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Estimates of Average Inelastic Deformation Demands for Regular Steel Frames by the Endurance Time Method

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Abstract. *The Endurance Time (ET) method is a new dynamic pushover procedure in which structures are subjected to gradually intensifying acceleration functions and their performance is assessed based on the length of the time interval that they can satisfy required performance objectives. In this paper, the accuracy of the Endurance Time method in estimating average deformation demands of low and medium rise steel frames using ETA20f series of ET acceleration functions has been investigated. The precision of the ET method in predicting the response of steel frames in nonlinear analysis is investigated by considering a simple set of moment-resisting frames. An elastic-perfectly-plastic material model and a bilinear material model with a post-yield stiffness equal to 3% of the initial elastic stiffness have been considered. For frames with an elastic-perfectly-plastic material model, which are $P-\Delta$ sensitive cases, the ET analysis for the maximum interstory drift ratio somewhat underestimates the nonlinear response history analysis results. The difference between the results of the ET analysis and the nonlinear response history analysis for the material model with 3% post-yield stiffness is acceptable. The consistency of the base shears obtained by the two methods is also satisfactory. It is shown that, although the results of the ET analysis are not exactly consistent with the results of ground motions analysis, the ET method can clearly identify the structure with a better performance even in the case of structures with a relatively complicated nonlinear behavior.*

Keywords: *Nonlinear response history analysis; Dynamic pushover; Endurance time method; Performance-based seismic engineering.*

INTRODUCTION

The concept of Performance Based Seismic Engineering (PBSE) is gaining increased interest among researchers and practitioners [1]. PBSE includes the concept that designs should be capable of satisfying various performance objectives under a spectrum of design ground motions ranging from minor to severe. Due to inherent randomness in ground shaking, lack of knowledge in the precise definition of the structure's characteristics, and an inability to model the actual behavior accurately, the estimation of seismic performance entails significant uncertainty [2].

Quantification of seismic demands for performance assessment implies the statistical and probabilistic evaluation of Engineering Demand Parameters (EDPs), i.e. story drifts, floor acceleration etc. as a function of ground motion Intensity Measures (IMs), i.e. peak ground acceleration, spectral acceleration at the first-mode period etc. Sensitivity of the relationship between EDPs and IMs to important structural and ground motion characteristics should also be studied [3]. Several research efforts have focused on the evaluation of demands for both Single and Multiple Degrees Of Freedom (SDOF and MDOF) systems in which displacement demands from nonlinear response history analyses have been quantified as a function of a normalized strength or ground motion intensity level [4-10].

Current structural engineering practice estimates seismic demands by the nonlinear static procedure or pushover analysis detailed in the Federal Emergency Management Agency (FEMA-356) or Applied Technology Council (ATC-40) guidelines [11,12]. The seismic demands are computed by nonlinear static analysis

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of the structure subjected to monotonically increasing lateral forces with an invariant or variant height-wise distribution until a target value of roof displacement is reached. This roof displacement value is determined from the earthquake-induced deformation of an inelastic Single-Degree-Of-Freedom system derived from the pushover curve [10].

Another promising tool for estimating inelastic deformation demands is Incremental Dynamic Analysis (IDA). In IDA, the seismic loading is scaled and different nonlinear dynamic analyses are performed to estimate the dynamic performance of the global structural system [13]. By using this method, EDPs of the structures can be obtained at different intensity measures and therefore the performance of the structures can be reviewed more precisely. The large number of nonlinear dynamic analyses needed for the IDA method and the different performances of the structure to the different records applied to it are the main drawbacks of this method in practical applications.

The Endurance Time method is basically a simple dynamic pushover test that tries to predict the EDPs of structures at different IMs by subjecting them to some predesigned intensifying dynamic excitations. These predesigned excitations in the ET method are called "acceleration functions" in this paper in order to clearly identify them from ground motions and simulated accelerograms that are usually compatible with ground motions. ET acceleration functions are designed in a manner that their intensity increases with time. In order to practically apply the ET method as a tool for the design and assessment of structures, ET acceleration functions should preferably represent different earthquake hazard levels at different times, as far as possible. For this purpose, the concept of response spectra has been taken advantage of in developing ET acceleration functions [14,15]. Optimization techniques are used in order to create a set of ET acceleration functions with the property of having a response spectra that proportionally intensifies with time, while remaining compatible to a pre-specified target response spectra curve. A detailed procedure for generating ET acceleration functions is described in the following section. Because of the increasing demand of the ET acceleration function, structures gradually go through elastic to yielding and nonlinear inelastic phases, finally leading to global dynamic instability.

Earlier studies have shown that in the linear seismic analysis of structures, the ET method can reproduce the results of codified static and response spectrum analysis procedures with acceptable accuracy [14]. The compliance and level of accuracy of this method in the nonlinear seismic analysis of SDOF structures has also been investigated. In this paper, the accuracy and consistency of the ET method in estimating the average inelastic deformation demands of regular steel frames on stiff soil are examined. These studies are required in order to provide a basis for practical application of the ET method in steel frames seismic assessment and design problems. To reach this goal, a set of ground motions is selected and their average response spectrum is calculated. This spectrum is set to be the target response spectrum for generating a set of acceleration functions used in this study (i.e. ETA20f series). A set of steel moment-resisting frames with a different number of stories was used in this study. This set consists of underdesigned, properly designed and overdesigned frames to examine the capability of the ET method in differentiating dissimilar structures. An Elastic-Perfectly-Plastic (EPP) material model and a bilinear material model with a post-yield stiffness equal to 3% of the initial elastic stiffness (STL) are used to study the nonlinear behavior of the frames. The results computed with the ET method were compared to the results of nonlinear response history analyses. A procedure is described to find an equivalent time in the ET analysis to compare its results with the results of the nonlinear response history analysis. Mean values and dispersions of the results obtained by two methods are compared for different frames. Finally, the potential application of the ET method for seismic rehabilitation of structures is explained by using dampers at different stories of a sample structure.

For frames with an EPP material model, which are $P-\Delta$ sensitive cases, estimations of an ET analysis for a maximum interstory drift ratio are less than nonlinear response history analysis results. A nonlinear response history analysis of flexible structures that are subjected to large displacements may be severely influenced by the $P-\Delta$ effects. For these cases, the maximum interstory drift ratio becomes very sensitive to ground motions that are relatively strong. Therefore, for these ground motions, $P-\Delta$ effects destabilize the structure and increase the maximum interstory drift ratio drastically resulting in the average value of this parameter to become unreliable and scattered. As will be shown later, average values of deformation demands cannot be reliably predicted by the ET method in these cases. Unlike real earthquakes, ET acceleration functions used in this study have quite similar characteristics. Consequently, the dispersion of the results of nonlinear response history analyses for these frames is high, but in ET analysis, the dispersion of the results is much less. However, it should be noted that the deformations resulted from the destabilizing effect of $P-\Delta$ in EPP models are usually beyond the drift levels that are practically important for design and are of little significance. The consistency between the results of the ET analysis and the nonlinear response history analysis for the material model with 3% post-yield stiffness is satisfactory. These frames are much less sensitive to $P - \Delta$ effects when subjected to strong ground motions. The consistency of the base shears obtained by two methods is acceptable for both material models. As will be shown, ET can be considered as a useful approximate analysis procedure that provides a practical tool in intermediate levels of the design process by drastically reducing the number of required nonlinear time-history analyses at each step of the design refinement.

CONCEPT OF ET METHOD

The Endurance Time method is a simple dynamic pushover procedure that predicts the seismic performance of structures by analyzing their resilience when subjected to predesigned intensifying dynamic excitations. In this method, numerical (or experimental) models of structures are subjected to intensifying acceleration functions. Major structural responses, such as displacements, drift ratios, stresses or other appropriate EDPs are monitored up to the desired limiting point where the structure collapses or failure criteria are met. Each specific time in the ET analysis can be correlated to a specific IM that expresses an earthquake hazard level. PBSE consists of a selection of different building performance levels for different seismic hazard levels. In ET analysis, the equivalent time for each seismic hazard level can be defined and the performance of the structure until that time can be compared by predefined performance objectives. Based on each different structural performance level, such as Immediate Occupancy (IO), Life Safety (LS), Collapse Prevention (CP) and the design or rehabilitation objectives, the corresponding equivalent time can be defined and alternative designs can be compared at these milestone times representing different excitation levels in each single analysis.

