A Fuzzy-Based Optimal Generation Rescheduling and Load Shedding Algorithm

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During an emergency condition, a power system operator may be confronted with the difficult task of Load Shedding and Generation Reallocation (LSGR) decisions. Based on Fuzzy Linear Programming (FLP), an optimization LSGR problem is developed and proposed in this paper. The objective function consists of terms of load curtailments and deviations in generation schedules. All networks constraints, as well as load dependency on frequency and voltage, modeling of tap-changing and phase-shifting transformers are taken into account. The objective function and constraints coefficients are uncertain, which are represented by fuzzy numbers. Thus, a fuzzy environment is prepared and a FLP approach is developed to solve the LSGR optimization problem more realistically and effectively. The method is successfully applied to a test system, where the load curtailments and deviations from the nominal states are to be minimized. The results of various cases of fuzzy and crisp modes of the problem are demonstrated.

INTRODUCTION

One of the main tasks of a power system control center is to serve as many customers as possible in an emergency condition. During this period, the objective may be considered as the safe operation combined with minimal generation rescheduling and load shedding of the system. In doing so, power plant productions are minimally disturbed from their respective economical operating points and minimum amount of load is dropped, if necessary.

In the published literature, the topic of load shedding and generation reallocation has received some attention. The load shedding problem is presented in [1-4]. The generation reallocation is addressed in [5,6] and a combination of the above problems (LSGR) is discussed in [7-10]. The techniques addressed by the last category suffer from the following drawbacks: frequency variations were not considered in [7]; reactive losses were not taken into account in [8]; the voltage and frequency characteristics of loads were not considered at all in [9]; and finally, in [10], the problem was so formulated to alleviate only line overloads. To overcome these difficulties, a Linear Programming (LP) approach to solve a LSGR problem in its entire complexity has been proposed in [11]. The objective function was considered to have terms of load curtailments and deviations in generation schedules, with crisp coefficients. The constraints were power system variables limitations with hard boundaries.

The difficulty of solving the LSGR problem still remains, because it is inconvenient for the

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decision maker to precisely assign the values of LSGR coefficients of the objective function. Also, most of the upper and lower limits of the constraints are not sharp but rather soft and flexible. Thus, it seems more appropriate to utilize an FLP approach to solve the LSGR optimization problem. To solve the LSGR problem more realistically, the present paper proposes a fuzzy linear programming approach. Objective function coefficients as well as most of constraints boundaries are considered to be fuzzy numbers. A fuzzy environment is thus developed and the resulting optimization problem is solved using a powerful linear programming approach.

The paper structure is as follows. The crisp formulation of the LSGR problem is examined first. A review of fuzzy linear programming methods is outlined next. Then, the proposed fuzzy LSGR optimization based on the FLP [12] is described. For illustration purposes, the results on a typical small power system are then demonstrated. Some including remarks are finally provided.

CRISP FORMULATION

As mentioned earlier, an algorithm was developed in [11], in which optimal LSGR problem was solved in its entire complexity by a linear programming approach. The method is briefly reviewed in this section.

Objective Function

To minimize customers dissatisfaction due to load curtailment and to ensure minimum deviation from economical operating conditions, the optimization problem during emergency situation can be formulated as follows [4,7,8,11]:

$$J=\sum_i a_i\cdot\Delta PG_i^2+\sum_i b_i\cdot\Delta QG_i^2+\sum_i c_i\cdot\Delta PL_i^2+\sum_i d_i\cdot\Delta QL_i^2\,, \qquad (1)$$

where $PG(PL)$ and $QG(QL)$ represent active and reactive powers of a generator (a load) and Δ represents deviation from nominal value. The objective function should be linearized (the details are provided in Appendix A).

