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Stability Analysis of a Window-Based High-Speed Hierarchical Rate Allocation Algorithm

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Providing the stability of any rate allocation algorithm is a challenging issue in current high-speed networks. Some researchers, such as Kelly, Massoulié, Vinnicombe and Johari, have shown the stability of their rate-based rate allocation algorithms using different approaches. Some other researchers have investigated the stability of the second-order, rate-based, rate allocation algorithms under some simplifying constraints. Mo et al. have proved the stability of the first-order, window-based rate allocation algorithms, using control theory concepts, for a wide range of fairness criteria. In the current work, the stability property of a second-order, high-speed and window-based rate allocation strategy has been investigated using the Lyapunov approach. Simulation results verify the stability of the proposed method under a general network scenario.

INTRODUCTION

Generally speaking, there exist two main approaches for modeling network traffic in data networks. In the window-based approach, network traffic is modeled as a discrete flow of packets traversing the network. In the rate-based approach, network traffic is considered as fluid flow and the continuous behavior of the traffic is important. Some researchers, such as Low [1] and Mo [2], have adopted the 1st perspective of the traffic and others, such as Golestani [3], Kelly [4] and Johari [5], have used the 2nd point of view for developing and analyzing proper rate allocation algorithms.

Hierarchical models of rate allocation are also investigated in [6] and their suitability for reducing the overhead in communication networks is discussed therein.

Fairness in rate allocation is an essential part of any rate allocation strategy. There are plenty of fairness criteria, such as max-min [7], proportional [4] and minimum potential delay [8] fairness. Selecting a fairness criterion depends on the network design strategy.

Another important feature of a rate allocation algorithm is its stability. Some researchers have adopted a rate-based point of view of network traffic for their stability analysis. For example, Kelly et al., in [4], have investigated the stability of their proposed method using the Lyapunov approach. Johari et al. [5] and Massoulié [9] have completed the work of Kelly by incorporating round trip delays in their stability analysis.

As the Kelly algorithm is based on the 1st order gradient descent method, it suffers from low convergence speed, so, Goudarzi et al. have proved the stability of the 2nd order and high-speed rate allocation algorithms with the assumption of a unique round trip delay for all users [10]. Moreover, Vinnicombe has used the robust control theory for deriving the conditions under which a set of Internet-like communication networks, incorporating a form of his so-called TCP-like congestion control, remain stable [11].

Some other researchers, such as Mo et al. [2], have used the more realistic window-based approach for stability analysis of the 1st order rate allocation algorithms by introducing an appropriate Lyapunov function.

In this paper, the same approach as that of Mo et al. is followed for investigating the stability property of a 2nd order hierarchical window-based rate allocation algorithm.

The paper is organized as follows. First, some related works emphasizing the work of Kelly are reviewed. Then, the high-speed hierarchical method is introduced and reviewed more closely. After that, the stability of the method is proven and, finally, the

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simulation results are presented and some conclusions are discussed.

BACKGROUND

Consider a network with set J of resources or links and set $\mathfrak{R}$ of users and let c_j denote the finite capacity of link $j \in J$. Each user, r, has a fixed traffic route, R_r, which is a nonempty subset of J. Also, a zero-one routing matrix, A, is defined, where $A_{rj} = 1$, if link j is in user r's traffic route, R_r, and $A_{rj} = 0$, otherwise. When the allocated rate to the user is x_r, user r receives utility $U_r(x_r)$. Utility $U_r(x_r)$ is an increasing, strictly concave and continuously differentiable function of x_r, over the range $x_r \geq 0$. Furthermore, it is assumed that the utilities are additive, so that the aggregate utility of rate allocation $\chi = (x_r, r \in \mathfrak{R})$ is: $\Sigma_{r\in\mathfrak{R}} U_r(x_r)$. This is a reasonable assumption, since these utilities are those of independent network users. It is assumed that user utilities are logarithmic.

Kelly's formulation of the proportionally-fair rate allocation would be:

$$x_r[n+1]=\left\{x_r[n]+k_r.\left(\omega_r-x_r[n].\sum_{j\in R_r}\mu_j[n]\right)\right\}^+, \tag{1}$$

where:

$$\mu_j[n] = p_j\left(\sum_{s:j\in R_s} x_s[n]\right), \qquad \{x\}^+ \cong \max(0,x). \tag{2}$$

Parameter k_r controls the speed of convergence in Equation 1. $p_j(y)$ is the amount that link 'j' penalizes its aggregate traffic 'y'. It is a non-negative, continuous increasing function and tends to infinity as aggregate rate 'y' tends to link capacity c_j. Given λ_r, user r selects an amount that it is willing to pay per unit time, ω_r, and receives a rate, $x_r = \omega_r/\lambda_r$.

