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Reliability Analysis of Bridge Structures for Earthquake Excitations

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Abstract. *In this paper, a numerical approach to the reliability analysis of prestressed reinforced concrete long span bridges is presented. A bridge is modeled by finite element software and the analysis is performed in time domain by considering the bridge material nonlinearity. The considered random variables are: Specific strength of concrete, yield stress of steel bars, yield stress of prestressed bars, all sectional dimensions, structural damping ratio, effective depth of steel bars and the magnitude and PGA of earthquake. In this study, the reliability of a bridge structure is evaluated under earthquake excitations. For this purpose, the First-Order Second-Moment (FOSM) method is used. In this method, the mean value and standard deviation of the random variables are considered for evaluating structural reliability. The proposed procedure is applied to evaluate the reliability of an existing prestressed arch concrete bridge located in Bandar-e-Anzali in Iran. Bandar-e-Anzali is a very high-risk earthquake zone. The results of the study show that the structural damping ratio, magnitude and PGA of earthquakes have a significant effect on the variation of reliability in the structure, while variations in the dimensions of the structure have little effect on the reliability index.*

Keywords: *Structural reliability; Non-linear analysis; Arch bridge; Prestressed concrete structures.*

INTRODUCTION

Most existing bridges were designed using deterministic approaches. Due to the randomness of the parameters involved in the design procedure, the deterministic approach does not have a complete view regarding bridge responses under various limit states. Therefore, in order to get a reasonable response from the structure, it seems to be necessary to analyze the structure using probabilistic methods, so that the reliability analysis of these structures, which takes into account the randomness of the involved parameters, would be able to break down the failure risk of such structures. The reliability analysis of structures, when under static loads, has been investigated by many researchers, but the reliability analysis of these structures, when under dynamic loads, has not been further investigated.

Biondini et al. [1] evaluated the reliability of a prestressed concrete arch bridge structure, when under live and dead loads, using the Monte Carlo method. They modeled and analyzed the bridge using a finite element approach. Loads and mechanical and geometrical properties were selected as random variables. The results were presented as graphs that show different reliability for different limit states. Nowak et al. [2] evaluated the reliability of a prestressed concrete bridge, when under live and dead loads, based on three codes (Spanish Norma IAP-98, 1998; ASHTO LRFD, 1998 and ENN 1991-3 Euro Code). Reliability indices were evaluated using the trial method. The mechanical and geometrical properties of sections were selected as random variables. Enrique Castill et al. [3] compared the three most common methods applied for evaluating the reliability analysis of structures (Level 1, Level 2 and Level 3 Reliability Methods). They applied these methods for a single supported beam and compared the results. It is observed that the Level 3 method provides exact results, while the results of the Level 2 method, when the random variables have normal distribution, are exact (the second exact should be opposite to the first, ie inexact). They considered the mechanical and geometrical properties of the beam cross-sections as random variables.

Kiureghian and Taylor [4] are the first researchers to couple the first order second moment (FOSM) reliability analysis with the finite element method (FEM). Haukaas [5] also performed research work on

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the "finite element reliability and sensitivity methods for performance-based engineering". He developed a modern and comprehensive computational framework for a nonlinear finite element reliability analysis. Moreover, much advanced research work on this subject has been reported in the literature [6-12]. Frangopol and Imai [13] studied the reliability of suspension bridges located in Japan, when under wind and earthquake loads. They applied spectral analysis and evaluated the reliability of the structure. The mechanical and geometrical properties of the sections were taken as the random variables.

Because of the lack of attention paid to the reliability analysis of bridge structures under earthquake excitations, in the current study, this subject is presented. The reliability of the bridge is evaluated using the FOSM method. For a numerical example, the Ghazian Bridge located in Bandr-e-Anzali, Iran, was selected. For evaluating system reliability, the bridge was modeled using finite element software and analyzed during many earthquakes. The mechanical and geometrical properties of the bridge; structural damping ratio and earthquake loadings, are considered as random variables.

The numerical results of the study show that variation of the structural damping ratio has a significant effect on bridge reliability.

ASSUMPTIONS

In this study, the following assumptions are made:

1. The bridge is modeled as a continuous beam with 2D-beam elements and, therefore, the effects of bridge piers and abutments on bridge response and reliability are ignored;
2. The bridge is analyzed only for the vertical component of earthquake ground acceleration;
3. Earthquake excitation is modeled as a normally distributed stationary stochastic random process. The bridge is analyzed from 50 earthquake acceleration records, from which the mean values and standard deviations of the bridge's response are calculated;
4. The Probability Density Functions (PDFs) of the random variables, which are not available in the literature, are assumed to be normal;
5. The bending failure mode of the bridge is considered as the dominant failure mode of the bridge;
6. No spatial variation is considered in ground acceleration; it means that it is asumed that all bridge piers and abutments are subjected to the same support excitation.

THEORY OF THE PROBLEM

The governing differential equation of the system can be expressed as [14]:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{P_{\text{eff}}(t)\}, \tag{1}$$

where $[M]$ is the lumped mass matrix; $[C]$ is the structural damping matrix; $[K]$ is the stiffness matrix; $\{P_{\text{eff}}(t)\}$ is the effective force vector of the earthquake; and $\{u\}$ is the displacement vector.

The effective force vector of the earthquake can be written as [14]:

$$\{P_{\text{eff}}(t)\} = -[M]\{r\}\ddot{u}_g(t), \tag{2}$$

where $\ddot{u}_g(t)$ is the vertical component of the ground acceleration applied to the piers and abutments of the bridge; and $\{r\}$ is the influence vector, with all elements equal to unity. It is assumed that $\ddot{u}_g(t)$ is the same for all piers and abutments of the bridge.

In order to evaluate the bridge response, Equation 1 is solved using a standard modal transformation for around 50 records of earthquake vertical accelerations.

RELIABILITY ANALYSIS

Bending failure is the most common failure mode in bridge structures, in comparison with the other failure modes. Therefore, in this study, bending failure mode is focused on for investigation. The bridge is modeled as a continuous beam with 2D-Beam elements. The bridge dead load is considered as a distributed load along the span of each element. The vertical components of ground earthquake accelerations are considered as earthquake loadings. The earthquake excitation is modeled as a normally distributed stationary stochastic random process. The bridge model was analyzed using a finite element approach, utilizing the nonlinear version of SAP2000 software [15], and the structural reliability was evaluated using the FOSM method, utilizing the computer program developed by the authors.

In order to calculate bridge reliability, the limit state equation is defined as [15,3]:

$$g = R - S, \tag{3}$$

in which, R is the resistance of the structure and S is the load effect on the structure. A percentage of the ultimate bending strength of the bridge cross-sections is considered as the resistance of the structure, R, and the bending moments caused by earthquake loadings in the bridge elements were taken as the load effects, S.

A bridge was analyzed under the vertical components of many earthquake accelerations (around

50 records) and its responses, such as displacements and bending moments, etc. were calculated. Then, from the obtained results, the mean values and standard deviations of the bridge responses are calculated.

Referring to Equation 3, by definition, the structure is said to be failed if $R < S$. Therefore, the probability of failure of structure P_f is expressed as [16]:

$$P_f = P(R < S) = P(R - S < 0) = P\left(\frac{R}{S} < 1\right). \quad (4)$$

Now, by considering R and S as random variables, P_f can be written as [16]:

$$P_f = 1 - R_0 = 1 - \int_{-\infty}^{\infty} f_S(s)\left[1 - F_R(s)\right] ds$$

$$= \int_{-\infty}^{\infty} f_S(s) F_R(s) ds. \quad (5)$$

Or:

$$P_f = 1 - \int_{-\infty}^{\infty} f_R(r) F_s(r) dr, \quad (6)$$

where R_0 is the reliability of the structure; f_s and F_s are the Probability Density Function (PDF) and Cumulative Distribution Function (CDF) of S, respectively; and f_R and F_R are those of R.

