{25}$$

$$G_z = \mathrm{Re}\ \mathrm{Pr} \frac{D}{l}. \tag{26}$$

G_z is called the Graetz number. Terms μ_b and μ_w are the viscosity of the oil computed at the oil bulk mean temperature of oil and at the winding wall temperature, respectively. Re is the Reynolds number and Pr is the Prandtl number. The relative importance of natural and forced cooling is indicated by the factor f_r=Gr/Re^2. If $f_r \approx 1$, then both of the cooling modes have to be considered. At a lower value of this factor, natural cooling can be ignored. The oil viscosity is an important property, which depends on temperature. The formula to consider the oil temperature variation is given in the following equation [9,10]:

$$\mu = \alpha . \exp \left(\frac{\gamma}{T_m} \right), \tag{27}$$

where:

$\alpha = 0.0000013573$ (kg.m^{-1}.s^{-1}),

$\gamma = 2797.3$ (K).

The viscosity was calculated at the mean oil and wall temperature. Initially, the winding surface temperature is not known, so a starting guess for winding surface temperature has to be made. After calculating the value of h, the temperature difference $(T_w - T_{\mathrm{oil}})$ is to agree with the assumed value. To calculate the winding wall temperature at different surfaces, only the bottom oil temperature is necessary. In this paper, the bottom oil rise over the ambient temperature has been calculated as:

$$\theta_u = \theta_{fl} . \left(\frac{I_r^2 R + 1}{R + 1} \right)^n, \tag{28}$$

where θ_u is the bottom oil temperature rise over the ambient temperature; θ_{fl} is the full load bottom oil temperature rise over the ambient temperature obtained from an off-line test; R is the ratio of load loss at rated load to no-load loss. The variable, I_r, is the ratio of the specified load to rated load:

$$I_r = \frac{I}{I_{\mathrm{rated}}}. \tag{29}$$

The exponent n depends upon the cooling state. The loading guide recommends the use of $n = 0.8$ for natural convection and $n = 0.9 - 1.0$ for forced cooling.

Assumptions have been made in the calculation of htc in the ducts provided in the disc-type windings under Oil-Forced (OF) modes of heat transfer (DOF and NDOF). The cooling in OF mode is due to mixed mode (natural and forced) convection. While the oil-flow velocity in both vertical and horizontal ducts has been assumed equal, we are in the DOF mode. If the flow velocity in horizontal ducts is assumed negligible compared to the velocity in the vertical ducts, we are in NDOF mode. In case of the DOF mode, the htc was assumed as a function of both heat flux through the surface and the oil-flow velocity. The mechanism of heat transfer in this mode of cooling is the same in both axial and radial directions of the disc. In case of the NDOF mode, the htc in the vertical duct has been estimated by the same formula as for DOF. However, the convection of heat in the horizontal ducts has been assumed as a purely natural type.

FINITE ELEMENT SOLUTION OF HEAT EQUATION FOR THREE-PHASE TRANSFORMER WINDINGS

For finite element solution of the heat conduction equation, a software program using the MATLAB code has been provided. The model has been validated on a transformer of rating 32 MVA. The Cu-loss (at T_{amb}=25°C) in the winding per disc/layer of the above transformer has been tabulated in Table 1. The result of temperature distribution in the x, y plan from the LV layer 1 winding in the 1 p.u. load, with Re = 750, has been shown in Figure 1. It can be pointed out that, for LV layer 1 winding, the maximum temperature location is around 90 to 95% of the winding height from the bottom and at about 50% of the radial thickness of the layer.

The result of the temperature distribution from LV layer 2 winding has been shown in Figure 2. It can be pointed out that for LV layer 2 winding, the maximum temperature location is the same as LV

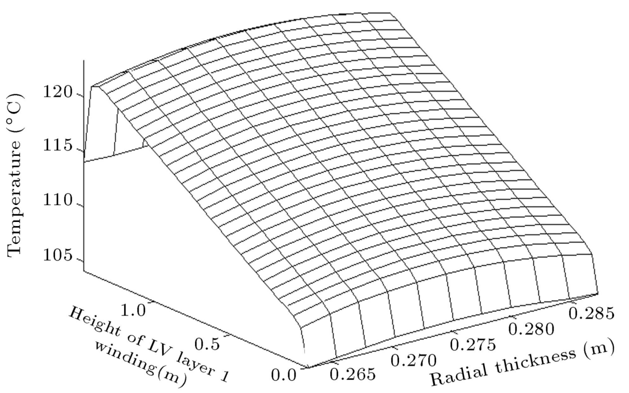

Figure 1. Temperature distribution of LV layer 1 winding at T_{amb} = 25°C with Re = 750.