One of the interpretations is that, by using Equation 1, the system tries to equalize ω_r with $x_r[n]$; $\sum_{j\in R_r} \mu_j[n]$ adjusting the $x_r[n]$ value. Systems 1 and 2 show that the unique equilibrium, x_r^*, is the solution of the following equation [4]:

$$\omega_r = x_r^*.\sum_{j\in R_r} p_j\left(\sum_{s:j\in R_s} x_s^*\right), \qquad r\in\mathfrak{R}. \tag{3}$$

HIGH-SPEED ALGORITHM

The high-speed algorithm is composed of a two-level hierarchical structure [6]. First, see an example by considering Figure 1. Let's assume that the network is consisted of 11 elastic sources [12] that are included in four source virtual users. Dotted lines show the boundaries of the virtual users and thick lines show the aggregate flow of each virtual user that is traversing through the common links (these links are denoted by letters L6, L7 and L8). Each source (destination) of information is denoted by 's' ('d') and, as mentioned before, the rate associated with each (source, destination) pair is denoted by 'x'. Links are unidirectional and in Figure 1, links 6, 7 and 8 constitute common links.

As Kelly has shown in [4], stabilized rates of users

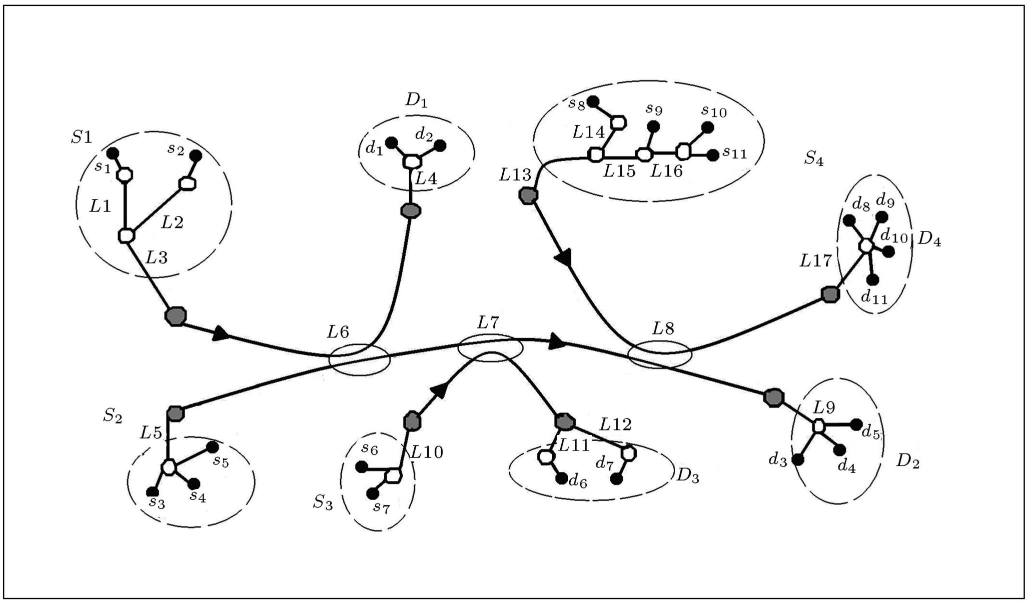

Figure 1. A sample network with two levels of hierarchy.

are:

$$x_r^* = \omega_r/\lambda_r^*, \qquad r \in \mathfrak{R},$$

where:

$$\lambda_r^* = \sum_{j \in R_r} p_j \left(\sum_{u: j \in R_u} x_u^* \right).$$

Since it is assumed that congestion may only occur in the common links, it may be considered that only λ_r^* is affected by these links and is approximated by:

$$\lambda_r^* \cong \sum_{\substack{j \in R_r \\ \& j \in \text{Common-links}}} p_j \left(\sum_{u: j \in R_u} x_u^* \right). \tag{4}$$

For example, for users 's_1' and 's_2' in Figure 1, one would have:

$$x_1^* = \frac{\omega_1}{\lambda_1^*}, \qquad x_2^* = \frac{\omega_2}{\lambda_2^*}. \tag{5}$$

Define:

$$\Lambda_1^* \triangleq p_6 \left(\sum_{u: L_6 \in R_u} x_u^* \right),$$

where Λ_1^* is the aggregate penalty of users 's_1' and 's_2' (λ_1^* and λ_2^*) in common links (link '6' in this case).

Then, at the equilibrium point, the aggregate rate of virtual user 1 is:

$$x_1^* + x_2^* = \frac{\omega_1}{\lambda_1^*} + \frac{\omega_2}{\lambda_2^*} \cong \frac{\omega_1 + \omega_2}{\Lambda_1^*}. \tag{6}$$

In another words, virtual user 1 might be regarded as a user with logarithmic utility function ($\Omega_1 \log(\chi_1)$), in which $\Omega_1 = \omega_1 + \omega_2$.