Constraints

The objective function should be minimized provided that the following constraints are met:

a) Generation active and reactive power constraints:

$$PG_i^{\min}\le PG_i\le PG_i^{\max}\quad i=1,\ldots,NG\,, \qquad (2)$$

$$QG_i^{\min}\le QG_i\le QG_i^{\max}\quad i=1,\ldots,NG\,. \qquad (3)$$

b) Load active and reactive power constraints:

$$PL_i^{\min}\le PL_i\le PL_i^{\max}\quad i=1,\ldots,NB\,, \qquad (4)$$

$$QL_i^{\min}\le QL_i\le QL_i^{\max}\quad i=1,\ldots,NB\,. \qquad (5)$$

c) Bus voltage magnitude constraints:

$$V_i^{\min}\le V_i\le V_i^{\max}\quad i=1,\ldots,NB\,. \qquad (6)$$

d) Transformer tap constraints:

$$t_i^{\min}\le t_i\le t_i^{\max}\quad i=1,\ldots,NT\,. \qquad (7)$$

e) Phase shifter constraints:

$$\phi_i^{\min}\le \phi_i\le \phi_i^{\max}\quad i=1,\ldots,NP\,. \qquad (8)$$

f) System frequency constraint:

$$F^{\min}\le F\le F^{\max}. \qquad (9)$$

g) Line flow angle stability constraints:

$$0\le|\delta_i-\delta_j|\le\Psi_{ij}^{\max}\quad i,j=1,\ldots,NL\,. \qquad (10)$$

h) Load flow constraints:

$$PG_i(F)-PL_i(V,F)-P_i(V,\delta)=0\quad i=1,\ldots,NB\,, \qquad (11)$$

$$QG_i(F)-QL_i(V,F)-Q_i(V,\delta)=0\quad i=1,\ldots,NB\,. \qquad (12)$$

The active power of generators is adjusted by the static response of a governor expressed by:

$$PG_i = PG_{\text{set}_i} - \frac{P_{R_i}}{R_i} \cdot \Delta F \,. \qquad (13)$$

Also the load dependency on voltage and frequency has to be considered [13]:

$$PL_i = PL_{\text{set}_i} \cdot (1 + k_{p_i} \cdot \Delta F) \cdot \left[p_{p_i} + p_{c_i} \cdot \left(\frac{V_i}{V_{LB_i}}\right)^{N1} + p_{z_i} \cdot \left(\frac{V_i}{V_{LB_i}}\right)^2 \right], \qquad (14)$$

$$QL_i = QL_{\text{set}_i} \cdot (1 + k_{q_i} \cdot \Delta F) \cdot \left[q_{p_i} + q_{c_i} \cdot \left(\frac{V_i}{V_{LB_i}}\right)^{N2} + q_{z_i} \cdot \left(\frac{V_i}{V_{LB_i}}\right)^2 \right]. \qquad (15)$$

Load flow equations should be properly modified to take the frequency effect into account. For LP formulation, each constraint should be linearized. For instance, the linearized version of generation active power constraints is given by:

$$PG_i^{\min} - PG_i^{\circ} \leq \Delta PG_i \leq PG_i^{\max} - PG_i^{\circ} \,, \qquad (16)$$

where index $\circ$ denotes initial value and $\Delta PG_i = PG_i - PG_i^{\circ}$. Quite similar relationships apply to other constraints.

Finally, the LP optimization problem can be expressed as:

min linearized form of Equation 1

subject to:

linearized forms of constraints in Equations 2–12 (17)

Algorithm

A simple flow chart of the iterative algorithm of the optimization problem is shown in Figure 1. The algorithm is so designed that initially only generation reallocation is tried for solving the problem. Provided that it is unsuccessful, both optimum load shedding and generation reallocation will be enabled.

