Transaction B: Mechanical Engineering
Vol. 16, No. 4, pp. 354–362

Research Note

SCIENTIA IRANICA

Similarity Solution in the Study of Flow and Heat Transfer Between Two Rotating Spheres with Constant Angular Velocities

A. Jabari Moghadam[1] and A. Baradaran Rahimi[1,*]

Abstract. *The similarity solution of the steady-state motion and heat transfer of a viscous incompressible fluid contained between two concentric spheres, maintained at different temperatures and rotating about a common axis with different constant angular velocities, is considered. The resulting flow pattern, temperature distribution and heat transfer characteristics are presented for various cases. Aside from the energy equation, the same results as previous works are obtained for Navier-Stokes equations, but with less computational complexities.*

Keywords: *Similarity solution; Steady-state flow and heat transfer; Concentric rotating spheres; Constant angular velocities.*

INTRODUCTION

A similarity solution for the motion of an incompressible viscous fluid and its heat transfer in a rotating spherical annulus is considered numerically, when the spheres are concentric and their angular velocities about a common axis of rotation are constant. Such motions may be described in terms of a pair of coupled non-linear partial differential equations in three independent variables and the energy equation is linear when the velocity field is known.

Available theoretical work concerning such problems is primarily of the boundary-layer or singular-perturbation character considered by Howarth [1], Proudman [2], Lord & Bowden [3], Fox [4], Greenspan [5], Carrier [6] and Stewartson [7]. The first numerical study of time-dependent viscous flow between two rotating spheres has been presented by Pearson [8], in which the case of one (or both) spheres is given an impulsive change in angular velocity, starting from a state of either rest or uniform rotation. Munson and Joseph [9] have considered the case of the steady motion of a viscous fluid between concentric rotating spheres using perturbation techniques for small values of Reynolds number and a Legendre polynomial expansion for larger values of Reynolds number. In both studies, the viscous dissipation terms have been neglected. Thermal convection in rotating spherical annuli has been considered by Douglass, Munson and Shaughnessy [10], in which the steady forced convection of a viscous fluid contained between two concentric spheres (which are maintained at different temperatures and rotate about a common axis with different angular velocities) is studied. Approximate solutions to the governing equations are obtained in terms of a regular perturbation solution valid for small Reynolds numbers and a modified Galerkin solution for moderate Reynolds numbers. Viscous dissipation is neglected in their study and all fluid properties are assumed constant. A study of viscous flow in oscillatory spherical annuli has been done by Munson and Douglass [11], in which a perturbation solution valid for slow oscillation rates is presented and compared with experimental results. Another interesting work is the study of the axially symmetric motion of an incompressible viscous fluid between two concentric rotating spheres done by Gagliardi et al. [12]. This work involves the study of the steady state and transient motion of a system consisting of an incompressible, Newtonian fluid in an annulus between two concentric, rotating, rigid spheres. The primary purpose of their research is to study the use of an approximate analytical method for analyzing the transient motion of the fluid in the

1. *Department of Mechanical Engineering, Ferdowsi University of Mashhad, Mashhad, P.O. Box 91775-1111, Iran.*
*. *Corresponding author. E-mail: rahimiab@yahoo.com*

Received 6 May 2007; received in revised form 16 September 2007; accepted 2 March 2008

annulus and spheres, which is started suddenly due to the action of prescribed torques. Similar cases include the study of Jen-Kang Yang et al. [13] and the finite element study by Ni and Negro [14]. These problems include the case where one or both spheres rotate with prescribed constant angular velocities and the case in which one sphere rotates due to the action of an applied constant or impulsive torque.

State-of-the-art work on the similarity solution of flow and heat transfer problems is abundant in the literature of which some recent examples can be found in [15-34].

A similarity solution in the study of the steady-state motion and heat transfer of an incompressible viscous fluid filling the annuli of two concentric spheres rotating with constant angular velocities has not been considered in the literature. In the present study, a numerical solution of the similarity equations of steady-state momentum and energy equations are solved for viscous flow between two concentric rotating spheres maintained at different temperatures and rotating with constant angular velocities. Aside from the energy equation, the same results as existing in the literature for Navier-Stokes equations are obtained, but with less computational complexities. Such rotating containers are used in engineering designs like centrifuges and fluid gyroscopes and are also important in geophysics. Other applications of the geometric configuration used in this problem are in meteorological instrumentations, where such apparatus and equipment are used to obtain quantitative information about weather.

PROBLEM FORMULATION

The geometry of the spherical annulus considered is indicated in Figure 1. A Newtonian, viscous incompressible fluid fills the gap between the inner and outer spheres, which are of radii R_i and R_o, with constant surface temperatures, T_i and T_o, and which rotate about a common axis with angular velocities, Ω_i and Ω_o, respectively. The components of the velocity in directions r, θ and ϕ are v_r, v_θ and v_ϕ, respectively. These velocity components for incompressible flow and in a meridian plane satisfy the continuity equation and are related to stream function ψ and angular momentum function, Ω, in the following manner:

$$\nu_r = \frac{\psi_\theta}{r^2 \sin\theta}, \quad \nu_\theta = \frac{-\psi_r}{r\sin\theta}, \quad \nu_\phi = \frac{\Omega}{r\sin\theta}. \tag{1}$$

Since the flow is assumed to be independent of the longitude, ϕ, the non-dimensional Navier-Stokes equations and energy equation can be written in terms of the stream function and the angular velocity function as follows:

$$\frac{\psi_\theta\Omega_r - \psi_r\Omega_\theta}{r^2\sin\theta} = \frac{1}{(\mathrm{Re})}D^2\Omega, \tag{2}$$