In general, the above integrals should be calculated using numerical methods. If R and S are statistically independent and normally distributed, then g also would be a normally distributed random variable. In this case, the mean value and standard deviation of g are defined as [16]:

$$\mu_g = \mu_R - \mu_S, \quad (7)$$

$$\sigma_g = \left(\sigma_R^2 + \sigma_S^2\right)^{\frac{1}{2}}, \quad (8)$$

where μ_g, μ_R and μ_S are mean values of g, R and S, respectively; and σ_g, σ_R and σ_S are the standard deviations of g, R and S, respectively. Then, the probability of failure, P_f, is defined as [16,3]:

$$P_f = P(g < 0) = F_g(0) = \Phi\left(\frac{0 - \mu_g}{\sigma_g}\right), \quad (9)$$

$$P_f = \Phi\left[\frac{\mu_S - \mu_R}{\sqrt{\sigma_R^2 + \sigma_S^2}}\right], \quad (10)$$

where Φ is the standard normal distribution function (with $\mu = 0$ and $\sigma = 1$) and F_g is the CDF of g. Now, if $\beta = \frac{\mu_g}{\sigma_g}$ is defined as the reliability index, then:

$$P_f = \Phi(-\beta). \quad (11)$$

From which [16,2]:

$$\beta = -\Phi^{-1}(P_f). \quad (12)$$

In this study, a percentage of the ultimate strength of the bridge cross-section is considered as the bridge resistance, R. R itself is a function of the number of basic variables, which, in this study, are considered as the following [1]:

(a) f_c': Specific strength of concrete;

(b) f_y: Yield stress of steel bars;

(c) A_s: Cross sectional area of steel bars;

(d) f_{yp}: Yield stress of prestressed steel bars;

(e) All geometrical dimensions of the bridge cross-section;

(f) The structural damping ratio of the bridge.

Uncertainties Considered in the Analysis

The uncertainties in the geometric dimensions and material properties of the bridge, plus construction defects, finally lead to uncertainties in the mass and stiffness properties of the bridge. The other uncertainties that are considered include: Knowledge uncertainties in the structural damping ratio, earthquake Peak Ground Acceleration (PGA) and earthquake magnitude. All these uncertainties affect the dynamic characteristics of the bridge and, hence, its dynamic response. Uncertainties arising from earthquake loading itself are incorporated by modeling the earthquake as a normally distributed stationary stochastic random process, and the bridge is analyzed from the vertical components of many earthquake ground acceleration records. Then, from the results obtained, the mean values and standard deviations of the bridge responses are evaluated. More explanations of random variables and their statistical values are provided in the numerical study section of the paper.

Failure Model

In reliability analysis, each structure can be modeled as a system with one or more component. Generally, the systems are divided into three forms: Series, parallel and mixed systems (Figure 1):

1. The series system works satisfactorily if all components work satisfactorily. The block diagram for this system is shown in Figure 1a and the reliability

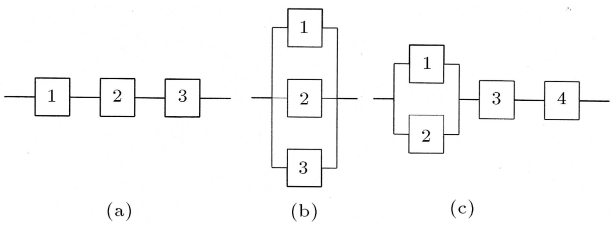

Figure 1. Structural systems in reliability analysis; a) series system, b) parallel system and c) mixed system.

is calculated as below. Let:

A_i = the event wherein component i works satisfactorily,
P_{ss} = probability of survival of the system,
P_{fs} = probability of failure of the system,
P_{ss} = $1 - P_{fs}$.

As every component should function satisfactorily for the system to be reliable, so;

$$P_{ss} = P\left(A_1 \cap A_2 \cap \cdots \cap A_n\right). \tag{13}$$

If the events, A_i, are assumed to be independent, the above equation simplifies to:

$$P_{ss} = [P(A_1) \times P(A_2) \times \cdots \times P(A_n)]$$

$$= \prod_{i=1}^{n}(1 - P_{fi}), \tag{14}$$

in which, P_{fi} is the probability of failure of event A_i.

2. Parallel systems survive even if one component has failed. The system fails to function satisfactorily only when all components of the system have failed to function satisfactorily. The block diagram for this system is given in Figure 1b and the reliability of the system is given by:

$$P_{ss} = 1 - P_{fs} = 1 - P(A_1^C \cap A_2^C \cap \cdots \cap A_n^C), \tag{15}$$

where A_i^c= the event wherein component i does not function satisfactorily. If events A_i^c are independent, Equation 15 simplifies to:

$$P_{ss} = 1 - \left[P(A_1^C) \times P(A_2^C) \times \cdots \times P(A_n^C)\right]$$

$$= 1 - \prod_{i=1}^{n} P_{fi}. \tag{16}$$

3. Mixed systems consist of series and parallel systems. These systems divide into some subsystems. A subsystem can be a series or parallel system. A mixed system is shown in Figure 1c. The reliability of this system also can be calculated using a combination of the above two systems.

In the present study, bridge reliability is calculated for ultimate conditions, explained in the following.

Ultimate Strength Condition

In this condition, the margin equation is expressed as [1]:

$$g = \alpha M_u - M_s, \tag{17}$$

where M_u is the ultimate strength moment of the critical section; α is a coefficient showing the percentage value of M_u considered in the analysis; and M_s is the critical section moment caused by earthquake loading. For each reinforced concrete section, it can be written that [17,18]:

$$x = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_s} \times d, \tag{18}$$

where x is the distance between the neutral axis of the section and the last compressed point of the concrete, d is the effective depth of the steel bars, ε_s is the strain in the steel bars, caused by M_s, and ε_{cu} is the ultimate compressive strain in the concrete.

The equilibrium equation for a reinforced concrete section with compression steel can be expressed as [17,18]:

$$T_S = T_S' + C_c, \tag{19}$$

in which T_S is the tensile force in the tension bars, T_S' is the compressive force in the compression bars and C_c is the compressive force in the concrete, all expressed as the following [17,18]:

$$T_S = \phi_s f_y A_s, \qquad T_S' = \phi_s f_S' A_S', \qquad C_c = \phi_c f_c' A_c, \tag{20}$$

in which ϕ_s is the reduction coefficient for steel bar yield stresses and ϕ_c is that of the concrete specific strength, f_y is the yield stress of steel bars, f_S' is the existing stress in the compression steel bars, f_c' is the specified compressive strength of the concrete, A_S is the cross-sectional area of the tensile steel bars, A_S' is the cross-sectional area of the compression steel bars and Ac is the area of the compressive concrete part of the cross-section.