If one denotes the aggregate rate of virtual user 1 with χ_1, at the equilibrium point, one has:

$$\chi_1^* = \frac{\omega_1 + \omega_2}{\Lambda_1^*}. \tag{7}$$

By considering Equations 5 and 7 and the assumption that $\lambda_1^* \cong \Lambda_1^*$, the following holds:

$$x_1^* \cong \frac{\omega_1}{\Omega_1} . \chi_1^*. \tag{8}$$

Now, in mathematical terms, let $\Sigma \cong \{\Sigma_i | i = 1, 2, \cdots, Q\}$ and $\Delta \cong \{\Delta_i | i = 1, 2, \cdots, Q\}$ be the sets that represent the virtual sources and virtual destinations, where Q represents the number of virtual sources (destinations). For example, in Figure 1 one has $Q = 4$ and $\Sigma_3 = \{s_6, s_7\}$, $\Delta_3 = \{d_6, d_7\}$.

If the rate associated with virtual user 'i' at iteration 'n' is denoted by '$\chi_i[n]$' and the rate of end users (as mentioned before) is denoted by the small letter 'x', the algorithm behaves in the following manner:

At the beginning, the algorithm works in the first level of hierarchy and allocates rates to the virtual sources using some high-speed algorithm (such as the Jacobi method). Then, each virtual user assigns some proportions of its rate to each end-user within the virtual user. Afterwards, by defining a temporary variable 'w', each user updates its corresponding 'w' parameter and when these new parameters are sent back to the virtual users, the first-level algorithm repeats its computations.

If the assumption in Equation 4 is true, when the system is in the vicinity of equilibrium point, users' rates are close to the optimal values. It will be shown that, by repeating this procedure, the rates will converge to the optimal rates. It must be emphasized here that the 'w' parameters, which are updated in the algorithm by end-users, have not the interpretation of users' willingness to pay (in contrast with what is discussed in [4] about "ω") and are merely temporary variables. The rate assignment by virtual user 'i' to a user, 'u', located within virtual user 'i', is:

$$x_u[n+1] = \chi_i[n] . \frac{w_u[n]}{W_i[n]}, \qquad n = 0, 1, 2, \cdots,$$

$$i = 1, 2, ..., Q, \qquad u \in i, \tag{9}$$

where notation '$u \in i$' means that user 'u' is located within virtual user 'i' and:

$$W_i[n] \triangleq \sum_{u \in i} w_u[n]. \tag{10}$$

Updating $\chi_i[n]$ in Equation 9 is as Jacobi iteration [7] ($i = 1, 2, \cdots, Q$):

$$\chi_i[n+1] =$$

$$\left\{ \chi_i[n] + K_i . \frac{W_i[n] - \chi_i[n] . \Lambda_i[n]}{\Lambda_i[n] + \chi_i[n] . \frac{\partial}{\partial \chi_i(t)} \Lambda_i(t) \Big|_{t=n}} \right\}^+, \tag{11}$$

where $\chi_i[0] = \varepsilon \cong 0$, $\forall i$ and, also:

$$\Lambda_i[n] \triangleq \sum_{\substack{j \in R_{S_i} \\ \& j \in \text{Common-links}}} p_j \left(\sum_{u: j \in R_u} \chi_u[n] \right).$$

Each 'w' parameter is updated in time scale, which is much larger than that of x using the following relation:

$$w_u[n+1]=$$

$$\begin{cases}\left\{w_u[n]+\alpha_u.\left(\frac{\frac{\omega_u}{\lambda_u[n]}-\frac{w_u[n]}{\Lambda_i[n]}}{\frac{\omega_u}{\lambda_u[n]}}\right)\right\}^+ & \text{for } n=0,N,2N,...\\ w_u[n] & \text{otherwise}\end{cases}$$

$$i=1,2,\cdots,Q, \qquad u\in i, \tag{12}$$

where $w_u[0]=\omega_u$ (the user-logarithmic utility function parameter), $u\in i$, $i=1,2,\cdots,Q$ and N is some large positive integer.

α_u is some positive constant ($0<\alpha_u<\delta_u, \forall i, u\in i$) that controls the convergence speed in Equation 12 and $\delta_u>0$ is an upper bound for α_u.

Equation 11 is, in fact, a form of the projected Jacobi method, as Bertsekas et al. have defined in [7]. The idea behind Equation 12 is that users try to adjust their final rates, which are assigned to them by a first-level algorithm, i.e. $(w_u[n]/\Lambda_i[n])$, to Kelly's rate, i.e. $(\omega_u/\lambda_u[n])$, by changing their 'w' parameters. The stability property of this algorithm is discussed in [10].

Now, we start to describe the previous rate-based equations in window-based form.

Define CWND$[n]$ as the window size for the aggregate window (the window associated with the virtual user) and cwnd$[n]$ as windows size for every source in time 'n'.

