

# A Simulation on the Propagation of Supernova Cosmic Particles in a Fractal Medium

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**Abstract.** *In this paper, we present a Mont Carlo simulation by using the inhomogeneous diffusion equation suggested by Lagutin et al. in a fractal interstellar medium for a supernova, as being the main source of Galactic cosmic rays in an energy range up to  $10^{16}$  eV. When the numerical results of this simulation are compared with the predicted energy density of cosmic rays from the supernova model of Erlikyn and Wolfendale (EW), they show a strong consistency with the observed experimental parameters of cosmic rays, i.e. Galactic radial gradient, percentage of total supernova energy transfer to cosmic particles etc., which is also confirmation of a supernova being of cosmic ray origin.*

**Keywords:** *Cosmic rays; Supernova; Particle propagation; Fractal medium;*

## INTRODUCTION

This simulation not only confirms that the interstellar medium has a fractal structure [1,2] with  $\alpha < 2$  [3], but also improves the defect of supernova models, especially the way of propagation of cosmic particles produced by a supernova.

In this work, it is assumed that all cosmic rays in the mentioned energy range are of supernova origin and the spatial distribution of a supernova is given by the following formula:

$$F(R, Z) = \left(\frac{R}{R_0}\right)^a \exp\left(-b\left(\frac{R}{R_0} - 1\right) - \frac{Z}{H_z}\right).$$

Also, the number of cosmic particles produced by supernova impulses in the galaxy is given by:

$$N(r, E) = \frac{2^{-\alpha} S_0 \Gamma\left(\frac{3-\alpha}{2}\right)}{\pi^{\frac{3}{2}} D_0 r^{3-\alpha} \Gamma\left(\frac{\alpha}{2}\right)} E^{-p-\delta}. \quad (1)$$

## METHOD OF SIMULATION

The number of produced supernova in this region for the effective time of  $T$  years, considering that a supernova explosion happens once per 100 years, can

be calculated from:

$$N = 0.11 \times \int_{4500}^{12500} (R/8500)^a \exp(-b((R/8500)-1)) \\ \times 2\pi R dR / \int_1^{15000} (R/8500)^a \exp(-b((R/8500)-1)) \\ \times 2\pi R dR \times 10^{-2} \times T.$$

In this section, the simulated galactic space is considered to be 4.5 to 12.5 Kpc from the galactic center.

'0.11' is considered as the value of the galactic space correction coefficient.

The next step is to calculate the constant coefficient, named  $A$ , by normalizing the number of supernova to one (actually, by using this method, we want to determine the role of one supernova in different parts of the galaxy):

$$A \times \int_1^{1000} e^{-Z/200} dZ \int_1^{15000} (R/8500)^a \\ \exp(-b((R/8500)-1)) 2\pi R dR = 1.$$

It should be noted that this formula, as mentioned before, is the radial distribution of a supernova in the galactic disk with cylindrical symmetry.

The limits of the integral on variable  $Z$  are from 1 to 1000 pc, because the number of supernovae decreases

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with altitude exponentially. So, there is no efficient supernova higher than 1000 pc. Substituting  $a = 1.69 \pm 0.22$  and  $b = 3.33 \pm 0.37$  [4,5] and solving the integral, the value of constant 'A' is calculated.

Then, the value of  $k\alpha$  is obtained by normalization to the total particle energy of one supernova as follows:

$$A \int_1^{1000} A^{-Z/200} dZ \int_1^{4000} (0.39 \times (R + 8500/8500))^a \exp(-b((R + 8500/8500) - 1) + 0.61 \times (8500 - R/8500))^a \times \exp(-b((8500 - R) - 1) \times N(r, \alpha) k_\alpha \pi R dR = 1.8 \times 10^{48}.$$

It should be mentioned that the first part in this formula has the constant coefficient 0.39, which is related to  $F(R + 8500)$ , and the second part is multiplied to 0.61, which is referred to as  $F(8500 - R)$ . Actually, these two constants show the different radial distribution of a supernova in two assumed semicircles.