Basically, the longer that the structure can endure imposed excitations, it is judged to have better performance. In practice, the analysis or experiment need not be continued until the real collapse of the structure. Any convenient performance parameter, such as maximum drift, stress ratio and plastic rotation can be considered, and the analysis or experiment can be commenced until the desired level of excitation has been covered [16].

One of the most important issues in successful implementation of the procedure is the determination of a suitable acceleration function, so that the results from ET analysis (or testing) can be correlated reliably well with the response of structures subjected to earthquakes. The concept of response spectra can be used in producing the intensifying ET acceleration functions for this purpose [14,15]. Optimization techniques are used in order to create a set of ET acceleration functions with response spectrum that proportionally intensify with time, while remaining compatible to a pre-specified template response spectra curve as far as possible. This means that the response spectrum of any window of these acceleration functions, from $t_0 = 0$ to $t_1 = t$, resembles that of the target spectrum with a scale factor that is proportional with time, (t) [14]. Even though other strategies can also be used to define intensifying ET acceleration functions, the stated method seems to provide a well suited acceleration function for the purposes of this study. To apply the ET method for PBSE, it is suitable to generate acceleration functions whose response spectra are compatible with the response spectra of different hazard levels. Generally, seismic hazard due to ground shaking is defined for any earthquake hazard level using spectral response acceleration. The response spectra for different hazard levels can be used for the generation of ET acceleration functions. The optimization procedure for generating ET acceleration functions is drastically time-consuming, but once a set of acceleration functions based on earthquake hazard levels of a seismic code is generated, it can be used easily for any structure.

Although current sets of ET acceleration functions are generated based on linear response spectra, their performance in estimating the nonlinear response of SDOF systems has been satisfactory. In this research, the accuracy of the ET analysis in estimating average inelastic deformation demands is examined by comparing its results with nonlinear response history analysis results. It should be noted that it is not the aim of the ET method to estimate the response of structures to each individual earthquake. The response spectra used for the generation of ET acceleration functions are representative of the average response of the structures subjected to a set of earthquakes. ET analysis is aimed to predict this average response. For design purposes, the response spectrum function can be adjusted to properly reflect the level of dispersion by applying statistical procedures. ET acceleration functions can then be generated based on these design spectrum. In this study, however, the average response spectrum from seven earthquakes is directly used in order to investigate the accuracy and consistency of ET analyses and directly compare them with the results of traditional time-history analyses.

ET acceleration functions that have been used in this research are designed in such a way that their response spectrum remains proportional to that of the average of seven strong motions recorded under a stiff soil condition. These acceleration functions are generated to be compatible with some ground motions to facilitate the comparison of the results of ET analysis and nonlinear response history analysis. Dynamic properties of these acceleration functions will be discussed in the next sections.

SPECIFICATIONS OF STEEL FRAMES, GROUND MOTIONS AND ACCELERATION FUNCTIONS

A set of steel moment-resisting frames with a different number of stories were studied. This set consists of two-dimensional regular generic frames with 3, 7 and 12 stories. These frames have either a single bay or three bays. The generic frames of this study are based on the models developed by Estekanchi et al. [14]. These frames are designed according to the AISC-ASD design code [17]. To facilitate the comparative studies, frames are designed for different levels of lateral load, named "Standard", "Underdesigned" and "Overdesigned". Standard frames have been designed, according to the recommendations of the INBC for a high seismicity area [18]. Frames are designed with the aid of an equivalent static procedure. The name of these frames ends with the letter S. Underdesigned frames have been designed assuming one half of the codified base shear as the design lateral load. The name of these frames ends with the letter W. Overdesigned frames have been designed for twice the standard lateral load. The name of these frames ends with the letter O. Frame masses are considered to be the same for these three kinds of frame. A response modification factor of $R = 6$ is used for the design of the frames and the interstory drift ratio is limited to 0.005 for all of them per code requirements. Usually, in standard and overdesigned frames, the story drift turns out to be the controlling design criteria. In underdesigned frames, element forces control the design. The geometry and section properties of the frames with seven stories and one bay are depicted in Figure 1. Some of the specifications of all frames are shown in Table 1.

The Elastic-Perfectly-Plastic material model and the bilinear material model with a post-yield stiffness equal to 3% of the initial elastic stiffness are used to study the nonlinear behavior of the frames. The EPP model has been used widely in previous investigations and, therefore, represents a benchmark to study the effect of hysteretic behavior. Furthermore, recent studies have shown that this is a reasonable hysteretic model for steel beams that do not experience lateral or local buckling or connection failure [19]. For more realistic nonlinear behavior, the STL material model is also used in this study. To apply these material models in the analysis, the OPENSEES beam-column element with nonlinear distributed plasticity is utilized [20]. This element is used for the beams and columns of the frames to account for the nonlinearity for both of them. Only one (horizontal) component of the ground motion has been considered, while dynamic soil-structure interaction is neglected. $P - \Delta$ effects have been included in the analysis. A viscous damping of 5%, as customary for these types of frame, has been applied in the analyses. This value of damping is consistent with the value used for generating codified response spectra and ET acceleration functions.

To investigate the accuracy of the ET method in estimating the nonlinear response of ground motions, a set of ET acceleration functions (ETA20f)

Table 1. Specifications of the frames.

Frames	Number of Stories	Number of Bays	Mass Participation Mode 1	Fundamental Period (sec)	Design Base Shear (KN)
FM03B1RGW	3	1	90.98%	1.20	59.7
FM03B1RGS	3	1	88.03%	0.89	116.32
FM03B1RGO	3	1	85.15%	0.60	244.92
FM03B3RGW	3	3	88.57%	1.25	179.3
FM03B3RGS	3	3	85.71%	0.89	362.17
FM03B3RGO	3	3	85.64%	0.61	729.26
FM07B1RGW	7	1	81.18%	2.03	101.38
FM07B1RGS	7	1	80.60%	1.43	204.78
FM07B1RGO	7	1	80.56%	0.99	414.92
FM07B3RGW	7	3	81.25%	2.05	302.34
FM07B3RGS	7	3	80.92%	1.44	609.77
FM07B3RGO	7	3	80.40%	0.97	1233.41
FM12B3RGW	12	3	79.32%	2.89	399.2
FM12B3RGS	12	3	78.43%	2.05	804.38
FM12B3RGO	12	3	75.17%	1.30	1631.52

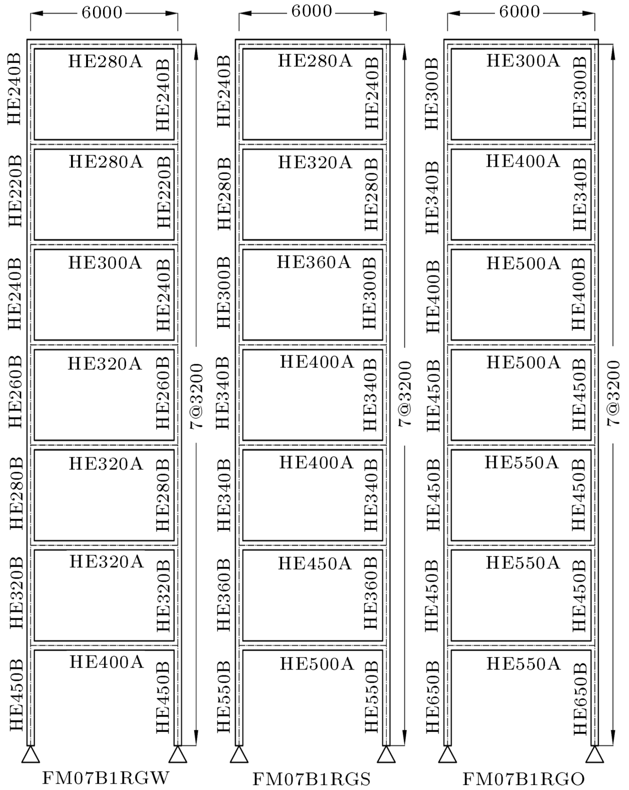

Figure 1. Schematics of the frames with seven stories and one bay.

