

Optimal Design of Geometrically Nonlinear Space Trusses Using an Adaptive Neuro-Fuzzy Inference System

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Abstract. An efficient methodology is proposed to optimize space trusses considering geometric nonlinearity. The optimization task is performed by a continuous Particle Swarm Optimization (PSO). Design variables are cross sectional areas of the trusses and their weights are also taken as the objective function. Design constraints are defined to restrict nodal displacements and element stresses and prevent the overall elastic instability of the structures during the optimization procedure. In order to reduce the computational effort of the optimization process, an Adaptive Neuro Fuzzy Inference System (ANFIS) is employed to approximate the nonlinear analysis of the structures instead of performing via a time consuming Finite Element Analysis (FEA). The presented ANFIS is compared with a Back Propagation Neural Network (BPNN) and appears to produce a better performance generality for evaluating structure design values. Test example results demonstrate the computational advantages of the suggested methodology for optimum design of geometrically nonlinear space trusses.

Keywords: Space truss; Geometric nonlinearity; Particle swarm optimization; Approximation concepts; Adaptive neuro fuzzy inference system.

INTRODUCTION

Due to the fact that material cost is one of the major factors in the construction of a structure, it is preferable to reduce it by minimizing the weight of the structural system. All of the methods used for minimizing the weight intend to achieve an optimum design having a set of design variables under certain design criteria. A great development of structural optimization took place in the early 60's when programming techniques were used in the minimization of structure weight. From then on, various general approaches have been developed and adopted for structural optimization [1-6]. Moreover, one of the main difficulties of structural optimization methods is that they need a great number of structural analyses to achieve an optimal solution. This deficiency may increase when nonlinear analysis must be implemented. This inherent nature of optimization methods can impose much computational effort on the process. Over the last years, some

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function approximation techniques, such as Sensitivity Analysis (SA), Multiple Regression Analysis (MRA) and Artificial Neural Networks (ANN), have been employed to approximate structural analysis instead of direct implementation [7-8]. However, the Adaptive Neuro-Fuzzy Inference System (ANFIS), which has been widely utilized for different purposes, such as prediction [9-12], knowledge discovery, medical decision making and disease diagnosis has not been tested yet for structural optimization. It is the first time that ANFIS has been tested and its performance compared with that of ANN in approximating nonlinear analysis for the design optimization of space trusses.

In this study, a novel application of ANFIS is incorporated into the design optimization of geometrically nonlinear space trusses. The optimization is carried out by a continuous Particle Swarm Optimization (PSO). The cross sectional areas of the structures are taken as design variables and the weights of trusses are selected as the objective function. The constraints involved here include limits on nodal displacements, element stresses and the collapse load of the structures. Some ANFIS models are built to predict the critical design values of the structures instead of direct evaluation by an accurate analysis. Thus, the optimization

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process can be performed using an approximate analysis estimated by ANFIS models. In order to assess the effectiveness of the proposed methodology, some benchmark space trusses are optimized. Numerical results reveal the efficiency of the suggested approach for the optimal design of structures.

GEOMETRIC NONLINEAR ANALYSIS OF SPACE TRUSSES

In the presence of large deflections, the compatibility equations (strain to displacement relationships) are not linear and geometrical nonlinearity becomes important. In such cases, although the strains are small and the material behaves linearly, the response of the structure becomes nonlinear [13].

In this study, a finite element model, taking into account the geometric nonlinearity of space trusses, including large deflection capabilities, is employed. In this model, a 3-D truss element is used, where the element is a uniaxial tension-compression element with three degrees of freedom at each node. In this case, since the axial strain is a nonlinear function of the element displacements, the structure stiffness is dependent on unknown nodal displacements and axial stresses. Obviously, the solution of the displacements cannot be obtained in a single step. Instead, the analysis is carried out by an incremental method combined with some iterative equilibrium corrections at every step. In this work, using the Newton-Raphson method of solution, the following steps are used:

1. Form tangent stiffness matrix of structure, \mathbf{K}_t , consisting of geometric stiffness matrix, \mathbf{K}_{t1} , and initial stress matrix, \mathbf{K}_{σ} , with the latest values of nodal displacements and element stresses as [13]:

$$\mathbf{K}_{t} = \mathbf{K}_{t1} + \mathbf{K}_{\sigma} = \sum_{i=1}^{ne} \int_{v} \mathbf{B}_{nl}^{T} E \mathbf{B}_{nl} dv$$
$$+ \sum_{i=1}^{ne} \int_{v} \frac{d\mathbf{B}_{nl}^{T}}{dU} \sigma dv, \qquad (1)$$

where \mathbf{B}_{nl} is the incremental strain-incremental displacement matrix and E is the elasticity modulus. Also $U^T = \{u_1, v_1, w_1, u_2, v_2, w_2\}$ is the nodal displacement vector of the truss element and σ is the element stress. To start the process in iteration 1 of load step 1, the linear stiffness matrix is used, assuming that the structure behaves linearly.

2. Solve the incremental displacement vector of the structure as:

$$\Delta \delta = \mathbf{K}_t^{-1} (\Delta F + \psi), \tag{2}$$

where ΔF is part of the load vector to be applied at the current increment (to be used only at the first iteration of a load step) and ψ is the residual force vector. Use zero values for ψ at the first iteration of the first load step.

3. Add the incremental displacements, $\Delta \delta$, to the total displacement vector, δ :

$$\delta = \delta + \Delta \delta. \tag{3}$$

- 4. Calculate the element strain based on the latest estimate of the displacements.
- 5. Calculate the total element stress using the linear elastic stress-strain relation:

$$\sigma = E\varepsilon. \tag{4}$$

6. Calculate the element internal force vector, F_i , as:

$$F_i = \int_{v} \mathbf{B}_{nl}^T \sigma dv. \tag{5}$$

Repeat steps 4 through 6 for all elements.

7. Calculate the residual force vector as:

$$\psi = \sum_{i=1}^{ne} F_i - F_e, \tag{6}$$

where vector F_e contains the cumulative external forces.

- 8. If the norm of ψ is less than a prescribed small value, $\|\psi\| \leq \text{CTOL}$, the current increment has converged and, thus, go to step 9. Otherwise, the current increment has not converged and so the equilibrium correction should be applied by repeating steps 1 through 8.
- 9. If all the load steps are done, stop. Otherwise, set ΔF = incremental loads to be applied at the next increment and repeat steps 1 through 9.

FORMULATION OF OPTIMIZATION PROBLEM

In the optimal design problem of geometrically nonlinear space trusses, the aim is to minimize the weight of the truss under constraints on stresses, displacements and ultimate load. This optimization problem can be expressed as follows:

Minimize:

$$w(x_1, \cdots, x_n, \cdots, x_{ng}) = \sum_{n=1}^{ng} x_n \sum_{m=1}^{nm} \gamma_m l_m.$$
 (7)

Subject to:

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$$\lambda_a \le \lambda_u,\tag{8}$$

$$\sigma_i \le \sigma_{\text{all},i}, \qquad i = 1, 2, \cdots, ne, \tag{9}$$

$$\delta_j \le \delta_{\text{all},j}, \qquad j = 1, 2, \cdots, nj, \tag{10}$$

where x_n , γ_m and l_m are the cross sectional area of members belonging to group n, the weight density and the length of the mth element in this group, respectively; ng and nm are the total number of groups in the structure and the number of members in group n, respectively; ne and nj are the total number of the elements and nodes in the truss, respectively; λ_a and λ_u are the applied and the ultimate load factors, respectively; σ_i and δ_j are stress in the *i*th element and displacement of the *j*th node, respectively. Also, $\sigma_{\text{all},i}$ and $\delta_{\text{all},j}$ are allowable stress in the *i*th member and allowable deflection of the *j*th node, respectively.

It should be noted that Equation 8 is defined to prevent the overall instability of the structure during the optimization process. In the optimum design of space trusses, considering nonlinearity effects, the allowable stress for a tension member is simply taken as the yield stress of steel and the allowable stress for a compression member is obtained according to the AISC code [14] as follows:

$$\sigma_{\text{all}} = \pi^2 E / \lambda^2 \quad \text{for } \lambda \ge C_c,$$

$$\sigma_{\text{all}} = \sigma_y (1 - 0.5\lambda^2 / C_c^2) \quad \text{for } \lambda < C_c, \quad (11)$$

where E, σ_y and λ are the modulus of elasticity, the yield stress of steel and the slenderness ratio, respectively. Also, in the above equation, $C_c = \sqrt{2\pi^2 E/\sigma_y}$ is the slenderness ratio dividing the elastic and inelastic buckling regions.