By referring to Figure 2, it can be written that:

$$M_u = C_c \times \left(d - \frac{a}{2}\right) + T_S' \times (d - d'), \tag{21}$$

from which, using the statistical theory, it is calculated

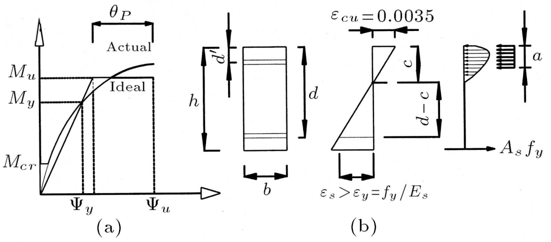

Figure 2. Stress distribution on cross section. a) Moment-Curvature; b) Ultimate condition.

that [16]:

$$\mu_M = \mu_{C_C} \times \left(\mu_d - \frac{\mu_a}{2}\right) + \mu_{T_{S'}} \times (\mu_d - \mu_{d'}), \quad (22)$$

$$\sigma_1 = \sigma_C^2 \left(\mu_d - \frac{\mu_a}{2}\right)^2, \qquad \sigma_2 = \sigma_{T'_S}^2 (\mu_d - \mu_{d'})^2,$$

$$\sigma_3 = \left(-\frac{1}{2}\mu_{C_c}^2\right)^2 \sigma_a^2, \qquad \sigma_4 = \sigma_d^2 (\mu_{C_C} + \mu_{T'_S})^2,$$

$$\sigma_5 = (-\mu_{T'_S})^2 \sigma_{d'}^2,$$

$$\sigma_M = (\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 + \sigma_5^2)^{\frac{1}{2}}, \quad (23)$$

in which μ_i and σ_i are the mean value and standard deviation of the corresponding variable.

In reinforced concrete structures, the rotation of each cross-section is limited to a specific value, defined as θ_P, and called the plastic rotation of the section. If a section rotation exceeds the value of θ_P, that section is said to be failed, in the form of rotation failure mode. In the following, the required equations for computing the sections plastic rotation, θ_P, are presented.

Plastic Rotation

The curvature of the section at the start of yielding is defined as Ψ_y (Figure 2), and is presented as below:

$$\Psi_y = \frac{\varepsilon_y}{d(1-k)}, \quad (24)$$

in which, ε_y is the yield strain of the steel bars, d is the depth of the tensile bars and k is a coefficient [18,19].

The yielding moment, M_y, of the section is given as:

$$M_y = A_s f_y \left(d - \frac{kd}{3}\right). \quad (25)$$

The maximum bending moment or ultimate bending moment, M_u, is also given as:

$$M_u = A_s f_y \left(d - \frac{\beta_1 c}{2}\right), \quad (26)$$

in which, β_1 is a coefficient and c is the distance between the last compressive fiber in the compressive concrete section and the neutral axis [20].

The maximum curvature of the section is given by:

$$\Psi_u = \frac{\varepsilon_{cu}}{c}. \quad (27)$$

Then, the plastic rotation of the section can be expressed as:

$$\theta_P = \left(\Psi_u - \Psi_y \frac{M_u}{M_y}\right) l_P, \quad (28)$$

where, l_P is the length of the plastic hinge, given by:

$$l_P = 0.5d + 0.05z, \quad (29)$$

in which, z is the distance between the neutral axis and the location of the maximum bending moment.

NUMERICAL STUDY

For the numerical example, a three-span prestressed concrete arch bridge located in Bandar-e-Anzali, I.R. Iran, is chosen. A typical cross-section of this bridge is shown in Figure 3, for which the structural data are also shown in Table 1. In this three-span bridge, the main span is 125 meters in length, the left span is 55.35 meters and the right span is 45.05 meters in length. In order to analyze the bridge using the finite-element model, a nonlinear version of SAP2000 software is utilized by considering 28 finite elements in the main span, 14 elements in the left span and 10 elements in the right span. For each span, element length, node number and element number are shown in Figures 4 to 6. The elements' positions are also shown in Figures 4a to 4d. Figure 4a shows the element position on the right side of the middle span. For example, element "3L" is the third element on the left side of the middle span. Also, Figures 4b, 4c and 4d show the elements' positions for the right side of the middle span, those of the left span and those of the right span, respectively. Figure 5 shows the elements' length and number for side spans and Figure 6 shows those of the middle span.

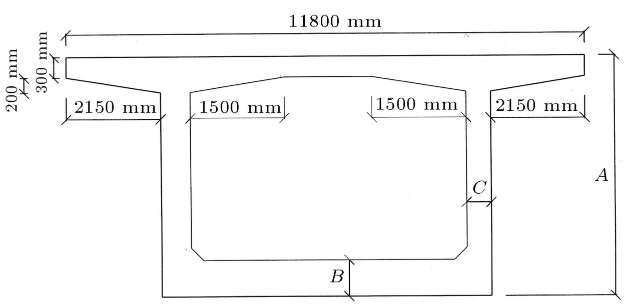

Figure 3. Bridge cross-section.

Table 1. Bridge cross-sectional data (according to the notations shown in Figure 3).

Section Position	A (mm)	B (mm)	C (mm)
1R, 1L	6180	800	650
2R, 2L	5880	700	500
3R, 3L	5570	500	500
4R, 4L	5240	450	500
5R, 5L	4940	350	500
6R,6L	4670	300	500
7R, 7L	4420	300	500
8R, 8L	4210	300	500
9R, 9L	4020	300	500
10R, 10L	3860	300	500
11R, 11L	3730	300	500
12R, 12L	3630	300	500
13R, 13L	3500	300	500
14R, 14L	3500	300	500
15R, 15L	3500	300	500

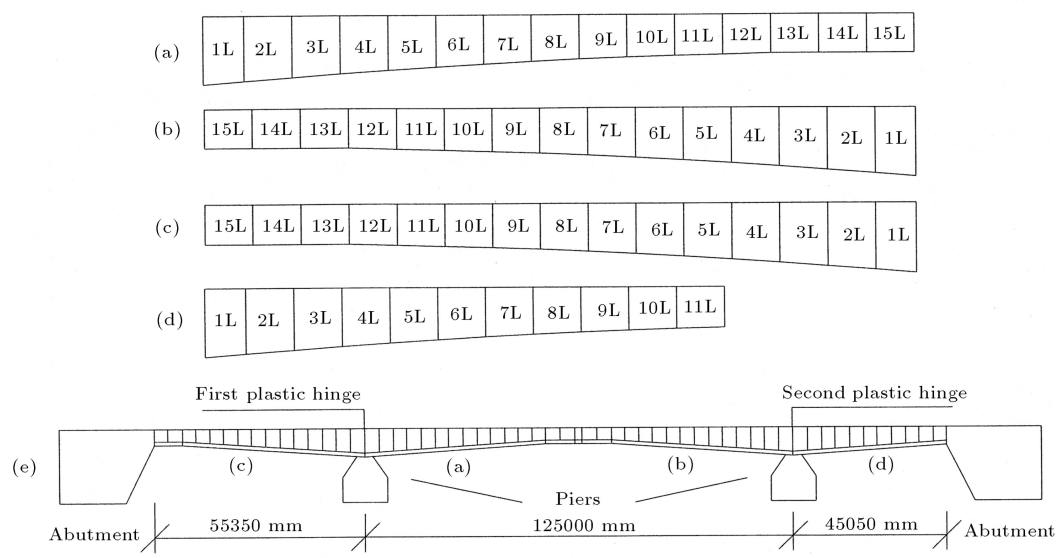

Figure 4. Bridge longitudinal view and the elements positions.