are generated using the average response spectrum of ground motions. To reach this goal, 20 accelerograms that are recorded on site class C, as defined by the NEHRP and used in FEMA 440, are selected [21]. From these ground motions, 7 records whose response spectra shapes were more compatible with the response spectrum of soil type II of INBC standard 2800 are selected (Table 2) [22]. These 7 accelerograms are scaled to produce a response spectrum that is compatible with the INBC standard 2800 spectrum. Finally, the average of the pseudo acceleration spectrum of these scaled accelerograms is obtained and smoothed. The smoothed spectrum is used as the target spectrum in generating new ET acceleration functions. As can be

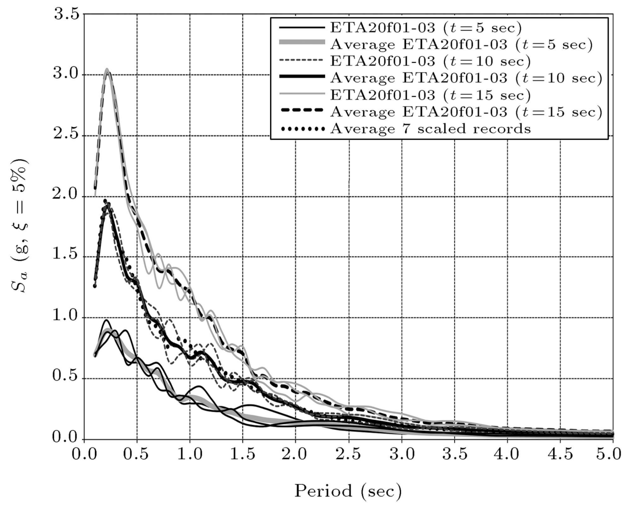

Figure 2. Total acceleration response spectra of ETA20f series acceleration functions for $\xi = 5\%$ at different times.

seen in Figure 2, the response spectrum of a window of ET acceleration functions from $t_0 = 0$ to $t_1 = 10$, i.e. $t \in [0, 10]$, matches reasonably well with the average response spectrum of the seven strong motion records. It is important to note that the ET response spectra remain proportional to the target spectra from seven ground motions at all times, e.g. it is 0.5 and 1.5 times the target spectra at $t = 5$ sec and $t = 15$ sec, respectively. A sample acceleration function generated in this way is shown in Figure 3. To compare the results of ET analysis with earthquakes, a set of ground motions (GM1) are used. The set consists of 7 records that are used for the generation of the ETA20f set of acceleration functions.

To be consistent with the seismic codes, the GM1 set of ground motions should be scaled. All frames are analyzed as planar structures subjected to a single horizontal component of ground motion. Therefore, records are scaled individually rather than scaling them as pairs. Most codes stipulate that the ground motions be scaled such that the average of the ordinates of the 5 percent-damped linear response spectra does not fall below the design spectrum for the period range $0.2\,T_i$ to

Table 2. Description of GM1 set of ground motions used in this study.

Date	Earthquake Name	Magnitude (Ms)	Station Number	Component (deg)	PGA (cm/s^2)	Abbreviation
06/28/92	Landers	7.5	12149	0	167.8	LADSP000
10/17/89	Loma Prieta	7.1	58065	0	494.5	LPSTG000
10/17/89	Loma Prieta	7.1	47006	67	349.1	LPGIL067
10/17/89	Loma Prieta	7.1	58135	360	433.1	LPLOB000
10/17/89	Loma Prieta	7.1	1652	270	239.4	LPAND270
04/24/84	Morgan Hill	6.1	57383	90	280.4	MHG06090
01/17/94	Northridge	6.8	24278	360	504.2	NRORR360

1.5 T_i where T_i is the fundamental period of vibration of each frame modeled as a linear system. Here, scale factors are obtained in a way that the ground motion spectrum matches the ASCE-7 spectrum in the mentioned range [23]. Scale factors obtained by this method for the GM1 set are shown in Table 3 for each frame.

COMPARISON BETWEEN THE RESULTS OF ET ANALYSIS AND NONLINEAR RESPONSE HISTORY ANALYSIS

For each set of ground motions and acceleration functions, the mean value and standard deviation of the specified EDP can be calculated. For example, for a set of ground motions, the mean value and standard deviation of the EDP can be calculated by the following equation:

$$\overline{\mathrm{EDP}}_{ex} = \frac{1}{n}\sum_{i=1}^{n}\mathrm{EDP}_{ex,i}, \tag{1}$$

$$\sigma_{ex} = \sqrt{\frac{\sum_{i=1}^{n}(\mathrm{EDP}_{ex,i} - \overline{\mathrm{EDP}}_{ex})^2}{n-1}}. \tag{2}$$

In this equation, $\mathrm{EDP}_{ex,i}$ is the value of EDP for a ground motion, n is the number of ground motions in the set, $\overline{\mathrm{EDP}}_{ex}$ is the mean value of EDP for the set and σ_{ex} is the standard deviation of it. Similar values can be calculated for the set of acceleration functions.

An important question is how the results of two methods can be compared. Results of ET analysis are obtained through time and as mentioned before in this method, the time is correlated with IM. Therefore, different values of EDP are calculated for different values of IM in an ET analysis. To establish a relation between the results of the ET method and any other method, the IM value of the other method should be found in the ET analysis. Therefore, a procedure should be defined to find an equivalent time in the ET analysis in which the IM values of the two methods are equal.

Many quantities have been proposed to characterize the intensity of a ground motion record. In the ET method, intensity increases through time and, therefore, scalable IMs can be conveniently used for this method. Common examples of scalable IMs are Peak Ground Acceleration (PGA), Peak Ground Velocity (PGV) and Spectral Acceleration at the structure's first-mode period ($S_a(T_1)$). In this research, first-mode spectral acceleration ($S_a(T_1)$) is used as IM to obtain the equivalent time. Most frames used in this study are first-mode dominated structures that are sensitive to the strength of the frequency content near their first-mode frequency, which is well characterized by $S_a(T_1)$, but not by PGA. Moreover, researchers show that $S_a(T_1)$ produces a lower dispersion over the full range of EDP values in IDA analysis [13]. Certainly, other IMs can be used easily to calculate the equivalent time. The equivalent time can be calculated for a single record or a set of records. To compare the results of a set of records with the results of ET analysis, the

Table 3. Scale factors of GM1 set for different frames.