Optimization Method

There are many techniques that can be applied to solve the optimization problem formulated above. Among the solution techniques available, Particle Swarm Optimization (PSO) proved to be robust, effective and easy to apply [6,15]. The particle swarm optimization has been inspired by the social behavior of animals such as fish schooling, insect swarming and bird flocking. It involves a number of particles, which are initialized randomly in the search space of an objective function. These particles are referred to as a swarm. Each particle of the swarm represents a potential solution of the optimization problem. The particles fly through the search space and their positions are updated based on the best positions of individual particles in each iteration. The objective function is evaluated for each particle and the fitness values of particles are obtained to determine which position in the search space is the best. This is achieved by transforming the constrained design problem into an unconstrained one by employing a penalty function method. In this study, the formulation proposed by Rajeev and Krishnamoorthy [16] is used to calculate the violation of constraints. This formulation for design constraints given in Equations 8 to 10 is expressed as follows:

$$g_{\lambda} = \max\left(\frac{\lambda_a}{\lambda_u} - 1, 0\right),$$
 (12)

$$g_{\sigma} = \sum_{i=1}^{ne} \max\left(\frac{\sigma_i}{\sigma_{i,\text{all}}} - 1, 0\right), \qquad (13)$$

$$g_{\delta} = \sum_{j=1}^{n_j} \max\left(\frac{\delta_j}{\delta_{j,\text{all}}} - 1, 0\right).$$
(14)

Therefore, the unconstrained form of the problem or fitness function, ϕ , can be defined as:

$$\phi = w[1 + \bar{r}(g_{\sigma} + g_{\delta} + g_{\lambda})], \qquad (15)$$

where w and \bar{r} are the objective function given in Equation 7 and the coefficient of penalty function, respectively. After calculating the fitness function, the particle position and velocity are updated by the following equation in each iteration:

$$V_i^{k+1} = \rho^k V_i^k + c_1 r_1 (P_i^k - X_i^k) + c_2 r_2 (P_g^k - X_i^k),$$
(16)

$$X_i^{k+1} = X_i^k + V_i^{k+1}, (17)$$

where X_i^k and V_i^k represent the position and the velocity vectors of the *i*th particle in the *k*th iteration, respectively; P_i^k is the best previous position of the *i*th particle and P_g^k is the best global position among all the particles in the swarm; r_1 and r_2 are two uniform random sequences generated from interval [0,1]; c_1 and c_2 are the cognitive and social scaling parameters, respectively, and ρ^k is the inertia weight used to discount the previous velocity of the particle preserved. The velocity vector, V_i , is limited to a maximum value, V_i^{max} , and a minimum value, V_i^{min} .

APPROXIMATION OF NONLINEAR ANALYSIS USING ANFIS

The Adaptive Neuro-Fuzzy Inference System (ANFIS) represents a useful neural network approach for the solution of function approximation problems. In this study, ANFIS is utilized to approximate the geometrically nonlinear analysis of space trusses. In this manner, design values, such as nodal displacements, element stresses and ultimate load factors of the structures, can be predicted using some ANFIS models. So, lengthy nonlinear analysis is not needed to be performed during the optimization process.

ANFIS Structure

The ANFIS is a multilayer feed-forward network, which uses neural network learning algorithms and fuzzy reasoning to map inputs into an output. Indeed, it is a Fuzzy Inference System (FIS) implemented in the framework of adaptive neural networks. For simplicity, a typical ANFIS architecture with only two inputs leading to four rules and one output for the first order Sugeno fuzzy model is expressed [17,18]. It is also assumed that each input has two associated Membership Functions (MFs). It is evident that this architecture can be easily generalized to our preferred dimensions. For a first-order Sugeno fuzzy model, a typical rule set with four fuzzy if-then rules can be expressed as:

Rule 1: if In_1 is A_1 and In_2 is B_1

then
$$f_{11} = p_{11}In_1 + q_{11}In_2 + r_{11}$$
,

Rule 2: if In_1 is A_1 and In_2 is B_2

then
$$f_{12} = p_{12}In_1 + q_{12}In_2 + r_{12}$$
,

Rule 3: if In_1 is A_2 and In_2 is B_1

then
$$f_{21} = p_{21}In_1 + q_{21}In_2 + r_{21}$$
,

Rule 4: if In_1 is A_2 and In_2 is B_2

then
$$f_{22} = p_{22}In_1 + q_{22}In_2 + r_{22},$$
 (18)

where A_1 , A_2 , B_1 and B_2 are labels for representing membership functions for the inputs, In_1 and In_2 , respectively. Also, p_{ij} , q_{ij} and $r_{ij}(i, j = 1, 2)$ are parameters of the output membership functions.

