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Application of Particle Swarm Optimization to Optimal Design of Cascade Stilling Basins

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Abstract. *This paper employs the Particle Swarm Optimization (PSO) method to solve the problem of the optimal design of cascade stilling basins. PSO is a relatively recent heuristic search method whose mechanism is inspired by the swarming or collaborative behavior of biological populations. The objective of this research is to minimize the total construction cost of cascade stilling basins, which is a function of height of the falls and length of stilling basins, while fulfilling the hydraulic and topographical criteria. To illustrate the application of PSO, a benchmark design is taken from the work of Vittal and Porey [1] on a cascade stilling basin for the Tehri Dam, India.*

Keywords: *Particle swarm optimization; Cascade stilling basins; Global optimization.*

INTRODUCTION

Energy dissipation below hydraulic structures, like dams, barrages across rivers and control weirs on canals, is accomplished conventionally by single-fall hydraulic jump-type stilling basins and roller buckets on trajectory buckets. However, in the case of high head dams, the kinetic energy at the toe of the spillways is very high and the tailwater depths available in the river are often inadequate for the former two devices. Narrow and curved gorges, consisting of fractured rock, prohibit the adoption of the last. In such situations, especially for earth and rock fill dams, a system of falls cascading down the side of a valley, with a stilling basin below each fall, can be used as an alternative spillway. The first design method for this system was developed by Vittal and Porey [1]. In this method, the height of the lowest fall is determined by the available river tailwater depth at design discharge, whereas the number of preceding equal-height falls are determined by the available distance for the spillway sections and stilling basins. Thus, it seems to be empirical to some extent and does not lead to an optimal design, so, it is necessary to use an alternative approach to obtain an optimal solution. In this paper, a PSO algorithm is proposed for the optimal design of cascade stilling basins. The results of the PSO model are compared with the results generated from the Vittal and Porey method. It is also shown that the PSO approach can produce better designs than those of the Vittal and Porey method, in almost all the cases considered. This paper is organized in five major sections. First, the PSO algorithm is summarized. Second, the Vittal and Porey method is explained. Next, an optimization model of cascade stilling basins, using the PSO algorithm, is introduced, and a case study and conclusions are presented in the last two sections.

PARTICLE SWARM OPTIMIZATION

In 1995, Russell Eberhart and James Kennedy [2] applied a model to the problem of finding optima in a search space, which can be compared to a flock of birds looking for a food source, and created the PSO algorithm. The literature describing the application of PSO to water engineering is not abundant. Gill et al. [3] described a multi-objective optimization approach using PSO for parameter estimation in hydrology.

Suribabu [4] applied a PSO for deriving operation policies for maximum hydropower generation. Also, in 2006, Suribabu and Neelakantan [5] used PSO to the optimal design of water distribution networks. Finally, Meraji and Afshar [6] used the algorithm to the reservoir operation of the Dez Dam in Iran. Also, they combined the PSO optimizer with an SWMM simulator

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to develop a model for the optimal design of a flood control system.

In PSO, a collection of particles, called a "swarm", move around in search space looking for the best solution to an optimization problem. All particles have their own velocity that drives the direction they move in. This velocity is affected by both the position of the particle with the best fitness and each particle's own best fitness. Fitness refers to how well a particle performs. In a flock of birds, this might be how close a bird is to a food source; in an optimization algorithm, fitness is a function of the objective function. Each particle's location is given by the parameters of the given optimization problem and a particle moves around in search space by adapting and changing these parameter values. At each time step, the particle's fitness is measured by observing the parameter values (location) of the particle. A particle keeps track of the best position it has reached so far (called the personal best position) and is also aware of the position of the overall best particle at a certain time step (called the globally best position). At each time step, the particle tries to adapt its velocity (i.e. speed and direction) to move closer to both the globally best position and the personal best position, in order to try and improve its fitness. Two variants of the PSO algorithm were developed; one with a global neighborhood and one with a local neighborhood. According to the global variant, each particle moves towards its best previous position and towards the best particle in the whole swarm. On the other hand, according to the local variant, each particle moves towards its best previous position and towards the best particle in its restricted neighborhood [2]. In the following paragraphs, the global variant is exposed (the local variant can be easily derived through minor changes).

