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Designing an Optimum Acceptance Sampling Plan Using Bayesian Inferences and a Stochastic Dynamic Programming Approach

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Abstract. *In this paper, we use both stochastic dynamic programming and Bayesian inference concepts to design an optimum-acceptance-sampling-plan policy in quality control environments. To determine the optimum policy, we employ a combination of costs and risk functions in the objective function. Unlike previous studies, accepting or rejecting a batch are directly included in the action space of the proposed dynamic programming model. Using the posterior probability of the batch being in state p (the probability of non-conforming products), first, we formulate the problem into a stochastic dynamic programming model. Then, we derive some properties for the optimal value of the objective function, which enable us to search for the optimal policy that minimizes the ratio of the total discounted system cost to the discounted system correct choice probability.*

Keywords: *Quality inspection; Acceptance sampling plan; Bayesian inference; Stochastic dynamic programming.*

INTRODUCTION AND LITERATURE REVIEW

There are various methods of inspection in quality control for improving the quality of products [1]. Acceptance sampling plans are practical tools for quality assurance applications involving quality contracts on product orders. The sampling plans provide the vendor and buyer with decision rules for product acceptance, in order to meet the present product quality requirement. With the rapid advancement of manufacturing technology, suppliers require their products to be of high quality with very low fraction non-conformities, often measured in parts per million. Unfortunately, traditional methods of calculating fraction non-conformities no longer work, since some samples of reasonable size probably contain no non-conforming product items [2].

Pearn and Wub [2] introduced an effective sampling plan based on a process capability index, C_{pk}, to deal with product acceptance determination for low fraction non-conforming products. The proposed new sampling plan was developed based on an exact sampling distribution rather than approximation. Practitioners can use this proposed method to determine the number of required inspection units and the critical acceptance value, and make reliable decisions in product acceptance.

Klassen [3] proposed a credit-based acceptance sampling system. The credit of the producer was defined as the total number of items accepted since the last rejection. In this system, the sample size of a lot depends on a simple function of lot size and credit, and results in a guaranteed upper limit on outgoing quality that is much smaller than for an isolated lot inspection.

On the one hand, the classical model of acceptance sampling by variables for individual lots, is not suitable [4-6]. On the other hand, it may be considered as an approximation for large lots. Investigating the approximation error for small lots, Seidel [4], by use of risk functions, showed that models with unknown variances perform better than those with known variances. For very small lots, it is often better to use a test based on an unknown variance, even if the variance is known. Because, by estimating the variance, under certain conditions, one obtains a test statistic that

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follows more closely the fraction nonconformities in the lot.

Chun and Rinks [7] assumed that the fraction non-conforming product is a random variable that follows a Beta distribution. They derived not only modified producer and consumer risks, but, also, the Bayes producer and consumer risks.

Tagaras [8] studied the joint process control and machine maintenance problem of a Markovian deteriorating machine. Assuming that sampling and preventive maintenance were performed at fixed intervals, he searched the best $\overline{X}$ control chart limits, preventive maintenance intervals and sampling intervals, in order to numerically minimize the average related maintenance and quality control costs.

Kuo [9] developed an optimal adaptive control policy for joint machine maintenance and product quality control. He included interactions between machine maintenance and product sampling in a search for the best machine maintenance and quality control strategy for a Markovian deteriorating, state unobservable, batch production system. However, unlike previous studies in this area, he did not impose mandatory fixed sample sizes and fixed sampling intervals on the system. Instead, he let the dynamic programming mechanism dictate the best sample size and sampling epoch based on the current state of the system. He derived several properties of the optimal value function, which helped to find the optimal value function and identify the optimal policy more efficiently in the value iteration algorithm of the dynamic programming.

In this paper, in order to reject or accept a batch in a quality inspection problem, we propose an adaptive optimal policy. This policy is derived based upon stochastic dynamic programming and Bayesian estimation approaches that develop an optimal framework for the decision-making process at hand.