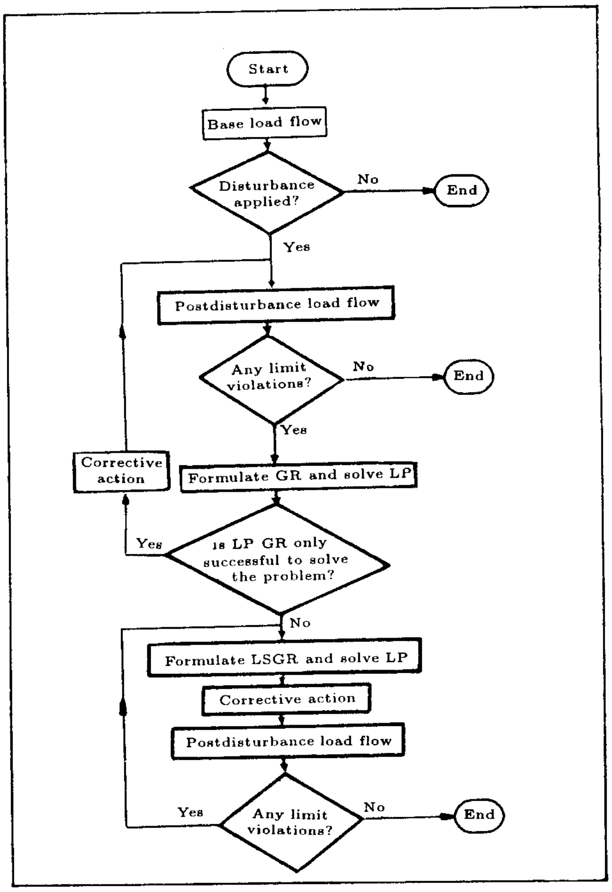

Figure 1. Algorithm flow chart.

A REVIEW OF FUZZY LINEAR PROGRAMMING METHODS

After the introduction of the well-known theory of the "fuzzy sets", Zadeh and Bellman proposed the idea of "decision making in a fuzzy environment" [14]. This work was a basis for a later work by Zimmermann [15] who used fuzzy set theory to formulate and solve a multi-objective function optimization problem as a fuzzy version of the linear programming. Later, some methods were presented on the multicriteria LP problem with fuzzy coefficients of the objective functions and constraints [16-18]. Carlsson and Korhonen [16] proposed a method to solve the Multi-objective FLP (MFLP) with all of the fuzzy coefficients. The model is finally changed to a parametric programming, however the method did not consider equality constraints. Slowinski [17] presented another method to solve the MFLP where in his approach, equality constraints were not considered, too. Also, in the fuzzy model, the number of inequality constraints in the final programming model were changed to two times the original constraints. Thus, this method is unsuitable for problems with many constraints. Rommelfanger [18] derived another method that has the same difficulties as the method proposed in [17].

A suitable method has been presented in [12] for solving the MFLP, where coefficients of the objective functions and the constraints are fuzzy numbers. Assuming the aspiration levels for particular criteria to be fuzzy and based on the comparison of fuzzy numbers, the original problem is transformed into a multicriteria linear fractional program. Fuzzy equality constraints are considered by the author. In this method, the number of inequality constraints are not changed. The details of the method are discussed in this section. A useful application of this fuzzy method is to define a fuzzy environment for solving the fuzzy LSGR problem in power systems. This topic is discussed in the fourth section, with the benefits also being illustrated.

Multi-Objective Fuzzy Linear Programming Formulation

Since the fuzzy LSGR analysis presented here is based on the MFLP method developed by Roubens [12], and for the sake of clarity, the method is reviewed in this section. In order to reveal the specific structure of the problem to be solved, the following standard form shall be considered:

$$\min_X \tilde{Z}_j = \tilde{C}_j \cdot X \quad j = 1, \dots, K$$

subject to:

$$I) \begin{cases} \tilde{a}_i \cdot X \leq \tilde{b}_i & i = 1, \dots, m \\ \tilde{a}_i \cdot X = \tilde{b}_i & i = m+1, \dots, n \end{cases} , \tag{18}$$

where X is the vector of L decision variables, $\tilde{C}_1, \dots, \tilde{C}_k$ are vectors of fuzzy cost coefficients $\tilde{c}_{jl}, l = 1, \dots, L$, corresponding to criteria $j = 1, \dots, K$, a_i is the ith row of the fuzzy coefficients matrix $(\tilde{a}_{il})$ and $\tilde{b}_i$ is the corresponding fuzzy right-hand side of the ith constraint.