On the other hand, the right hand side of the formula shows 10 percent of the total transferred energy to the particles from a supernova explosion.

In this formula,  $N(r, \alpha)$  is the number of particles at distance 'r' from the source, which is the solution of the steady state case of a diffusion equation:

$$D(-\Delta)^{\frac{\alpha}{2}} N(r, E) = S(r, E).$$

Green's function,  $G(r, E; E_0)$ , satisfies the equation [3]:

$$D(-\Delta)^{\frac{\alpha}{2}} G(r, E; E_0) = \delta(E - E_0)\delta(r).$$

Here,  $(-\Delta)^{\frac{\alpha}{2}}$  is fractional Laplacian called "Riss operator" [6]. The solution of Green's function equation can be found by means of a Fourier transformation which is as follows:

$$F^\Lambda(-\Delta)^{\frac{\alpha}{2}} G(r, E; E_0) = |k|^\alpha G^\sim(k, E; E_0).$$

Applying the inverse Fourier transformation, we will obtain [3]:

$$G(r, E; E_0) = \delta(E - E_0)/(2\pi)^3 \int dy \int dk \exp(-ikr - D|y|^2 y).$$

Finally, using Green's function, we can find the solution of a steady state case of a diffusion equation or Equation 1.

After calculating  $k_\alpha$ , the energy density of cosmic rays can be obtained from:

$$I = Ak_\alpha e^{-Z/200} N(r, \alpha) [0.39 \times F(8500 + r) + 0.61 \times F(8500 - r)] \times 2.13 \times 10^{-44}.$$

It should be noted that in our simulation energy density  $I$  is a function of  $Z, r, \alpha, T$  and a percentage of the supernova transfer energy to cosmic rays.

Then, we produce two random variables,  $Z$  and  $r$  (the former refers to the simulated distance of a supernova from the galactic surface and the latter is the galactic radius), which are given by:

$$r = 4000\zeta_i,$$

$$Z = 1000\zeta_i,$$

where  $\zeta_i$  is a random number between zero and one.

Testing different values of parameters such as  $\alpha$ , in a range of 0.5 to 2,  $T$ , from  $10^4$  to  $10^7$  years, the percentage of transfer energy to particles, from 1% to 10%, and by using these parameters in a computer program, we will get the energy density values of cosmic rays.

To compare the simulated energy density of cosmic rays with experimental results, we use one of the suitable models of Erykin and Wolfendale that gives the energy spectrum of cosmic rays.

Now, for calculating energy density, we divide the EW spectrum (Figure 1) [7], into four parts (AB, BC, CD, DE). Then, by means of formula  $\varepsilon = 4\pi/c \int IE dE$ , the energy density of each part is obtained. To find every part's energy density, the equation of each line is obtained. Then, by considering the logarithmic form of each line and using the energy density formula, we have:

$$\text{For AB} \rightarrow y = x - 3.9$$

$$\Rightarrow \log I \varepsilon^3 = \log \varepsilon - 3.9 \Rightarrow I \varepsilon = 10^{-3.9} \varepsilon^{-1}.$$

The limits of the following integral are from  $10^{4.4}$  to

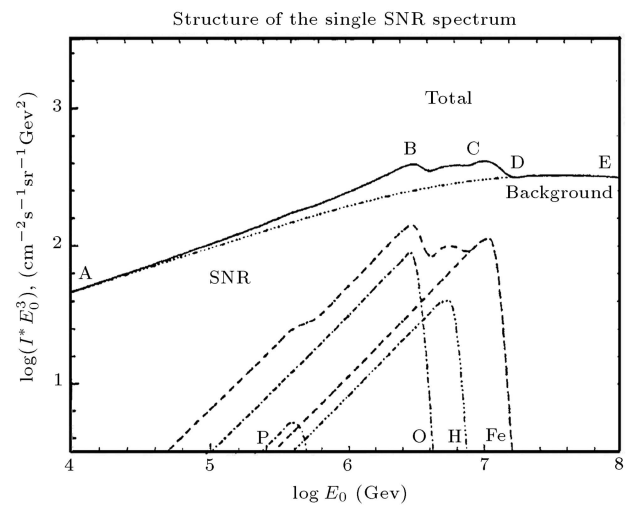


Figure 1. The structure of an EW model supernova cosmic rays spectrum [7].