The rest of the paper is organized as follows. First, the model is presented. Then, its application by a numerical example is demeonstrated. At the end, the paper is concluded.

THE MODEL

Stochastic dynamic programming is one of the most powerful techniques used to model the stochastic behavior of decision-making processes [10]. In acceptance sampling plans, in cases where we are to decide between accepting and rejecting a batch, we are in a stochastic state and can never surely say that a batch is acceptable or should be rejected. Since the stochastic state of the process may be dynamic, we may use the concept of stochastic dynamic programming to model an acceptance-sampling plan. Before doing so, we first need to have some notations and definitions.

Notations, Assumptions and Definitions

We will use the following notations and definitions in the rest of the paper.

'p' is defined as the fraction non-conformation of the product. In situations in which little is known, priori, relative to what the data has to tell us about p, we employ the likelihood function. Furthermore, it is possible to express the unknown parameter, p, in terms of a metric $f(p)$, so that the corresponding likelihood is data translated. This means that the likelihood curve for $f(p)$ is completely determined, a priori, except for its location, which depends on the data yet to be observed. Then, in order to express that we know little a priori relative to what the data is going to tell us, it may be said that we are almost equally willing to accept one value of $f(p)$ as another. This state of indifference may be expressed by taking $f(p)$ as locally uniform, and the resulting priori distribution is called non-informative for $f(p)$, with respect to the data.

According to Jeffreye's rule [11], assuming $\phi(p) = -\underset{x|p}{E}\left[\frac{\partial^2 \log p(x|p)}{\partial p^2}\right]$ for the observed data, x, the informative for p should be chosen, so that, locally, $f(p) = \phi^{0.5}(p)$. Specifically, for the case of Beta distribution, the information measure is $\phi(p) \approx p^{-1}(1-p)^{-1}$. Hence, $f(p) \approx p^{-0.5}(1-p)^{-0.5}$ and we can conclude that $f(p)$, as a non-informative prior distribution, is Beta (0.5, 0.5) [12].

Then, using Bayesian inference, we can easily show that the posterior probability density function of p is:

$$f(p) = \frac{\Gamma(\alpha+\beta+1)}{\Gamma(\alpha+0.5)\Gamma(\beta+0.5)} p^{\alpha-0.5}(1-p)^{\beta-0.5}, \quad (1)$$

where α is the number of non-conforming products and β is the number of conforming products in all of the past stages of the decision-making process.

m:	the total number of products in a batch,
k_i:	the sample size at stage i,
λ:	the discount factor,
R:	defined as the cost of rejecting a batch,
C:	the cost of having one non-conforming product shipped to the customer,
$V_n(p)$:	the cost associated with p when there are n remaining stages to make the decision,
$W_n(p)$:	defined as the probability of correct decision associated with p, when there are n remaining stages to make the decision,
d_n:	the upper threshold for p. If the probability of having a non-conforming product is more than d_n, then, we reject the batch,

d'_n: defined as the lower threshold for p. If the probability of having a non-conforming product is less than d'_n, we accept the batch,
δ_1: the minimum acceptable level of the batch quality (Accepted Quality Level (AQL)),
δ_2: defined as the minimum rejectable level of the batch quality (Lot Tolerance Proportion Defective (LTPD)),
CS: the event of making the correct decision,
ε_1: the probability of type-one error in making a decision,
ε_2: the probability of type-two error in making a decision.

Derivations

Consider a process that is observed at the time points of possible states that are countable and are labeled as nonnegative integers $0, 1, 2, \cdots$. After observing the state of the process, an action must be taken. We let A (assumed finite) represent the set of possible actions.

If the process is in state i at stage n of the decision-making process and action a is chosen, then, independent of the past, two things occur:

(i) We receive an expected reward;

(ii) The next state of the system, (j), is chosen, according to the transition probabilities.