It is assumed that $\tilde{C}_j, \tilde{a}_i, \tilde{b}_i$ have components which are trapezoidal fuzzy numbers defined as:

$$\tilde{c}_{jl} = (c_{jl}^L, c_{jl}^U, \sigma_{c_{jl}}^L, \sigma_{c_{jl}}^U) , \tag{19}$$

$$\tilde{a}_{il} = (a_{il}^L, a_{il}^U, \sigma_{a_{il}}^L, \sigma_{a_{il}}^U) , \tag{20}$$

$$\tilde{b}_i = (b_i^L, b_i^U, \sigma_{b_i}^L, \sigma_{b_i}^U) , \tag{21}$$

where the representation for a trapezoidal fuzzy number $\tilde{m}$ is the quadruple $(m^L, m^U, \sigma_m^L, \sigma_m^U)$ of parameters of its membership function $\mu_{\tilde{m}}(x)$ defined by:

$$\mu_{\tilde{m}}(x) = \begin{cases} 1 - (m^L - x)/\sigma_m^L & \text{if } m^L - \sigma_m^L \leq x \leq m^L \\ 1 & \text{if } m^L \leq x \leq m^U \\ 1 - (x - m^U)/\sigma_m^U & \text{if } m^U \leq x \leq m^U + \sigma_m^U \\ 0 & \text{if otherwise} , \end{cases} \tag{22}$$

where m^L and m^U are the left and right main values, σ_m^L and σ_m^U are the left and right spreads of $\tilde{m}$, respectively. Figure 2 shows the membership function of $\tilde{m}$ more clearly. To complete the problem formulation, it is assumed that for each criterion j, the decision maker is able to define a corresponding fuzzy goal, denoted by $\tilde{g}_j = (\bar{g}_j, \bar{g}_j, o, \sigma g_j^U)$. If unknown, the main value of the goal $\bar{g}_j = g_j^L = g_j^U$ might be obtained from unicriteria minimization of each criterion with crisp coefficients c_{jl} equal to $1/2(c_{jl}^L + c_{jl}^U)$, and the right spread being obtained as a given percentage of difference $g_j^\star - \bar{g}_j$ (usually 100% [15]), where $g_j^\star$ corresponds for instance to the maximum value of g_j obtained for other criteria. In other words, the following K fuzzy linear programming problems are first solved to obtain the optimal solutions $X^{\star(j)}$:

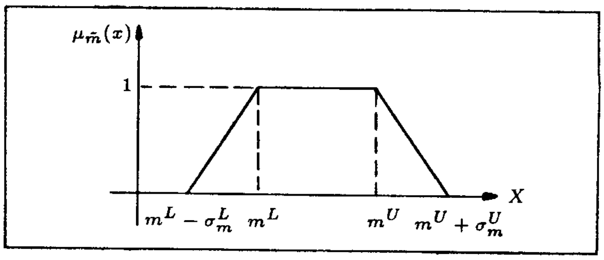

Figure 2. Membership function of $\tilde{m}$.

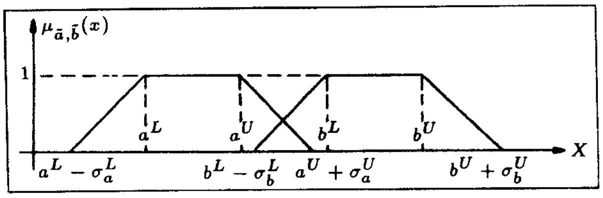

Figure 3. Membership function of $\tilde{a}$, $\tilde{b}$.