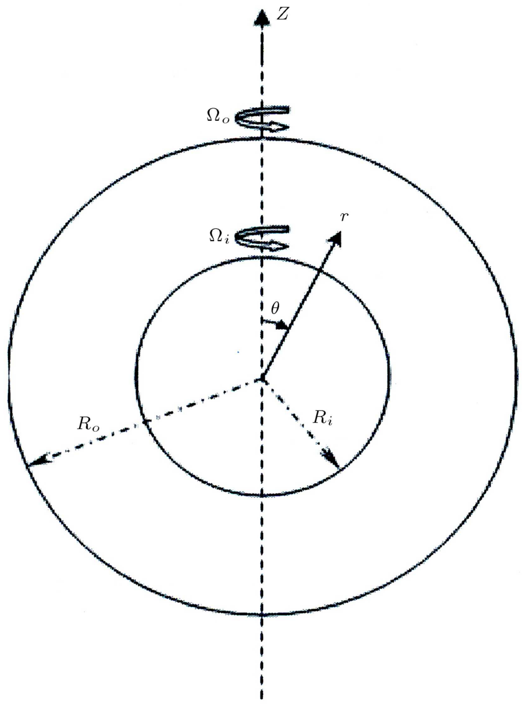

Figure 1. Spherical annulus.

$$\frac{2\Omega}{r^3\sin^2\theta}[\Omega_r r\cos\theta - \Omega_\theta\sin\theta] - \frac{1}{r^2\sin\theta}[\psi_r(D^2\psi)_\theta - \psi_\theta(D^2\psi)_r] + \frac{2D^2\psi}{r^3\sin^2\theta}[\psi_r r\cos\theta - \psi_\theta\sin\theta] = \frac{1}{(\mathrm{Re})}D^4\psi, \tag{3}$$

$$\nu_r\frac{\partial T}{\partial r} + \frac{\nu_\theta}{r}\frac{\partial T}{\partial\theta} = \frac{1}{(Pe)}\left[\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2} + \frac{\cot\theta}{r^2}\frac{\partial T}{\partial\theta}\right] + (\mathrm{EK})(\text{Dissipation terms}), \tag{4}$$

in which:

$$\text{Dissipation terms} = \left\{2\left[\left(\frac{\partial\nu_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial\nu_\theta}{\partial\theta} + \frac{\nu_r}{r}\right)^2 + \left(\frac{\nu_r}{r} + \frac{\nu_\theta}{r}\cot\theta\right)^2\right] + \left[r\frac{\partial}{\partial r}\left(\frac{\nu_\theta}{r}\right) + \frac{1}{r}\frac{\partial\nu_r}{\partial\theta}\right]^2 + \left[\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{\nu_\phi}{\sin\theta}\right)\right]^2 + \left[r\frac{\partial}{\partial r}\left(\frac{\nu_\phi}{r}\right)\right]^2\right\}, \tag{5}$$

and the non-dimensional quantities of Reynolds number (Re), Prandtl number (Pr), Peclet number (Pe) and Eckert number (Ek) are defined as:

$$\begin{aligned} &\mathrm{Re} = \frac{\Omega_o R_o^2}{\nu}, && \mathrm{Pr} = \nu/\alpha, \\ &\mathrm{Pe} = \mathrm{Re.Pr} = \frac{\Omega_o R_o^2}{\alpha}, && \mathrm{Ek} = \frac{\nu\Omega_o}{c_P(T_o - T_i)}. \end{aligned} \tag{6}$$

The following non-dimensional parameters have been used in the above equations and then the asterisks have been omitted:

$$\begin{aligned} &r^* = \frac{r}{R_o}, && \psi^* = \frac{\psi}{R_o^3\Omega_o}, \\ &\Omega^* = \frac{\Omega}{R_o^2\Omega_o}, && T^* = \frac{T-T_i}{T_o-T_i}. \end{aligned} \tag{7}$$

The non-dimensional boundary and initial conditions for the above governing equations are:

$$\theta = 0 \to \{\psi = 0, \quad D^2\psi = 0, \quad \Omega = 0\}, \quad \frac{\partial T}{\partial \theta} = 0,$$

$$\theta = \pi \to \{\psi = 0, \quad D^2\psi = 0, \quad \Omega = 0\}, \quad \frac{\partial T}{\partial \theta} = 0,$$

$$r = \frac{R_i}{R_o} = b \to \begin{cases} \psi = 0, \quad \psi_r = 0, \quad \Omega = \Omega_{io} b^2 \sin^2\theta \\ T = 0 \end{cases}$$

$$r = 1 \to \begin{cases} \psi = 0, \quad \psi_r = 0, \quad \Omega = \sin^2\theta \\ T = 1 \end{cases} \tag{8}$$

where:

$$D^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} - \frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta},$$

$$\Omega_{io} = \Omega_i/\Omega_o. \tag{9}$$

These governing equations, along with their related boundary conditions, are to be changed into differential equations with related appropriate boundary conditions by using similarity parameters and are then to be solved numerically in the following sections.

SIMILARITY VARIABLES

Using the following similarity parameters, one independent variable can be omitted:

$$\eta = \frac{(r-b)\sin^2\theta}{1 - r + b\sin^2\theta}, \tag{10}$$

$$\begin{cases} \Omega = \sin^2\theta F(\eta) \\ \psi = \sin^2\theta G(\eta) \\ T(r,\theta) = H(\eta) \end{cases} \tag{11}$$

