Transaction A: Civil Engineering
Vol. 16, No. 1, pp. 43–49

SCIENTIA
IRANICA

Identification of Inelastic Shear Frames Using the Prandtl-Ishlinskii Model

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Abstract. *In this paper, a new method is proposed for identification of inelastic shear frame structures with hesteresis, using data collected on their dynamic response. It uses the Prandtl-Ishlinskii rate independent model for hysteresis, which was originally used in the field of plasticity and ferromagnetism. The proposed identification method is capable of identifying the mass, damping and restoring force of a frame structure, which can be used in forming the equations of motion of the frame. By solving the equations of motion, the dynamic response is predicted. The method is based on the combined use of Quadratic Programming (QP) and Genetic Algorithms (GA). First, assuming a set of Prandtl-Ishlinskii constants, the QP is used to find the best frame parameters that can be used in its equations of motion to predict its dynamic response with the minimum of error compared to the real data collected on its dynamic response, while the GA is used to find the best Prandtl-Ishlinskii constants for more reduction in error. The method has been applied to different frames with bilinear nonlinearity where the results show the high capability of the method. Two examples, a Single and a Multi Degree Of Freedom (SDOF and MDOF) frame, are included in the paper.*

Keywords: *Prandtl-Ishlinskii model; Identification; Inelastic behavior; Structural dynamic; Earthquake.*

INTRODUCTION

Identification of hysteretic systems is a problem widely encountered in the field of structural dynamics. For example, the inelastic nonlinear behavior of a structure is usually seen during strong ground earthquake excitation. Due to their hysteretic behavior, the restoring forces not only relate to the current state variables, but also to the history of state variables. This phenomenon complicates the modeling and identification of hysteretic systems. Many research studies have been reported in the literature about either the modeling or identification of hysteretic systems. Some noteworthy research on this problem are mentioned in Brokate [1], Iwan [2], Bouc [3], Wen [4,5], Benedettini et al. [6], Chassiakos et al. [7], Sato and Qi [8], Smyth et al. [9] and Kosmotopoulos et al. [10].

In this study, the authors have utilized the Prandtl-Ishlinskii model in the equations of motion of shear frames, in order to identify their dynamic parameters, such as mass, damping and restoring forces, based on the measurement of displacement, velocity and acceleration at a number of its degrees of freedom, as well as the external excitations. The new identification method also uses the Genetic Algorithm (GA) to find the Prandtl-Ishlinskii constants. The proposed method has been tested for a number of frames, including Single and Multi Degree of Freedom (SDOF and MDOF) inelastic frames with bilinear nonlinearity, where the results assert the capability of the method. The method has been firstly developed for the SDOF shear frames, but then, generalized to the multi degree of freedom (MDOF) shear frames. As examples, a one story and three-story shear frame, considering a different restoring force for each story, have been used in the assessment of the proposed method, where the identification results have been satisfactory.

In the following sections, first, a brief explanation of the stop operator, the Prandtl-Ishlinskii model and the equations of motion of the SDOF frames, are included. Then, the essentials of identification methods, followed by a brief explanation of the GA, are presented. Finally, the examples and discussion of results are reported.

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Received 31 May 2006; received in revised form 17 January 2007; accepted 7 February 2007

STOP OPERATOR

Prandtl has introduced an elasto-plastic model called the "stop operator" [11]. The model can be explained by means of a heavy body connected to a spring, which can move freely on a horizontal surface, as depicted in Figure 1a. In this model, the spring stiffness is assumed to be:

$$k = 1. \tag{1}$$

Denoting x = displacement of point A, z = displacement of the body or stop operator drift and, assuming Columb's friction exists between the body and the horizontal surface, the differential equation of the stop operator is written as follows [11]:

$$f = x - z, \tag{2a}$$

$$\dot{z} = \begin{cases} \dot{x} & \text{for } |x-z| = r \text{ and } \dot{x}(x-z) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{2b}$$

where f is spring force and r = threshold > 0.