For a concrete bridge, the following values are assumed:

$$\mu_{f'_C} = 322 \frac{\mathrm{N}}{\mathrm{mm}^2}, \qquad \sigma_{f'_C} = 5 \frac{\mathrm{N}}{\mathrm{mm}^2}, \text{ (Table 2).}$$

Steel bars are also assumed to be AIII-type with yield stress $f_y = 400 \frac{\mathrm{N}}{\mathrm{mm}^2}$ and standard deviation $\sigma_{f_y} = 30 \frac{\mathrm{N}}{\mathrm{mm}^2}$ (Table 2). As shown in Table 2, both f'_C and f_y are assumed to be log normally distributed random variables. The modulus of elasticity for steel bars is assumed to be about [18].

$$E_s = 2 \times 10^5 \frac{\mathrm{N}}{\mathrm{mm}^2}.$$

The mean value of the cross-sectional area of the steel bars (μ_{A_S}) is assumed to be the nominal cross-section area and its standard deviation is also assumed to be $\sigma_{A_S} = 0.025 \times \mu_{A_S}$. Its PDF is also assumed to be normal [1]. The structural damping ratio is also a random variable, assumed to be normally distributed,

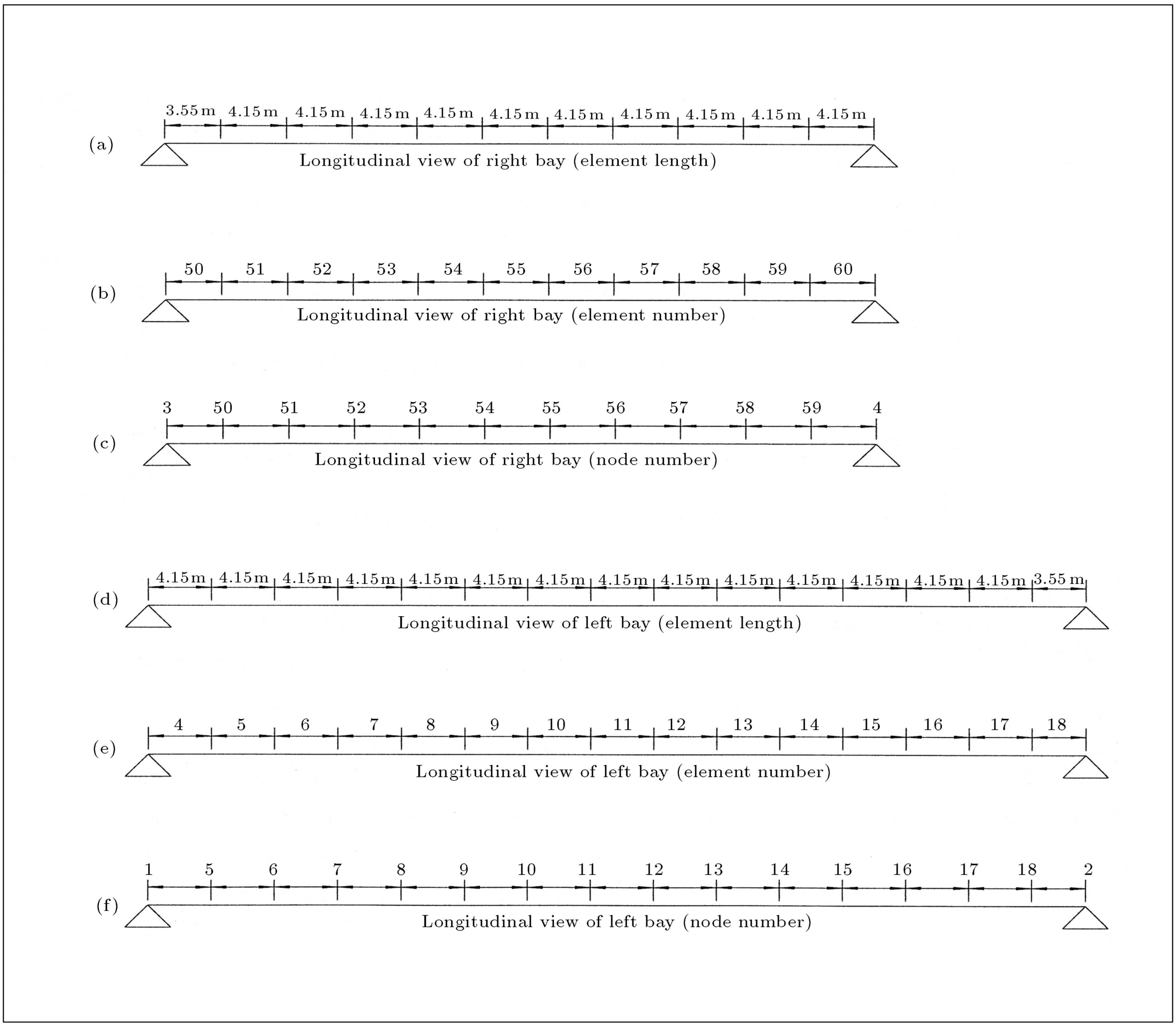

Figure 5. Bridge sides (left and right) bays.

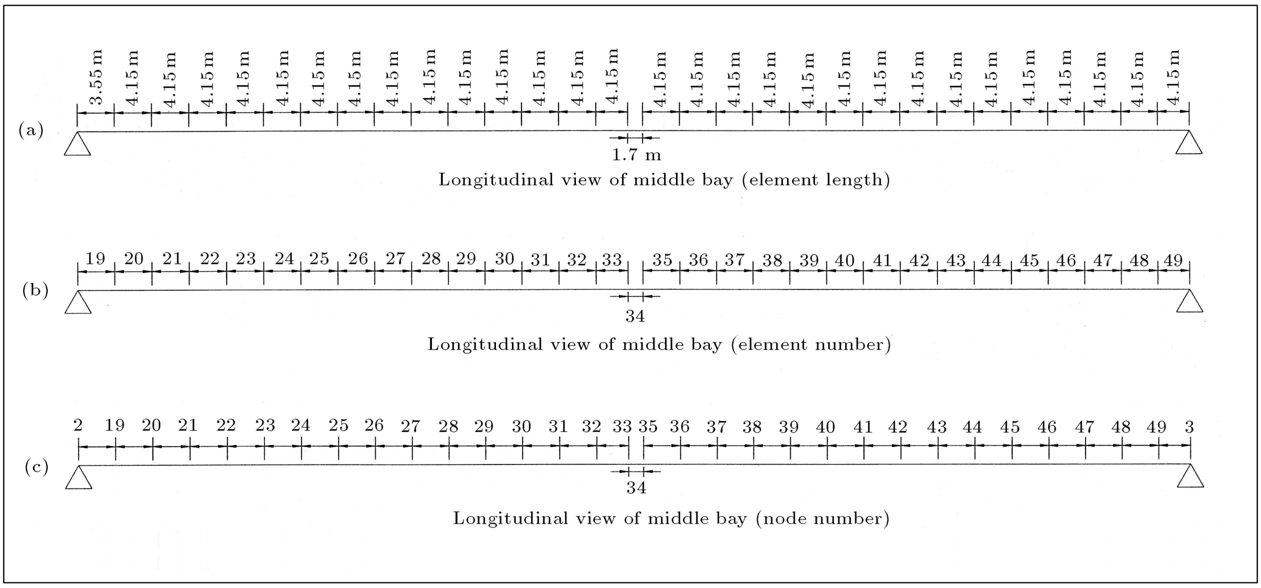

Figure 6. Bridge middle bay.

Table 2. Random variables and their required data.