$\bar{d}_r$ is the user 'r' propagation delay and its round trip time is RTT_r. Using Little's theorem, we have [7]:

$$\chi_i[n]=\frac{\text{CWND}_i[n]}{\text{RTT}_i[n]}, \qquad i=1,2,\cdots,Q. \tag{13}$$

Using the same approach as Walrand et al. Equation 11 can be written in the following form:

$$\text{CWND}_i[n+1]=\Bigg\{\text{CWND}_i[n]+K_i.\text{RTT}_i[n]$$

$$.\left(W_i[n]-\frac{\text{CWND}_i[n]}{\text{RTT}_i[n]}.d_i[n]\right)/D_i[n]\Bigg\}^+, \tag{14}$$

where:

$$D_i[n]\triangleq\Bigg|d_i[n]+\frac{\text{CWND}_i[n]}{\text{RTT}_i[n]}$$

$$.\left(\frac{d_i[n]-d_i[n-1]}{\frac{\text{CWND}_i[n]}{\text{RTT}_i[n]}-\frac{\text{CWND}_i[n-1]}{\text{RTT}_i[n-1]}}\right)\Bigg|,$$

$$d_i[n]=\text{RTT}_i[n]-\bar{d}_i. \tag{15}$$

Similarly, $\bar{d}_i$ is virtual user 'i's propagation delay and its corresponding round trip time is RTT_i.

STABILITY ANALYSIS

Before proving the stability of the proposed window-based method, the window-based Equations 16-17 are transformed to their continuous time equivalents. So, by considering Equation 16, we can write:

$$\frac{dw_i(t)}{dt}=\Bigg\{K_i.\text{RTT}_i(t)$$

$$.(P_i(t)-\frac{w_i(t)}{\text{RTT}_i(t)}.(\text{RTT}_i(t)-\bar{d}_i))/D_i(t)\Bigg\},$$

$$i=1,2,...,Q, \tag{16}$$

where:

$$w_i(t)\triangleq\text{CWND}_i(t),$$

and:

$$P_i(t)\triangleq W_i(t).$$

Also:

$$\chi_i(t)=\frac{w_i(t)}{\text{RTT}_i(t)},$$

and:

$$d_i(t)=\text{RTT}_i(t)-\bar{d}_i. \tag{17}$$

We define:

$$s_i=w_i-\chi_i\bar{d}_i-P_i. \tag{18}$$

Theorem

The function $V(w)=\frac{1}{2}\sum_i s_i^2$ is a Lyapunov function for the system of Equations 16. The unique value, w, minimizing $V(w)$, is a stable point; the system to which trajectories converge.

Proof

From Equations 18-20 it can be shown that:

$$\frac{dw_i(t)}{dt}=-K_i\text{RTT}_i(t)s_i(t)/D_i(t). \tag{19}$$

By differentiating $V(w)=\frac{1}{2}\sum_i s_i^2$, we have:

$$\frac{dV(w)}{dt}=\sum_{i=1}^{Q}\left(s_i(t)\frac{ds_i(t)}{dt}\right). \tag{20}$$

However,

$$\frac{ds_i(t)}{dt}=\frac{dw_i(t)}{dt}-\bar{d}_i\frac{d\chi_i(t)}{dt}-\frac{dP_i(t)}{dt}. \tag{21}$$

We have:

$$\frac{d\chi_i(t)}{dt} = K_i \frac{P_i(t) - \chi_i(t)(\mathrm{RTT}_i(t) - \overline{d}_i)}{D_i(t)}$$

$$= -K_i \frac{s_i(t)}{D_i(t)}. \tag{22}$$

From Equations 19, 21 and 22, we have:

$$\frac{ds_i(t)}{dt} = -K_i \mathrm{RTT}_i(t) \frac{s_i(t)}{D_i(t)} + K_i \overline{d}_i \frac{s_i(t)}{D_i(t)} - \frac{dP_i(t)}{dt}. \tag{23}$$

In addition, $\frac{dP_i(t)}{dt} = 0$, so, Equation 20 can be rewritten as follows:

$$\frac{dV(w)}{dt} = \sum_i -K_i s_i^2(t) \left(\frac{\mathrm{RTT}_i(t) - \overline{d}_i}{D_i(t)} \right). \tag{24}$$

It is clear that:

$$\frac{\mathrm{RTT}_i(t) - \overline{d}_i}{D_i(t)} \geq 0.$$

Thus, we have:

$$\frac{dV(w)}{dt} \leq 0.$$

Consequently, $V(w)$ is a Lyapunov function and $w^* = (w_i^*)$, $i = 1, 2, \cdots, Q$ (where, $w_i^* = \chi_i^*.\overline{d}_i^* + P_i^*$) is a stable point of the system, to which all the trajectories converge. P_i^* is also the so-called total backlog, as mentioned in [2].

SIMULATION RESULTS

Consider the network topology of Figure 2, which is composed of 87 elastic users and 94 links. Gray nodes are the network's backbone boundary.

A similar approach to that of Alpcan and Başar [13] has been adopted for simulating the rates allocated to the users with different propagation delays. The OPNET discrete-event simulator has been used. It has been assumed that those users whose numbers are multiples of 5 (such as 5, 10, 15, $\cdots$) act as background variable-rate traffic. The bottleneck links are links 11, 15, 17, 47, 48, 49 and 91, but their capacity is selected to be 800 kbps. Other link capacities are selected to be 800 Mbps. All the links' propagation delays are set to 5 ms. It is assumed that sources have data for sending at all times (greedy sources). All the links' buffer sizes are set to 100 packets and, so, loss occurs in the network.