Suppose that the search space is D-dimensional, then, the ith particle of the swarm can be represented by a D-dimensional vector, $X_i = (x_{i1}, x_{i2}, \cdots, x_{iD})^T$. The velocity (position change) of this particle can be represented by another D-dimensional vector, $V_i = (v_{i1}, v_{i2}, \cdots, v_{iD})^T$. The best previously visited position of the ith particle is denoted as $P_i = (p_{i1}, p_{i2}, \cdots, p_{iD})^T$. Define g as the index of the best particle in the swarm (i.e., the gth particle is the best) and let the superscripts denote the iteration number, then, the swarm is manipulated according to the following two equations [2]:

$$v_{id}^{n+1} = v_{id}^n + cr_{1,i,d}^n (p_{id}^n - x_{id}^n) + cr_{2,i,d}^n (p_{gd}^n - x_{id}^n), \qquad (1)$$

$$x_{id}^{n+1} = x_{id}^n + v_{id}^{n+1}, \qquad (2)$$

where $d = 1, 2, \cdots, D$; $i = 1, 2, \cdots, N$, and N is the size of the swarm; c is a positive constant, called the acceleration constant; $r_{1,i.d}$, $r_{2,i,d}$ are random numbers, uniformly distributed in $[0, 1]$; and $n = 1, 2, \cdots$ determines the iteration number.

Equations 1 and 2 define the initial version of the PSO algorithm. Since there is no actual mechanism for controlling the velocity of a particle, it was necessary to impose a maximum value, $V_{d,\max}$, on it (i.e. $-V_{d,\max} \leq V_{id}^{n+1} \leq V_{d,\max}$). If the velocity exceeded this threshold, it was set equal to $V_{d,\max}$. This parameter proved to be crucial, because large values could result in particles moving past good solutions, while small values could result in insufficient exploration of the search space. The value of $V_{d,\max}$ is usually chosen to be $K \times X_{d,\max}$ with $0.1 \leq k \leq 1.0$, where $X_{d,\max}$ is the upper bound of the search space of particles in the dth dimension [7]. This lack of a control mechanism for the velocity resulted in low efficiency for PSO, compared to EC techniques [8]. Specifically, PSO located the area of the optimum faster than evolutionary computation techniques, but once in the region of the optimum, it could not adjust its velocity step size to continue the search at a finer grain. The aforementioned problem was addressed by incorporating a weight parameter for the previous velocity of the particle. Thus, in the latest versions of the PSO, Equations 1 and 2 are changed to the following ones [9,10]:

$$v_{id}^{n+1} = \chi(wv_{id}^n + c_1 r_{1,i,d}^n (p_{id}^n - x_{id}^n) + c_2 r_{2,i,d}^n (p_{gd}^n - x_{id}^n)), \qquad (3)$$

$$x_{id}^{n+1} = x_{id}^n + v_{id}^{n+1}, \qquad (4)$$

where w is called inertia weight; c_1 and c_2 are two positive constants, called cognitive and social parameter, respectively; and χ is a constriction factor, which is used alternatively to w and to limit velocity. The role of these parameters is discussed in the next section.

The Parameters of PSO

The role of the inertia weight, w, in Equation 3, is considered critical for the PSO's convergence behavior. The inertia weight is employed to control the impact of the previous history of velocities on the current one. Accordingly, the parameter, w, regulates the trade-off between the global and local exploration abilities of the swarm. A large inertia weight facilitates global exploration (searching new areas), while a small one tends to facilitate local exploration, i.e. fine-tuning the current search area. A suitable value for the inertia weight, w, usually provides a balance between global and local exploration abilities and, consequently, results in a reduction of the number of iterations required to locate the optimum solution. Initially, the inertia weight was constant. However, experimental results indicated that it is better to initially set the inertia to a large value, in order to promote global exploration of the search space, and gradually decrease it to get more refined solutions. Thus, Shi and Eberhart [9,10] made a significant improvement in the performance of the PSO, with a linearly varying inertia weight over the iterations, which linearly varies from $w_{\max}$ at the beginning of the search to $w_{\min}$ at the end. Thus, the following weighting function is usually utilized in Equation 3:

$$w = w_{\max} - \frac{(w_{\max} - w_{\min}) \times n}{\text{iter}_{\max}}, \tag{5}$$

where $w_{\max}$ and $w_{\min}$ are the maximum and minimum value of inertia weight, respectively, n is the current iteration number and $\text{iter}_{\max}$ is the maximum iteration number. The parameters c_1, c_2 in Equation 3 are not critical for PSO's convergence. However, proper fine-tuning may result in faster convergence and alleviation of local minima. An extended study of the acceleration parameter in the first version of PSO is given in [11]. As default values, $c_1 = c_2 = 2$ were proposed, but experimental results indicate that $c_1 = c_2 = 0.5$ might provide even better results. Recent work reports that it might be even better to choose a larger cognitive parameter, c_1, than a social parameter, c_2, but with $c_1 + c_2 \leq 4$ [12]. The parameters r_1 and r_2 are used to maintain the diversity of the population and they are uniformly distributed in the range [0, 1]. The constriction factor, χ, controls the magnitude of the velocities, in a way similar to the $V_{d,\max}$ parameter, resulting in a variant of PSO different from the one with the inertia weight.