Random Variable	Standard Deviation	Mean Value	Distribution Function*
f'_C	5 N/mm^2	32.2 N/mm^2	LN
f_{yp}	100 N/mm^2	600 N/mm^2	LN
f_y	30 N/mm^2	400 N/mm^2	LN
Sectional dimensions	50 mm	Nominal dimensions	N
Depth of steel bars	5 mm	Nominal dimensions	N
Cross-section area of steel bars (A_s)	0.025 As	Nominal dimensions	N
Damping ratio	3%	5%	N
Earthquake PGA	0.017 g	0.125 g	N
Earthquake magnitude	2 Richter	6 Richter	N

* LN = log-normal, N = normal

with a mean value of 5% (Table 2). The other statistical data are given in Table 2.

In the following, a reliability analysis of the bridge under an ultimate strength condition is presented. This procedure is performed for two cases. In the first case it is performed without considering the bars prestressing effect and, in the second case, it is performed by considering the bars prestressing effect. For computing the reliability of the structure, the mean value and standard deviation of the resistant moment of the bridge cross-sections are necessary.

Reliability Analysis of Bridge for Ultimate Strength Condition Without Bars Prestressing Effect

In this condition, a specific percentage of the ultimate moment of the bridge cross-section is considered as the resistance, R, and the mean value and standard deviation of the moments, caused by earthquake, in the bridge element are considered as the load effect, S.

The bridge critical sections are shown in Figures 5c, 5f and 6c (Node number). For these critical sections, by applying Equations 19 to 29, the mean values and standard deviations of the resisting moments and plastic rotations are computed. All results are presented in Table 3. The reliability analysis is accomplished without considering the bars prestressing effect. The mean values and standard deviations of the resistent moments and moments caused by earthquake loadings, at the bridge critical sections are presented in Figures 7 and 8, respectively. The bridge is subjected to vertical components of many earthquake accelerations (50 earthquakes) [20,21]. Some of the most famous of these earthquakes are the El-Centro earthquake (USA), the Kobe earthquake (Japan), the Newhall earthquake (USA), the Zanjiran earthquake (Iran) and the Sarein earthquake (Iran). The magnitude of these earthquakes varies from 5 to 7.7 Richter.

In order to perform a reliability analysis, the bridge block diagram, consisting of its failure modes, is constructed and is shown in Figure 9. The bridge system consists of piers, abutments and a deck. But, because of the very low probability of failure of the bridge piers and abutments, in this study, the effects of piers and abutments on bridge reliability are ignored

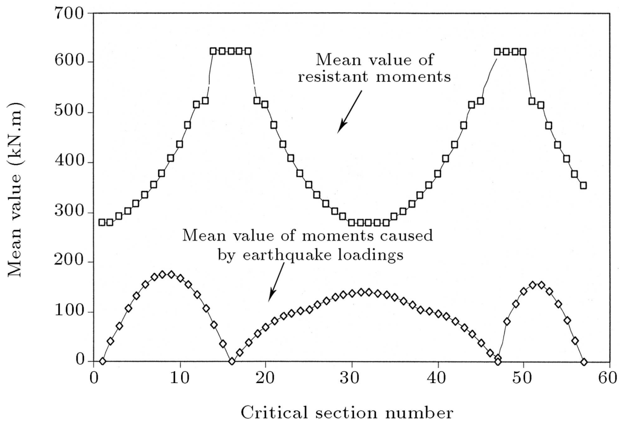

Figure 7. Mean value of resistant moments and moments caused by earthquake (without bars prestressing effect).

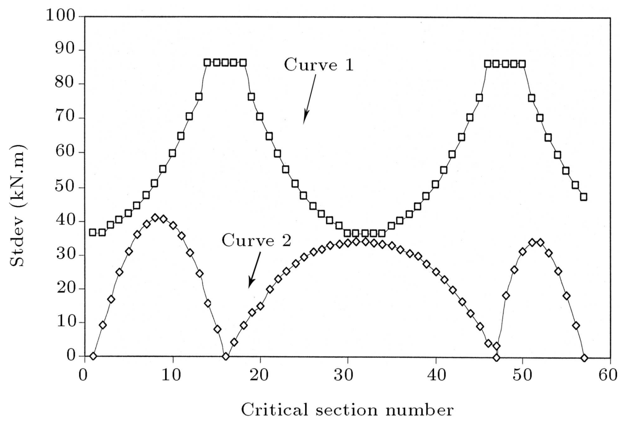

Figure 8. Standard deviation of resistant moment (curve 1), and moments caused by earthquake (curve 2) (without bars prestressing effect).

Table 3. Mean values and standard deviation of resistant moments and plastic rotation.

Section Position	Mean Value of Resistant Moments (KN.m)	Standard Deviation of Resistant Moments (KN.m)	θ_P (rad)
1R, 1L	621.41	80.06	$3.822*10^{-3}$
2R, 2L	522.3	76.154	$4.039*10^{-3}$
3R, 3L	514	70.611	$5.780*10^{-3}$
4R, 4L	474.074	64.84	$6.020*10^{-3}$
5R, 5L	435.835	59.7	$7.880*10^{-3}$
6R, 6L	407.166	55.17	$9.068*10^{-3}$
7R, 7L	378.641	51.05	$9.190*10^{-3}$
8R, 8L	355.067	47.65	$9.290*10^{-3}$
9R, 9L	334.042	44.61	$9.389*10^{-3}$
10R, 10L	316.651	42.1	0.0102
11R, 11L	302.509	40.1	0.0114
12R, 12L	291.792	38.53	0.0103
13R, 13L	277.979	36.54	0.0104
14R, 14L	277.979	36.54	0.0104
15R, 15L	277.979	36.54	0.0104

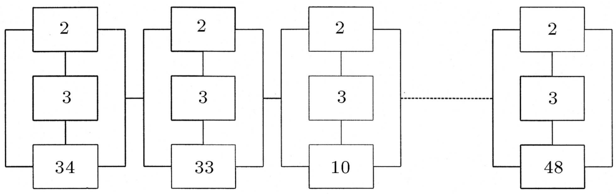

Figure 9. Block diagram for failure modes.

and, therefore, a bridge system is considered consisting of only the bridge deck as a continuous beam. Thus, noting the fact that this three-span continuous beam fails, if three hinges occur, a block diagram of the bridge failure modes is constructed (Figure 9).

In Figure 9, the numbers in blocks show the bridge critical cross-section numbers. For example, 34 means the 34th critical section (see Figure 6). Block model of, for example, the bridge system, consists of 54 subsystems connected together in the form of a series and each subsystem consists of 3 plastic hinged sections connected together in the form of a parallel (Figure 9). The probability of failure of each failure mode is presented in Table 4. The probability of survival or reliability of a series system is expressed as [16]:

$$P_{ss} = 1 - \sum_{i=1}^{n} P_{fi}, \qquad (30)$$

where, P_{ss} is the probability of survival of the system and P_{fi} is the probability of failure of the ith subsystem.

In order to evaluate the reliability of the bridge, the ultimate strength condition is considered. The results of the reliability analysis show that the first two plastic hinges occur at bridge section numbers 2 & 3 (shown in Figure 4, over the middle supports of the bridge). After forming these two plastic hinges, by forming more plastic hinges, a mechanism is formed, after which, the failure modes of the bridge can be extracted, which are shown in Table 4. By using Equation 30, considering the failure modes shown in Table 4 and the block diagram shown in Figure 9, the reliability of the bridge under the considered condition is evaluated as:

$$P_{fs} \approx 0.02196, \qquad P_{ss} = 1 - 0.02196 = 0.97804.$$

Reliability Analysis of Bridge for Ultimate Strength Condition by Considering Bars Prestressing Effect

The mean value and standard deviation of the ultimate moments (evaluated using Equations 18-23 and the moments caused by earthquakes (obtained by analyzing the bridge conditions under many earthquakes) are shown in Figures 10 and 11.