$\min_X g_j = C_j \cdot X$

subject to:

$$I) \begin{cases} \tilde{a}_i \cdot X \leq \tilde{b}_i & i = 1, \ldots, m \\ \tilde{a}_i \cdot X = \tilde{b}_i & i = m+1, \ldots, n \ . \end{cases} \quad (23)$$

Then, the following computation is performed:

$$g_j^{(1)} = C_j \cdot X^{\star(1)}, \ldots, g_j^{(j)} = C_j \cdot X^{\star(j)}, \ldots, g_j^{(K)} = C_j \cdot X^{\star(K)} \ . \quad (24)$$

Finally, $\bar{g}_j, g_j^\star$ and σg_j^U are calculated as:

$$\begin{cases} \bar{g}_j = g_j^{(j)} \\ g_j^\star = \max\limits_{i,\, i \neq j}(g_j^{(i)}) \\ \sigma_{g_j}^U = \text{percentage (usually 100\%) of } g_j^\star - \bar{g}_j \ . \end{cases} \quad (25)$$

***Remark* 1**

It should be mentioned that Problem 23 is solved by the method developed in the following subsection. It will be observed that Problem 23 is a special case of Equation 18.

Fuzzy Constraints with Fuzzy Coefficients

In this section, the question "What happens to a linear constraint $a_i \cdot X = b_i$ or to the linear inequality $a_i \cdot X \leq b_i$ when the coefficients a_{il} and b_i become fuzzy numbers?" will be answered. It has close relations with the comparison of fuzzy numbers which has been extensively studied [16–19].

Suppose $\tilde{a} = (a^L, a^U, \sigma_a^L, \sigma_a^U)$ and $\tilde{b} = (b^L, b^U, \sigma_b^L, \sigma_b^U)$. Now, for expressing equality and inequality relations between $\tilde{a}$ and $\tilde{b}$, and with the account of Figure 3, first $hgt(\inf \tilde{b} \cap \sup \tilde{a})$ shall be defined.

***Definition* 1**

The function hgt is defined as the non-negative height of the intersection of the increasing left-hand side for $\mu_{\tilde{b}}(x)$ and the decreasing right-hand side for $\mu_{\tilde{a}}(x)$, given by:

$$hgt(\inf \tilde{b} \cap \sup \tilde{a}) = \max\left\{ \frac{a^U - b^L}{\sigma_a^U + \sigma_b^L} + 1, 0 \right\} = \begin{cases} \geq 1 & \text{if } a^U \geq b^L \\ \leq 1 & \text{if } a^U \leq b^L \ . \end{cases} \quad (26)$$

***Definition* 2**

The grade of possibility of dominance (PD) of $\tilde{a}$ over $\tilde{b}$ (introduced by Dubois and Prade [19] which represents the fuzzy extension for $\tilde{a} > \tilde{b}$) is defined by:

$$PD(\tilde{a}, \tilde{b}) = \max_{x,y} \min_{x \geq y} [\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)] = \min[1, hgt(\inf \tilde{b} \cap \sup \tilde{a})] \ . \quad (27)$$

***Remark* 2**

One may consider the grade of possibility of dominance of $\tilde{a}$ over $\tilde{b}$ for the relation $\tilde{a} >_\theta \tilde{b}$ as the degree of satisfaction of the relation $>$ between $\tilde{a}$ and $\tilde{b}$.