A go back n method has been used for re-sending the packets that are doubly acknowledged. The links' scheduling discipline is FIFO. As in TCP, the Slow-Start method is used for initializing the rate allocation.

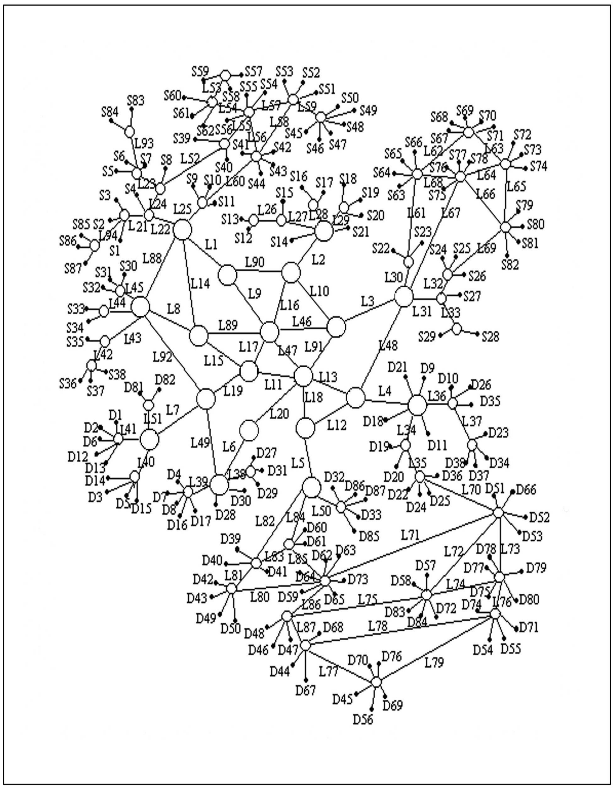

Figure 2. Simulated network topology.

Receivers' window sizes are set to unity and sender window sizes in Jacobi and Kelly methods are updated according to Relations 13-15 and the following relations, respectively:

$$\mathrm{cwnd}_r[n+1] = \Bigg\{ \mathrm{cwnd}_r[n] + k_r.\mathrm{RTT}_r[n]. \left(\omega_r - \frac{\mathrm{cwnd}_r[n]}{\mathrm{RTT}_r[n]}.d_r[n] \right) \Bigg\}^+, \tag{25}$$

where $d_r[n] = \mathrm{RTT}_r[n] - \overline{d}_r$.

$k_r = K_r = 0.0003$ has been used and user utility parameters are summarized in Table 1.

It is important to say that, as congestion occurs only in common links, the rate allocation algorithm is only consisted of Equations 9 and 11. Equation 12 has no effect on the rate allocation algorithm.

The simulation results for users in Figure 2 are depicted in Figures 3 to 6. In these figures, the proposed second order method has been compared with Kelly's method and TCP. It can be verified that, while maintaining stability, the proposed method outperforms that of Kelly in convergence speed.

Another outstanding feature of the proposed rate allocation strategy is that the user rates in the proposed

Table 1. Users' utility parameters.

ω	**User**	ω	**User**	ω	**User**	ω	**User**
0.03	67	0.04	45	0.04	23	0.05	1
0.025	68	0.07	46	0.07	24	0.05	2
0.025	69	0.03	47	0.025	25	0.03	3
0.03	70	0.025	48	0.03	26	0.03	4
0.05	71	0.025	49	0.02	27	0.04	5
0.05	72	0.03	50	0.05	28	0.07	6
0.03	73	0.05	51	0.03	29	0.03	7
0.03	74	0.05	52	0.03	30	0.025	8
0.04	75	0.03	53	0.04	31	0.025	9
0.07	76	0.03	54	0.07	32	0.03	10
0.03	77	0.04	55	0.025	33	0.02	11
0.025	78	0.07	56	0.03	34	0.05	12
0.025	79	0.03	57	0.02	35	0.03	13
0.03	80	0.025	58	0.05	36	0.03	14
0.05	81	0.025	59	0.03	37	0.04	15
0.05	82	0.03	60	0.03	38	0.07	16
0.03	83	0.05	61	0.07	39	0.025	17
190.03	84	0.05	62	0.023	40	0.03	18
0.04	85	0.03	63	0.05	41	0.02	19
0.07	86	0.03	64	0.05	42	0.05	20
0.03	87	0.04	65	0.03	43	0.03	21
		0.07	66	0.03	44	0.03	22

method and that of Kelly, have less fluctuation, with respect to TCP. Also, the rate allocation is TCP friendly, because none of the allocated rates in the Jacobi or Kelly methods are greater than their corresponding TCP rate allocations.