The hysteresis diagram of a stop operator is shown in Figure 1b. The input-output behavior of the stop operator can also be described in analytical form for a given piecewise monotonic input, x. Defining $t_i = i\Delta t$ and assuming x is monotonic in each interval $[t_i, t_{i+1}]$, then:

$$f(t) = \varepsilon_r[x(t)], \tag{3}$$

where r is Prandtl parameter, defined in Equation 2b, and ε_r is a symbol representing the stop operator [1], which is described by induction as:

$$f(t_{i+1}) = \min\{-r, \max\{r, [x(t_{i+1}) - x(t_i) + f(t_i)]\}, \tag{4a}$$

$$f(t_{-1}) = x(t_{-1}) = 0. \tag{4b}$$

PRANDTL-ISHLINSKII MODEL

Ishlinskii [11] developed a more general model from the stop operator called the Prandtl-Ishlinskii model. As a matter of fact, this model is a weighted superposition of a number of stop operators. The model can be formulated as follows:

$$f = \sum_{j=1}^{q+1} w_j \varepsilon_{r_j}[x], \tag{5}$$

where w_j is the weight corresponding to stop operator j, $j = 1, 2, \cdots, q+1$. The differential equation of the Prandtl-Ishlinskii model, based on Equations 2, is:

$$f(t) = \sum_{j=1}^{q+1} w_j (x - z_j), \tag{6a}$$

$$\dot{z}_j = \begin{cases} \dot{x} & \text{for } |x-z_j| = r_j \text{ and } \dot{x}(x-z_j) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{6b}$$

$$r_{q+1} = \infty. \tag{6c}$$

According to Equations 6, the Prandtl-Ishlinskii model is a rate independent model that can generate nested loops in its hysteresis diagram, where each hysteresis loop is odd symmetric, with respect to the center point of the loop.

EQUATION OF MOTION OF SDOF FRAMES WITH PRANDTL-ISHLINSKII

The equation of motion of a SDOF frame, like the one shown in Figure 2, is comprised of four parts: The inertia, damping and restoring force on the left hand side, and the excitation on the right hand side:

$$m\ddot{x} + c\dot{x} + f(t) = F(t), \tag{7}$$

where t = time, $\ddot{x}$ = acceleration, $\dot{x}$ = velocity, x = displacement, m = mass, c = damping, $f(t)$ = restoring force and $F(t)$ = external excitation. The restoring force in Equation 7 can be rewritten by using the Prandtl-Ishlinskii model, according to Equations 6,

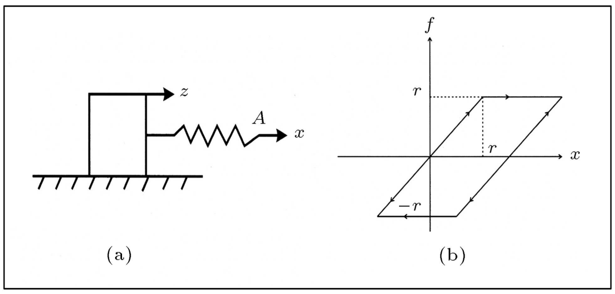

Figure 1. Prandtl's model of elasto-plasticity. (a) Stop operator; (b) The corresponding hysteresis diagram.

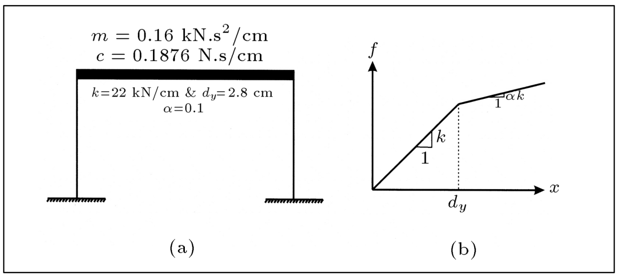

Figure 2. A SDOF frame. (a) Geometry; (b) Bilinear model.

as follows:

$$m\ddot{x}+c\dot{x}+\sum_{j=1}^{q+1} w_j(x-z_j)=F(t), \tag{8a}$$

$$\dot{z}_j=\begin{cases}\dot{x} & \text{for } |x-z_j|=r_j \text{ and } \dot{x}(x-z_j)>0\\ 0 & \text{otherwise}\end{cases} \tag{8b}$$

where w_j and z_j are weight and drift of the jth stop operator. The unknown parameters of Equations 8 are: m, c, w_j's and r_j's. After the free parameters of Equations 8 are determined, the response of the frame under any external excitation can be analyzed by solving Equations 8 by an integration scheme, such as the Runge-Kutta method.