Some authors have developed sequential analysis inference, in combination with an optimal stopping problem, to determine the probability of making correct decisions. One such research is a new approach in the probability distribution fitting of given statistical data that Eshragh and Modarres [13] named the Decision On Belief (DOB). In this decision-making method, a sequential analysis approach is employed to find the best underlying probability distribution of the observed data. Moreover, Eshragh and Niaki [14] applied the DOB concept as a decision-making tool in the response surface methodology. In this article, we use the concept of DOB to model the problem.

We may model an acceptance sampling process as an optimal stopping problem, in which, at each stage of the decision-making process, we take a sample from a batch and, based on the information obtained from the sample, we decide whether to accept or reject the batch or to continue to take more samples.

We mentioned that the probability distribution of the fraction non-conformities could be modeled by Bayesian inference as a Beta distribution, with parameters $\alpha + 0.5$, $\beta + 0.5$. Hence, $P(p \geq d_n)$ shows the probability of rejecting a batch and $P(p \leq d'_n)$ shows the probability of accepting a batch. Then, assuming $d_n \geq d'_n$ (to ensure that the probability is not negative), by use of the total probability theorem, $[1 - P(p \geq d_n) - P(p \leq d'_n)]$ shows the probability of neither accepting nor rejecting a batch and, hence, continuing to take more samples.

If we define n to be the index of the decision-making stage and p to be the state variable, then, $RP(p \geq d_n)$ shows the cost when we reject the batch, $CmpP(p \leq d'_n)$ represents the cost when we accept the batch and $\lambda V_{n-1}(p)$ shows the cost when we continue to the next stage. It is obvious that we need the discount factor, λ, to evaluate the cost of the next stage at the current stage (according to the approach of stochastic dynamic programming). Hence, we can define the stochastic dynamic equation of the cost as follows:

$$\begin{aligned} E(\text{cost}) = {} & E(\text{cost}|\text{Reject})P(\text{Reject}) \\ & + E(\text{cost}|\text{Accept})P(\text{Accept}) \\ & + E(\text{cost}|\text{Neither accepting nor rejecting}) \\ & \quad P(\text{Neither accepting nor rejecting}). \end{aligned} \tag{2}$$

Then, the cost associated with p, when there are n remaining stages to make the decision, is:

$$\begin{aligned} V_n(p) = {} & RP(p \geq d_n) + CmpP(p \leq d'_n) \\ & + (1 - P(p \geq d_n) - P(p \leq d'_n))\lambda V_{n-1}(p), \end{aligned} \tag{3}$$

and we need to minimize it over d_n and d'_n. In other words, the stochastic dynamic equation of the cost is $\min_{d_n, d'_n} \{V_n(p)\}$. However;

$$\begin{aligned} P(CS) = {} & P(CS|\text{Reject})P(\text{Reject}) \\ & + P(CS|\text{Accept})P(\text{Accept}) \\ & + P(CS|\text{Neither accepting nor rejecting}) \\ & \quad P(\text{Neither accepting nor rejecting}), \end{aligned} \tag{4}$$

where:

$$P(CS|\text{Reject}) = \int_{\delta_2}^{1} f(p)dp,$$

$$P(CS|\text{Accept}) = \int_{0}^{\delta_1} f(p)dp.$$

Then, the probability of a correct decision associated with p, when there are n remaining stages to make the decision, is:

$$\begin{aligned} W_n(p) = {} & P(p \geq d_n)\int_{\delta_2}^{1} f(p)dp + P(p \leq d'_n)\int_{0}^{\delta_1} f(p)dp \\ & + (1 - P(p \geq d_n) - P(p \leq d'_n))\lambda W_{n-1}(p), \end{aligned} \tag{5}$$

and we need to maximize it over d_n and d'_n. In other words, we can define the stochastic dynamic equation of making the correct decision as $\max_{d_n, d'_n}\{W_n(p)\}$.