Then, by performing a reliability analysis (using Equations 13-16), the probability of the survival of the

Table 4. failure modes and their probability of failure.

Sections	Mode No.	failure Mode	β	P_f
2,3,34	1	Mechanism failure	2.791	0.00264
2,3,33	2	Mechanism failure	2.793	0.00260
2,3,10	3	Mechanism failure	2.804	0.00250
2,3,32	4	Mechanism failure	2.826	0.00235
2,3,35	5	Mechanism failure	2.826	0.00235
2,3,11	6	Mechanism failure	2.853	0.00217
2,3,9	7	Mechanism failure	2.939	0.00164
2,3,6	8	Rotation failure	3.047	0.00115
2,3,31	9	Mechanism failure	3.0995	0.000970
2,3,36	10	Mechanism failure	3.1000	0.000968
2,3,12	11	Rotation failure	3.116	0.000921
2,3,8	12	Mechanism failure	3.354	0.000400
2,3,37	13	Rotation failure	3.3638	0.000380
2,3,30	14	Mechanism failure	3.3638	0.000380
2,3,13	15	Mechanism failure	3.556	0.000190
2,3,38	16	Rotation failure	3.6951	0.000110
2,3,29	17	Mechanism failure	3.6951	0.000110
2,3,14	18	Mechanism failure	4.0518	0.000030
2,3,7	19	Mechanism failure	4.0706	0.0000235
2,3,28	20	Rotation failure	4.07635	0.0000230
2,3,39	21	Rotation failure	4.0806	0.0000225
2,3,54	22	Mechanism failure	4.103	0.0000207
2,3,55	23	Mechanism failure	4.2000	0.0000133
2,3,53	24	Mechanism failure	4.3620	0.0000065
2,3,27	25	Rotation failure	4.4954	0.00000356
2,3,56	26	Mechanism failure	4.5790	0.0000024
2,3,40	27	Rotation failure	4.6954	0.0000014
2,3,15	28	Mechanism failure	4.7419	0.0000013
2,3,26	29	Rotation failure	4.7889	0.00000085
2,3,41	30	Rotation failure	4.7889	0.00000085
2,3,52	31	Mechanism failure	4.8093	0.000000793
2,3,51	32	Mechanism failure	5.0286	0.000000240
2,3,25	33	Rotation failure	5.1104	0.000000160
2,3,42	34	Rotation failure	5.1107	0.000000160
2,3,57	35	Mechanism failure	5.3247	0.0000000510
2,3,24	36	Rotation failure	5.4051	0.000000330
2,3,43	37	Rotation failure	5.4051	0.000000330
2,3,16	38	Rotation failure	5.4563	0.000000260
2,3,23	39	Rotation failure	5.7907	0.0000000037
2,3,44	40	Rotation failure	5.7907	0.0000000037
2,3,46	41	Rotation failure	6.0525	0.00000000077
2,3,50	42	Mechanism failure	6.1295	0.00000000046
2,3,45	43	Rotation failure	6.1301	0.00000000040
2,3,22	44	Rotation failure	6.158	0.00000000037
2,3,17	45	Rotation failure	6.1661	0.00000000036
2,3,5	46	Rotation failure	6.3337	0.00000000013
2,3,58	47	Mechanism failure	6.3761	0.00000000010
2,3,21	48	Rotation failure	6.4408	0.000000000077
2,3,49	49	Mechanism failure	6.7253	0.000000000009
2,3,47	50	Rotation failure	6.7505	0.0000000000079
2,3,20	51	Rotation failure	6.7519	0.0000000000070
2,3,18	52	Rotation failure	6.7908	0.0000000000060
2,3,19	53	Rotation failure	7.0074	0.00000000000128
2,3,48	54	Rotation failure	7.0086	0.00000000000120

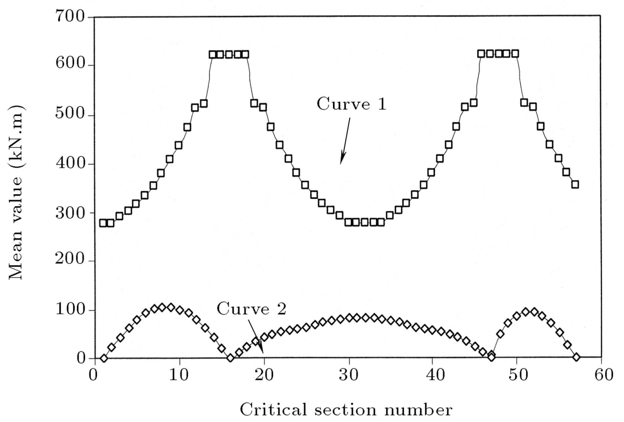

Figure 10. Mean value of ultimate moments (curve 1), and moments caused by earthquake (curve 2) (with bars prestressing effect).

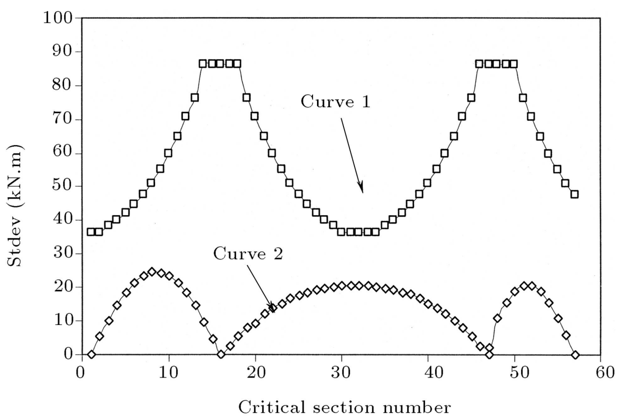

Figure 11. Standard deviation of ultimate moments (curve 1), and moments caused by earthquake (curve 2) (with bars prestressing effect).

bridge is obtained as:

$$P_{fs} \approx 0.000025,$$

and

$$P_{ss} = 1 - 0.000025 = 0.999975.$$

It is seen that the reliability of the bridge significantly increases by considering the bars prestressing effect.

SENSITIVITY ANALYSIS

Since bridge reliability depends on many random variables, for which the probability distributions are not exactly known, in this part of the study, the effects of a variation of these random variables on the bridge reliability index, β, are investigated. In the following, the numerical results are presented only by considering the bars prestressing effect.

Effect of Variation of Ultimate Moment on Reliability Index

In general, under an ultimate condition, the reliability of the bridge highly depends on the ultimate moments (M_u) of the bridge cross section. In this condition, a percentage of the ultimate moment is considered as the threshold level for evaluating bridge reliability. Figures 12 and 13 show the effects of a variation in the value of $Cov(M_u)$ on the reliability index for the damping ratios of 5% and 8%, respectively, for different values of threshold level. It is seen from the figures that β decreases with an increase in the value of the coefficient of variation of the ultimate moment $[Cov(M_u)]$. Moreover, the sensitivity of the reliability index to the variation of $Cov(M_u)$ for high threshold levels is much more than that of low threshold levels.