IDENTIFICATION METHOD

For any given set of r_j values, $j=1,2,\cdots,q+1$, Equation 8a can be rewritten in discrete form as follows:

$$m\ddot{x}(t_i)+c\dot{x}(t_i)+\sum_{j=1}^{q+1} w_j\varepsilon_{r_j}[x(t_i)]=F(t_i),$$

$$i=1,2,..N, \tag{9}$$

where N is the number of sampling points of the response. Equation 9 can be written in discrete form as:

$$[A]\{\theta_r\}=\{B\}, \tag{10}$$

where:

$$\{\theta_r\}=[m,c,w_1,\cdots,w_{q+1}]_r^T, \tag{11}$$

$$\{\ddot{x}\}=[\ddot{x}_1,\ddot{x}_2,\cdots,\ddot{x}_N]^T, \tag{12}$$

$$\{\dot{x}\}=[\dot{x}_1,\dot{x}_2,\cdots,\dot{x}_N]^T, \tag{13}$$

$$[X]=[X_{ij}],$$

$$i=1,2,\cdots N \text{ and } j=1,2,\cdots,q+1, \tag{14a}$$

$$X_{ij}=\varepsilon_{r_j}[x_i], \tag{14b}$$

$$[A]=[\{\ddot{x}\},\{\dot{x}\},[X]], \tag{15}$$

$$\{B\}=[F_1,F_2,\cdots,F_N]^T. \tag{16}$$

Knowing $\ddot{x}$, $\dot{x}$, x and F for all the instances $i=1,2,\cdots,N$, the Mean Square Error (MSE) corresponding to any vector $\{\theta_r\}$, which is a quadratic function of $\{\theta_r\}$, is defined as:

$$MSE\ (\theta,r)=\frac{1}{N}\{(e_r)\}\{e_r\}^T, \tag{17}$$

where:

$$\{e_r\}=\{B\}-[A]\{\theta r\}. \tag{18}$$

It is now desired to find the vector $\{\theta_r\}=\{\theta_r^*\}$, where the value of $MSE\ (\theta,r)$ is minimum. This is an unconstrained minimization problem having only one local minimum, which is also the global minimum, because the function, $MSE\ (\theta,r)$, is quadratic. According to [12], the answer is:

$$\{\theta_r^*\}=([A]^T[A])^{-1}[A]^T\{B\}. \tag{19}$$

And the corresponding $MSE\ (\theta,r)$ is:

$$MSE^*(r)=\frac{1}{N}\{e^*(r)\}\{e^*(r)\}^T, \tag{20a}$$

where:

$$\{e^*(r)\}=\{B\}-[A]\{\theta_r^*\}. \tag{20b}$$

It is noteworthy that the vector of optimum values, $\{\theta_r^*\}$ and $MSE^*(r)$, are unique, because $MSE\ (\theta,r)$, which is a quadratic function of $\{\theta_r\}$, has only one relative minimum, which is also the global minimum.

Since the $MSE^*(r)$ is a function of r, it is now desired to find the set of r_j values, $j=1,2,\cdots,q$ (noticing $r_{q+1}=\infty$), for which the $MSE^*(r)$ is the minimum. However, in this case, it is not generally possible to assume an analytic form for the $MSE^*(r)$ function. There might be different algorithms available to solve this unconstraint optimization problem. In this paper, the real Genetic Algorithm (GA), as explained in the next section, has been used, where each chromosome (string or individual) is identified by an ordered set of genes as follows:

$$\text{chromosome}=[r_1,r_2,\cdots,r_q], \tag{21}$$

where:

$$0<r_i<|x|_{\max},\qquad i=1,2,\cdots,q, \tag{22}$$

and $|x|_{\max}$ is the maximum of the absolute value of the displacement. The fitness function for a chromosome is defined as:

$$\text{Fitness}=\begin{cases}M & \text{if } |r_i-r_j|=0\ \&\ i\neq j\\ \text{MSE}^*(r) & \text{otherwise}\end{cases} \tag{23}$$

where M is a large penalty value introduced to avoid the repetition of the same gene values in a chromosome. If the gene values are repeated, then, the matrix, $[A]^T[A]$, becomes singular, which in turn does not let the optimum weights be calculated from Equation 19.