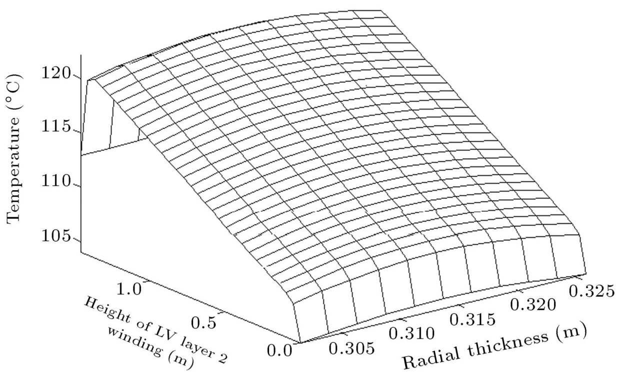

Figure 2. Temperature distribution of LV layer 2 winding at $T_{\mathrm{amb}} = 25°\mathrm{C}$ with Re = 750.

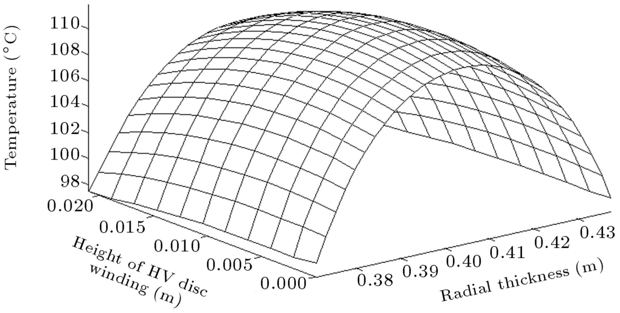

Figure 3. Temperature distribution of HV disc winding in NDOF mode at $T_{\mathrm{amb}} = 25°\mathrm{C}$ with Re = 750.

layer 1 winding. Temperature distribution from a disc of HV winding has been shown in Figures 3 and 4. It may be observed that the maximum temperature occurs in the neighborhood of 30-35% of the axial and 50% of the radial thickness of the disc in NDOF

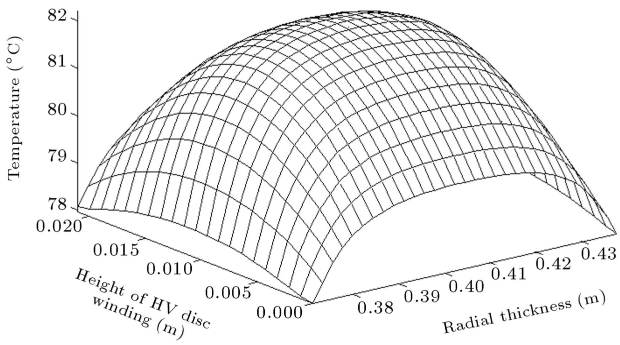

Figure 4. Temperature distribution of HV disc winding in DOF mode at $T_{\mathrm{amb}} = 25°\mathrm{C}$ with Re = 750.

mode and 50-55% of the axial and 50% of the radial thickness of the disc in DOF mode. Magnitudes of HST at different loading and OF modes are given in Table 2. In this work, different rates of forced oil circulation are taken into account. Figures 5 to 8 show variations of local HST with load and Reynolds number in different windings and different OF modes. Tables 2 and 3 show a comparison of the proposed method with the boundary value method, using the finite integral transform used in [10] and the experimental test in [9].

It is clear from Table 2 that the HST in the case of the NDOF mode is greater than that of the corresponding DOF mode and so is the average temperature rise. From Figures 5, 6 and 8, it is also borne out that an increase in the amount of cooling, by way of increased oil flow speed, would provide a reduced winding temperature, which is obvious. However, an increased oil flow has a lesser effect on HST in case of the NDOF mode rather than the DOF mode (Figures 7 and 8). From the results of this investigation, it is clear

Table 1. Characteristics of transformer windings.

Transformer Rating (MVA)	Winding Type	Cu-Loss (W)	Dimensions (mm) a,b,l	Remarks
32	LV layer 1	7579	263,287.5,1460	Per layer
32	LV layer 2	8536	301.5,326,1460	Per layer
32	HV disc	570	371,438,21	Per disc

Table 2. HST(°C) magnitudes at $T_{\mathrm{amb}} = 25°\mathrm{C}$ with Re = 750.

Load (p.u.)	HST (°C)-LV Layer 1	HST (°C)-LV Layer 2	HST (°C)-HV Disc		Global HST (Proposed)	HST-Ref. [10]
			NDOF	DOF		
0.8	95	94	87	67	95	92
0.9	108	107	99	74	108	104
1.0	123	122	112	82	123	117
1.1	140	138	126	91	140	134
1.2	158	156	141	101	158	152
1.3	177	175	157	112	177	183
1.4	199	197	174	124	199	200
1.5	222	220	192	137	222	229

Table 3. HST°C locations (in LV winding) at $T_{\text{amb}} = 25°$C.