As can be seen from Figure 1, the architecture of a typical ANFIS consists of five layers, which perform different actions in the ANFIS and are detailed below.

Layer 1

All the nodes in this layer are adaptive nodes. They generate membership grades of the inputs. The outputs of this layer are given by:

$$O_{A_i}^1 = \mu_{A_i}(In_1), \qquad i = 1, 2,$$

 $O_{Bj}^1 = \mu_{Bj}(In_2), \qquad j = 1, 2,$ (19)

where In_1 and In_2 are inputs and A_i and B_j stand for appropriate MFs, which can be triangular, trapezoidal, Gaussian functions or other shapes. In the current study, the Gaussian MFs defined below are utilized:

$$\mu_{A_i}(In_1, \sigma_i, c_i) = \exp\left(-\frac{(In_1 - c_i)^2}{2\sigma_i^2}\right), \qquad i = 1, 2,$$
$$\mu_{B_j}(In_2, \sigma_j, c_j) = \exp\left(-\frac{(In_1 - c_j)^2}{2\sigma_j^2}\right), \qquad j = 1, 2,$$
(20)

where $\{\sigma_i, c_i\}$ and $\{\sigma_j, c_j\}$ are the parameters of the MFs, governing the Gaussian functions. The parameters in this layer are usually referred to as premise parameters.

Layer 2

The nodes in this layer are fixed nodes labeled \prod indicating that they perform as a simple multiplier. The outputs of this layer are represented as:

$$O_{ij}^2 = W_{ij} = \mu_{A_i}(In_1)\mu_{B_j}(In_2), \qquad i, j = 1, 2.$$
(21)

Layer 3

The nodes in this layer are also fixed nodes labeled N indicating that they play a normalization role in the network.



Figure 1. A typical ANFIS architecture for a two-input Sugeno model with four rules.

The outputs of this layer can be represented as:

$$O_{ij}^{3} = \overline{W}_{ij} = \frac{W_{ij}}{\sum_{i=1}^{2} \sum_{j=1}^{2} W_{ij}}, \qquad i, j = 1, 2,$$
(22)

which are called normalized firing strengths.

Layer 4

Each node in this layer is an adaptive node whose output is simply the product of the normalized firing strength and a first-order polynomial (for a first order Sugeno model). Thus, the outputs of this layer are given by:

$$O_{ij}^{4} = \overline{W}_{ij} f_{ij} = \overline{W}_{ij} (p_{ij} I n_1 + q_{ij} I n_2 + r_{ij}),$$

 $i, j = 1, 2.$ (23)

Parameters in this layer are referred to as consequent parameters.

Layer 5

The single node in this layer is a fixed node labeled \sum , which computes the overall output as the summation of all incoming signals, i.e.:

$$Out = O^{5} = \sum_{i=1}^{2} \sum_{j=1}^{2} \overline{W}_{ij} f_{ij}$$
$$= \sum_{i=1}^{2} \sum_{j=1}^{2} \overline{W}_{ij} (p_{ij} In_{1} + q_{ij} In_{2} + r_{ij})$$
$$= \sum_{i=1}^{2} \sum_{j=1}^{2} [(\overline{W}_{ij} p_{ij}) In_{1} + (\overline{W}_{ij} q_{ij}) In_{2} + (\overline{W}_{ij} r_{ij})],$$
(24)

where the overall output, "Out", is a linear combination of the consequent parameters when the values of the premise parameters are fixed.