VITTAL & POREY DESIGN PROCEDURE

The design of cascade stilling basins was first introduced by Vittal and Porey in 1987 [1]. In the following paragraphs, the considerations and procedure for the design of cascade stilling basins, as well as the necessary relationships for design, are presented. The procedure for the design of cascade stilling basins can be summarized as:

1. Determination of the height and the length of the terminal fall and the proportioning of a suitable stilling basin for it;
2. Determination of the number and nature of the preceding falls;
3. Determination of the height of the raised crest for the preceding falls.

Terminal Fall

The height of terminal fall H_t (the difference in the levels of the terminal crest and river bed in Figure 1) is determined, such that the post jump depth of flow for hydraulic jump formation at the design discharge is equal to the tailwater depth available in the river. This will avoid deep excavation of the river bed, which would be expensive and might induce dangerous landsliding of the valley slope.

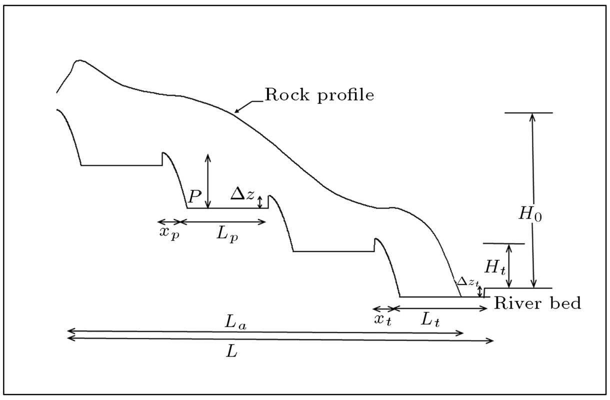

Figure 1. Longitudinal section of cascade of falls.

$$H_t = \frac{g y_{td}^4}{7.80 q_d^2}, \tag{6}$$

where q_d is the unit design discharge, y_{td} is the tailwater depth at the design discharge and g is the acceleration due to gravity. The deficiency or excess of tailwater at partial discharge can be known by comparing the Free-Jump-Height Curve (FJHC) for the terminal fall with the Tailwater Rating Curve (TWRC) of the river. In the event of a tailwater excess, the stilling basins need not be depressed, whereas, in the event of a deficiency, the floor will be lowered by Δz_t, equal to the maximum difference in the ordinates of FJHC and TWRC at partial discharge. With the drop in the floor level, the height of the terminal crest above the stilling basin floor, P_t, will now be replaced by $H_t + \Delta z_t$. The length of the stilling basin for the terminal fall will vary according to the Froud number [13]:

$$L_t = \begin{cases} 4.25 y_{2d} & F_{r1} \geq 4.5 \\ 2.80 y_{2d} & F_{r1} < 4.5 \end{cases} \tag{7}$$

where L_t is the length of the stilling basin for terminal fall, y_{2d} is the post jump depth of flow at the design discharge and the Froude number is the pre jump Froude number for the last cascade, which may be computed from Equation 8 by trial and error [13]:

$$\frac{g^{\frac{1}{2}} P_t^{\frac{3}{2}}}{q} = \left(\frac{1}{2} \text{Fr}_1^{\frac{4}{3}} + \text{Fr}_1^{-\frac{2}{3}} - \frac{1}{2^{\frac{1}{3}} C^{\frac{2}{3}}} \right)^{\frac{3}{2}}, \tag{8}$$

where C is the discharge coefficient.

Preceding Falls

The length required to accommodate all the spillway section and stilling basins, (L), is:

$$L = (N-1)(x_p + L_p) + (x_t + L_t), \tag{9}$$

in which x_t and x_p are the base widths of the spillway sections; L_t and L_p are the lengths of the stilling basins for the terminal fall and preceding falls, respectively; and N is the number of cascades. Adopting the ogee profile given by [13], one obtains the following:

$$\frac{y}{h_{0d}} = 0.50\left(\frac{x}{h_{0d}}\right)^{1.85}, \tag{10}$$

in which x and y are the coordinates of the spillway profile and h_{0d} is the total head over crest at the design discharge. The following equations can be written for x_t and x_p:

$$x_t = 1.455 h_{0d}\left(\frac{P_t}{h_{0d}}\right)^{\frac{1}{1.85}}, \tag{11}$$

$$x_p = 1.455 h_{0d}\left(\frac{P}{h_{0d}}\right)^{\frac{1}{1.85}}. \tag{12}$$

The equation suggested by Poggi [14] for the floor length of stilling basins without appurtenance can be adopted for a cascade system:

$$L_p = 6(y_2 - y_1), \tag{13}$$

where y_1 and y_2 are the pre-jump and post-jump depths of flow, respectively.