Effect of Variation of Structural Damping Ratio on Reliability Index

In this part of the study, the effect of the structural damping ratio ξ on the bridge reliability index is con-

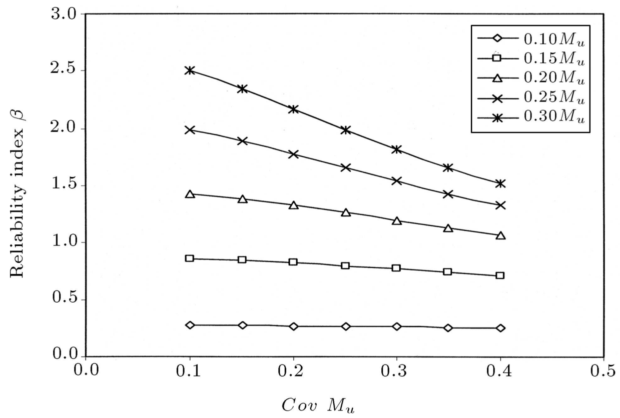

Figure 12. Effect of variation in the resisting moment on reliability index β for $\xi = 5\%$.

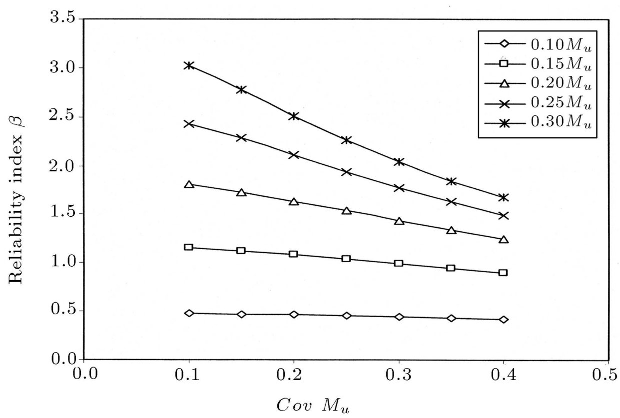

Figure 13. Effect of variation in the resisting moment on reliability index β for $\xi = 8\%$.

ducted. Figure 14 shows the effect of a variation in the structural damping ratio on the reliability index. In the reliability limit state equation, a percent of the ultimate resistant moment of the bridge's last failed section in a mechanism path is considered as the resistance of the structure, R. Also, the internal moments in the bridge elements caused by earthquakes are considered as the load effect, S. Here, also, the results are presented for different values of the threshold level. It is seen from the figure that the reliability index, β, increases with an increase in the value of ξ. Also, the reliability index increases with an increase in the value of the threshold level (the percentage value of ultimate moment which is considered as bridge resistance). Furthermore, the rate of increase in the value of β for low values of the threshold level is higher than that of the high values of the threshold level. When the threshold level becomes higher, the variation of the reliability index, β, versus ξ, almost becomes a smooth curve.

Effect of Variation in Effective Depth of Steel Bars on Reliability Index

The depth of the steel bars has also a significant effect on the reliability index of the bridge. This effect is shown in Figures 15 and 16 for the structural damping ratio taken at about 5% and 8%, respectively. In both figures, the reliability index, β, of the example bridge is traced versus the standard deviation of the effective depth of the steel bars. The figures show that β decreases with an increase in the value of the standard deviation (Stdev) of the effective depth, d. Again, at high threshold levels, the sensitivity is greater.

Effect of Variation in Specific Strength of Concrete and Yield Stress of Steel Bars on Reliability Index

Figure 17 shows the effect of a variation in the standard deviation of the concrete specific strength, f'_C (Stdev f'_C), on the bridge reliability index, β, for different values of the threshold level. The figure shows that the reliability index, β, decreases by increasing the value of Stdev (f'_C), but the sensitivity of β to the variation of Stdev (f'_C) for low values of the threshold level, is more than that of the high threshold level.

The effect of variation in the standard deviation

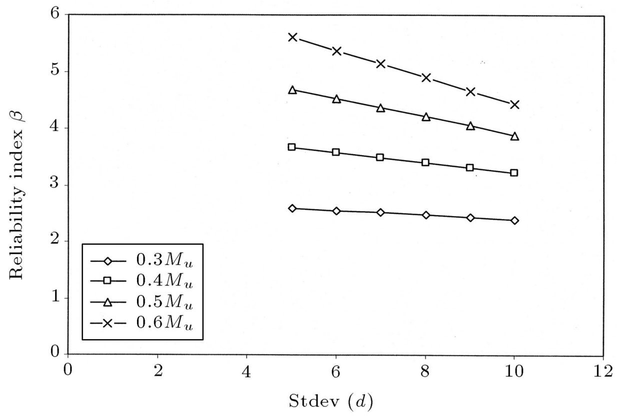

Figure 15. Effect of variation in effective depth of steel bars on the reliability index β for $\xi = 5\%$.

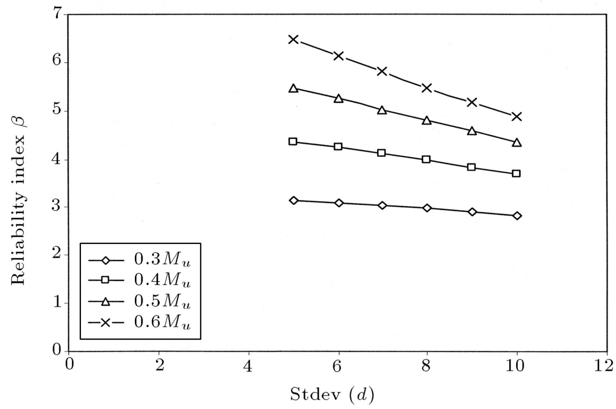

Figure 16. Effect of variation in effective depth of steel bars on the reliability index β for $\xi = 8\%$.

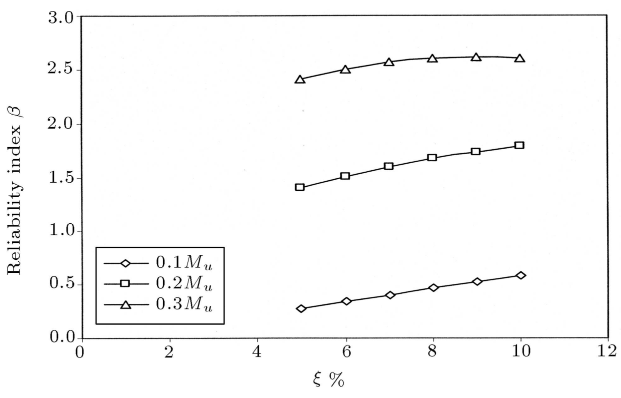

Figure 14. Effect of damping ratio variation on the reliability index β.

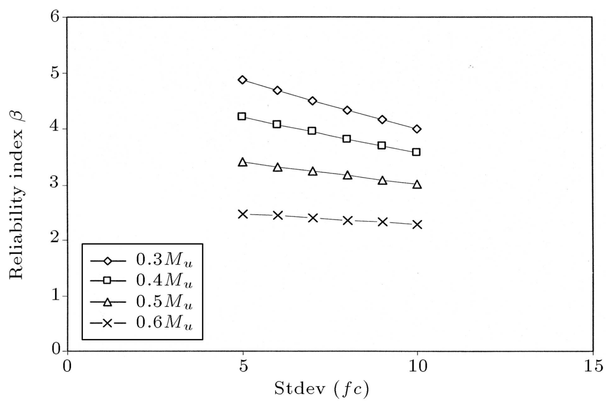

Figure 17. Effect of variation in specific strength of concrete on the reliability index β for $\xi = 5\%$.

of the yield stress of the steel bars (Stdev (f_s)) is also shown in Figure 18. The figure shows that the bridge reliability index, β, is not very sensitive to the variation of Stdev (f_s).