Since we are to minimize the objective function given in Equation 3 and maximize the objective function of Equation 5 simultaneously, based on the ratio of the cost to the probability of a correct decision criterion, we combine these two equations in one, as follows:

$$H_n(p) = \frac{V_n(p)}{W_n(p)}, \tag{6}$$

and it is obvious that this function should be minimized.

Theorem

The optimal values of d_n and d'_n in Equation 6 are in the boundary limits of d_n and d'_n.

Proof

We take the first derivatives of $H_n(p)$ in Equation 6, with respect to d_n and d'_n and set them both equal to zero. That is,

(a):

$$\frac{\partial H_n(p)}{\partial d_n} = 0 \Rightarrow$$

$$\frac{W_n(p)(-f(d_n))(R-\lambda V_{n-1}(p))}{(W_n(p))^2} - \frac{V_n(p)(-f(d_n))\left(\int_{\delta_2}^1 f(p)dp - \lambda W_{n-1}(p)\right)}{(W_n(p))^2} = 0,$$

(b):

$$\frac{\partial H_n(p)}{\partial d'_n} = 0 \Rightarrow$$

$$\frac{W_n(p)(f(d'_n))\left(m\frac{\alpha+0.5}{\alpha+\beta+1}C - \lambda V_{n-1}(p)\right)}{W_n(p))^2} - \frac{V_n(p)(f(d'_n))\left(\int_0^{\delta_1} f(p)dp - \lambda W_{n-1}(p)\right)}{(W_n(p))^2} = 0.$$

In other words,

$$\text{(a)} \qquad \frac{W_n(p)}{V_n(p)} = \frac{\left(\int_{\delta_2}^1 f(p)dp - \lambda W_{n-1}(p)\right)}{(R-\lambda V_{n-1}(p))}, \tag{7}$$

$$\text{(b)} \qquad \frac{W_n(p)}{V_n(p)} = \frac{\left(\int_0^{\delta_1} f(p)dp - \lambda W_{n-1}(p)\right)}{\left(m\frac{\alpha+0.5}{\alpha+\beta+1}C - \lambda V_{n-1}(p)\right)}. \tag{8}$$

As Equations 7 and 8 share a unique left hand side, their right hand sides must be equal. However, we notice that, in general, they cannot be equal, concluding that, at most, one of the derivatives in either (a) or (b) can be equal to zero. Suppose the derivative in (a) is equal to zero. In this case, the derivative in (b) is not equal to zero and we conclude that the optimal values of d'_n are at its boundary limits. However, if we expand Equation 7, we will have:

$$P(p \le d'_n) =$$

$$\begin{bmatrix} \lambda W_{n-1}(p)(R-\lambda V_{n-1}(p)) \\ -\lambda V_{n-1}(p)\left(\int_{\delta_2}^1 f(p)dp - \lambda W_{n-1}(p)\right) \end{bmatrix}$$

$$\begin{bmatrix} (Cmp-\lambda V_{n-1}(p))\left(\int_{\delta_2}^1 f(p)dp - \lambda W_{n-1}(p)\right) \\ -\left(\int_0^{\delta_1} f(p)dp - \lambda W_{n-1}(p)\right)(R-\lambda V_{n-1}(p)) \end{bmatrix}^{-1}.$$

This is a contradiction, because we showed that the optimal value of d'_n is at its boundary limits. Therefore, we conclude that none of the derivatives in Equations 7 and 8 is equal to zero and, hence, the optimal values of d_n and d'_n are at their boundary limits. For situations in which the derivative in (b) is equal to zero, the reasoning is similar.