The momentum and energy equations become:

$$c_1(F'G - G'F) = \frac{1}{Re}(c_2F'' + c_3F' + c_4F), \tag{12}$$

$$\begin{aligned} &(d_1F' + d_2F)F + (d_3G' + d_5G'')G' \\ &\quad + (d_4G' + d_6G + d_7G''' + d_8G'')G \\ &\quad = \frac{1}{Re}(d_9G' + d_{10}G + d_{11}G''' + d_{12}G'' \\ &\quad + d_{13}G''''), \end{aligned} \tag{13}$$

$$\begin{aligned} (e_1\nu_r + e_2\nu_\theta)H' &= \frac{1}{\mathrm{Pe}}(f_1H'' + f_2H') \\ &+ \mathrm{Ek.\ (dissipation\ terms)}, \end{aligned} \tag{14}$$

where the c and d coefficients are functions of r and θ. Maple software has been used to do all the algebra producing the following coefficients in a compact form as:

$$\lambda = (-1 + r - b\sin^2\theta),$$

$$\gamma = (-1 + r - b + b\cos^2\theta),$$

$$c_1 = 2\cos\theta\sin^2\theta\ (1 - b\cos^2\theta)\lambda^2,$$

$$\begin{aligned} c_2 &= \sin^2\theta\{-b^2r^2\cos^6\theta + b(2+b)r^2\cos^4\theta \\ &+ [4r^4 - 8(1+b)r^3 + (3+14b+4b^2)r^2 \\ &- 8b(1+b)r + 4b^2]\cos^2\theta + r^2\}, \end{aligned}$$

$$\begin{aligned} c_3 &= -2\lambda\{b[2r^2 - (1+b)r + b]\cos^4\theta \\ &+ [5r^3 - (8b+11)r^2 + (2b^2+12b+5)r - 5b \\ &- 2b^2]\cos^2\theta - r^3 + (3+2b)r^2 - (1+3b+b^2)r \\ &+ b(1+b)\}, \end{aligned}$$

$$c_4 = -2\lambda^4,$$

$$\begin{aligned} d_1 &= 2r^2\lambda^6\sin^2\theta\cos\theta\{br\sin^2\theta + 2r^2 \\ &- (3b+1)r + 2b\}, \end{aligned}$$

$$d_2 = -4r^2\cos\theta\lambda^8,$$

$$\begin{aligned} d_3 &= 4\sin^2\theta\lambda^3\{[2r^3b^2 - b^2(1+b)r^2 + rb^3]\cos^6\theta \\ &+ [-2b(b+4)r^3 + b(3b+5-b^2)r^2 + b^2(5b-1)r \\ &- 4b^3]\cos^4\theta - [20r^5 - (65+47b)r^4 + (63 \\ &+ 30b^2 + 129b)r^3 - (124b + 5b^3 + 20 + 79b^2)r^2 \\ &+ b(13b^2 + 40 + 70b)r - 20b^2 - 8b^3]\cos^2\theta \end{aligned}$$

$$+4r^5-(21+11b)r^4+(23+10b^2+37b)r^3$$
$$-(25b^2+8+37b+3b^3)r^2+b(19b+12+7b^2)r$$
$$-4b^2(1+b)\}\cos\theta,$$

$$d_4=-4\lambda^4\{rb^3\cos^8\theta-[6r^2b^2-4b^2(1+b)r$$
$$+8b^3]\cos^6\theta-[22r^3b-2b(21b+22)r^2$$
$$+b(70b+18b^2+23)r-32b^2-24b^3]\cos^4\theta$$
$$-[24r^4-(65b+71)r^3+(62+154b+58b^2)r^2$$
$$-(112b+104b^2+20b^3+18)r+48b^2+24b^3$$
$$+24b]\cos^2\theta+8r^4-(23b+27)r^3+$$
$$+(22+54b+22b^2)r^2-(38b^2+6+37b+7b^3)r$$
$$+8b^3+8b+18b^2\}\cos\theta,$$

$$d_5=-4\cos\theta\sin^4\theta\lambda^2[r^2b^2(3r^2$$
$$-2(1+b)r+b)]\cos^6\theta+[4r^5b-b(11b+14)r^4$$
$$+2b(4+3b^2+11b)r^3-b^2(10+9b)r^2$$
$$+4rb^3]\cos^4\theta+[-8r^6+(24+22b)r^5$$
$$-(62b+21+20b^2)r^4+(64b+60b^2+6$$
$$+6b^3)r^2+b^2(24+22b)r-8b^3]\cos^2\theta$$
$$-2r^5+(1+4b)r^4-2b(b+3)r^3$$
$$+b(3+4b)r^2-2rb^2),$$

$$d_6=16\gamma^8\cos\theta,$$

$$d_7=2r\lambda^2\cos\theta\sin^4\theta(b\cos^2\theta-1)\{b^2r^2\cos^6\theta$$
$$-b(b+2)r^2\cos^4\theta-[4r^4-8(1+b)r^3$$
$$+(4b^2+3+14b)r^2-8b(1+b)r+4b^2]\cos^2\theta-r^2\},$$

$$d_8=4\sin^2\theta\lambda^3\{r^2b^3\cos^8\theta+[5r^3b^2-b^2(3b+4)r^2$$
$$+rb^3]\cos^6\theta+[5r^4b-b(7b+15)r^3$$
$$+b(5+2b-b^2)r^2+b^2(10b+6)r-8b^3]\cos^4\theta$$
$$+[-8r^5+(19b+15)r^4-(14b^2+39b+2)r^3$$
$$-(2-3b^3-41b^2-31b)r^2-b(11b^2+7$$
$$+37b)r+8b^2+8b^3]\cos^2\theta+r^4-(2b+6)r^3$$
$$+(b^2+4b+2)r^2-b(b+1)r\}\cos\theta,$$

$$d_9=8r\lambda^3\{\cos^{10}\theta rb^4+b^3[4r^2-4(1+b)r-b]\cos^8\theta$$
$$+[6r^3b^2-b^2(15b+18)r^2+b^2(6b^2+9+14b)r$$
$$+4b^4+4b^3]\cos^6\theta+[-14r^4b+(63b+28b^2)r^3$$
$$-(78b+3b^3+102b^2)r^2+(26b+30b^3+120b^2$$
$$-4b^4)r-41b^2-32b^3-6b^4]\cos^4\theta+[10r^5-(49$$
$$+17b)r^4-(2b^2-48b-90)r^3-(60b-11b^3$$
$$-30b^2+64)r^2-(38b^3-b^4+43b^2-16-42b)r$$
$$+14b^2-10b+4b^4+28b^3]\cos^2\theta-2r^5+(7b+9)r^4$$
$$-(22+8b^2+27b)r^3+(16+34b+3b^3+18b^2)r^2$$
$$-(16b+4+2b^3+14b^2)r+3b^2+2b-b^4\},$$