Although, in Figures 5 and 6, it appears that the proposed method achieves less link capacity utilization, for comparing link utilization among multiple algorithms, the aggregate rate traversing through the bottleneck links 11, 15, 17, 47, 48, 49 and 91 must be compared. These aggregate utilizations are depicted in Figures 7-13. As can be verified, the proposed algorithm has a higher convergence rate in comparison with Kelly's and can better utilize the bottleneck link capacities. In Figure 14, the Goodput of the three algorithms has been compared for 87 users and, then, the surface beneath the curves has been calculated as a performance metric for overall network utilization.

This metric is 26890, 36232 and 38849 for TCP,

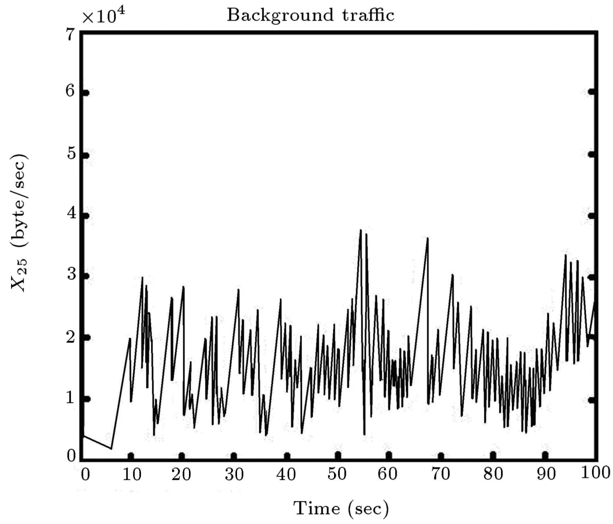

Figure 3. Background traffic 25.

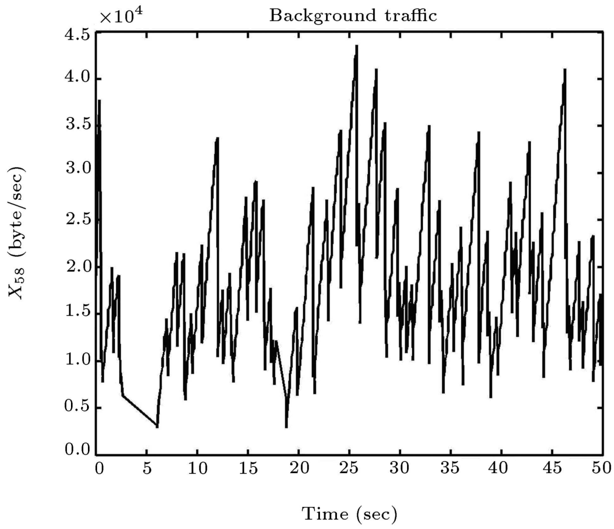

Figure 4. Background traffic 58.

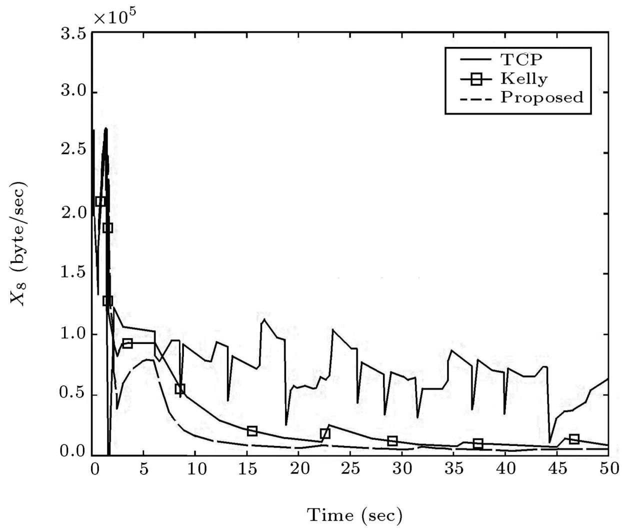

Figure 5. Rate allocated to user 8.

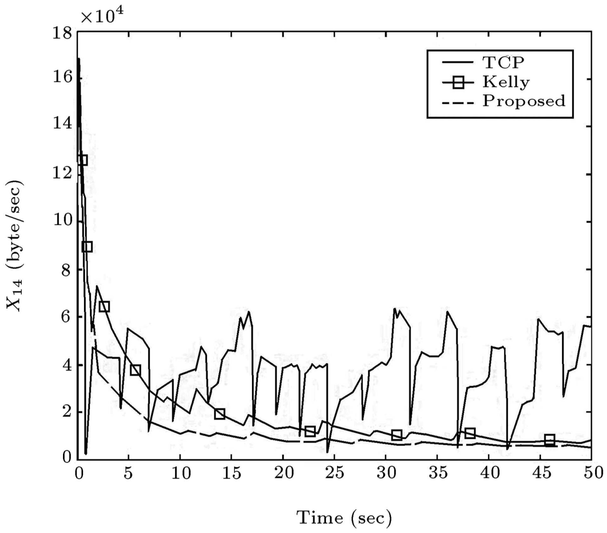

Figure 6. Rate allocated to user 14.