The height of the crest above the stilling basin floor, (P), for the preceding falls of equal height can be calculated from Equation 14 [1]. Assuming a known value of N, Equation 14 can be solved for P by trial and error:

$$P = \frac{H_0 - H_t}{N} + 1.671\frac{q_d^{\frac{1}{2}} P^{\frac{1}{4}}}{g^{\frac{1}{4}}} - \left(\frac{q_d}{C\sqrt{2g}}\right)^{\frac{2}{3}} + 0.179\frac{q_d}{g^{\frac{1}{2}} P^{\frac{1}{2}}}. \tag{14}$$

To force the jump, a control or crest, preferably of ogee profile, is placed at the end of the floor. The required height of crest Δz for jump formation at the design discharge is given by:

$$\Delta z = P - \frac{H_0 - H_t}{N-1}. \tag{15}$$

Here, P is computed from Equation 14, H_0 is the total fall and H_t is the height of the terminal fall.

OPTIMAL DESIGN OF CASCADE STILLING BASINS

The aim of the PSO model is to minimize the construction cost of the system by changing the design variables, i.e. height of falls and length of stilling basins, while fulfilling the topographical and hydraulic criteria. The design principles used in the optimization model are those of the Vittal and Porey method, with the difference that, in the Vittal and Porey method, the length and the height of the preceding falls are equal, thus, it is not necessarily optimal. However, the PSO model can choose different values for the height and length of the falls in order to design a system with optimum cost. The optimization model can be mathematically stated as follows:

$$\text{Minimize } f = \sum_{i=1}^{N} (f_1(P_i, \ell_i) + f_2(P_i, \ell_i)), \tag{16}$$

where f is the total cost of construction, which is a function of the design variables, and f_1 and f_2 are the concrete and excavation costs of the ith cascade, respectively, P_i is the height of the ith fall, l_i is the length of the ith fall, and N is the total number of cascades. The general constraints of the system are topographical and hydraulic as explained in the following paragraphs.

Topographical Constraints

$$\sum_{i=1}^{N} (L(i) + x(i)) = L_a, \tag{17}$$

$$H_0 - \sum_{i=1}^{N} (P_i - \Delta z(i)) - \Delta z_t = 0, \tag{18}$$

where L_a is the total length available, which is the horizontal distance between the center point of the first fall and the terminal point of the last basin; Δz_t is equal to the maximum difference in the ordinates of FJHC and TWRC at partial discharge; and $\Delta z(i)$ is the height of the crest for the ith fall. The required height of crest $\Delta z(i)$ for jump formation at the ith fall and the design discharge is defined as follows [1]:

$$\Delta z(i) = 1.671\frac{q_d^{0.5} P_i}{g^{\frac{1}{4}}} - \left(\frac{q_d}{C\sqrt{2g}}\right)^{\frac{2}{3}} + 0.179\frac{q_d}{g^{\frac{1}{2}} P_i^{\frac{1}{2}}},$$

$$\Delta z(N) = 0, \tag{19}$$

where C is the discharge coefficient, q_d is the unit design discharge and P_i is the height of the ith fall.

Hydraulic Constraints

Maximum and Minimum Height of the Fall

$$P_{\min} \le P_i \le P_{\max}. \tag{20}$$

$P_{\max}$ and $P_{\min}$ are the maximum and minimum allowable heights of the falls, respectively, which have been calculated using the maximum and minimum pre-jump Froude numbers of the flow in the corresponding stilling basins. For the range of Froud numbers of the incoming flow between 4.5 and 9, a stable and well-balanced jump occurs. Turbulence is confined to the main body of the jump and the water surface downstream is comparatively smooth [13]. Thus, $\mathrm{Fr}_{1,\max} = 9$ and $\mathrm{Fr}_{1,\min} = 4.5$:

$$P_{\max} = \frac{q_d^{\frac{2}{3}}}{g^{\frac{1}{3}}}\left(\frac{1}{2}\mathrm{Fr}_{1\max}^{\frac{4}{3}} + \mathrm{Fr}_{1_{\max}}^{\frac{-2}{3}} - \frac{1}{2^{\frac{1}{3}}C^{\frac{2}{3}}}\right), \tag{21}$$

$$P_{\min} = \frac{q_d^{\frac{2}{3}}}{g^{\frac{1}{3}}}\left(\frac{1}{2}\mathrm{Fr}_{1_{\min}}^{\frac{4}{3}} + \mathrm{Fr}_{1_{\min}}^{\frac{-2}{3}} - \frac{1}{2^{\frac{1}{3}}C^{\frac{2}{3}}}\right). \tag{22}$$