In order to determine the boundary limits of d_n and d'_n, we use the concepts of first and second type errors. First type error shows the probability of rejecting the batch when the fraction non-conforming of the batch is acceptable and second type error is the probability of accepting the batch when the fraction non-conforming of the batch is not acceptable. Then, on the one hand, if $p \le \delta_1$, the probability of rejecting the batch will be smaller than ε_1 and, on the other hand, in cases where $p \ge \delta_2$, the probability of accepting the batch will be smaller than ε_2. Hence, in cases where the expected value of p is δ_1, we will have:

$$\begin{cases} E(p) = \frac{\alpha}{\alpha+\beta} = \delta_1 \\ \alpha+\beta = \sum_{i=1}^n k_i \end{cases} \Rightarrow \begin{cases} \alpha = \delta_1 \sum_{i=1}^n k_i \\ \beta = (1-\delta_1)\sum_{i=1}^n k_i \end{cases}$$

and the probability of rejecting the batch is obtained by:

$$P(p \ge d_n) = \int_{d_n}^1 \frac{\Gamma\left(\sum_{i=1}^n k_i\right)}{\Gamma\left(\delta_1 \sum_{i=1}^n k_i\right)\Gamma\left((1-\delta_1)\sum_{i=1}^n k_i\right)} \times p^{\delta_1\left(\sum_{i=1}^n k_i\right)-1}(1-p)^{(1-\delta_1)\left(\sum_{i=1}^n k_i\right)-1} dp \le \varepsilon_1.$$

In other words, $F_p(1) - F_p(d_n) \leq \varepsilon_1$, where $F_p(d_n)$ is the cumulative probability distribution function of p, evaluated at d_n.

Knowing that $F_p(1) = 1$ implies $F_p(d_n) \geq 1 - \varepsilon_1$, if we define t_1 to be the boundary limit of d_n, since $F_p(d_n)$ is an increasing function, we have:

$$d_n \geq F_p^{-1}(1 - \varepsilon_1) = t_1. \tag{9}$$

In a similar way, when $E(p) = \delta_2$, we have:

$$\begin{cases} E(p) = \frac{\alpha}{\alpha+\beta} = \delta_2 \\ \\ \alpha + \beta = \sum\limits_{i=1}^{n} k_i \end{cases} \Rightarrow \begin{cases} \alpha = \delta_2 \sum\limits_{i=1}^{n} k_i \\ \\ \beta = (1 - \delta_2) \sum\limits_{i=1}^{n} k_i \end{cases}$$

and the probability of rejecting the batch is obtained by:

$$P(p \leq d'_n) = \int_0^{d'_n} \frac{\Gamma\left(\sum\limits_{i=1}^{n} k_i\right)}{\Gamma\left(\delta_2 \sum\limits_{i=1}^{n} k_i\right) \Gamma\left((1-\delta_2) \sum\limits_{i=1}^{n} k_i\right)} \times p^{\delta_2\left(\sum\limits_{i=1}^{n} k_i\right)-1} (1-p)^{(1-\delta_2)\left(\sum\limits_{i=1}^{n} k_i\right)-1} dp \leq \varepsilon_2.$$

In other words, $F_p(d'_n) - F_p(0) \leq \varepsilon_2$.

In this case, $F_p(0) = 0$ implies that $F_p(d'_n) \leq \varepsilon_2$. Accordingly, if we define t_2 to be the boundary limit of d'_n, we have:

$$d'_n \leq F_p^{-1}(\varepsilon_2) = t_2. \tag{10}$$

Now, since the optimal values of d_n and d'_n are at the boundary limits, in order to make the optimum decision, we can consider the decision tree given in Equation 11 to make a decision. In this equation, $\frac{\alpha+0.5}{\alpha+\beta+1}$ is the mean of the Beta distribution for p, given in Equation 1.