Frames	Scale Factors							Equivalent
	LPAND270	LADSP000	MHG06090	LPGIL067	LPLOB000	NRORR360	LPSTG000	Time (sec)
FM03B1RGW	2.89	3.97	1.74	2.35	2.63	1.11	1.61	10.79
FM03B1RGS	2.55	3.66	1.61	2.12	1.87	1.15	1.76	10.26
FM03B1RGO	2.36	3.64	1.79	1.91	1.60	1.31	1.87	9.63
FM03B3RGW	2.92	4.02	1.77	2.40	2.78	1.11	1.60	10.96
FM03B3RGS	2.61	3.68	1.61	2.14	1.93	1.14	1.75	10.36
FM03B3RGO	2.36	3.62	1.72	1.99	1.68	1.20	1.87	9.48
FM07B1RGW	3.51	4.58	2.30	3.15	4.59	1.25	1.59	13.13
FM07B1RGS	3.08	4.18	1.92	2.58	3.29	1.13	1.55	11.47
FM07B1RGO	2.76	3.75	1.62	2.20	2.14	1.10	1.71	10.64
FM07B3RGW	3.63	4.70	2.35	3.41	5.05	1.27	1.59	13.51
FM07B3RGS	3.13	4.22	1.96	2.65	3.49	1.13	1.54	11.67
FM07B3RGO	2.69	3.71	1.61	2.17	2.04	1.12	1.72	10.57
FM12B3RGW	4.26	5.46	2.82	4.06	6.24	1.49	1.78	15.36
FM12B3RGS	3.57	4.64	2.33	3.28	4.81	1.26	1.59	13.28
FM12B3RGO	2.96	4.07	1.82	2.47	2.99	1.12	1.57	11.16

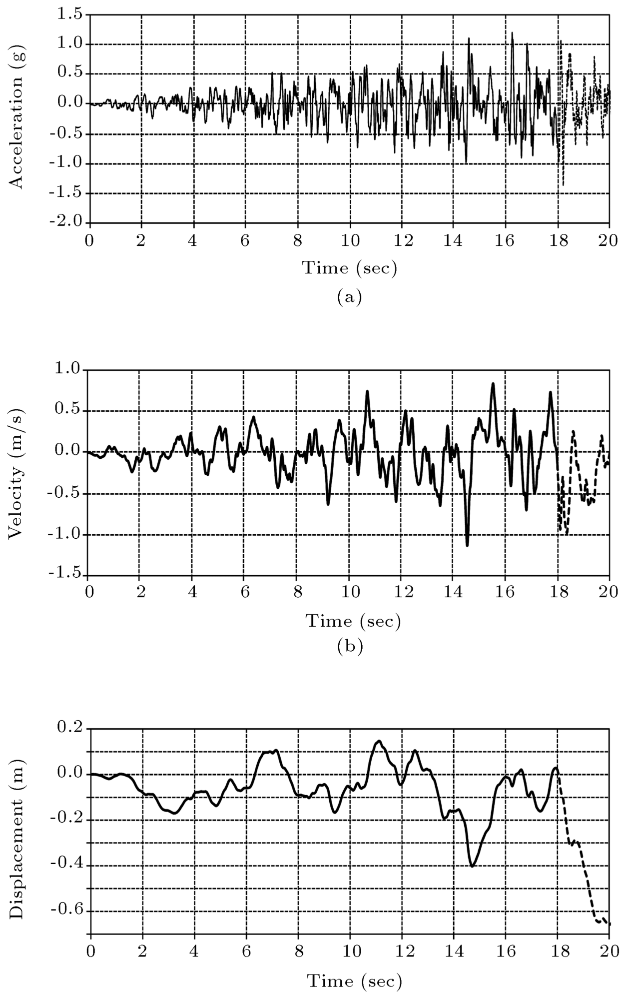

Figure 3. a) ETA20f02 acceleration function; b) ETA20f02 velocity function; and c) ETA20f02 displacement function.

average of the first-mode spectral acceleration of the records ($S_{a,\mathrm{Ave}}$) should be calculated. Furthermore, the value of the smooth response spectrum used for the generation of ET acceleration functions at the first-mode period (T_1) is calculated ($S_{a,\mathrm{ET}}$). Finally, the equivalent time is obtained by the following equation:

$$t_{\mathrm{eq}} = \frac{S_{a,\mathrm{Ave}}}{S_{a,\mathrm{ET}}} \times 10. \tag{3}$$

Constant 10 is used in this equation because the response spectrum of the ET acceleration function at $t = 10$ seconds matches the target smooth response spectrum. Equivalent times of each frame for the GM1 set of records are shown in Table 3.

As seen in Table 3, for frames that have long periods, the equivalent time is larger than 10 seconds, which is the target time in the generation of ET acceleration functions. The main reason for obtaining such a large equivalent time is the scaling procedure used for the records. For example, the equivalent time for a FM12B3RGW frame is 15.36 sec. The period of this frame is 2.89 sec and, therefore, the scaling procedure is done for the range of 0.578 to 4.335 sec. In this range, the smooth spectrum used for the generation of ET acceleration functions is less than the ASCE-7 spectrum (Figure 4). Therefore, large scale factors are used for this frame (Table 3). It can be concluded that for a better comparison of the results of ET analysis with the results of other methods, the response spectrum used for the generation of acceleration functions should be consistent with the spectrum used in other methods in a wide range of periods (e.g. 0 to 5 sec).

All the frames have been subjected to the ETA20f set of acceleration functions and GM1 set of ground motions. The response data summarized in this research are part of a comprehensive database on EDPs acquired for the previously defined generic frames. The discussion presented here focuses on maximum interstory drift ratios for frames. This EDP is relevant to structural damage, if the damage is dominated by the maximum story deformation over the height and is a measure of damage to nonstructural components. For $P-\Delta$ sensitive structures, the maximum interstory drift ratio is the most relevant EDP for global collapse assessment, because dynamic instability is controlled by the story in which the story drift grows most rapidly [3]. To examine the consistency of the IM of nonlinear response history analysis and IM obtained from ET analysis, the base shears of the frames obtained by two methods are compared.

The average of maximum interstory drift ratios for ET acceleration functions through time and values of equivalent time are presented for FM03B1RG frames in Figure 5. These results are obtained considering the EPP material model for the frames. It should be noted that ET analysis results are usually presented by

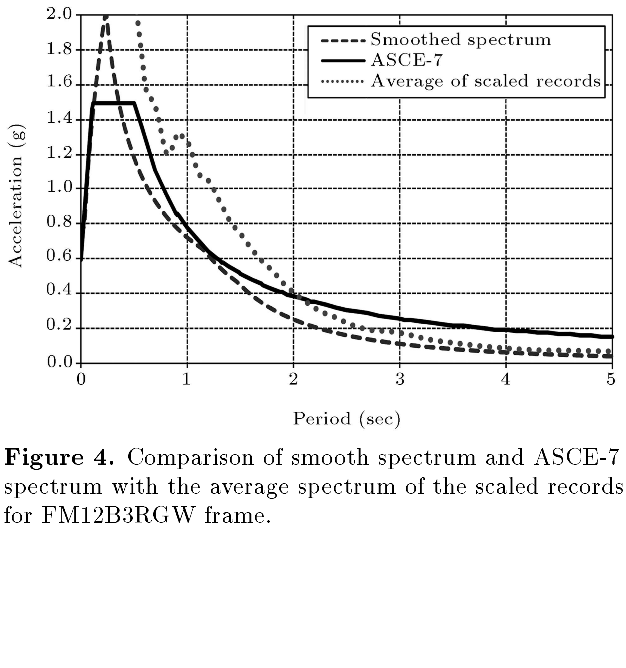

Figure 4. Comparison of smooth spectrum and ASCE-7 spectrum with the average spectrum of the scaled records for FM12B3RGW frame.

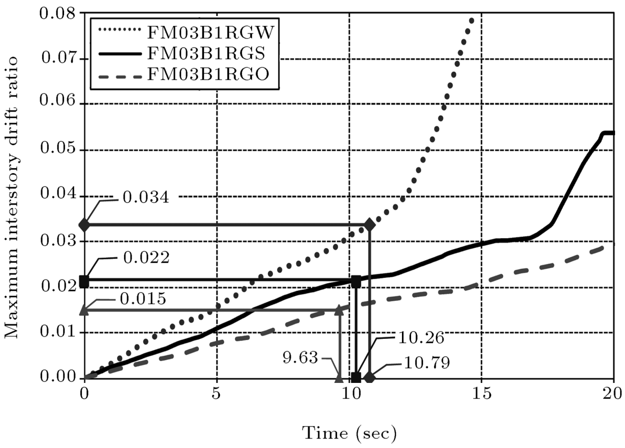

Figure 5. ET maximum interstory drift ratio curves for FM03B1RG frames with EPP material model and values of t_{eq} of nonlinear response history analysis.

increasing curves, where the y coordinate at each time value, t, corresponds to the maximum absolute value of the required parameter in the time interval, $[0, t]$, as given in Equation 4:

$$\Omega(f(t)) \equiv \max(\mathrm{Abs}(f(\tau) : \tau \in [0, t]), \tag{4}$$

where Ω is the Max-Abs operator as defined above and $f(t)$ is the response history, such as base shear, interstory drift, damage index or other parameters of interest.