Minimum Length of Stilling Basins

$$l_i \ge l_{i,\min}, \tag{23}$$

where $l_{i,\min}$ is the minimum allowable length of the falls and is determined based on the so-called throw length of the jet and the necessary length of a hydraulic jump.

$$l_{i,\min} = 6(y_{2,i} - y_{1,i}). \tag{24}$$

with:

$$h_{0D} = \left(\frac{q_d}{C\sqrt{2g}}\right)^{\frac{2}{3}}, \tag{25}$$

$$y_{1,i} = \left(\frac{q_d}{g^{\frac{1}{2}}\mathrm{Fr}_{1i}}\right)^{\frac{2}{3}}, \tag{26}$$

$$y_{2,i} = \frac{y_{1,i}}{2}\left(\sqrt{1 + 8\mathrm{Fr}_{1i}^2} - 1\right). \tag{27}$$

$y_{1,i}$ and $y_{2,i}$ are the pre-jump and post-jump depths of flow, respectively; $\mathrm{Fr}_{1,i}$ is the pre-jump Froude number in the ith fall; and C is the discharge coefficient.

Minimum Height of Terminal Crest Above Stilling Basins Floor

$$P(N) \ge H_t + \Delta z_t. \tag{28}$$

The basin must be made deep enough to provide for the full post-jump depth of flow (or some greater depth, to include a factor of safety) at maximum spillway design discharge. A tailwater depth greater than the required post-jump depth is conducive to the formation of a so-called drowned jump (with the drowned jump, instead of achieving a good-type dissipation by intermingling of the upstream and downstream flows, the incoming jet plunges to the bottom and carries along the entire length of the basin floor at high velocity). The above constraint assures that the required post-jump depth will be always greater than the tailwater depth.

The problem's constraints now can be written in standard form as:

$$g_1 = H_0 - \sum_{i=1}^{N}(P_i - \Delta z(i)) - \Delta z_t = 0, \tag{29}$$

$$g_2 = L_a - \sum_{i=1}^{N}(L(i) + x(i)) = 0, \tag{30}$$

$$g_3 = 1 - \frac{P_i}{P_{\max}} \le 0, \tag{31}$$

$$g_4 = 1 - \frac{P_i}{P_{\min}} \le 0, \tag{32}$$

$$g_5 = 1 - \frac{l_i}{l_{i\min}} \le 0, \tag{33}$$

$$g_6 = 1 - \frac{H_t + \Delta z_t}{P(N)} \le 0. \tag{34}$$

CASE STUDY

In this section, both the Vittal and Porey method and the proposed PSO model are used for the design of the Tehri dam spillway and the results are compared. The Tehri dam is an earth and rockfill dam, 260.50 m high, on the river Bhagirathi, a tributary of the river Ganga valley of the central Himalayan region of India. The spillway is located on the right abutment of the dam. At the dam site, the exposed rocks are alternate bands of weak quartzites and phyllites. Various alternatives for the type of spillway were considered. A single-stage hydraulic jump-type stilling basin involves a velocity of 66.00 m/s in the basin and a 15.00 m riverbed excavation to make up for the deficiency of tailwater depth. A chute spillway, followed by a ski-jump bucket, throws trajectories on the hill slopes of the narrow valley, and the rocks cannot withstand significant impact. Further, the situation of the hills, due to the spray, may result in sheet landslides. Thus, a spillway with a cascade of falls and stilling basins will be adopted. A 95.00 m wide control structure, consisting of five bays of 16.00 m each, separated by 3.75 m thick piers, is provided with a full-reservoir level of 818.00 m. Other design data are listed in Table 1.

Table 1. Design data for Tehri dam.

Characteristic	Datum
Design discharge, Q_d	11000.00 m^3/s
Total fall, H_0	218.00 m
River level at exit	600.00 m
Length of spillway crest at lower falls	95.00 m
Tailwater depth at design discharge, y_{td}	29.20 m
Distance available between first crest and exit, L_a	778 m

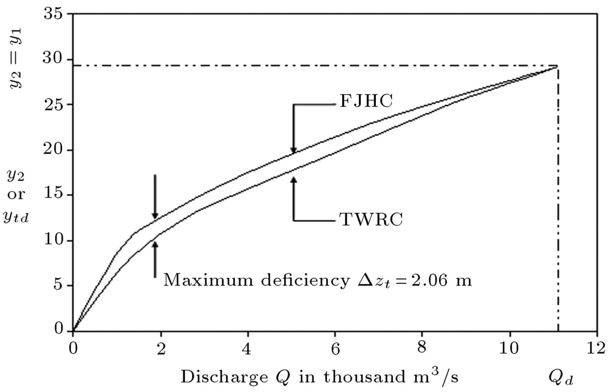

Figure 2. TWRC and FJHC for terminal fall - Tehri dam spillway.