$$d_{10}=-8r\gamma^8,$$

$$d_{11}=-4r\lambda\sin^2\theta\{[-4r^4b^3+(b^4+b^3)r^3-r^2b^4]\cos^{10}\theta$$
$$-[17r^5b^2-(30b^3+37b^2)r^4+(7b^4+11b^2$$
$$+31b^3)r^3-(3b^4+7b^3)r^2-4rb^4]\cos^8\theta-[4r^6b$$
$$-(46b+42b^2)r^5+(82b+154b^2+64b^3)r^4-(129b^3$$
$$+141b^2+23b^4+31b)r^3+(39b^4+96b^3+47b^2)r^2$$
$$-(28b^3+28b^4)r+12b^4]\cos^6\theta+[36r^7-(116b$$
$$+152)r^6+(119b^2+432b+223)r^5-(131+614b$$
$$+419b^2+34b^3)r^4+(638b^2+147b^3-5b^4+419b$$
$$+27)r^3-(233b^3-b^4+441b^2+103b)r^2+(156b^3$$
$$+4b^4+112b^2)r-36b^3]\cos^4\theta-[12r^7-(56+48b)r^6$$
$$+(78+190b+72b^2)r^5-(248b^2+48b^3+262b$$
$$+30)r^4+(150b^3+335b^2+12b^4+2+159b)r^3$$
$$-(36b^4+30b+199b^2+178b^3)r^2+(36b^4+88b^3$$
$$+40b^2)r-12b^3(1+b)]\cos^2\theta-r^5+(2b+5)r^4$$
$$-(1+3b+b^2)r^3+b(b+1)r^2\},$$

$$d_{12} = 4r\lambda^2\{-68r^3b + (38b+2)r^2 + (-28b^2 - 8b)r$$
$$+ [-4r^3b^3 + (4b^3 + 4b^4)r^2 + 2rb^4]\cos^{10}\theta$$
$$+ [-15r^4b^2 + (24b^2 + 18b^3)r^3 + (8b^3 - 3b^4$$
$$- 3b^2)r^2 + (-20b^3 - 14b^4)r - b^4]\cos^8\theta + [-22r^5b$$
$$+ (132b + 112b^2)r^4 + (-114b^3 - 378b^2 - 198b)r^3$$
$$+ (366b^2 + 16b^4 + 70b + 234b^3)r^2 + (-210b^3$$
$$- 12b^4 - 110b^2)r + 24b^4 + 74b^3]\cos^6\theta + [71r^6$$
$$+(-168b - 320)r^5 + (594b + 55b^2 + 510)r^4 + (-840b$$
$$- 326 - 118b^2 + 82b^3)r^3 + (251b^2 - 190b^3$$
$$- 31b^4 + 74 + 610b)r^2 + (-256b^2 + 52b^4 + 166b^3$$
$$- 160b)r - 42b^4 - 46b^3 + 71b^2]\cos^4\theta + [-50r^6$$
$$+ (240 + 154b)r^5 + (-584b - 146b^2 - 400)r^4$$
$$+ (818b + 244 + 30b^3 + 434b^2)r^3 + (-86b^3$$
$$- 526b - 536b^2 - 52 + 12b^4)r^2 + (-22b^4 + 90b^3$$
$$+ 298b^2 + 120b)r - 50b^2 - 34b^3 + 16b^4]\cos^2\theta$$
$$- 16r^5 + (34 + 50b)r^4 + (-14 - 58b^2)r^3 + (30b^3$$
$$+ 66b^2)r^2 + (-26b^3 - 6b^4)r + 3b^2 + 6b^3 + 3b^4$$
$$- \cos^{12}\theta r^2b^4 + 3r^6 - 12r^3b^3 + 18r^4b^2 + 3r^2b^4$$
$$- 12r^5b\},$$

$$d_{13} = -r\sin^4\theta\{b^2r^2\cos^6\theta - (2b + b^2)r^2\cos^4\theta$$
$$- [4r^4 - 8(1+b)r^3 + (3 + 14b + 4b^2)r^2$$
$$- 8b(1+b)r + 4b^2]\cos^2\theta - r^2\}^2,$$

$$e_1 = \frac{(1 - b\cos^2\theta)\sin^2\theta}{\lambda^2},$$

$$e_2 = \frac{(r-b)(1-r)\sin 2\theta}{r\lambda^2},$$

$$f_1 = \frac{r^2\sin^4\theta(1-b\cos^2\theta)^2 + \sin^2 2\theta[(r-b)(1-r)]^2}{r^2\lambda^4},$$

$$f_2 = \frac{2\sin^2\theta(1 - b\cos^2\theta)(1 + b\sin^2\theta)}{r\lambda^3}$$

$$+ \frac{2(r-b)(1-r)[(\lambda + 2b\sin^2\theta)\cos 2\theta - b\sin^4\theta]}{r^2\lambda^3}$$

$$+ \frac{2(r-b)(1-r)\cos^2\theta}{r^2\lambda^2}, \tag{15}$$

with boundary conditions:

$$\eta = 0 \rightarrow G(\eta) = 0, \quad G'(\eta) = 0, \quad F(\eta) = \Omega_{io}b^2, \quad H = 0,$$

$$\eta = \frac{1}{b} - 1 \rightarrow G(\eta) = 0, \quad G'(\eta) = 0, \quad F(\eta) = 1, \quad H = 1. \tag{16}$$

The differential Equations 12-14, along with boundary conditions (Equations 16), constitute a closed form system, which is solved numerically with less complexity compared to the initial system of partial differential equations. Note that the formulation of the problem this way enables one to find the functions $\psi(r,\theta)$ and $\Omega(r,\theta)$ at any desired point within the flow field independently, without having to solve for the whole region.