Because of the statistical characteristics and dispersion of ET analysis results in a nonlinear range, the resulted curves are serrated. Sometimes, the value of the response does not pass the maximum value experienced before in a long time interval. Therefore, the resulted ET curve has a constant value in that interval. In this research, a moving average procedure is used to reduce the serrated nature of the ET curves in a nonlinear range.

Looking through a typical ET curve can reveal significant information about a structure. During the initial phase of the excitation, the structure behaves linearly until it reaches a certain point where a plastic hinge is created, i.e. plastic behavior. By increasing the intensity of the acceleration function through time, the structure experiences more plastic deformations until it reaches the collapse limit. Occurrence of the collapse for a structure through an ET analysis is dependent on its lateral stiffness. For example, a FM03B1RGW frame experiences significantly high displacements after 12 second. But, the two other frames do not collapse before the 20th second, as can be seen in Figure 5.

Maximum interstory drift ratios of FM03B1RG frames with an EPP material model obtained from a nonlinear response history analysis are presented in Figure 6. As can be seen in Figure 6, all the records rank the structures based on their lateral stiffness. This result can be seen in ET curves too. Maximum interstory drift ratios of a FM03B1RGW frame for NRORR360 and LADSP000 records are far beyond the average values of other records. In a nonlinear response history analysis, such exceptions can significantly change the average value of the response. It will be discussed later that the main cause of the differences between the results is the $P-\Delta$ effect. As can be seen in Table 4, the ET method underestimates the results of this frame. It means that the ET method could not estimate these exceptions well.

Figures 7 and 8 show the interstory drift ratio response history of the FM03B3RGW frame for the LPAND270 record and the ETA20f02 acceleration function, respectively. Most of the records and all ET acceleration functions anticipate that the maximum

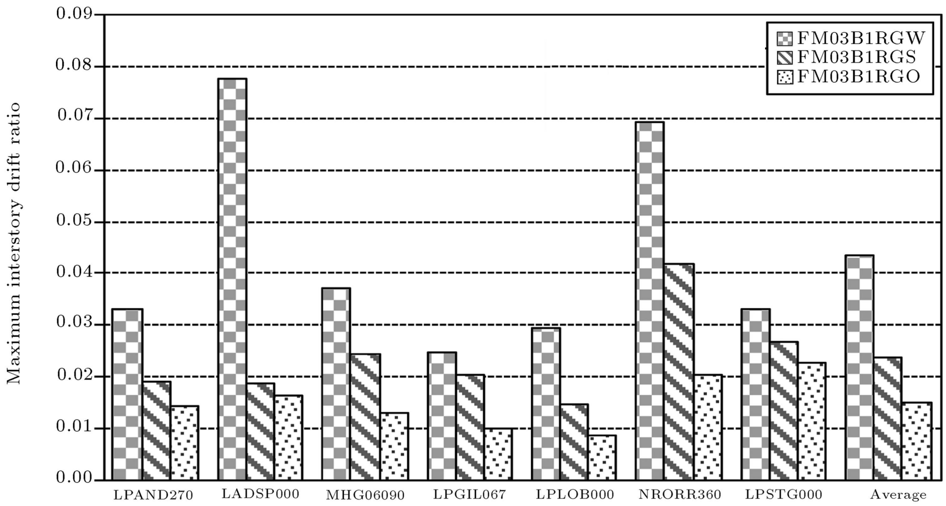

Figure 6. Maximum interstory drift ratios of the accelerograms and their average for FM03B1RG frames with EPP material model.

Table 4. Comparison between the results of nonlinear response history analysis and ET analysis for different frames with EPP material model.

Frames	Maximum Interstory Drift Ratio				Base Shear (KN)	
	Average		Standard Deviation		Average	
	Nonlinear Time History Analysis	ET Analysis	Nonlinear Time History Analysis	ET Analysis	Nonlinear Time History Analysis	ET Analysis
FM03B1RGW	0.043	0.034	0.021	0.004	218.1	217.8
FM03B1RGS	0.024	0.021	0.009	0.003	323.0	335.6
FM03B1RGO	0.015	0.015	0.005	0.001	505.3	558.3
FM03B3RGW	N.A.	0.057	N.A.	0.022	N.A.	543.6
FM03B3RGS	0.045	0.025	0.037	0.003	912.5	933.7
FM03B3RGO	0.017	0.014	0.007	0.002	1424.7	1513.3
FM07B1RGW	0.032	0.026	0.011	0.003	284.9	286.6
FM07B1RGS	0.023	0.022	0.004	0.005	433.6	446.5
FM07B1RGO	0.015	0.014	0.002	0.002	729.3	733.2
FM07B3RGW	N.A.	0.037	N.A.	0.007	N.A.	738.1
FM07B3RGS	0.025	0.023	0.003	0.005	1256.4	1274.4
FM07B3RGO	0.016	0.015	0.003	0.003	2098.2	2109.0
FM12B3RGW	0.032	0.025	0.004	0.004	1002.5	991.6
FM12B3RGS	0.029	0.022	0.010	0.002	1465.1	1481.6
FM12B3RGO	0.015	0.015	0.003	0.001	2898.1	2983.1

interstory drift ratio is obtained in the second story for this frame. The difference between the results of this story and the others is significant in high intensity measures.

As another example, the average of maximum interstory drift ratios for ET acceleration functions through time are presented for FM07B1RG frames with an EPP material model in Figure 9. As can be seen, the general trend of the results is like the FM03B1RG frames results, but there are some exceptions too. ET results show that between $t = 9$ to 13 seconds, the maximum interstory drift ratio of the FM07B1RGW frame is less than that obtained for the FM07B1RGS frame. This estimation of the ET method can be checked by a nonlinear response history analysis. To do so, a reduction scale factor is applied to the GM1 set to change the equivalent time of the FM07B1RGW frame to the equivalent time of the FM07B1RGS frame. Finally, the FM07B1RGW frame is analyzed for the GM1 set with this reduction scale factor. Now, the results of FM07B1RGW and FM07B1RGS frames can be compared at $t_{\mathrm{eq}} = 11.47$ seconds. At this time,

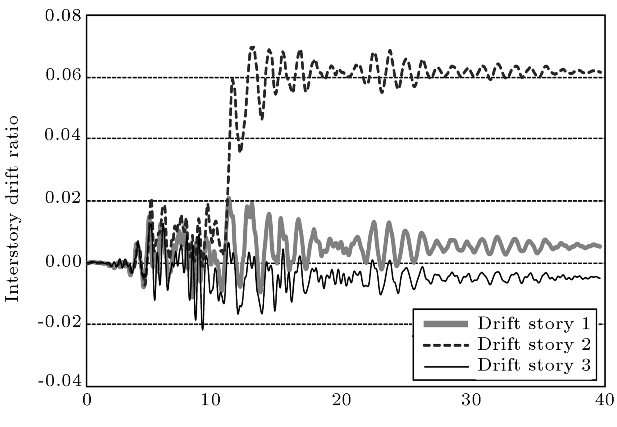

Figure 7. Interstory drift ratio response history of FM03B3RGW frame for LPAND270.

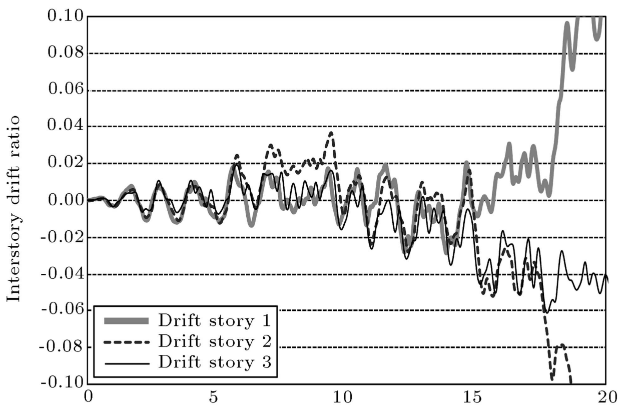

Figure 8. Interstory drift ratio response history of FM03B3RGW frame for ETA20f02.