***Definition* 3**

$\tilde{a}$ is smaller than $\tilde{b}$ with degree of satisfaction θ [13], defined by:

$$\tilde{a} \leq_\theta \tilde{b} \quad \text{iff } PD(\tilde{a}, \tilde{b}) \leq \theta \ , \quad (28)$$

similary,

$$\tilde{a} \approx_\theta \tilde{b} \text{ iff } PD(\tilde{a}, \tilde{b}) \geq \theta \text{ and } PD(\tilde{b}, \tilde{a}) \geq \theta \ . \quad (29)$$

Based on notations $\tilde{a}$ and $\tilde{b}$, and the above definition, a possible interpretation for $\tilde{a} \leq_\theta \tilde{b}$ and $\tilde{a} \approx_\theta \tilde{b}$ are obtaind as:

$$\tilde{a} \leq_\theta \tilde{b} \quad \text{iff } a^L - (1-\theta) \cdot \sigma_a^L \leq b^U + (1-\theta) \cdot \sigma_b^U \ , \quad (30)$$

$$\tilde{a} \approx_\theta \tilde{b} \quad \text{iff } \begin{cases} b^U + (1-\theta) \cdot \sigma_b^U \geq a^L - (1-\theta) \cdot \sigma_a^L \\ a^U + (1-\theta) \cdot \sigma_a^U \geq b^L - (1-\theta) \cdot \sigma_b^L \ . \end{cases} \quad (31)$$

Thus, based on Equations 30 and 31, fuzzy relations of $\tilde{a} \leq_\theta \tilde{b}$ and $\tilde{a} \approx_\theta \tilde{b}$ can be changed to crisp relations.

Also, $\tilde{a}_i \cdot X$ can be expressed by ([19]):

$$\tilde{a}_i \cdot X = \sum_l \tilde{a}_{il} \cdot x_l = \tilde{a}_{i1} \odot x_1 + \cdots + \tilde{a}_{il} \odot x_l$$
$$= \left(\sum_l a_{il}^L \cdot x_l, \sum_l a_{il}^U \cdot x_l, \sum_l \sigma_{a_{il}}^L \cdot x_l, \sum_l \sigma_{a_{il}}^U \cdot x_l \right) , \quad (32)$$

where $\odot$ represents the fuzzy product operator [19]. Then, from the Equation 30, the fuzzy inequality $\tilde{a}_i \cdot X \leq_\theta \tilde{b}_i$ can be interpreted as:

$$\tilde{a}_i \cdot X \leq_\theta \tilde{b}_i \Rightarrow \sum_l [a_{il}^L - (1-\theta) \cdot \sigma_{a_{il}}^L] \cdot x_l \leq b_i^U + (1-\theta) \cdot \sigma_{b_i}^U , \quad (33)$$

and the fuzzy equality $\tilde{a}_i \cdot X \approx_\theta \tilde{b}_i$ corresponds to the system of crisp inequalities:

$$\tilde{a}_i \cdot X \approx_\theta \tilde{b}_i \Rightarrow \begin{cases} \sum_l [a_{il}^U + (1-\theta) \cdot \sigma_{a_{il}}^U] \cdot x_l \geq b_i^L - (1-\theta) \cdot \sigma_{b_i}^L \\ \sum_l [a_{il}^L - (1-\theta) \cdot \sigma_{a_{il}}^L] \cdot x_l \leq b_i^U + (1-\theta) \cdot \sigma_{b_i}^U \end{cases} \quad (34)$$

Finally, it can be concluded that by using Equations 33 and 34, the fuzzy constraints of Equation 18 may be changed to the crisp constraints.

Fuzzy Objective Function and MFLP Model

Consider the following fuzzy objectives:

$$\min \tilde{C}_j \cdot X = \sum_l \tilde{c}_{jl} \cdot x_l = \left(\sum_l c_{jl}^L \cdot x_l, \sum_l c_{jl}^U \cdot x_l, \sum_l \sigma_{c_{jl}}^L \cdot x_l, \sum_l \sigma_{c_{jl}}^U \cdot x_l \right) . \quad (35)$$