As seen in Figure 2, the TWRC is entirely below the FJHC at partial discharge, and the maximum tailwater deficiency of 2.06 m occurs at 1960 m^3/s. Hence, $\Delta z_t = 2.06$ m.

Considering the site condition and its type, there exist just two types, $N = 3$ and $N = 4$, in the feasible space. In the following, the results of the design by the Vittal and Porey and PSO methods are compared (all dimensions in meters). It is notable that, in this study, the concrete and excavation costs per cubic meter are considered to be 180000 and 23100, respectively.

PSO has several explicit parameters, whose values can be adjusted to produce variations in the way the algorithm searches the solution space. Shi and Eberhart [9,10] tried to examine the parameter selection of these parameters. According to their examination, the following parameters are appropriate and adopted in this paper:

$$W_{\max} = 0.9, \qquad W_{\min} = 0.4, \qquad c_1 = c_2 = 2.0.$$

In the following, the results are presented.

Table 2 compares the results obtained by the proposed PSO algorithm with that of Vittal and Porey. Table 3 shows the maximum, minimum, average and standard deviations of the solution costs obtained in 10 runs on a Pentium 4 with a CPU of 2.40 GHz and 512 MB of RAM. As mentioned above, for each swarm size, the model has been run ten times and the best solution is selected as an optimal cost, as presented in Table 4. As can be seen from Table 4, in both cases the PSO has produced superior results to the Vittal and Porey method. The savings offered by the PSO are about 22 and 17 percent for the number of cascades equal to 3 and 4, respectively. Figures 3 and 4 schematically compare the PSO and Vittal and Porey solutions for $N = 3$ and $N = 4$, respectively. As can be seen clearly in both cases, PSO has chosen a smaller height for the first cascade than that of Vittal and Porey.

There is no rule as to how many particles should be used to solve a specific problem. A large number of particles allow the algorithm to explore the search space faster; however, the fitness function needs to be evaluated for each particle, so the number of particles will have a huge impact on the speed at which the simulation will run. Here, a sensitivity analysis is carried out on the swarm size for a fixed number of

Table 2. Comparisons of the results obtained by PSO and vittal and porey method.

Cascade No.	P		L		XP		Δz	
	PSO	V.P.	PSO	V.P.	PSO	V.P.	PSO	V.P.
$N = 3$								
1	62.28	93.55	304.88	175.39	48.98	58.15	15.48	17.80
2	92.39	93.55	173.95	175.39	57.68	58.15	17.67	17.80
3	92.39	66.87	134.26	125.04	57.68	46.19	0	0
$N = 4$								
1	32.40	65.57	138.27	156.61	32.73	48.06	10.86	15.25
2	43.86	65.57	143.36	156.61	38.56	48.06	12.61	15.25
3	92.39	65.57	174.26	156.61	57.68	48.06	17.67	15.25
4	92.39	66.87	134.89	125.04	57.68	49.16	0	0

P: Height of crest above stilling basin floor; Δz: Rise of crest
L: Length of stilling basin; XP: Base width of spillway section

Table 3. Solution costs and average run time obtained in 10 runs.

Population	Maximum (10^6)	Minimum (10^6)	Average (10^6)	Standard Deviation (10^6)	CPU Time/Run (sec)
$N = 3$					
25	1063.4	979.1	1014.2	28.6	70
50	1143.7	986.9	1025.8	48.6	68
100	1080.9	982.8	1019.3	30.4	67
125	1051.9	976.8	1013.1	24.5	70
150	1063.6	940.1	1008.6	37.2	71
200	1054.9	983.5	1014.5	26.6	66
$N = 4$					
25	1025.5	930.3	976.8	31.3	87
50	1034.1	917.8	989.1	34.8	85
100	1025.1	922.1	966.5	31.1	88
125	1029.4	951.9	996.3	26.9	89
150	1049.1	914.8	985.7	48.5	87
200	1063.7	943.4	983.3	36.5	86

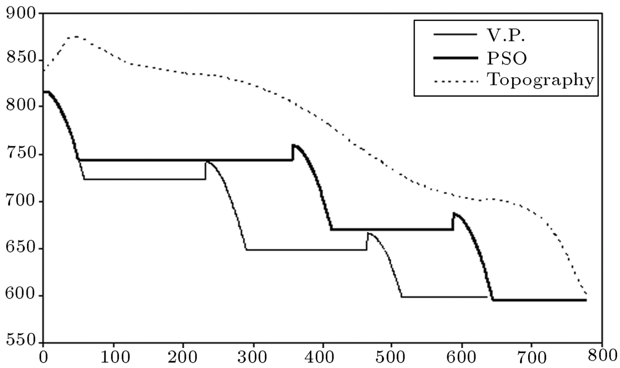

Figure 3. Comparison between PSO and Vittal and Porey designs ($N = 4$).