$10^{6.45}$  :

$$\varepsilon = 4\pi/c \int 10^{-3.9} E^{-1} dE = 0.86 \times 10^{-4}$$

$$\Rightarrow \varepsilon_{AB} = 0.86 \times 10^{-4}(\text{ev/cm}^{-3}).$$

Using this method for the rest of the parts, we get:

For BC  $\rightarrow y = -1.7 x + 13.52$

$$\Rightarrow \varepsilon_{BC} = 0.81 \times 10^{-4}(\text{ev/cm}^{-3}),$$

For CD  $\rightarrow a \approx 0, y = 2.3$

$$\Rightarrow \varepsilon_{CD} = 0.14 \times 10^{-4}(\text{ev/cm}^{-3}),$$

For DE  $\rightarrow y = -12.86 x + 93 \Rightarrow \varepsilon_{DE} \approx 0$

$$\Rightarrow \varepsilon_{EW} = \varepsilon_{AB} + \varepsilon_{BC} + \varepsilon_{CD} + \varepsilon_{DE}$$

$$= 1.81 \times 10^{-4}(\text{ev/cm}^{-3}).$$

Therefore, the total energy density of the EW model ( $\varepsilon_{EW} = 1.8 \times 10^{-4} \text{ ev/cm}^{-3}$ ) is obtained.

For comparing the simulation results with the EW model, all the supernova parameters and also the average, median and maximum of the simulated distribution of the energy density of cosmic rays are shown in Table 1. The row of the table which is most consistent with the energy density of the EW model ( $\log \varepsilon = \log(1.8 \times 10^{-4}) = -3.74$ ) is of our concern. At this row, we get  $\alpha = 1.8$ , which is our expected result, implying a fractal structure for ISM [8].

An example of a histogram that is not matched with our expected result is shown in Figure 2.

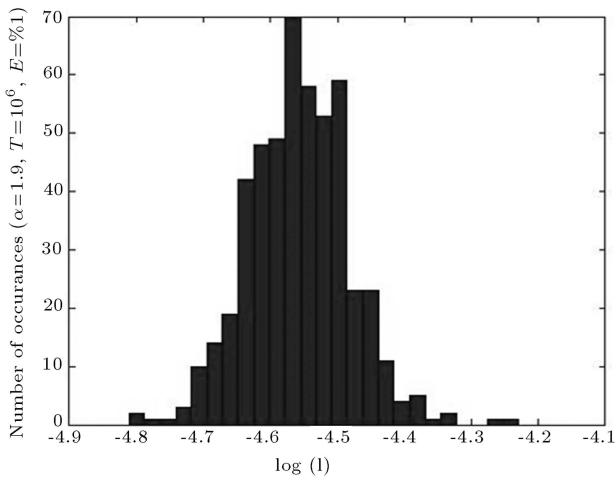
Figure 3 shows the most probable energy density with specific parameters, which are  $\alpha = 1.8, T = 7 \times 10^4$  years and 10% percent for the transmitted energy to cosmic rays.