Assume that the decision maker can specify fuzzy goals $\tilde{g}_j$ in form of fuzzy numbers $(\bar{g}_j, \bar{g}_j, o, \sigma_{gj}^U)$. Now, the best consistency between the goals and the objective functions should be achieved. In order to achieve this, first, the possibility of dominance of $\tilde{g}_j$ over $\sum \tilde{c}_{jl} \cdot x_l$ is computed by:

$$PD(\tilde{g}_j, \tilde{C}_j \cdot X) = h_j = 1 + \frac{\bar{g}_j - \sum_l c_{jl}^L \cdot x_l}{\sigma_{g_j}^U + \sum_l \sigma_{c_{jl}}^L \cdot x_l} . \quad (36)$$

Then, it is proposed to solve the following crisp parametric multicriteria linear fractional programming problem (MFLP). Given an aspiration level θ:

$$\max PD(\tilde{g}_j, \tilde{C}_j \cdot X) \quad j = 1, \ldots, K$$

subject to:

$$I) \begin{cases} PD(\tilde{a}_i \cdot X, \tilde{b}_i) \geq \theta & i = m+1, \ldots, n \\ PD(\tilde{b}_i, \tilde{a}_i \cdot X) \geq \theta & i = 1, \ldots, n . \end{cases} \quad (37)$$

In consequence, the following non-fuzzy mathematical programming problem equivalent to Equation 37 is obtained.

$$\min \frac{\sum_l c_{jl}^L \cdot x_l - \bar{g}_j}{\sum_l \sigma_{c_{jl}}^L \cdot x_l + \sigma_{g_j}^U} \quad j = 1, \ldots, K$$

subject to:

$$\sum_l [a_{il}^U + (1-\theta) \cdot \sigma_{a_{il}}^U] \cdot x_l \geq b_i^L - (1-\theta) \cdot \sigma_{b_i}^L \quad i = m+1, \ldots, n ,$$

$$\sum_l [a_{il}^L - (1-\theta) \cdot \sigma_{a_{il}}^L] \cdot x_l \leq b_i^U + (1-\theta) \cdot \sigma_{b_i}^U \quad i = 1, \ldots, n . \quad (38)$$

To solve the above fractional programming, we may further consider following change of variables:

$$Y = \frac{X}{\sum_l \sigma_{c_{jl}}^L \cdot x_l + \sigma_{g_j}^U} , \quad (39)$$

$$t = \frac{1}{\sum_l \sigma_{c_{jl}}^L \cdot x_l + \sigma_{g_j}^U} , \quad (40)$$

and the equivalent LP system is obtained:

$$\min \sum_l c_{jl}^L \cdot y_l - \bar{g}_j \cdot t \quad j = 1, \ldots, K ,$$

subject to:

$$\sum_l [a_{il}^U + (1-\theta)\cdot\sigma_{a_{il}}^U]\cdot y_l - [b_i^L - (1-\theta)\cdot\sigma_{b_i}^L]\cdot t \geq 0 \quad i=m+1,\ldots,n$$

$$\sum_l [a_{il}^L - (1-\theta)\cdot\sigma_{a_{il}}^L]\cdot y_l - [b_i^U + (1-\theta)\cdot\sigma_{b_i}^U]\cdot t \leq 0 \quad i=1,\ldots,n$$

$$\sum_l \sigma_{c_{jl}}^L \cdot y_l + \sigma_{g_j}^U \cdot t = 1 \,. \tag{41}$$

Finally the resulting Problem 41 is a multi-objective linear programming, equivalent to the MFLP problem formulated in Equation 18.

THE FUZZY LSGR OPTIMIZATION PROBLEM

Having described the LSGR problem in the second section, and outlined the MFLP method in the third section, it is now the stage to develop a fuzzy environment for the solution of load shedding and generation reallocation optimization problem.