The significant benefit of using the GA is that it is a zero-order algorithm, works with the fitness values directly and does not depend on its derivatives. A concise explanation of the GA is included in the next section. More details can be obtained from Goldberg [13].

GENETIC ALGORITHM

Genetic Algorithms (GAs) are stochastic optimization techniques, based on the concepts of biological evolutionary theory [13,14]. They consist of maintaining a population of chromosomes (individuals), which represent potential solutions to the optimization of a function, generally very complex. Each individual in the population has an associated fitness, indicating the utility or adaptation of the solution that it represents.

A GA starts off with a population of randomly generated chromosomes and advances towards better chromosomes by applying genetic operators, generally modeled on the genetic processes occurring in nature. During successive iterations, called generations, the chromosomes are evaluated as possible solutions. Based on these evaluations, a new population is formed using a mechanism of selection through applying the so called genetic operators, including crossover and mutation.

In this paper, a real code GA is utilized. Its outline is described in the following steps:

1. Randomly create an initial population of chromosomes;
2. Compute the fitness of the members of the current population and sort them according to their fitness values, with the fittest assigned as the first;
3. Select parents to mate and produce newborns based on their fitness values. Children are produced either by making random changes to a single parent (mutation) or by combining the genes of a selected pair of parents (crossover);
4. Replace the current population with the children to form the next generation;
5. If the stopping criteria are met, the algorithm will be stopped. Otherwise, go back to step 2.

Three kinds of children are produced in each generation: Elite children, crossover children and mutation children. Elite (cloned) children are the best individuals that are cloned from the current population to the next population. The number of elites, (N_{Elite}), crossover, ($N_{\text{Crossover}}$), and mutation, (N_{Mutation}), children are set in advance and, as a result, the population size will be $N_{\text{pop}} = N_{\text{Elite}} + N_{\text{Crossover}} + N_{\text{Mutation}}$. The following genetic operators are used.

Roulette-Wheel Selection

Chooses parents by simulating a roulette wheel, in which the area of the wheel is divided into smaller areas representing the individual's expectation of selection. The algorithm uses a random number to select one of the sections with a probability equal to its area. Each individual's expectation is computed by the following formula:

$$p_i = \frac{\frac{1}{\sqrt{R_i}}}{\sum_{k=1}^{N_{\text{pop}}} \frac{1}{\sqrt{R_k}}}, \qquad i = 1, 2, \cdots, N_{\text{pop}}, \tag{24}$$

where p_i is ith individual's expectation and R_i is the rank of the ith individual in the current population, noting that the rank of the best individual is 1.

Intermediate Crossover

Creates children by taking a weighted average of the parents. Each child is created from parent1 and parent2 by child = parent1 + rand $\times$ 0.5 $\times$ (parent2 - parent1), where rand is a random number in [0,1]. Therefore, for producing $N_{\text{Crossover}}$ children, $2N_{\text{Crossover}}$ parents must be selected by the selection operator.

Uniform Mutation

Uniform mutation is a two-step process. First, the algorithm selects N_{Mutation} individuals for mutation, where each gene has a probability rate of being mutated. In the second step, the algorithm replaces each selected gene by a random number, which is uniformly generated in the range defined for that gene.

Stopping Criteria

In this paper, the authors have used a simple stopping criterion. The algorithm stops whenever the generation number reaches a predefined maximum number of iterations, denoted by Max_{iter}.