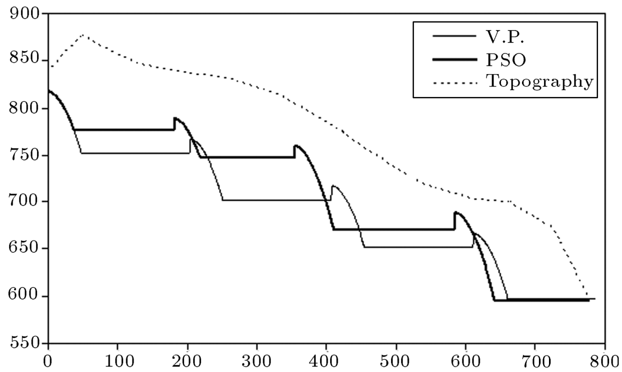

Figure 4. Comparison between PSO and Vittal and Porey designs ($N = 4$).

50,000 function evaluations. Figures 5 and 6 show the variations of the solution costs with the swarm sizes used for $N = 3$ and $N = 4$, respectively. It is seen that the best results are obtained with swarm sizes of 150 for the number of cascades equal to 3 and 4. Figures 7 and 8 show the convergence characteristics of the PSO for $N = 3$ and $N = 4$. It is clearly seen that the method has been effectively able to locate the best solutions within 20,000 function evaluations, long before the maximum number of function evaluations has been exhausted.

Table 4. Optimal unit cost of the designs (10^6).

N	PSO	V.P.
3	940	1194
4	914	1094

CONCLUSION

A Particle Swarm Optimization (PSO) algorithm is applied to the cascade stilling basins problem. A cascade system of falls with a stilling basin below each fall is well suited to energy dissipation below high head spillways. The previous design method of a cascade stilling basin was based on the hypothesis that the length and height of the preceding falls are equal; thus, it seems to be empirical, to some extent, and not necessarily optimal. While a lot of meta-heuristic algorithms have been developed for combinatorial optimization problems, PSO has been basically developed for continuous optimization problems. One particularly interesting aspect of the algorithm is that there are very few parameters to adjust. Also, PSO comprises a very simple concept and can be implemented in a few lines of computer code. It requires only primitive mathematical operators and is computationally inexpensive, in terms of both memory requirements and speed. The performance of PSO is compared to that of the Vittal and Porey method on a test example of the Tehri dam spillway. The result indicated that the optimization model is capable of significant savings in the cost of cascade stilling basins.

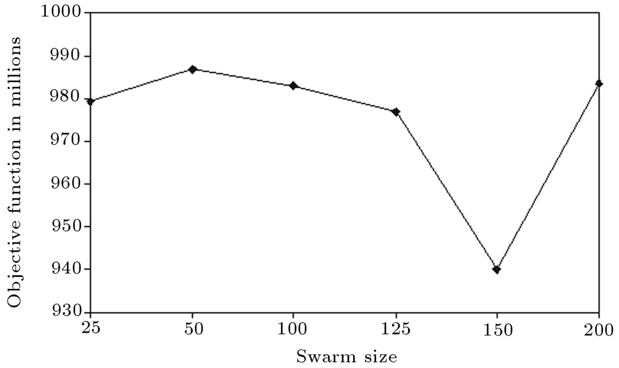

Figure 5. Sensitivity of PSO to swarm size ($N = 3$).

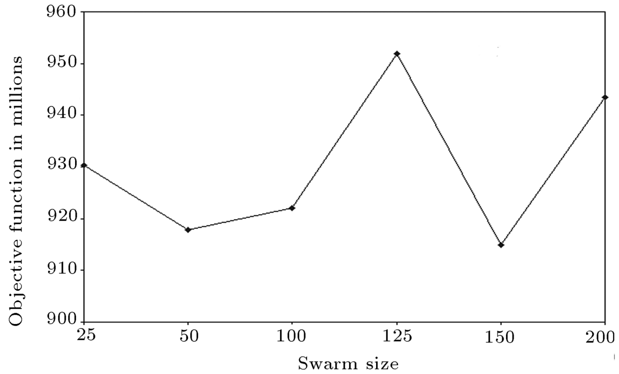

Figure 6. Sensitivity of PSO to swarm size ($N = 4$).