### CONCLUSION

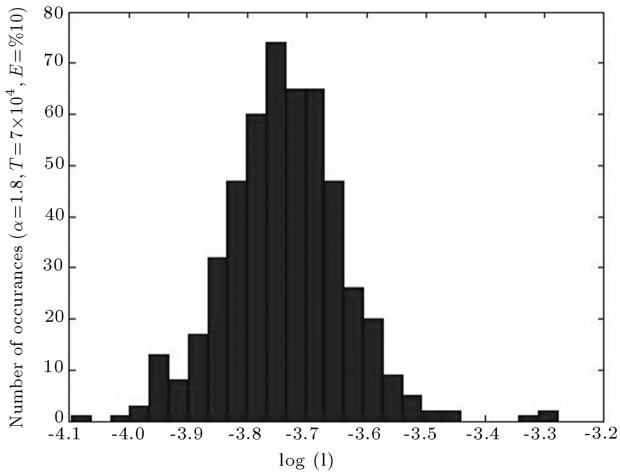
From the results of this simulation, the suitable interstellar medium for the propagation of cosmic rays that are produced by a supernova is fractal and non-homogeneous and, in this work, the important characteristics of cosmic rays in the galaxy such as the

**Table 1.** Supernova parameters.

Effective Time	Energy	$\alpha$	Mean	Median	Maximum	Error
$10^4$	$1.8 \times 10^{48} = \%10$	1.8	-4.613	-4.617	-4.6	0.234
$5 \times 10^4$	%10	1.7	-3.921	-3.926	(-3.8,-4)	0.118
$5 \times 10^4$	%10	1.8	-3.88	-3.89	(-3.9,-4)	0.109
$5 \times 10^4$	%10	1.9	-3.85	-3.86	-3.9	0.104
$5 \times 10^4$	%1	1.7	-4.924	-4.926	(-4.9,-5)	0.109
$5 \times 10^4$	%1	1.8	-4.88	-4.89	(-4.8,-5)	0.109
$6 \times 10^4$	%10	1.8	-3.810	-3.813	-3.8	0.100
$7 \times 10^4$	%10	1.6	-3.81	-3.82	-3.8	0.109
$7 \times 10^4$	%10	1.7	-3.775	-3.779	(-3.8,-3.7)	0.094
$7 \times 10^4$	%10	1.8	-3.739	-3.740	(-3.8,-3.7)	0.089
$7 \times 10^4$	%10	1.9	-3.708	-3.708	-3.7	0.089
$7 \times 10^4$	%1	1.8	-4.745	-4.749	(-4.7,-4.8)	0.089
$8 \times 10^4$	%10	1.6	-3.755	-3.757	(-3.6,-3.8)	0.104
$8 \times 10^4$	%10	1.7	-3.71	-3.72	(-3.7,-3.8)	0.089
$8 \times 10^4$	%10	1.8	-3.686	-3.688	(-3.6, -3.7)	0.083
$8 \times 10^4$	%10	1.9	-3.652	-3.653	(-3.6, -3.7)	0.083
$10^5$	%10	1.6	-3.657	-3.66	(-3.6, -3.7)	0.083
$10^5$	%10	1.7	-3.615	-3.618	(-3.6, -3.7)	0.083
$10^5$	%10	1.8	-3.58	-3.58	(-3.6, -3.7)	0.082
$10^5$	%10	1.9	-3.556	-3.558	(-3.5,-3.6)	0.070
$10^5$	%1	1.7	-4.615	-4.619	(-4.4,-4.6)	0.089
$10^5$	%1	1.8	-4.58	-4.59	(-4.6,-4.7)	0.077
$10^5$	%1	1.9	-4.55	-4.55	(-4.5,-4.6)	0.070



**Figure 2.** Example of a histogram that is not matched with our expected result.



**Figure 3.** Distribution of the energy density of supernova cosmic rays which happens statistically when 10% of the supernova energy goes to cosmic rays.

percentage of supernova transfer energy to cosmic rays, the effective time of SN in producing cosmic rays and also the proper energy density of cosmic particles are confirmed.

As a result, the strong consistency of this simulation in this paper with the observed parameter of cosmic rays confirms a diffusion of cosmic particles of supernova origin in the fractal medium and rejects their diffusion in a homogenous medium with Gaussian distribution ( $\alpha = 2$ ).

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