EXAMPLE 1

A SDOF Frame

It is desired to identify the structural parameters of the SDOF frame shown in Figure 2a. The restoring force-displacement relationship follows the bilinear nonlinearity model shown in Figure 2b. In a real test, the frame would be subjected to some known forces, F, and the response would be measured. Hence, the history of response and force, including $\ddot{x}$, $\dot{x}$, x and F, are recorded. In this study, the experiment has been replaced with numerical simulation, where the frame has been modeled, subjected to different excitations and analyzed by the Wilson method of integration. The identification method presented in this paper has then been applied to the results of the numerical simulation and the structure has been identified. The identification results have been compared to the assumed structural parameters for the assessment of the capability of the identification method.

The following excitation has been applied to the frame:

$$F(t) = 15t\sin(10t)\ \text{kN}, \tag{25}$$

where the corresponding response and hysteresis loop are shown in Figure 3. The acceleration, velocity and displacement signals and the corresponding hysteresis loop are shown in Figure 3. It has been shown that bilinear nonlinearity can be model by Equation 5 with $q = 1$ [1], therefore, in this example, it is enough to set $q = 1$. By applying the proposed identification method with $N_{\text{Elite}} = 2$, $N_{\text{Crossover}} = 14$, $N_{\text{Mutation}} = 4$, $\text{Max}_{\text{iter}} = 100$ and rate $= 0.04$, the following parameters have been obtained for this frame: $r_1 = 2.7985$ cm, $m = 0.1602$ kg, $c = 0.1878$ kN.s/cm, $w_1 = 19.8072$ kN/cm, and $w_2 = 2.2181$ kN/cm, where the errors of identification have been (0.1602-0.16)/0.16=0.12% for m and (0.1878-0.1876)/0.1876= 0.11% for c. The convergence progress of the identification method is depicted in Figure 4, where the convergence has been achieved only in 14 generations. The accuracy of identification of restoring force has been evaluated by using the obtained values for m, c, the Prandtl-Ishlinskii stop parameter, r_1, and weights w_1 and w_2, in the equation of motion of the SDOF frame (Equations 8), solving the equations by the Runge-Kutta method and, finally, comparing the results with the results obtained from solving the same equations but with the assumed target values for the parameters. Hence, the frame was subjected to 200% of the El Centro earthquake, as shown in Figure 5. The responses are the same for both the identified and target parameters.

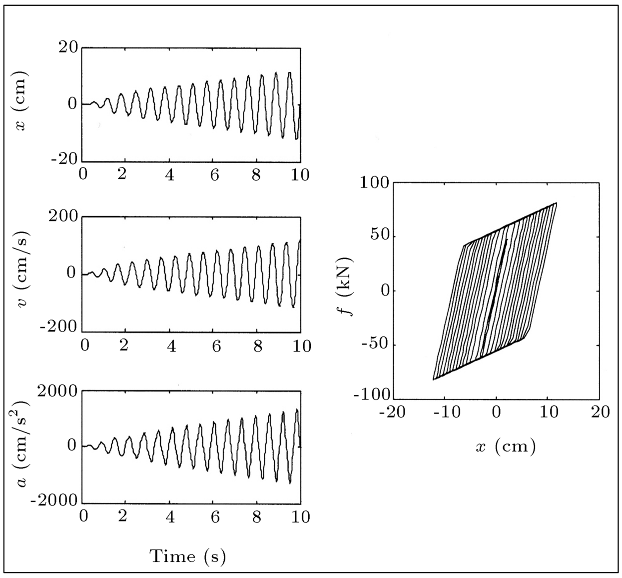

Figure 3. Response of the SDOF frame, excited by $F(t) = 15t\sin(10t)$ kN.

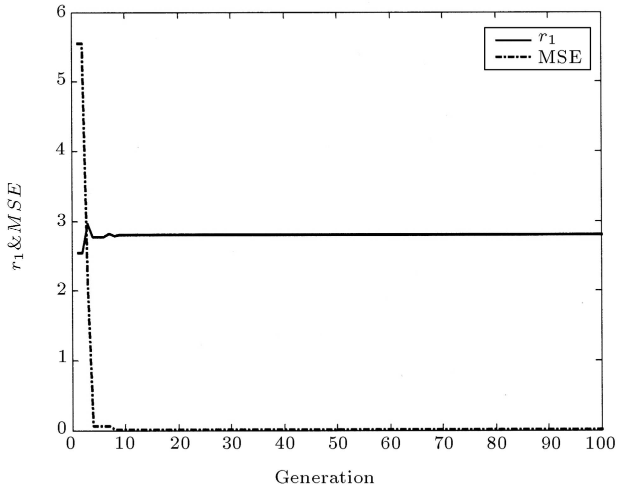

Figure 4. Convergence characteristics of the GA.