It can be observed that the ANFIS architecture has two adaptive layers: Layers 1 and 4. Layer 1 has modifiable parameters, $\{\sigma_i, c_i\}$ and $\{\sigma_i, c_i\}$, related to the input MFs. Layer 4 has modifiable parameters, $\{p_{ij}, q_{ij}, r_{ij}\}$, pertaining to the first-order polynomial. The task of the learning algorithm for this ANFIS architecture is to tune all the modifiable parameters to make the ANFIS output match the training data. Learning or adjusting these modifiable parameters is a two-step process, which is known as the hybrid learning algorithm. In the forward pass of the hybrid learning algorithm, the input membership function parameters are hold fixed, node outputs go forward until Layer 4 and the output membership function parameters are identified by the least squares method. In the backward pass, the output membership function parameters

are held fixed, the error signals propagate backward and the input membership function parameters are updated by the gradient descent method. The detailed algorithm and mathematical background of the hybrid learning algorithm can be found in [19].

In this study, for space truss problems, the cross sectional areas of the structures are selected as ANFIS inputs and nodal displacements; element stresses and the ultimate load factor can be separately considered as ANFIS output. For each input, two Gaussian membership functions are adopted and the maximum number of epochs in the training mode is set to 250.

TEST EXAMPLES

In order to assess the effectiveness of the proposed methodology, three illustrative truss examples with fixed geometries are optimized and the numerical results are compared with those reported in the literature. The optimization of the first example is described in detail, whereas more abbreviations are involved in the two other test examples. For all trusses, the elasticity modulus, yield stress and weight density are considered as $E = 210 \text{ kN/mm}^2$, $F_y = 240 \text{ N/mm}^2$ and $\gamma = 7850 \times 10^{-9} \text{ kg/mm}^3$, respectively. With the mentioned conditions, the optimum solution is achieved for two cases:

- a) Optimization using FEA,
- b) Optimization using an approximate analysis via ANFIS.

The specifications of PSO are given in Table 1. The optimization process is performed by a $core^{TM}$ 2 Duo 2GHz CPU and the time of all computations is evaluated in clock time.

One-Hundred-Twenty-Bar Space Truss

The space truss shown in Figure 2 is considered as the first example. It has 37 joints and 120 members, which are collected into seven different groups. The grouping of members is shown in the figure. The truss is subjected to a vertical loading of 60 kN at joint 1, 30 kN at joints 2-13 and 10 kN at joints 14-37 acting in the negative direction of the z-axis. The vertical

Table 1. The specifications of PSO method.

| Swarm size | 30 |
|------------------------------|------|
| Cognitive parameter | 2 |
| Social parameter | 2 |
| Minimum of inertia weight | 0.01 |
| Maximum of inertia weight | 0.90 |
| Maximum number of iterations | 100 |



Figure 2. One-hundred-twenty-bar space truss.

displacement of all joints is limited to 10 mm. The minimum size constraints are taken as 200 mm^2 .

Training and Testing the ANFIS Models

In order to select data for training the ANFIS models, for predicting the critical design values of the structure, seven cross sectional areas of the truss are selected as inputs, and the vertical displacements of joints 1 and 2 and the stresses of elements 25, 37 and 85, which govern on the design procedure, are taken as the outputs. For this meaning, a total number of 175 structures are randomly generated and the corresponding displacements and stresses of all generated trusses are evaluated using a conventional finite element analysis. Since each nonlinear analysis of the selected truss elapses after approximately 13 seconds; this process takes about 2275 seconds. Thereafter, truss samples are randomly split into two sets with 140 samples for training and 35 samples for testing, respectively. Then, with the 140 data sets, five ANFIS models are trained to approximate the mentioned design quantities.

In order to validate the trained ANFIS models and compare the results, a Back Propagation Neural Network (BPNN) is also employed to anticipate the design values of the truss. The BPNN uses design variables of the truss as the input vector and the mentioned design quantities as the output vector. The maximum number of epochs for BPNN training is also set to 2000 epochs. The Absolute Percentage Errors (APE) of the design values obtained by ANFIS and BPNN in the testing mode are shown in Figures 3 to 7. For further comparison, Relative Root-Mean-Squared Error (RRMSE), Mean Absolute Percentage Error (MAPE) and the absolute fraction of variance (R^2) , which arose during testing in ANFIS and BPNN, are also calculated by using the following equations:

RRMSE =
$$\sqrt{\frac{\frac{1}{n_t - 1} \sum_{i=1}^{n_t} (a_i - p_i)^2}{\frac{1}{n_t} \sum_{i=1}^{n_t} (a_i)^2}},$$
 (25)



Figure 3. Absolute percentage errors of displacement of joint 1.