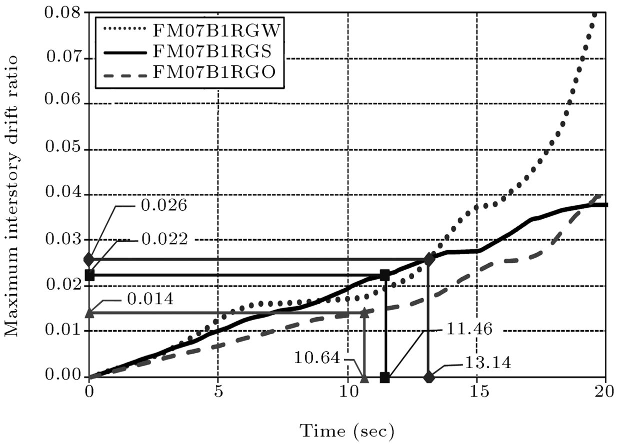

Figure 9. ET maximum interstory drift ratio curves for FM07B1RG frames with EPP material model and values of t_{eq} of nonlinear response history analysis.

the maximum interstory drift ratio of the FM07B1RGS frame obtained from the ET analysis is larger than the corresponding value of the FM07B1RGW frame. But, the results of the nonlinear response history analysis are vice versa. The maximum interstory drift ratio of the FM07B1RGS frame is 0.0226 and the corresponding value for the FM07B1RGW frame at $t_{\mathrm{eq}} = 11.47$ seconds is 0.024. It shows that when the EDPs of the frames are very close together, small differences in ET analysis curves might be a randomness effect and should not be interpreted as an indication that one structure has better performance over the other. In these cases, performance differences may actually be insignificant. A more refined analysis, using more ground motions and ET acceleration functions, are required if a definitive conclusion is to be made in such cases.

Figure 10 shows the average of the maximum interstory drift ratios of FM07B3RG frames with an

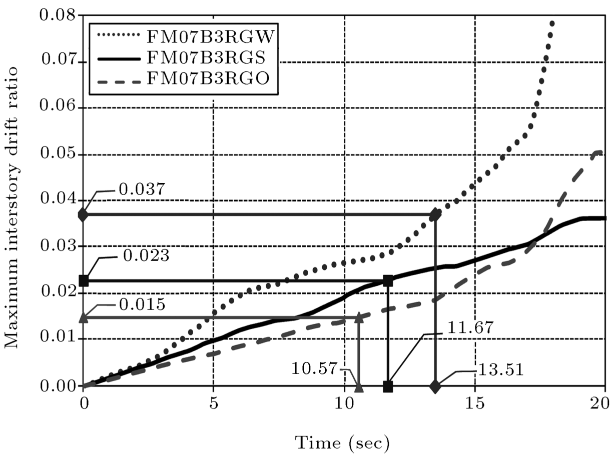

Figure 10. ET maximum interstory drift ratio curves for FM07B3RG frames with EPP material model and values of t_{eq} of nonlinear response history analysis.

EPP material model for ET acceleration functions through time. Like the previous examples, ET curves differentiate between three frames. Between $t = 18$ to 20 seconds, the maximum interstory drift ratios of the FM07B3RGO frame are larger than the corresponding value for the FM07B3RGS frame. This can be checked by doing a nonlinear response history analysis for the equivalent IM for this range of time. To do so, increasing scale factors are applied to the GM1 set for FM07B3RGO and FM07B3RGS frames. These scale factors change the equivalent time of these frames to 20 seconds. Maximum interstory drift ratios of FM07B3RGS and FM07B3RGO frames at $t_{\mathrm{eq}} = 20$ seconds are 0.0444 and 0.0663, respectively. This time, the results of the nonlinear response history analysis are consistent with the results of the ET analysis.

The average and standard deviation of maximum interstory drift ratios and the average of base shears obtained from a nonlinear response history analysis and the ET analysis of the frames with the EPP material model are compared in Table 4. It should be noted that for two cases of underdesigned frames, the nonlinear response history analyses of some ground motions did not converge and, therefore, no results are presented for them. In most frames, estimations of ET analysis for the maximum interstory drift ratio are less than nonlinear response history analysis results. The difference between the results is more in underdesigned frames which experience more nonlinearity in their analyses. Usually, in these frames, the dispersion of the results of nonlinear response history analyses is high. But, in frames that behave more linearly than others, the difference between the results of ground motions is less and the results match the results of the ET analysis better.

Table 4 shows that the consistency of the base shears obtained by two methods is acceptable. It means that the procedure to find the equivalent time in the ET analysis to match the IMs of the two methods works well. It should be noted that although the equivalent time tries to show consistency between the IMs of the two methods, this is done just for one period. Therefore, the average response spectrum of ET acceleration functions at $t = t_{\mathrm{eq}}$ and the average response spectrum of scaled accelerograms have some minor differences, which cause the inconsistency of the base shears in some frames like the FM03B1RGO and FM03B3RGO. The difference between the base shears obtained by two methods is more in overdesigned frames, and in overdesigned and properly designed frames, the ET method always overestimates the base shear.

Figure 11 shows the mean and mean plus and minus standard deviation (STDEV) of maximum interstory drift ratios of frames with the EPP material

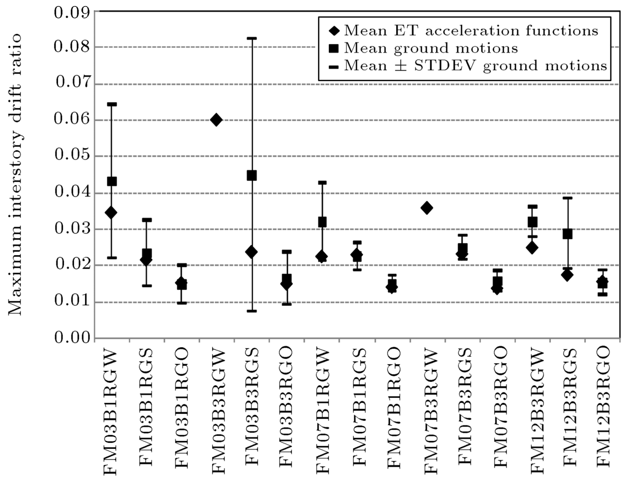

Figure 11. Maximum interstory drift ratios of the frames with EPP material model obtained by nonlinear response history analysis and ET analysis.

model obtained by nonlinear response history analyses. The average of maximum interstory drift ratios of the frames obtained by ET analysis is also shown in this figure. If this figure is compared with Table 4, it can be concluded that when the dispersion of the results is high, the difference between the results of the two methods is high either. For example, Figure 11 shows that the dispersion of the results of underdesigned frames is larger than properly designed and overdesigned frames. For these frames, Table 4 and Figure 11 show the maximum difference between interstory drift ratios obtained by two methods. For the FM03B3RGS frame where the dispersion is high, the difference between the results of the two methods is high either.

The $P-\Delta$ effect increases the dispersion of the results of nonlinear response history analyses. The nonlinear seismic response of steel moment-resisting frame structures, which are usually quite flexible, may be severely influenced by the structure $P-\Delta$ effect. This occurs especially when these structures are subjected to large displacements under severe ground motions. For structures in which this effect induces negative post-yield story stiffness, the responses become very scattered under severe ground motions [7].

Figure 12 compares the results of the nonlinear response history analysis of the FM03B1RGW frame with the EPP material model by considering and eliminating the $P-\Delta$ effect. In most of the accelerograms, the maximum interstory drift ratio obtained for the model, eliminating the $P-\Delta$ effect, is less than the model considering this effect. The largest difference between these results is obtained for LADSP000 and NRORR360 records. Also, the responses of the frame to these records are the largest in the GM1 set. By eliminating the $P-\Delta$ effect, the results of these records approach the average response for the GM1 set and the dispersion of the results reduces significantly.