COMPUTATIONAL PROCEDURE

The flow Equations 12 and 13 are not coupled with the energy Equation 14 and, therefore, need to be solved before the latter can be solved. These non-linear flow equations are solved numerically using finite difference approximations. A quasi-linearization technique is first applied to replace the non-linear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence. Then, a Crank-Nicolson algorithm is used to replace the different terms by their second-order central difference approximations [35]. An iterative scheme is used to solve the quasi-linear system of difference equations. An initial guess is chosen and the iterations are continued until convergence within the prescribed accuracy. Finally, the resulting block tri-diagonal system is solved using the generalized Thomas algorithm. Energy Equation 14 is a linear second-order ordinary differential equation with variable coefficients which are known from the solution of the flow equations. This equation is solved numerically, using central differences for the derivatives and the Thomas algorithm for the solution of the set of discretized equations. Convergence is assumed when the ratio of every one of the quantities for the last two approximations differ from unity by less than 10^{-5} at all values of the independent variable. Aside from the energy equation, the same results for Navier-Stokes equations are obtained as [8-9].

PRESENTATION OF RESULTS

If the bounding spherical surfaces were stationary, there would be no fluid motion and the temperature distribution would simply be a conduction distribution. Any rotation of the bounding spheres sets up a primary flow (ω) around the axis of rotation. This relative motion induces an unbalanced centrifugal force field which drives the secondary flows (ψ) in the meridian plane. Thus, if the bounding spheres are of unequal temperatures, this secondary flow produces forced convection within the annulus, resulting in a temperature distribution which is different from the pure conduction distribution. The relative magnitudes of the secondary flow and forced convection effects depend on the parameters involved including those concerning the geometry of the flow and those concerning the dynamics of the flow such as $\Omega_{io} = \Omega_i/\Omega_o$, $b = R_i/R_o$, Prandtl number and Reynolds number. These secondary flows, known as vortex, have a clockwise or counterclockwise motion, depending on whether the outer sphere or the inner sphere is dominant, as far as the secondary flow is concerned. Results for temperature fields are presented when the outer sphere is hotter than the inner one. The cases considered here are constant angular velocities and presentations are only at selected Ω_{io}.

The velocity fields and temperature distribution for the particular case of constant angular velocity $\Omega_{io} = -3.0$ (negative sign indicates rotation in different directions) are presented in Figure 2 for Reynolds number Re = 50, Pr = 10 and Ek = 0. As expected, the ψ contours show that the annulus space is under the effect of both spheres. The vortex close to the inner sphere is dominating the flow field since it is rotating three times faster than the outer sphere. The same type of dominating effect is shown in Figure 2b for the Ω function. In terms of velocity vectors, Figure 2c is displaying the same effect. Figure 2d is presenting the temperature field and showing that there is a large delay in heat transfer because of the rotation of the inner sphere.

Figures 3 to 5 present the same situations but for the case $\Omega_{io} = -1$ and for Re = 100, Pr = 1 and Ek = 0; for the case $\Omega_{io} = -2$ and Re = 250, Pr = 10 and Ek = 0; and for the case $\Omega_{io} = -1$ and Re = 500, Pr = 1 and Ek = 0. The effect of Reynolds number can be seen obviously in these figures in comparison with each other and also compared with Figure 2. A detailed physical discussion regarding the flow field and heat transfer characteristics can be presented using these figures.