Effect of Variation in the Area of Cross Section of Steel Bars on Reliability Index

Figure 19 shows the effect of the coefficient of variation of the steel bars (A_s) on the reliability index, β. This figure shows that the reliability index, β, is not very sensitive to the variation in CoV of A_s.

Effect of Earthquake Magnitude on Reliability Index

In the reliability analysis of the structures against earthquake loadings, the magnitude of the earthquake is one of the most important random variables to seriously affect the results of the analysis. So, the effect of this parameter on the bridge reliability index is studied and the results are shown in Figure 20. The reliability index, β, is evaluated for different values of the standard deviation of the involved random variables shown in Table 5. For example, type 2 shows the results of the reliability analysis for a second set of standard deviations assumed for different random variables (Table 5). As seen from the figure, the reliability index, β, significantly decreases with an increase in the value of the earthquake magnitude. Furthermore, it is seen from the figure that the effect of a variation in the standard deviations of other random variables on reliability index β for high values of earthquake magnitude, is less than that of low values of earthquake magnitude.

Effect of Earthquake PGA on Reliability Index

The effect of earthquake Peak Ground Acceleration (PGA) on reliability index β is shown in Figure 21. The figure is plotted for different values of the standard deviations of the considered parameters (Table 5). It can be seen from the figure that the reliability index decreases with an increase in the value of PGA.

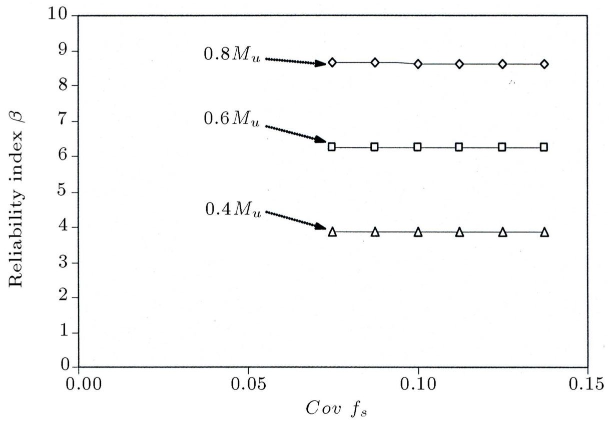

Figure 18. Effect of variation in yield stress of steel bars on the reliability index β for $\xi = 5\%$.

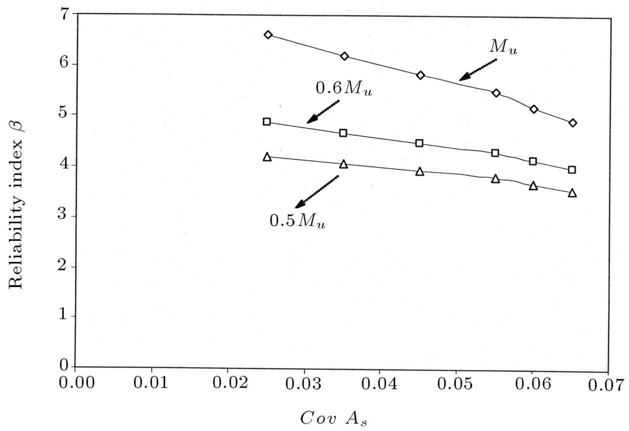

Figure 19. Effect of variation in cross section of steel bars on the reliability index β for $\xi = 5\%$.

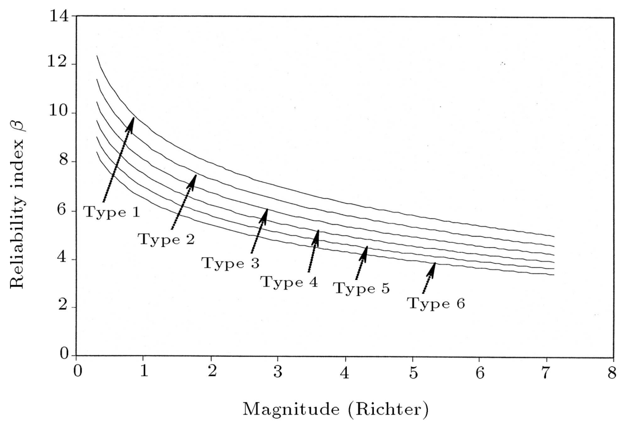

Figure 20. Magnitude vs reliability index β for various standard deviations.

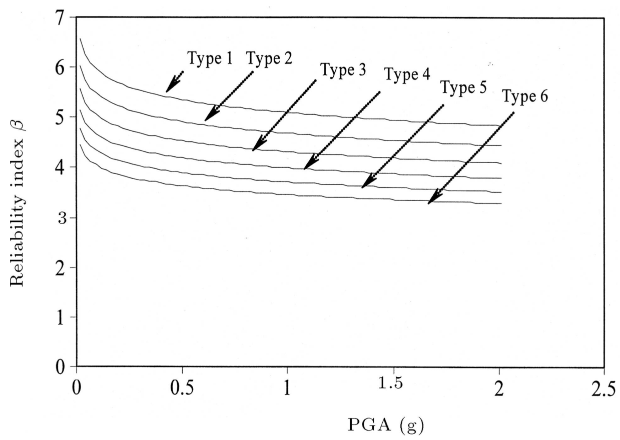

Figure 21. PGA vs reliability index β for various standard deviations.

Table 5. Random variables and their standard deviation range.

Random Variables	Type 1	Type 2	Type 3	Type 4	Type 5	Type 6
f'_C	5 N/mm^2	6 N/mm^2	7 N/mm^2	8 N/mm^2	9 N/mm^2	10 N/mm^2
f_{yp}	100 N/mm^2	110 N/mm^2	120 N/mm^2	130 N/mm^2	140 N/mm^2	150 N/mm^2
f_y	30 N/mm^2	35 N/mm^2	40 N/mm^2	45 N/mm^2	50 N/mm^2	55 N/mm^2
Cross-sectional dimension	50 mm	55 mm	60 mm	65 mm	70 mm	75 mm
Effective depth of steel bars	5 mm	6 mm	7 mm	8 mm	9 mm	10 mm
Cross section of steel bars	0.025 A_s	0.035 A_s	0.045 A_s	0.055 A_s	0.065 A_s	0.075 A_s
Structural damping ratio	0.5%	1%	1.5%	2%	2.5 %	3%
Earthquake PGA	0.0028 g	0.0056 g	0.0084 g	0.0112 g	0.014 g	0.017 g
Earthquake magnitude	0.5 Richter	1 Richter	1.5 Richter	2 Richter	2.5 Richter	3 Richter

CONCLUSION

In this paper, a numerical approach to the reliability analysis of prestressed concrete bridges is presented. The bridge is modeled using a finite element approach and analyzed utilizing a nonlinear version of SAP2000 software and its reliability is evaluated using the FOSM method. A three-span prestressed arch concrete bridge, located in Bandar-e-Anzali, Iran, is chosen as the numerical example. From the numerical results of the study, it is concluded that:

1. Variation in the threshold level has a significant effect on the reliability index, β;
2. Sensitivity of the reliability index, β, to the variation of the random variables at high threshold levels is much more than that for low threshold levels;
3. The structural damping ratio, ξ, has a significant effect on β. Reliability index β increases with an increase in the value of the structural damping ratio, ξ;
4. Reliability index, β, decreases with an increase in the values of the coefficient of variation (CoV) of M_u, earthquake PGA and standard deviation of the effective depth of the steel bars;
5. Reliability index β significantly decreases with the increase in the value of the earthquake magnitude.

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