Load (p.u.)	**Proposed (Radial Thickness%-Height%)**	**Ref. [9] (Radial Thickness%- Height%)**
0.8	50 / 95	44 / 94
0.9	50 / 94.8	45 / 95
1.0	50 / 94.6	45 / 94
1.1	50 / 94.5	45 / 93
1.2	50 / 94.5	45 / 93
1.3	50 / 94.3	49 / 91
1.4	50 / 94.2	49 / 91
1.5	50 / 94	52 / 90

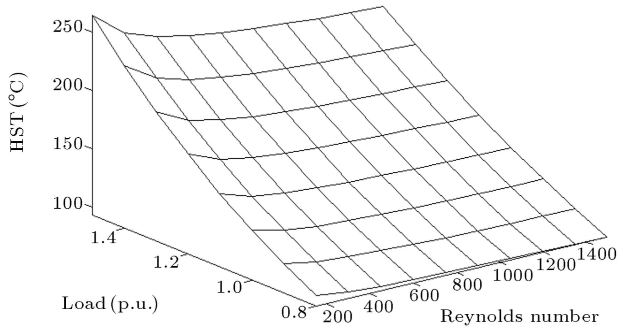

Figure 5. HST (local) distribution of LV layer 1 winding at $T_{\text{amb}} = 25°$C.

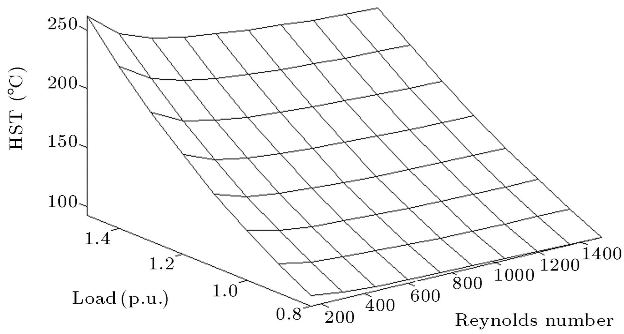

Figure 6. HST (local) distribution of LV layer 2 winding at $T_{\text{amb}} = 25°$C.

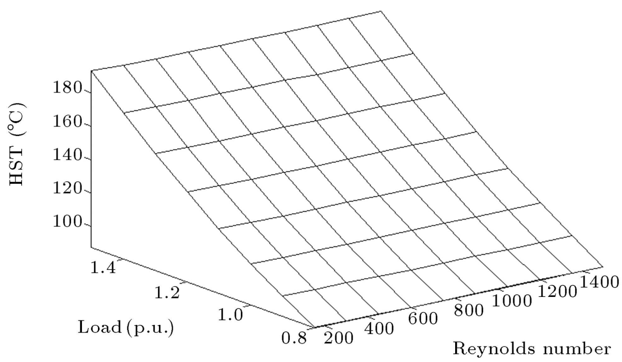

Figure 7. HST (local) distribution of HV disc winding in NDOF mode at $T_{\text{amb}} = 25°$C.

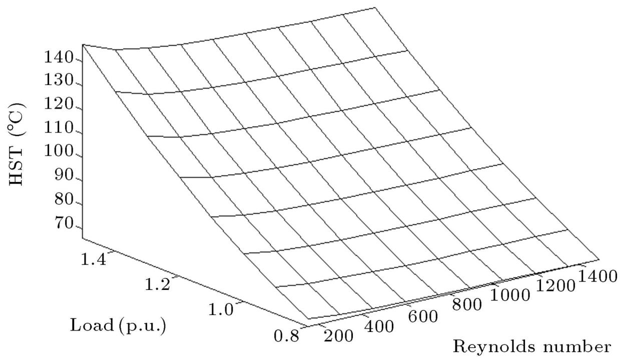

Figure 8. HST (local) distribution of HV disc winding in DOF mode at $T_{\text{amb}} = 25°$C.

that DOF cooling is more effective than the NDOF cooling mode as expected.

CONCLUSIONS

In this paper, an attempt has been made to suggest a method to improve the accuracy of temperature prediction of the hottest spot in a power transformer by solving the heat transfer partial differential equation (PDE), numerically. The purely numerical approach followed in this paper for evaluating the hot spot and its location seems to correspond reasonably well with results of calculations and actual tests and on site measurements [9,10]. The authors wish to point out that the IEEE loading guide and other similar documents offer relations for calculation of the HST, based on per-unit load. The formulations tend to ignore the possibilities of two transformers that are rated identical but have a different winding structure and a varying heat loss/unit volume. The method suggested by the authors gives due representation for this omission and, hence, is believed to give more accurate estimates. The thermal model presented here can predict the hot spot location with a reasonable degree of accuracy. The authors are currently working on a 3D model of a transformer for the estimation of temperature at different points of the transformer, knowing the load conditions. A two-dimensional model is suitable for estimation of the temperature at transformer windings and is not useful for the core; because in a two-dimensional case, we use the assumption that the core is symmetrical in all directions, which is not a correct assumption. If we want to estimate temperature distribution at the core of the power transformer, we must use a three-dimensional model.