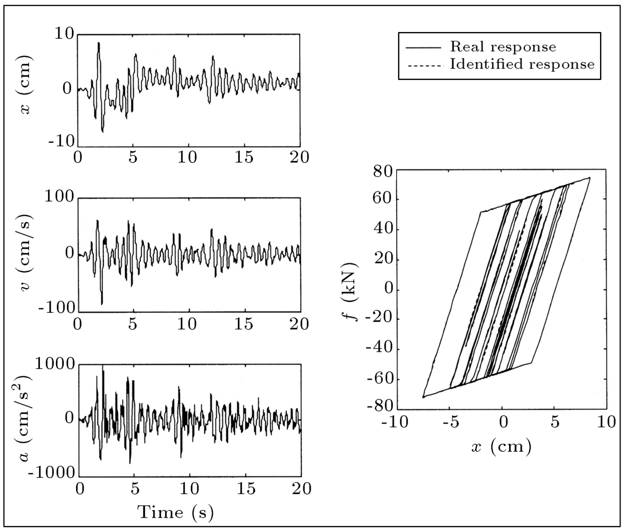

Figure 5. Comparison between target (real) and identified responses of the SDOF frame excited by 200% El Centro earthquake, where the difference is not significant.

EXAMPLE 2

A MDOF Frame

MDOF shear frames can also be identified by the method proposed in this paper. For example, a three-story shear frame and its parameters are shown in Figure 6. The set of equations of motion of this frame is:

$$m_3\ddot{x}_3 + c_3\dot{\delta}_3 + f_3 = F_3(t), \tag{26a}$$

$$m_2\ddot{x}_2 + c_2\dot{\delta}_2 + f_2 - c_3\dot{\delta}_3 - f_3 = F_2(t), \tag{26b}$$

$$m_1\ddot{x}_1 + c_1\dot{\delta}_1 + f_1 - c_2\dot{\delta}_2 - f_2 = F_1(t), \tag{26c}$$

where f_1, f_2 and f_3 are the restoring forces of the first, second and third story, respectively. The restoring

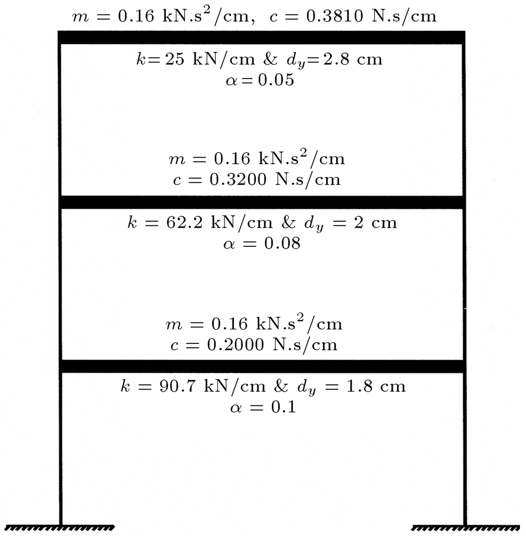

Figure 6. Three story frame of Example 2, where structural parameters are different for each story.

force, f_k, $k = 1, 2, 3$, can be obtained according to Equation 5 as follows:

$$f_k = \sum_{j=1}^{q+1} w_j^k \varepsilon_{r_j^k}[\delta_k]. \tag{27}$$

Using Equations 27 and 6:

$$f_k = \sum_{j=1}^{q+1} w_j^k (\delta_k - z_j^k), \tag{28a}$$

$$\dot{z}_j^k = \begin{cases} \dot{\delta}_k & \text{for } |\delta_k - z_j^k| = r_j^k \text{ and } \dot{\delta}_k(\delta_k - z_j^k) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{28b}$$

where δ_k and $\dot{\delta}_k$, $k = 1, 2, 3$ are the interstory displacement and velocity for the kth story, respectively. Equations 26 and 27 are utilized for identification, while Equations 26 and 28 are then used for analyzing the response of the identified frame.