1. $\dfrac{\partial H_n(p)}{\partial d_n} \leq 0, \qquad \dfrac{\partial H_n(p)}{\partial d'_n} \leq 0 \Rightarrow d_n = 1,$

$$d'_n = t_2 \Rightarrow \begin{cases} \text{if } \frac{\alpha+0.5}{\alpha+\beta+1} \leq t_2 \Rightarrow \text{ accept the batch} \\ \text{else continue to the next stage} \end{cases}$$

2. $\dfrac{\partial H_n(p)}{\partial d_n} \leq 0, \qquad \dfrac{\partial H_n(p)}{\partial d'_n} \geq 0 \Rightarrow d_n = 1,$

$$d'_n = 0 \Rightarrow \text{ continue to the next stage.}$$

3. $\dfrac{\partial H_n(p)}{\partial d_n} \geq 0, \qquad \dfrac{\partial H_n(p)}{\partial d'_n} \leq 0 \Rightarrow d_n = t_1,$

$$d'_n = t_2 \Rightarrow$$

$$\begin{cases} \text{if } t_1 \leq \frac{\alpha+0.5}{\alpha+\beta+1} \leq t_2 \rightarrow \text{ impossible, because } d_n \geq d'_n \\ \\ \text{if } t_2 \leq \frac{\alpha+0.5}{\alpha+\beta+1} \leq t_1 \rightarrow \text{ continue to the next stage} \\ \\ \text{if } t_1, t_2 \leq \frac{\alpha+0.5}{\alpha+\beta+1} \rightarrow \text{ reject the batch} \\ \\ \text{if } \frac{\alpha+0.5}{\alpha+\beta+1} \leq t_2, t_1 \rightarrow \text{ accept the batch} \end{cases}$$

4. $\dfrac{\partial H_n(p)}{\partial d_n} \geq 0, \qquad \dfrac{\partial H_n(p)}{\partial d'_n} \geq 0 \Rightarrow d_n = t_1,$

$$d'_n = 0 \Rightarrow \begin{cases} \text{if } \frac{\alpha+0.5}{\alpha+\beta+1} \geq t_1 \rightarrow \text{ reject the batch} \\ \\ \text{else continue to the next stage} \end{cases} \tag{11}$$

In order to identify the signs of the derivatives, the values for $W_n(p)$ and $V_n(p)$ are required. Moreover, to obtain these functions, the values of d_n and d'_n are needed. We showed that these values are at their boundaries, resulting in four cases, which are combinations of the values for d_n and d'_n, as $d_n = 1$, $d_n = t_1$, $d'_n = 0$ and $d'_n = t_2$. Then, we may evaluate the objective function, given in Equation 6, by these cases and pick the one with the lowest value.

In the next section, we provide a numerical example to illustrate the application of the proposed methodology.

NUMERICAL EXAMPLE

Suppose $n = 1$. This means that we are dealing with a one-stage decision-making process. For the next stages, we may use the recursive properties of $W_n(p)$ and $V_n(p)$. Moreover, suppose $\alpha = 1$, $\beta = 4$, $R = 100$, $m = 50$, $\lambda = 0.9$ and $C = 10$. Then, $k_1 = \alpha + \beta = 5$ and, using Equation 1, we have:

$$f(p) = \frac{\Gamma(6)}{\Gamma(1.5)\Gamma(4.5)} p^{0.5}(1-p)^{3.5} = \frac{256}{7\pi} p^{0.5}(1-p)^{3.5}.$$

In this case, assuming $\varepsilon_1 = 0.05$, $\varepsilon = 0.2$, $\delta_1 = 0.1$ and $\delta_2 = 0.2$, at stage one, we have:

$$H_1(p) = \frac{100P(p \geq d_1) + 125P(p \leq d'_1) + (1 - P(p \geq d_1) - P(p \leq d'_1))\lambda V_0(p)}{0.55P(p \geq d_1) + 0.2P(p \leq d'_1) + (1 - P(p \geq d_1) - P(p \leq d'_1))\lambda W_0(p)}.$$

Using Equations 9 and 10

$$P(p \geq d_n) = \int_{d_n}^{1} \frac{\Gamma(5)}{\Gamma(0.5)\Gamma(4.5)} p^{-0.5}(1-p)^{3.5} dp \leq 0.05, \quad (12)$$

$$P(p \leq d'_n) = \int_{0}^{d'_n} \frac{\Gamma(5)}{\Gamma(1)\Gamma(4)} p^{0}(1-p)^{3} dp \leq 0.2. \quad (13)$$

In order to calculate t_1 and t_2, we need to evaluate the integrals of Equation 12 numerically. This evaluation leads us to $t_1 = 0.36$ and $t_2 = 0.05$.