NOMENCLATURE

l	thickness of the disc or height of layer winding (m)
n	temperature rise exponent to bottom oil
C_p	specific heat of transformer oil ($\mathrm{J.kg^{-1}.K^{-1}}$)
k_{oil}	thermal conductivity of transformer oil ($\mathrm{W.m^{-1}.K^{-1}}$)
a	inner radius of the annular disc or layer (m)
b	outer radius of the annular disc or layer (m)
D_m	mean diameter of annular disc or layer of windings (m)
f_r	relative index of free-to-forced convection
G_z	Graetz number
Gr_{hf}	Grashof number based on constant heat flux
Gr	Grashof number based on temperature difference
k_1	thermal conductivity at inner cylinder surface ($\mathrm{W.m^{-1}.K^{-1}}$)
k_2	thermal conductivity at outer cylinder surface ($\mathrm{W.m^{-1}.K^{-1}}$)
k_3	thermal conductivity at bottom cylinder surface ($\mathrm{W.m^{-1}.K^{-1}}$)
k_4	thermal conductivity at top cylinder surface ($\mathrm{W.m^{-1}.K^{-1}}$)
h_1	heat transfer coefficient at inner surface ($\mathrm{W.m^{-2}.K^{-1}}$)
h_2	heat transfer coefficient at outer surface ($\mathrm{W.m^{-2}.K^{-1}}$)
h_3	heat transfer coefficient at bottom surface ($\mathrm{W.m^{-2}.K^{-1}}$)
h_4	heat transfer coefficient at top surface ($\mathrm{W.m^{-2}.K^{-1}}$)
f_1	boundary function at inner cylinder surface
f_2	boundary function at outer cylinder surface
f_3	boundary function at bottom cylinder surface
f_4	boundary function at top cylinder surface
k_{cu}	thermal conductivity of copper ($\mathrm{W.m^{-1}.K^{-1}}$)
k_{in}	thermal conductivity of insulation ($\mathrm{W.m^{-1}.K^{-1}}$)
m	oil temperature gradient along winding height ($\mathrm{K.m^{-1}}$)
Nu_m	mean Nusselt number
Nu	local Nusselt number
q_w	heat flux in ($\mathrm{W.m^{-2}}$)
$r_i's$	radius of insulation and conductor layers (m), $i = 1, 2, \ldots$
Ra_{hf}	Rayleigh number based on constant heat flux
Ra	Rayleigh number based on temperature difference
T_{amb}	ambient temperature (K)
T_b	temperature at the bottom of the winding (K)
T_{top}	temperature at the top of the winding (K)
T_m	temperature average of oil and winding surface (K)
t_{cu}	thickness of conductor (m)
t_{in}	thickness of insulation (m)
g	acceleration due to gravity ($\mathrm{m.s^{-2}}$)
R	loss ratio = load loss/no load loss
Re	Reynolds number
Pr	Prandtl number
Q	volumetric heat source function ($\mathrm{W.m^{-3}}$)
Q_0	volumetric heat source at ambient temperature ($\mathrm{W.m^{-3}}$)
k_r	thermal conductivity in radial direction ($k_r = k_1 = k_2$)
k_z	thermal conductivity in axial direction ($k_z = k_3 = k_4$)
K	referred to as equivalent thermal conductivity
LV	low voltage
HV	high voltage
p.u.	per unit
DOF	directed oil-forced cooling
NDOF	non-directed oil-forced cooling

Greek Symbols

β	coefficient of volumetric expansion of oil ($\mathrm{K^{-1}}$)

θ_u	ultimate bottom oil temperature rise over ambient temperature at rated load (K)
θ_{fl}	ultimate bottom oil temperature rise over ambient temperature at related load (K)
δ	characteristic dimension (m)
ρ	density of transformer oil (kg.m^{-3})
$f(\xi)$	correction factor for cylindrical curvature
α_c	temperature coefficient of electrical resistance (K^{-1})
μ	viscosity of oil (kg.m^{-1}.s^{-1})
μ_b	viscosity of oil at oil bulk mean temperature (kg.m^{-1}.s^{-1})
μ_w	viscosity of oil at winding wall temperature (kg.m^{-1}.s^{-1})

Subscripts

b	bulk property
m	mean value
w	wall value

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