CONCLUSION

A similarity solution for the problem of the flow and

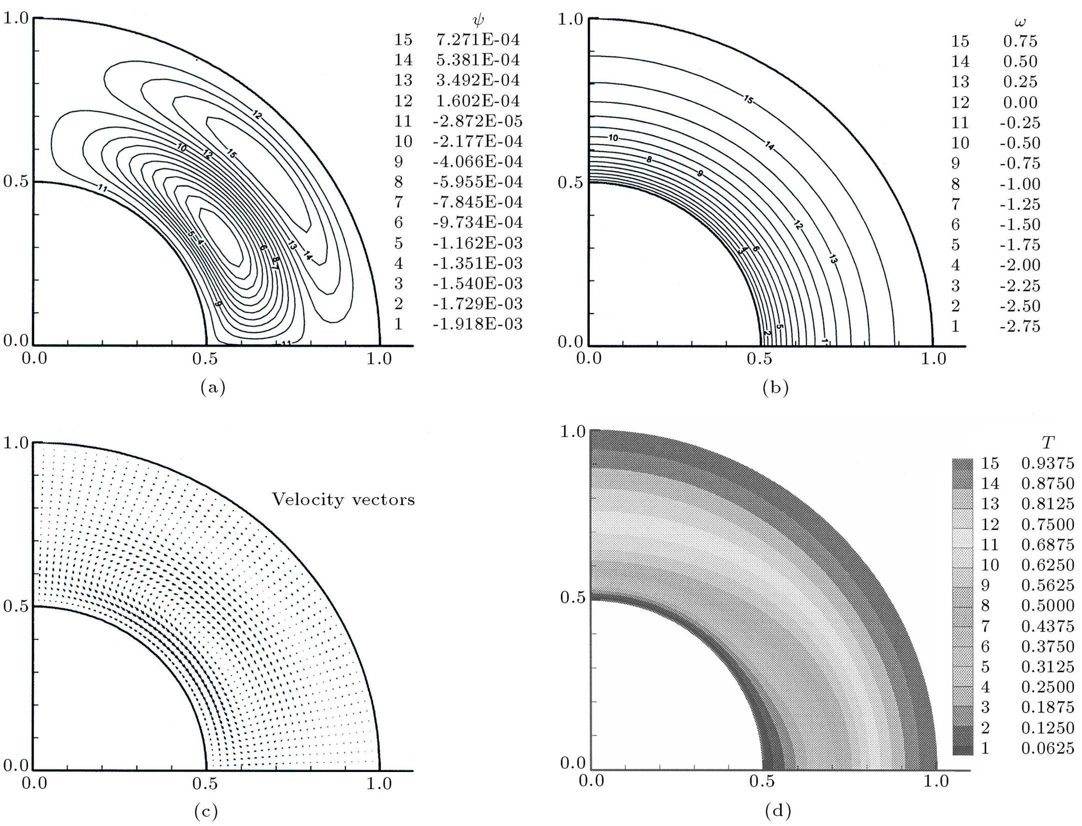

Figure 2. Flow and heat patterns for Re = 50, Pr = 10, Ek = 0 and $\Omega_{io} = -3$.

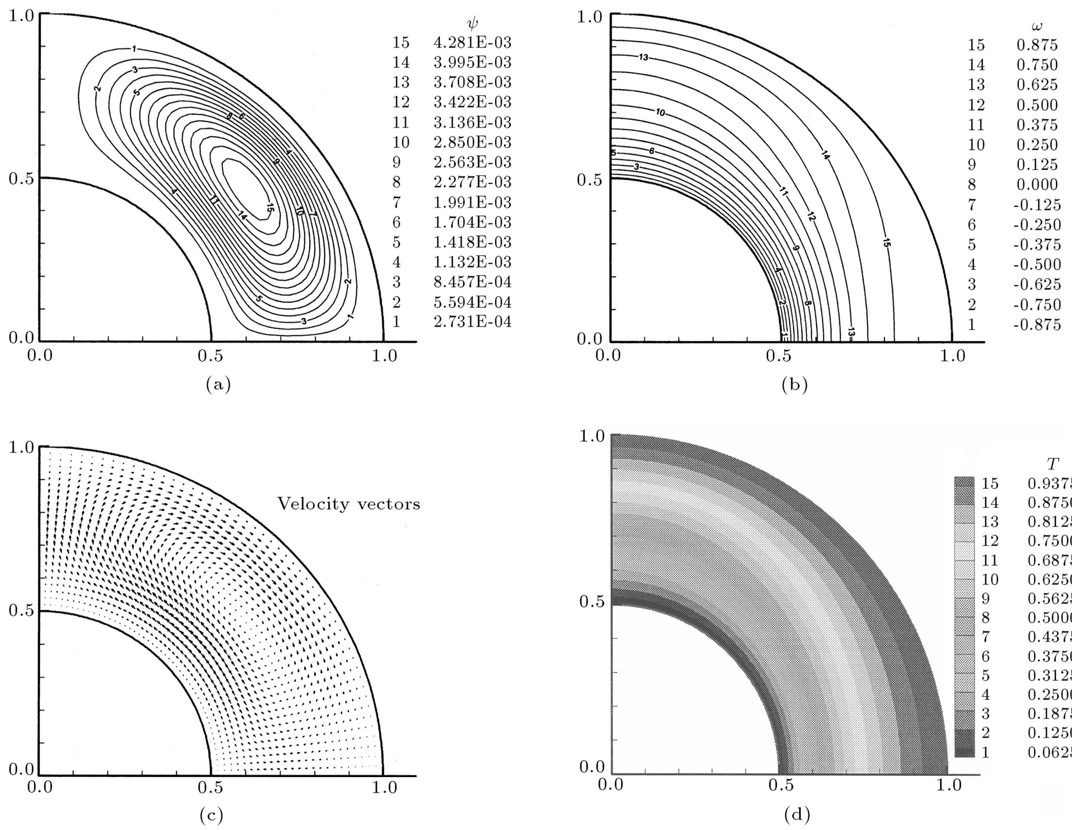

Figure 3. Flow and heat patterns for Re = 100, Pr = 1, Ek = 0 and $\Omega_{io} = -1$.