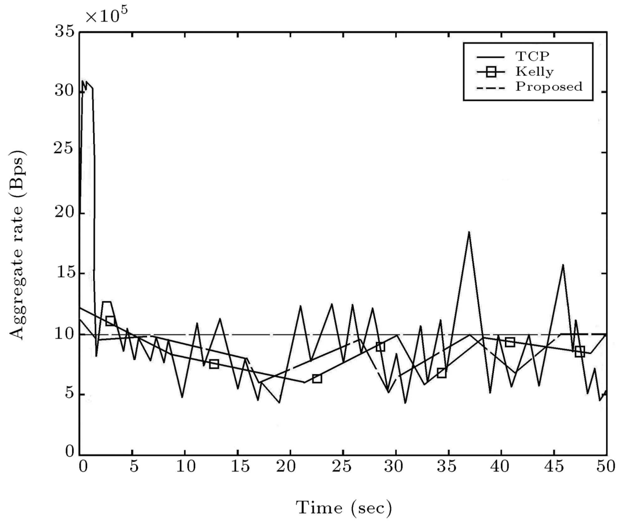

Figure 9. Aggregate rate through bottleneck 17.

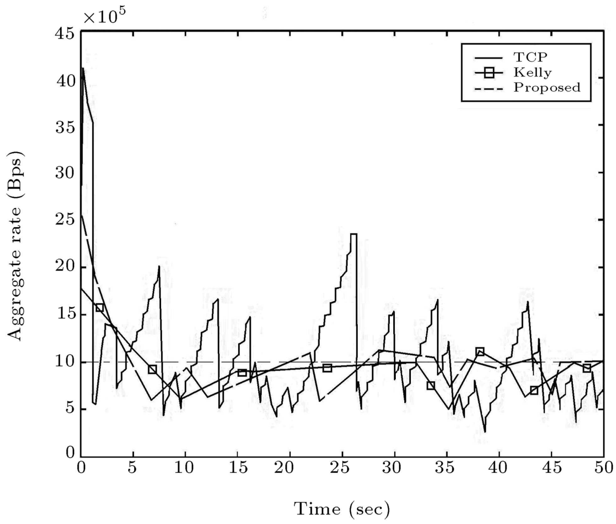

Figure 7. Aggregate rate through bottleneck 11.

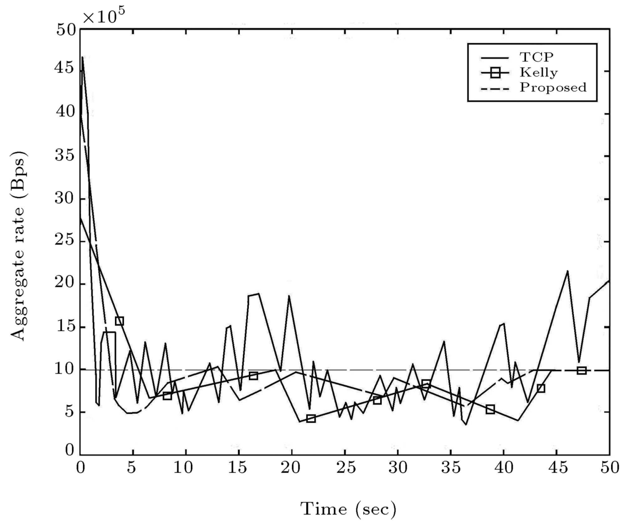

Figure 10. Aggregate rate through bottleneck 47.

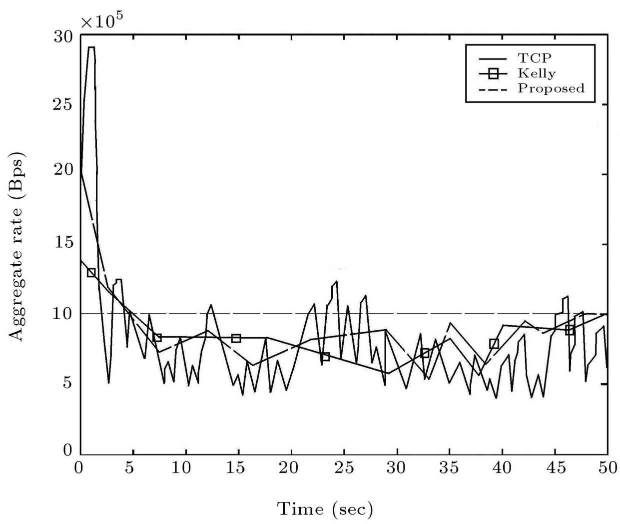

Figure 8. Aggregate rate through bottleneck 15.