Figure 4. Absolute percentage errors of displacement of joint 2.



Figure 5. Absolute percentage errors of stress of element 25.

$$MAPE = \frac{1}{n_t} \sum_{i=1}^{n_t} 100 \times \left| \frac{a_i - p_i}{a_i} \right|, \qquad (26)$$

$$R^{2} = 1 - \left(\frac{\sum_{i=1}^{n_{t}} (a_{i} - p_{i})^{2}}{\sum_{i=1}^{n_{t}} p_{i}^{2}}\right),$$
(27)

where a is actual value, p is predicted value and n_t is the number of testing samples. The smaller RRMSE and MAPE and larger R^2 mean a better performance



Figure 6. Absolute percentage errors of stress of element 37.



Figure 7. Absolute percentage errors of stress of element 85.

generality. The statistical parameters for design values of the structure found from testing in ANFIS and BPNN are compared in Table 2. All of the statistical values in this table demonstrate that the proposed ANFIS achieves a better performance than the BPNN. Therefore, ANFIS is a good choice for predicting the displacements and stresses of the truss. It is also observed that, besides the high-speed computing of the ANFIS model in comparison with the BPNN, the main advantage of the ANFIS is that its solution stability is high while BPNN exhibits different performances in each run. Nevertheless, the main disadvantage of ANFIS is to have only one output.

Optimization Results

The optimal solution is achieved by incorporating the trained ANFIS models into the optimization procedure and the results are listed in Table 3. The minimum weight sought is 7,194 kg while the value found in [20] is 7,587 kg. It is impressive to mention that the optimization takes about 11 seconds.

The optimal structure is also analyzed by an accurate finite element method and the percentage errors between the accurate design values and those predicted using ANFIS models are given in Table 4. It can be observed that the errors are acceptable.

| Statistical | Joint 1 | | Joint 2 | | Element 25 | | Element 37 | | Element 85 | |
|-------------|---------|-------|---------|-------|------------|-------|------------|-------|------------|-------|
| Parameters | BPNN | ANFIS | BPNN | ANFIS | BPNN | ANFIS | BPNN | ANFIS | BPNN | ANFIS |
| RRMSE | 0.089 | 0.075 | 0.059 | 0.045 | 0.014 | 0.009 | 0.009 | 0.008 | 0.016 | 0.016 |
| MAPE | 7.64 | 7.43 | 4.913 | 3.99 | 2.370 | 1.26 | 1.501 | 0.803 | 2.87 | 1.322 |
| R^2 | 0.992 | 0.994 | 0.997 | 0.998 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 |

Table 2. The statistical values found from testing in ANFIS and BPNN for 120-bar truss.

| Design Variables | ${ m Ref}\left[20 ight]({ m mm^2})$ | Present Study (mm ²) | | | |
|--|-------------------------------------|----------------------------------|-----------|--|--|
| | | FEA | ANFIS | | |
| A_1 | 1750 | 906 | 862. | | |
| A_2 | 4,556 | 4,710 | 4,670 | | |
| A_3 | 2,545 | 1,890 | $1,\!890$ | | |
| A_4 | 844. | 733 | 862. | | |
| A_5 | 2,230 | $2,\!600$ | 2,640 | | |
| A_6 | 1,596 | 1,171 | 862. | | |
| A_7 | 390. | 862 | 862. | | |
| Optimal weight (kg) | 7,587 | 7,419 | 7,194 | | |
| Maximum violated constraint | - | 0.00 | 0.041 | | |
| Optimization time (min.) | - | 650 | 0.18 | | |
| Required time for data generation (min.) | - | - | 38.0 | | |
| Training time (min.) | - | - | 3.60 | | |
| Overall time (min.) | _ | 650 | 41.78 | | |

Table 3. Optimization results for 120-bar space truss.

Table 4. The errors between design values of approximate analysis and FEA.

| Design Values | FEA | ANFIS | Error (%) |
|------------------------------|-------|-------|-----------|
| Displacement of joint 1 (mm) | 9.34 | 8.66 | 7.28 |
| Displacement of joint 2 (mm) | 10.41 | 9.91 | 4.77 |
| Stress of element 25 (Mpa) | 30.57 | 30.69 | 0.42 |
| Stress of element 37 (Mpa) | 28.96 | 29.11 | 0.54 |
| Stress of element 85 (Mpa) | 51.58 | 52.19 | 1.18 |

Fifty-Six-Bar Space Truss

The space truss shown in Figure 8 is considered as the second example. The members are collected into four different groups as shown in the figure. The external loading is taken as 45.5 kN in the *x*-direction at joints 1, 2, 5, 7, 9, 11, 13 and 15, and 91 kN in the negative *z*-direction at joints 1, 2, 3 and 4. The vertical displacements of all joints are restricted to 32 mm. The lower bounds of 500 mm² are used for these area variables.