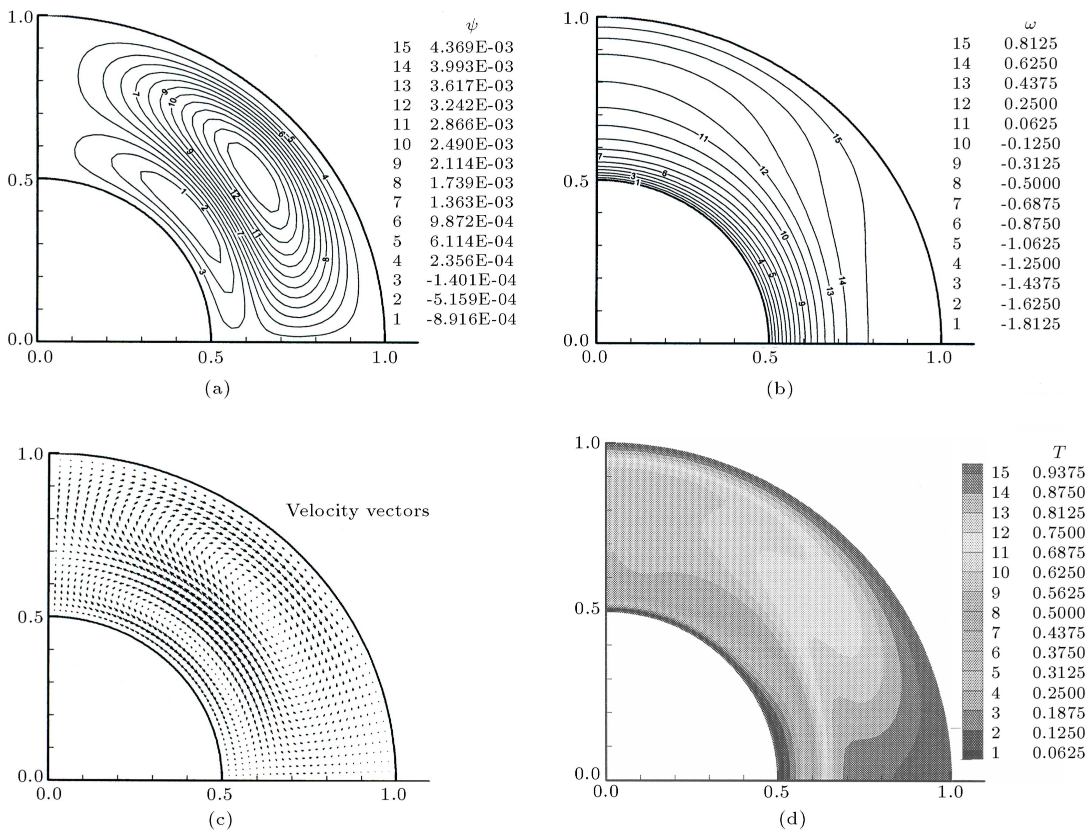

Figure 4. Flow and heat patterns for Re = 250, Pr = 10, Ek = 0 and $\Omega_{io} = -2$.

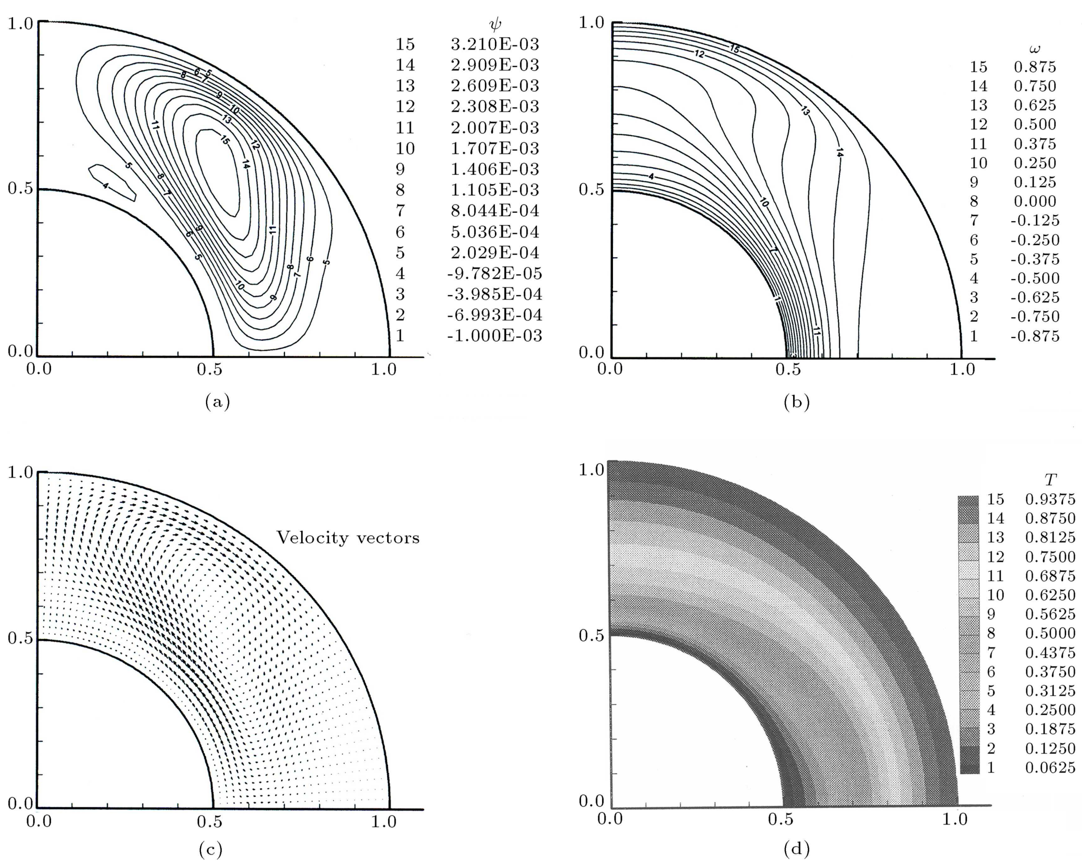

Figure 5. Flow and heat patterns for Re $= 500$, Pr $= 1$, Ek $= 0$ and $\Omega_{io} = -1$.

heat transfer of a viscous incompressible fluid within a rotating spherical annulus has been investigated when the spheres have constant angular velocities. The results, aside from the energy equation, are the same as in previous works [8-9] for Navier-Stokes equations, but with less computational complexities. Besides, the formulation of the problem this way enables one to find the functions $\psi(r,\theta)$ and $\Omega(r,\theta)$ at any desired point within the flow field independently, without having to solve for the whole region.