The same trend can be seen in the results of the ET analysis. Figure 13 compares the results of the ET analysis of the FM03B1RGW frame with the EPP material model by considering and eliminating the $P-\Delta$ effect. It can be seen that the curves are separated from each other at about $t = 11$ seconds. After this time, the $P-\Delta$ effect increases the maximum interstory drift ratio of the frame. As can be seen in the figure, the $P-\Delta$ effect changes the EDP after the equivalent time computed for the nonlinear response history analysis. In other words, effects of $P-\Delta$ on the results of the ET analysis and nonlinear response history analysis are not seen at the same IM. The

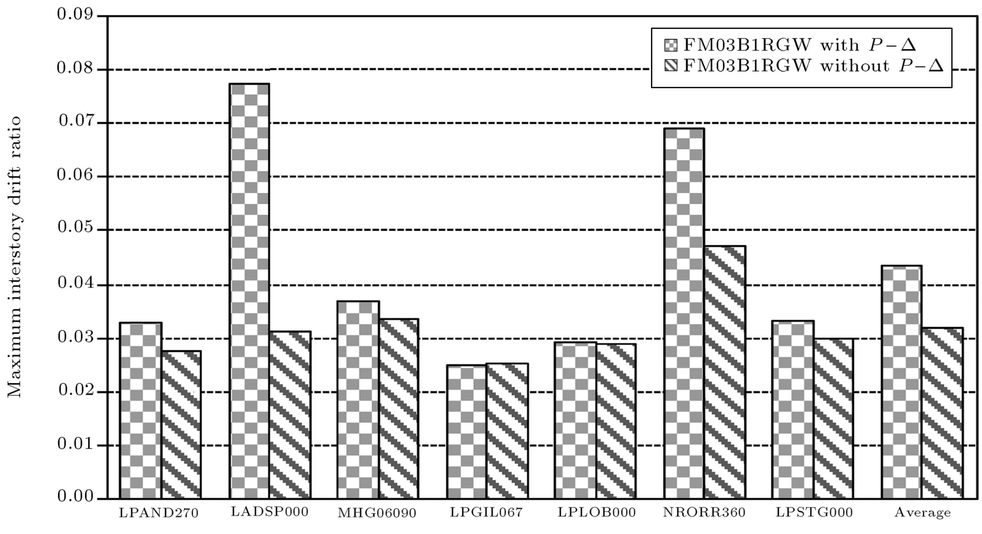

Figure 12. Maximum interstory drift ratios of the accelerograms and their average for FM03B1RGW frame with EPP material model with and without considering $P-\Delta$ effects.

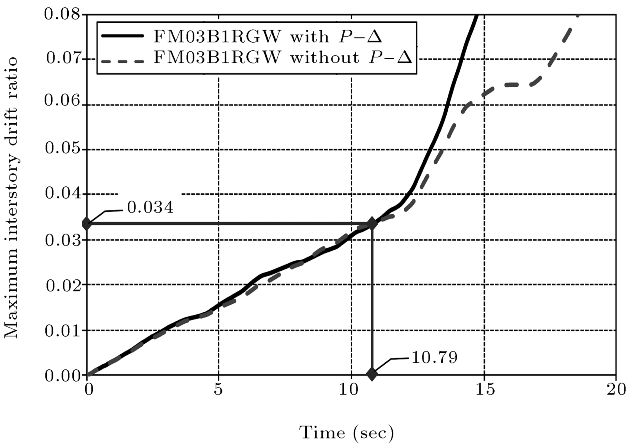

Figure 13. ET maximum interstory drift ratio curves for FM03B1RGW frame with EPP material model with and without considering $P - \Delta$ effects.

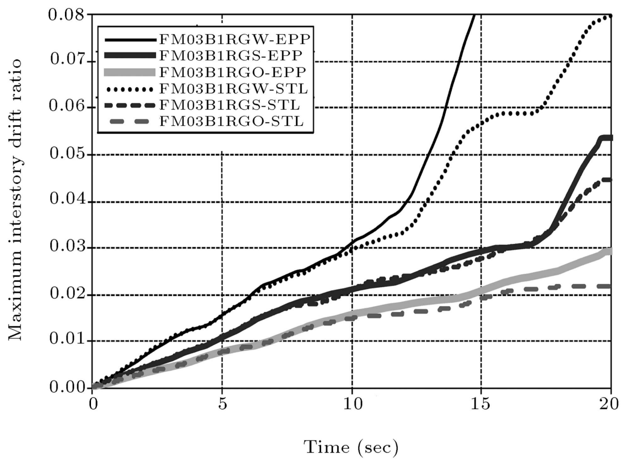

Figure 14. ET maximum interstory drift ratio curves for FM03B1RG frames with EPP and STL material model.

reason for this phenomenon is the difference in nature of acceleration functions and ground motions. For some of the ground motions, $P - \Delta$ effects increase the maximum interstory drift ratio drastically and the average value of this parameter is changed a lot. This is due to the fact that the characteristics of the ground motions are really different from each other. Unlike real earthquakes, ET acceleration functions have similar characteristics. Consequently, the dispersion of the results of nonlinear response history analyses for these frames is high but, in ET analysis, the dispersion of the results is not significant.

$P - \Delta$ effects are not critical if the effective stiffness at maximum displacement remains positive [7]. As mentioned before, the EPP material model was used for previous models. Because this material model has no strain hardening, the frames tend to reach the negative post-yield stiffness especially in underdesigned frames and, therefore, $P - \Delta$ effects can change their drifts a lot. If the STL material model that has 3% post-yield stiffness is used instead of the EPP material model, it can be guessed that the results of the ET analysis approach nonlinear response history analysis results.

Figure 14 compares ET maximum interstory drift ratio curves for FM03B1RG frames with the EPP and STL material model. As can be seen, by increasing the strain hardening of the material model, the $P - \Delta$ effects are decreased. If this figure is compared with Figure 13, it can be judged that ET results of the FM03B1RGW frame with the STL material model, considering $P - \Delta$ effects, are very similar to the result of this frame with the EPP material model eliminating $P - \Delta$ effects. The average and standard deviation of maximum interstory drift ratios, and the average of base shears obtained from the nonlinear response history analysis, and ET analysis of the frames with the STL material model are compared in Table 5. If this table is compared with Table 4, it can be seen that by considering 3% strain hardening, the differences between the results of the two methods are totally decreased. It can be concluded that for structures that are extremely sensitive to $P - \Delta$ effects, the results of ET analysis should be used with special care. To avoid underestimated values for EDPs, such as a maximum interstory drift ratio, it is better to define a limit for the lateral deformation of the structure. This limit should specify the onset of reaching negative post-yield story stiffness. The interstory drift ratio obtained at the maximum base shear of a pushover curve can be a good value for this limit.

Although the maximum interstory drift ratios and corresponding base shears are calculated at t_{eq}, these values do not necessarily happen at the same time. ET curves are obtained by the Max-Abs operator described in Equation 4 and perhaps the maximum value for the base shear is not obtained at the same time that the maximum value of the maximum interstory drift ratio is gained.

The results of the ET analysis reported in this research can be used in order to estimate the mean demand structures due to ground motions. However, a scatter of the results from earthquakes is not anticipated by the ET method. Therefore, a safety margin for expecting seismic demands should be defined. In spectral analysis, this issue is taken into account by using the mean spectrum plus the standard deviation of the ground motions. The same method can be used in ET analysis. In addition to generating ET acceleration functions based on a mean spectrum, another set of acceleration functions can be generated by assuming the mean spectrum plus the standard deviation as the target spectrum. The results of the ET analysis obtained for this set can be used as an upper estimation of the results of ground motions and it can be addressed as a safety margin in the ET analysis.

Table 5. Comparison between the results of nonlinear response history analysis and ET analysis for different frames with STL material model.