In this example, six ANFIS models are built to approximate the critical design values, including the horizontal displacements of joints 1 and 2 in an xdirection and the stresses of elements 24, 32, 53 and 56, respectively. For this meaning, a total number of 200 structures are randomly generated. Then, samples are randomly divided into two sets with 160 samples for training and 40 samples for testing, respectively. The statistical characteristics for the mentioned design values of the structure found from testing in ANFIS and BPNN are compared in Table 5. All of the statistical values in this table demonstrate that the proposed ANFIS achieves a good performance generality.

Optimization Results

The truss is optimized for mentioned conditions and the results are listed in Table 6. The optimal weight

| Statistical | Joi | nt 1 | Joi | nt 2 | Elem | ent 24 | Elem | ent 32 | Elem | ent 53 | Elem | ent 56 |
|-------------|-------|-------|-------|-------|-------|--------|-------|--------|-------|--------|-------|--------|
| Parameters | BPNN | ANFIS | BPNN | ANFIS | BPNN | ANFIS | BPNN | ANFIS | BPNN | ANFIS | BPNN | ANFIS |
| RRMSE | 0.016 | 0.014 | 0.020 | 0.020 | 0.065 | 0.018 | 0.013 | 0.012 | 0.050 | 0.043 | 0.064 | 0.038 |
| MAPE | 1.72 | 1.61 | 1.62 | 1.37 | 4.19 | 1.45 | 0.89 | 0.74 | 5.93 | 5.35 | 6.92 | 6.16 |
| R^2 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.997 | 0.998 | 0.998 | 0.998 |

Table 5. The statistical values found from testing in ANFIS and BPNN for 56-bar truss.

| Design Variables | ${\rm Ref} \ [20] \ ({\rm mm}^2)$ | Present Study (mm ²) | | |
|--|-----------------------------------|----------------------------------|--------|--|
| | | FEA | ANFIS | |
| A1 | 744 | 500 | 500 | |
| A_2 | 11,102 | 11,300 | 11,300 | |
| A_3 | 500 | 500 | 500 | |
| A_4 | 4,646 | 4,827 | 4,492 | |
| Optimal weight (kg) | 13,577 | 13,587 | 13,441 | |
| Maximum violated constraint | - | 0.00 | 0.024 | |
| Optimization time (min.) | - | 455 | 0.170 | |
| Required time for data generation (min.) | - | - | 43.33 | |
| Training time (min.) | - | _ | 3.00 | |
| Overall time (min.) | - | 455 | 46.50 | |

Table 6. Optimization results for 56-bar space truss.

Table 7. The errors between design values of approximate analysis and FEA.

| Design Values | FEA | ANFIS | Error (%) |
|------------------------------|-------|-------|-----------|
| Displacement of Joint 1 (mm) | 31.22 | 30.00 | 3.62 |
| Displacement of Joint 2 (mm) | 32.78 | 29.72 | 9.38 |
| Stress of element 24 (Mpa) | 61.04 | 59.35 | 2.77 |
| Stress of element 32 (Mpa) | 46.57 | 46.93 | 0.78 |
| Stress of element 53 (Mpa) | 73.12 | 69.88 | 4.43 |
| Stress of element 56 (Mpa) | 62.07 | 66.26 | 6.75 |

obtained in this study is 13,441 kg, while the weight found in [20] is 13,577 kg.

The errors between design values of the optimum solution predicted using ANFIS and those evaluated by an accurate analysis are also given in Table 7. It can be observed that the errors are small.

Fifty-Two-Bar Space Truss

The space truss of Figure 9 is considered as the third example. The joints are subjected to vertical loads in the negative direction of the z-axis, which are 150 kN at joints 6-13. The vertical displacements of all joints are restricted to 10 mm. The structure has 52 members,

which are collected into eight groups, as shown in Figure 9. The lower bounds of 200 mm^2 are used for these area variables.