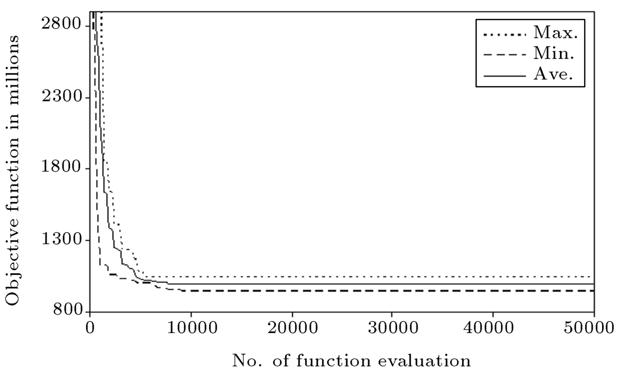

Figure 7. PSO convergence curve ($N = 3$).

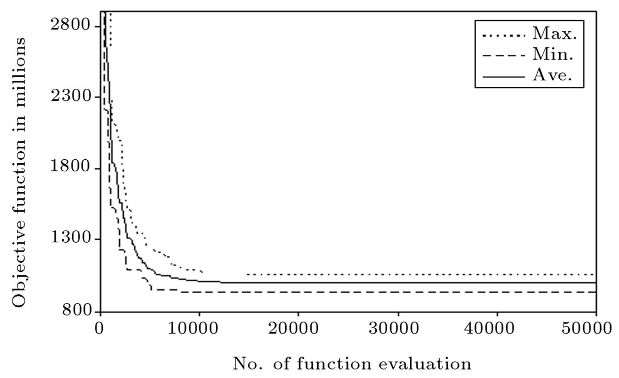

Figure 8. PSO convergence curve ($N = 4$).

REFERENCES

1. Vittal, N. and Porey, P.D. "Design of cascade stilling basins for high dam spillways", *ASCE, Journal of Hydraulic Division, ASCE*, **113**(9), pp. 225-237 (1987).
2. Eberhart, R. and Kennedy, J. "A new optimizer using particle swarm theory", *Proceedings of the Sixth International Symposium on Micro Machine and Human Science*, Nagoya, Japan, pp. 39-43, Piscataway, NJ: IEEE Service Center (1995).
3. Gill, M.K., Kaheil, Y.H., Khalil, A. and Kee, M.M. "Multiobjective particle swarm optimization for parameter estimation in hydrology", *Water Resources Research*, **42**(7), W07417 (2006).
4. Suribabu, C.R. "Particle swarm optimization techniques for deriving operation policies for maximum hydropower generation: A case study", *International Journal of Ecology and Development*, **4**(W06), pp. 66-85 (2006).
5. Suribabu, C.R. and Neelakantan, T.R. "Design of water distribution networks using particle swarm optimization", *Urban Water Journal*, Taylor and Francis Publishing, **2**(3), pp. 1-10 (2006).
6. Meraji, S.H., Afshar, M.H. and Afshar, A. "Reservoir operation by particle swarm optimization algorithm", *7th International Conference of Civil Engineering*, Tehran, Iran (2006).
7. Eberhart, R., Simpson, P. and Dobbins, R., *Computational Intelligence PC Tools*, Boston: Academic Press Professional, p. 491 (1996).
8. Angeline, P.J. "Evolutionary optimization versus particle swarm optimization", in *Evolutionary Programming VII*, V.W. Porto, N. Saravanan, D. Waagen and A.E. Eiben, Eds., Springer, pp. 601-610 (1998).
9. Shi, Y. and Eberhart, R. "Parameter selection in particle swarm optimization", *Proc. of the 1998 Annual Conference on Evolutionary Programming*, San Diego, pp. 591-600 (1998a).
10. Shi, Y. and Eberhart, R.C. "A modified particle swarm optimizer", *Proc. of IEEE International Conference on Evolutionary Computation (ICEC'98)*, Anchorage, Alaska, May 4-9, pp. 69-73 (1998b).
11. Kennedy, J. "The behavior of particles", in *Evolutionary Programming VII*, V.W. Porto, N. Saravanan, D. Waagen and A.E. Eiben, Eds., Springer, pp. 581-590 (1998).
12. Carlisle, A. and Dozier, G. "An off-the-shelf PSO", *Proceedings of the Particle Swarm Optimization Workshop*, pp. 1-6 (2001).
13. U.S. Bureau of Reclamation "Hydraulic design of stilling basins and bucket energy dissipaters", Engineering Monograph No. 25, U.S. Dep. of Interior, Bureau of Reclamation, Denver (1985).
14. Poggi, B. "Lo scaricatori a scala di stramazzi: Criteri di calcolo rilievei sperimentali", *L'Energia Elettrica*, **33**(1), pp. 33-40 (1956).