REFERENCES

1. Howarth, L. "Secondary flow about a sphere rotating in a viscous liquid", *Philosophy Magazine Series*, **7**(42), p. 1308 (1951).
2. Proudman, I. "The almost rigid rotation of viscous fluid between concentric spheres", *Journal of Fluid Mechanics*, **1**, p. 505 (1956).
3. Lord, R.G. and Bowden, F.P. "Using perturbations in dealing with the polar boundaries", *Proceeding of Royal Society A*, **271**, p. 143 (1963).
4. Fox, J. "Using singular perturbation in fluid around spheres", NASA TN D-2491 (1964).
5. Greenspan, H.P. "The theory of rotating fluids", *Journal of Fluid Mechanics*, **21**, p. 673 (1964).
6. Carrier, G.F. "Some effects of stratification and geometry in rotating fluids", *Journal of Fluid Mechanics*, **24**, p. 641 (1966).
7. Stewartson, K. "On almost rigid rotations", *Journal of Fluid Mechanics*, **26**(1), pp. 131-144 (1966).
8. Pearson, C. "A numerical study of the time-dependent viscous flow between two rotating spheres", *Journal of Fluid Mechanics*, **28**(2), pp. 323-336 (1967).
9. Munson, B.R. and Joseph, D.D. "Viscous incompressible flow between concentric rotating spheres. Part I: Basic flow", *Journal of Fluid Mechanics*, **49**, pp. 289-303 (1971).
10. Douglass, R.W., Munson, B.R. and Shaughnessy, E.J. "Thermal convection in rotating spherical annuli-1. Forced convection", *Int. J. Heat and Mass Transfer*, **21**, pp. 1543-1553 (1978).
11. Munson, B.R. and Douglass, R.W. "Viscous flow in oscillatory spherical annuli", *Journal of Physics of Fluids*, **22**(2), pp. 205-208 (Feb. 1979).
12. Gagliardi, J.C., Nigro, N.J., Elkouh, A.F. and Yang, J.K. "Study of the axially symmetric motion of an incompressible viscous fluid between two concentric rotating spheres", *Journal of Engineering Mathematics*, **24**, pp.1-23 (1990).
13. Jen-Kang Yang, Nigro, N.J. and Elkouh, A.F. "Numerical study of the axially symmetric motion of an

incompressible viscous fluid in an annulus between two concentric rotating spheres", *International Journal for Numerical Methods in Fluids*, **9**, pp. 689-712 (1989).

14. Ni, W. and Nigro, N.J. "Finite element analysis of the axially symmetric motion of an incompressible viscous fluid in a spherical annulus", *International Journal for Numerical Methods in Fluids*, **19**, pp. 207-236 (1994).

15. Magyari, E. "Analytical solutions for unsteady free convection in porous media", *Journal of Engineering Mathematics*, **48**(2), pp. 93-104 (February 2004).

16. Magyari, E., Pop, I. and B. Keller "New analytical solutions of a well-known boundary value problem in fluid mechanic", *Fluid Dynamics Research*, **33**(4), pp. 313-317 (October 2003).

17. Zaturska, M.B. and Banks, W.H.H. "New solutions for flow in a channel with porous walls and/or non-rigid walls", *Fluid Dynamics Research*, **33**(1-2), pp. 57-71 (July 2003).

18. Merkin, J.H. and Pop, I. "Mixed convection along a vertical surface: Similarity solutions for uniform flow", *Fluid Dynamics Research*, **30**(4), pp. 233-250 (April 2002).

19. Miyazaki, T. "Diffusion equation coupled to Burgers' equation", *Fluid Dynamics Research*, **2**(1), pp. 25-33 (August 1987).

20. Kambe, T. "A class of exact solutions of the Navier-Stokes equations", *Fluid Dynamics Research*, **1**(1), pp. 21-31 (August 1986).

21. Wang, C.Y. "Axisymmetric stagnation flow on a cylinder", *Q. Appl. Math.*, **32**, pp. 207-213 (1974).

22. Gorla, R.S.R. "Heat transfer in an axisymmetric stagnation flow on a cylinder", *Appl. Sci. Res.*, **32**, pp. 541-553 (1976).

23. Gorla, R.S.R. "Unsteady laminar axisymmetric stagnation flow over a circular cylinder", *Development in Mechanics*, **9**, pp. 286-288 (1977).

24. Gorla, R.S.R. "Nonsimilar axisymmetric stagnation flow on a moving cylinder", *International Journal of Science*, **16**, pp. 392-400 (1978).

25. Gorla, R.S.R. "Transient response behavior of an axisymmetric stagnation flow on a circular cylinder due to time-dependent free stream velocity", *Lett. Applied. Eng. Science*, **16**, pp. 493-502 (1978).

26. Gorla, R.S.R. "Unsteady viscous flow in the vicinity of an axisymmetric stagnation-point on a cylinder", *International Science*, **17**, pp. 87-93 (1979).

27. Wang, C.Y. "Shear flow over a rotating plate", *Applied Scientific Research*, **46**, pp. 89-96 (1989).

28. Cunning, G.M., Davis, A.M.J. and Weidman, P.D. "Radial stagnation flow on a rotating cylinder with uniform transpiration", *J. Eng. Math.*, **33**, pp. 113-128 (1998).

29. Jung, W.L., Mangiavacchi, N. and Akhavan, R. "Suppression of turbulence in wall-bounded flows by high-frequency spanwise oscillations", *Physics of Fluids*, **A4**, pp. 1605-1607 (1992).

30. Wang, C.Y. "Axisymmetric stagnation flow towards a moving plate", *American Institute of Chemical Engineering Journal*, **19**, p. 961 (1973).

31. Weidman, P.D. and Mahalingam, S. "Axisymmetric stagnation-point flow impinging on a transversely oscillating plate with suction", *Journal of Engineering Mathematics*, **31**, pp. 305-318 (1997).

32. Saleh, R. and Rahimi, A.B. "Axisymmetric stagnation-point flow and heat transfer of a viscous fluid on a moving cylinder with time-dependent axial velocity and uniform transpiration", *Journal of Fluids Engineering*, **126**, pp. 997-1005 (2004).

33. Rahimi, A.B. and Saleh, R. "Axisymmetric stagnation-point flow and heat transfer of a viscous fluid on a rotating cylinder with time-dependent angular velocity and uniform transpiration", *Scientia Iranica*, **12**(4), pp. 329-337 (2005).

34. Rahimi, A.B. and Saleh, R. "Similarity solution of unaxisymmetric heat transfer in stagnation-point flow on a cylinder with simultaneous axial and rotational movements", *Journal of Heat Transfer*, **130**, pp. 054502-1–054502-5 (December 2007).

35. Press, W.H., Flannery, B.P., Teukolsky, S.A. and Vetterling, W.T., *Numerical Recipes,The Art of Scientific Computing*, Cambridge University Press, Cambridge (1997).