For this example, since element stresses are governed on the design procedure, only the stresses of elements 21, 29, 30, 37 and 45 are approximated by the ANFIS. For this meaning, a total number of 200 sample structures are randomly generated. Then, samples are randomly divided into two sets; with 160 samples for training and the remaining samples for testing, respectively. The statistical characteristics for the mentioned design values of the structure found from testing in ANFIS and BPNN are compared in Table 8. All of the statistical values in the table

| Statistical | Element 21 | | Element 21 Element 29 | | Element 30 | | Element 37 | | Element 45 | |
|-------------|------------|-------|-----------------------|-------|------------|-------|------------|-------|------------|-------|
| Parameters | BPNN | ANFIS | BPNN | ANFIS | BPNN | ANFIS | BPNN | ANFIS | BPNN | ANFIS |
| RRMSE | 0.038 | 0.027 | 0.031 | 0.025 | 0.023 | 0.022 | 0.037 | 0.037 | 0.037 | 0.037 |
| MAPE | 3.59 | 2.15 | 4.87 | 2.32 | 2.68 | 2.26 | 3.40 | 2.07 | 3.40 | 2.07 |
| R^2 | 0.998 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.998 | 0.999 | 0.998 | 0.999 |

Table 8. The statistical values found from testing in ANFIS and BPNN for 52-bar truss.



Figure 8. Fifty-six-bar space truss.

reveal the good performance generality of the proposed ANFIS.

Optimization Results

The truss optimization results achieved by FEA and ANFIS are listed in Table 9. It is impressive to note that the overall computing time of the approximate optimization procedure is 0.07 times that required through FEA.

Optimum structures obtained using ANFIS are analyzed by FEA and the stress and displacement errors between them are given in Table 10. It can be observed that the errors are small.

CONCLUSION

An efficient methodology is presented for the optimization of geometrically nonlinear space trusses employing the Adaptive-Neuro Fuzzy Inference System (ANFIS)



Figure 9. Fifty-two-bar space truss.

for analysis approximation. The optimization algorithm used in this investigation is a Particle Swarm Optimization (PSO). In order to reduce the computational effort of the optimization process involving many nonlinear structural analyses, some ANFIS models are built to predict the nodal displacements and element stresses of trusses instead of computing by FEA. The present ANFIS models are compared with a Back Propagation Neural Network (BPNN) and it is proved to have a better performance generality than BPNN. Some illustrative test examples are considered to assess the effectiveness of the proposed method. The numerical results demonstrate the computational advantages of the suggested methodology when compared with those reported in the literature. It is also impressive to mention that the overall computing time including data generation, ANFIS modeling and optimization tasks is much lower than that needed by optimization using FEA while the errors are small.

| Design Variables | Prese | nt Study (mm ²) |
|--|-------|-----------------------------|
| | FEA | ANFIS |
| A_1 | 200 | 200 |
| A_2 | 200 | 200 |
| A_3 | 800 | 939 |
| A_4 | 800 | 800 |
| A5 | 2,866 | 2,609 |
| A_6 | 2,481 | 2,463 |
| A7 | 3,732 | 3,786 |
| A | 3,758 | 3,770 |
| Optimal weight (kg) | 8,672 | 8,615 |
| Maximum violated constraint | 0.00 | 0.00 |
| Optimization time (min.) | 650 | 0.180 |
| Required time for data generation (min.) | - | 43.33 |
| Training time (min.) | - | 2.00 |
| Overall time (min.) | 650 | 45.51 |

Table 9. Optimization results for 52-bar space truss.

Table 10. The errors between design values of approximate analysis and FEA.

| Design Values | FEA | ANFIS | Error (%) | Allowable Stress |
|------------------------------|-------|-------|-----------|------------------|
| Stress of element 21 (Mpa) | 37.04 | 34.33 | 7.42 | 37.83 |
| Stress of element 29 (Mpa) | 35.95 | 33.62 | 6.55 | 39.84 |
| Stress of element $30 (Mpa)$ | 38.97 | 39.33 | 1.42 | 39.84 |
| Stress of element 37 (Mpa) | 25.88 | 26.28 | 0.76 | 26.34 |
| Stress of element 45 (Mpa) | 25.15 | 26.18 | 3.76 | 26.19 |

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