Each of the equations in Equations 26, from (a) to (c), is in fact, the same as the equation of motion (Equation 7) for a SDOF frame. Equation 26a can be solved to find f_3, f_3 is substituted in Equation 26b to find f_2 and, then, f_2 is used with Equation 26c to find f_1. This method can be generalized by induction for a shear frame of n stories. Hence, the equation of motion of a shear frame can be summarized as:

$$m_n \ddot{x}_n + c_n \dot{\delta}_n + f_n = F_n(t), \tag{29a}$$

$$m_k \ddot{x}_k + c_k \dot{\delta}_k + f_k - c_{k+1}\dot{\delta}_{k+1} - f_{k+1} = F_k(t), \tag{29b}$$

where $k = 1, 2, \cdots, n-1$.

The problem is solved for the nth story first and then proceeds from the top to the bottom of the frame. In the three-story frame, therefore, the free parameters of the third story are tuned firstly and these parameters are used for identification of the second story and so on.

To collect appropriate data for identification, the frame shown in Figure 6 was excited by the following story forces:

$$F_1(t) = 50\sin(7t) \text{ kN}, \tag{30a}$$

$$F_2(t) = -75\sin(5t) \text{ kN}, \tag{30b}$$

$$F_3(t) = 50\sin(2.5t) \text{ kN}. \tag{30c}$$

The response of the frame was then recorded. Similar to the SDOF frame, mass (m), damping (c), r, and two weights, w_1 and w_2, were considered for each of the floors. The Least Square Error and GA were applied to the *MSE* and the story parameters were identified for each floor. Table 1 shows the identified parameters of the frame. The identified parameters are very close to the target parameters, which are specified in Figure 6. The displacement of the roof of the identified frame, excited by 200% of the El Centro earthquake, was compared with the real displacement in Figure 7. In this case too, similar to the SDOF frame, their error is not significant.

CONCLUSIONS

In this paper, the authors have proposed a useful identification method to determine the parameters of

Table 1. Tuned parameters.

k	m (kN.s^2/cm)	c (kN.s/cm)	r (cm)
1	0.1599	0.3876	1.7990
2	0.1600	0.3210	1.9994
3	0.1600	0.1996	2.7989

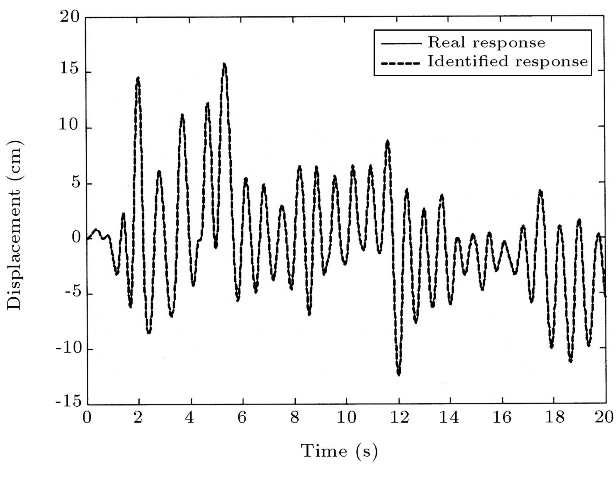

Figure 7. Comparison between the target (real) and identified time history response of the roof displacement of Example 2 under 200% El Centro earthquake.

shear frames with bilinear nonlinearity and the hysteretic response from the data collected on the forced vibration response of the frames. The method uses Genetic Algorithms (GA) and Quadratic Programming (QP). The main benefit of using GA is that it rapidly lets the identification method escape from the local minima and convergence to the final answer. Through numerical simulations, the method has been applied to both Single Degree Of Freedom (SDOF) and Multi-Degrees of Freedom (MDOF) frames, where the obtained identification results have been very accurate.

ACKNOWLEDGMENT

The authors would like to thank the Deputy of Higher Education and, also, the Research Deputy of Sharif University, Tehran, Iran for partially supporting this research.

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