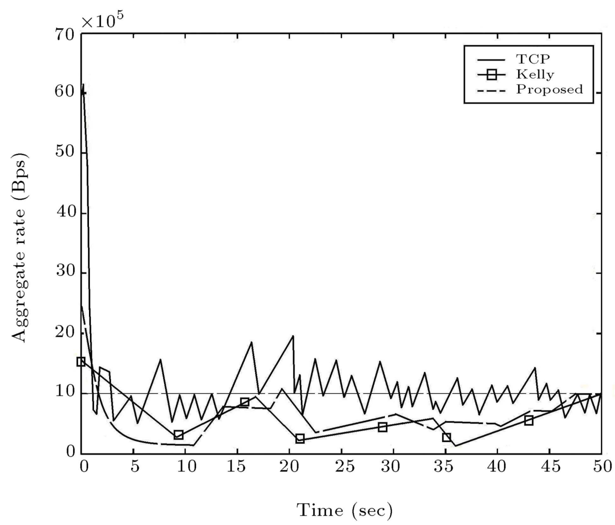

Figure 11. Aggregate rate through bottleneck 48.

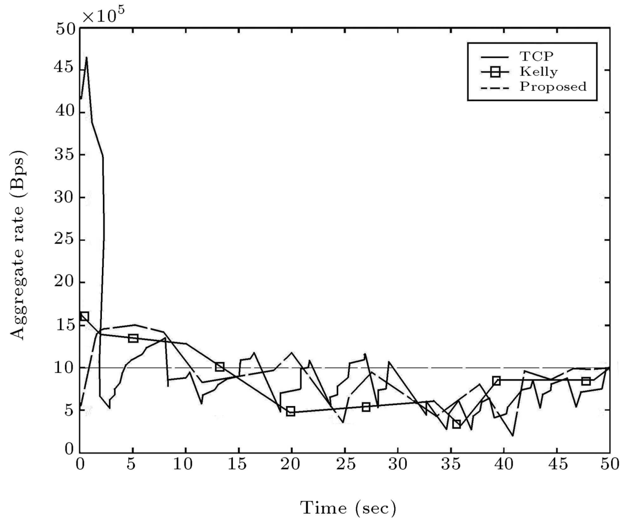

Figure 12. Aggregate rate through bottleneck 49.

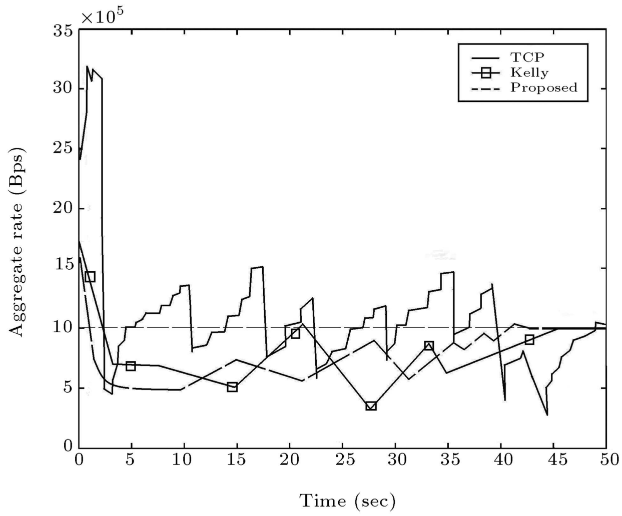

Figure 13. Aggregate rate through bottleneck 91.

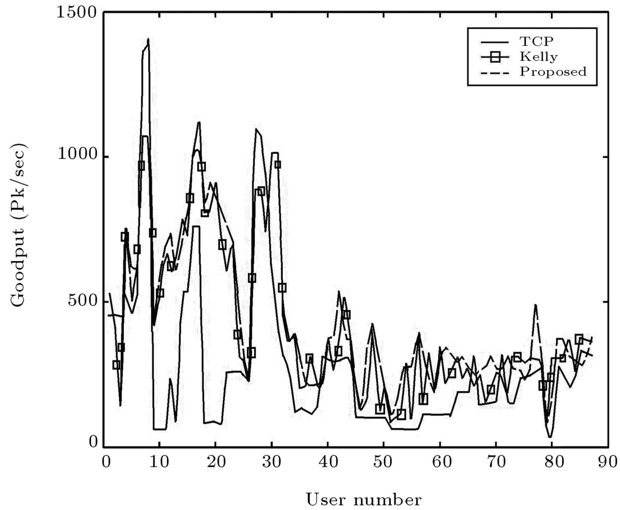

Figure 14. Goodput comparison between different methods.

Kelly's and the proposed algorithm, respectively. So, it can be verified that overall network utilization is better in the proposed algorithm, with respect to TCP and Kelly's algorithm (in the figures, each KBps is equivalent to 8 Kbps).

As Equations 13-15 use only the RTT and propagation delay of the connection, they can be implemented in an end-to-end manner, even in the current Internet.

CONCLUSION

In the current paper, the stability property of a high-speed second-order algorithm has been proven. The proposed method has also been compared with the conventional Kelly algorithm and TCP in the presence of background traffic. Simulation results show that the proposed method, in the presence of variable bit-rate background traffic, while maintaining stability, has less fluctuation, with respect to the TCP method.

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