Frames	Maximum Interstory Drift Ratio				Base Shear (KN)	
	Average		Standard Deviation		Average	
	Nonlinear Time History Analysis	ET Analysis	Nonlinear Time History Analysis	ET Analysis	Nonlinear Time History Analysis	ET Analysis
FM03B1RGW	0.034	0.031	0.006	0.003	227.6	230.8
FM03B1RGS	0.022	0.022	0.006	0.004	339.4	355.4
FM03B1RGO	0.014	0.014	0.004	0.001	523.9	568.3
FM03B3RGW	0.035	0.030	0.009	0.002	590.463	588.9
FM03B3RGS	0.025	0.022	0.006	0.003	964.0	1019.9
FM03B3RGO	0.015	0.014	0.005	0.002	1538.9	1579.8
FM07B1RGW	0.025	0.023	0.003	0.002	294.8	305.6
FM07B1RGS	0.021	0.020	0.003	0.004	470.7	466.0
FM07B1RGO	0.015	0.014	0.001	0.001	779.1	779.6
FM07B3RGW	0.036	0.033	0.006	0.003	790.679	795.9
FM07B3RGS	0.021	0.019	0.003	0.003	1338.8	1322.6
FM07B3RGO	0.015	0.014	0.002	0.001	2261.4	2295.8
FM12B3RGW	0.026	0.022	0.002	0.001	1036.0	1067.4
FM12B3RGS	0.022	0.019	0.004	0.001	1545.8	1619.9
FM12B3RGO	0.015	0.014	0.002	0.002	2917.5	3225.1

APPLICATION OF ET METHOD IN SEISMIC REHABILITATION OF BUILDINGS

The analysis of underdesigned, properly designed and overdesigned frames discussed in the previous section were for explanatory purposes and their relative performances could be guessed without advanced analysis. In order to further demonstrate the significance of ET analysis, the capability of the ET method in a more involved situation is demonstrated in this section.

As was shown in Table 4, the maximum interstory drift ratio of the FM03B3RGW frame with the EPP material model obtained by ET analysis is 0.057. The nonlinear response history analysis of this frame does not converge for two ground motions and it can be judged that the performance of this frame is not acceptable. Now let us assume that the maximum acceptable interstory drift ratio for this frame has been set to 0.04. In order to improve the frame's performance, a viscoelastic damper is to be used. The question to be answered is, at which story level should a damper with a damping constant of 2000 KN.sec/m be installed in order to result in the best performance? i.e. the least overall maximum interstory drift ratio.

ET analysis and nonlinear response history analysis are done for this frame by installing the damper in different stories. Results of the analysis are shown in Figure 15 and Table 6. Figure 15 shows that by installing the damper in the 3rd story, no significant

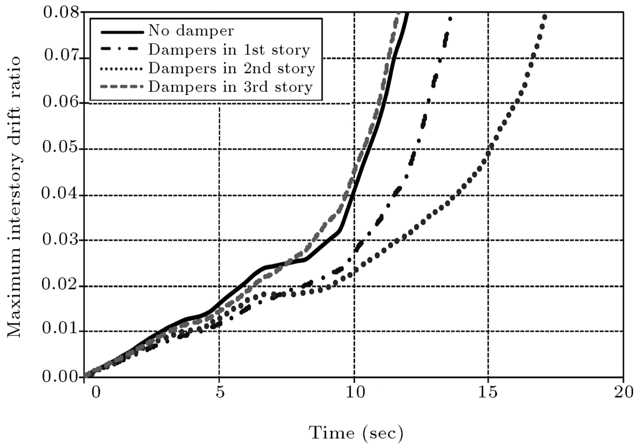

Figure 15. ET maximum interstory drift ratio curves for FM03B3RGW frames with different locations for dampers.

performance improvement is achieved. The same result is also obtained by comparing the values of maximum interstory drift ratios for different records obtained from nonlinear response history analyses. Analyses do not converge for LADSP000 and NRORR360 records for both cases. Another point that can be concluded from Figure 15 is that, by installing the damper in the 1st or 2nd story, the performance of the frame improves. The best performance is obtained when the damper is installed in the 2nd story. The performance of this case is really better in high IMs. Maximum interstory drift ratios of the frames having dampers in the 1st or 2nd story at the equivalent time are

Table 6. Comparison between the results of nonlinear response history analysis for FM03B3RGW frames with different locations for dampers.

Records	Maximum Interstory Drift Ratio			
	No Damper	Damper in 1st Story	Damper in 2nd Story	Damper in 3rd Story
LPAND270	0.0699	0.0401	0.0246	0.0534
LADSP000	N.A.	0.0219	0.0497	N.A.
MHG06090	0.0434	0.0534	0.0468	0.0456
LPGIL067	0.0244	0.0199	0.0250	0.0231
LPLOB000	0.0290	0.0241	0.0221	0.0287
NRORR360	N.A.	0.2797	0.0504	N.A.
LPSTG000	0.0444	0.0314	0.0335	0.0362

0.0344 and 0.0272, respectively. Again, the results of nonlinear response history analyses confirm the results of the ET analysis. Nonlinear response history analysis shows that the averages of maximum interstory drift ratios of the frames having dampers in the 1st or 2nd story are 0.0672 and 0.0360, respectively. The results of ET analysis somewhat underestimate the results of earthquakes again because of the $P-\Delta$ effects.

As mentioned before, one of the beneficial advantages of the ET method is its capability in differentiating between different structural systems with a minimum number of analyses. The previous example shows that although the results of the ET analysis are not exactly consistent with the results of ground motions analysis, the ET method can pinpoint the structure with a better performance, even in the case of structures with relatively complicated behavior. This can be put into good use in performance-based seismic design where a lot of trial designs should be checked in order to find the structure that meets the performance objectives in an optimal manner.

SUMMARY AND CONCLUSIONS

In most frames with an EPP material model, estimations of ET analysis for the maximum interstory drift ratio are less than nonlinear response history analysis results. The difference between the results is more in underdesigned frames, which experience more nonlinearity in their analysis. But, in frames that behave more linearly than others, the difference between the results of ground motions is less and the results match the results of ET analysis more closely. The consistency of the base shears obtained by two methods is reasonable. The procedure to find the equivalent time in the ET analysis to match the IMs of two methods is acceptable. The ET method is successful in locating the story with the maximum interstory drift ratio.

The dispersion of the results of the nonlinear response history analysis for the frames with the EPP material model that experience more nonlinearity is relatively high. The dispersion of results cannot be estimated using current sets of ET acceleration functions. When the dispersion of the results of the nonlinear response history analysis is high, ET analysis underestimates the maximum interstory drift ratio more significantly.

The main reason for the dispersion of the results of the nonlinear response history analysis is $P-\Delta$ effects. In cases where $P-\Delta$ effects are excluded, the results of the two methods closely match. The same phenomenon can be seen in the results of ET analysis, but effects of $P-\Delta$ on the results of ET analysis show up at a higher IM as compared to a nonlinear response history analysis. Frames with an EPP material model are more sensitive to $P-\Delta$ effects, because they develop a negative post-yield stiffness under severe ground motions. Therefore, for some of the ground motions, $P-\Delta$ effects increase the maximum interstory drift ratio drastically and the average value of this parameter is affected considerably making it unusable.

In STL material models that have 3% post-yield stiffness, the results of the ET analysis match the nonlinear response history analysis results with good precision.

It is shown that, although the results of the ET analysis are not exactly consistent with the results of the ground motions analysis in all cases of material properties and IMs, in most cases the ET method is quite successful in differentiating between structures (or design alternatives) with better performance, even in the case of relatively complicated structures. It should be noted that when the ET analysis estimation for the EDPs of the frames shows closely identical results, some reservations should be considered before drawing a conclusion. The differences observed in these cases may have been resulted from pure randomness effects. A detailed nonlinear response history analysis, using a relatively large set of relevant earthquakes, is needed for the final verification in cases where two systems show a very close response.

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