We will calculate the objective function for different possible values of d_1, d'_1 and then, we choose d_1, d'_1 that minimizes the objective function:

$$d_1 = 0.36,\ d'_1 = 0 \Rightarrow H_1(0.25) = \frac{23.4}{0.129} = 181.39,$$

$$d_1 = 1,\ d'_1 = 0.05 \Rightarrow H_1(0.25) = \frac{10.92}{0.017} = 642.35,$$

$$d_1 = 1,\ d'_1 = 0 \Rightarrow H_1(0.25) = \frac{0}{0} \Rightarrow \text{ no answer},$$

$$d_1 = 0.36,\ d'_1 = 0.05 \Rightarrow H_1(0.25) = \frac{34.34}{0.146} = 235.20.$$

Hence, the optimum values for d_1, d'_1 are $d_1 = 0.36$, $d'_1 = 0$ and, since the expected fraction non-conforming is equal to the mean of the Beta distribution with parameters $\alpha = 1.5$, $\beta = 4.5$, that is 0.25, we are in state four of the decision tree in Equation 11 and should continue sampling.

At stage two of the sampling process, let $k_2 = 7$, $\alpha = 4$ and $\beta = 8$. Then, $t_1 = 0.266$ and $t_2 = 0.101$ and:

$$d_1 = 0.266,\ d'_1 = 0.101 \Rightarrow H_1(0.34) = \frac{73.24}{0.62} = 118.13,$$

$$d_1 = 1,\ d'_1 = 0.101 \Rightarrow H_1(0.34) = \frac{1.97}{0.00011} = 17909.1,$$

$$d_1 = 1,\ d'_1 = 0 \Rightarrow H_1(0.34) = \frac{0}{0} \Rightarrow \text{ no answer},$$

$$d_1 = 0.266,\ d'_1 = 0 \Rightarrow H_1(0.34) = \frac{71.33}{0.62} = 115.05.$$

Since the minimum of the objective function occurs at $d_1 = 0.266$, $d'_1 = 0$ points, having $V_1(0.34) = 71.33$ and $W_1(0.34) = 0.62$, first, we obtain $V_2(0.34)$ and $W_2(0.34)$, based on Equations 3 and 5, respectively. Then, we calculate $H_2(0.34)$, using Equation 6 for different corner points as follows:

$$d_2 = 0.266,\ d'_2 = 0.101 \Rightarrow H_2(0.34) = \frac{90.94}{0.774} = 117.49,$$

$$d_2 = 1,\ d'_2 = 0.101 \Rightarrow H_2(0.34) = \frac{65.425}{0.555} = 118.60,$$

$$d_2 = 0.266,\ d'_2 = 0 \Rightarrow H_2(0.34) = \frac{89.71}{0.78} \Rightarrow 114.96,$$

$$d_2 = 1,\ d'_2 = 0 \Rightarrow H_2(0.34) = \frac{64.197}{0.558} = 115.04.$$

Since at the minimum value of the objective function ($d_2 = 0.266$, $d'_2 = 0$) $d_2 = 0.266 < 0.34$, we are in state four of the decision making tree and reject the batch.

CONCLUSION AND RECOMMENDATIONS FOR FUTURE RESEARCH

In this article, we applied a stochastic dynamic programming model to design an acceptance plan in quality inspection environments. In order to determine the optimal policy, we considered a cost function, in combination with a correct decision probability function, and tried to optimize the combined function. The important result of this method is that the boundary limits for decision-making thresholds, based on the system state that arises from the dynamic definition of the system, would change. For further research, we propose either to consider other objective functions or to employ other functions to model the state of the system. Moreover, we can employ the other functions to model the probability of correct choice when the batch is accepted or rejected.

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