In the real world, some of the equality and inequality constraints defining the feasible region of the problem may not be sharp but susceptible to soft and flexible boundaries. Similarly, some of the data and rules required for modeling the problem may be subject to some uncertainty and vagueness. In such cases the problem can be handled properly by fuzzy mathematical concepts and tools. To clarify this adaption more, specifically for the LSGR problem, the following points are considered:

I) In most of the classical solution methods of the optimization problems, the objective functions are made up of a set of variables with coefficients. There is not a clear idea for selecting the proper values of coefficients. The LSGR objective function is not exempted from this difficulty. The decision maker can hardly indicate the precise value of coefficients (a_i, b_i, c_i, d_i) in Equation 1 (see the second section). On the other hand, it is very inconvenient to assign the relation between active and reactive power generation and consumption deviations in Equation 1. Thus, a way to handle the problem is using the $\tilde{a}_i, \tilde{b}_i, \tilde{c}_i, \tilde{d}_i$ fuzzy numbers. Therefore, for every component of Equation 1 $(\tilde{\alpha}_i \cdot \Delta S_i^2)$ with fuzzy coefficients, the coefficients of Equation A.1 are transformed into fuzzy numbers. The fuzzy numbers $\tilde{\rho}_{i1}$ to $\tilde{\rho}_{i4}$ are defined from fuzzy number $\tilde{\alpha}_i$ as shown in Appendix B.

II) In the LSGR problem, the coefficients, as well as lower and upper limits of some of the dominant constraints are not sharp but rather soft and flexible. These constraints may be also handled more properly by the fuzzy constraints. Thus, the decision maker can consider each of coefficients and limits of constraints of Equation 17 in the form of fuzzy numbers represented by trapezoidal membership function.

III) In the LP problem, the solution lies on the boundaries of the feasible region. Thus, there may be cases where there is no feasible solution for the problem. However, due to the flexibility induced by the fuzzy objective function and constraints, the problem may possibly have an acceptable solution in FLP.

Finally, by considering the above reasons, it is very acceptable to explain the LSGR optimization problem (illustrated in the second section) based on fuzzy optimization method (given in the third section). Thus, the proposed model in Equation 17 (with the objective function and constraints coefficients in the form of fuzzy numbers) can be solved by using Equation 41.

SIMULATION RESULTS

Based on the fuzzy optimization technique described in the preceding sections, the results on the test power system of Figure 4 are provided in this section. The crisp and fuzzy data of the system are provided in Appendix C.

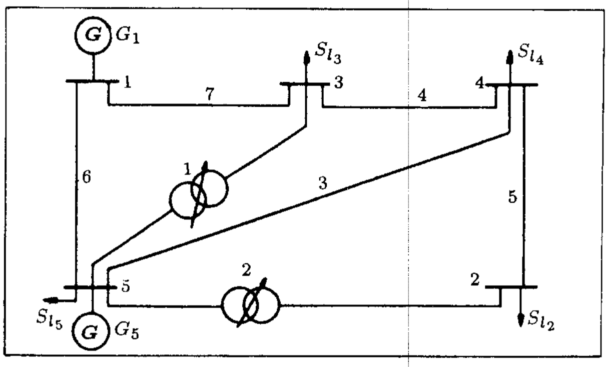

Figure 4. The 5-bus test system.

For simulation of the above power system, the following disturbances (faults) are considered: line outage, generation reduction, outage of generators, transmission overloads and load increase of buses. The results on two separate examples are obtained and provided in Tables 1 and 2. It is assumed that in predisturbance condition, the system is operating in its economical state. Also, ϕ_1 and ϕ_2 of the two phase-shifting transformers are adjusted to limit their respective active power transformers at 0.25

Table 1. The simulation results of Example 1.

	Case 1	**Case 2**	**Case 3**	**Case 4**	**Case 5**	**Case 6**
State	Prefault	Postfault	Postfault	Postfault	Postfault	Postfault
Method	–	–	C.G.R.*	F.G.R.**	C.G.R.	F.G.R.
LMCB*2**	No	No	No	No	Yes	Yes
PG1	1.325	1.508	1.497	1.481	1.566	1.498
PG5	0.400	0.222	0.234	0.249	0.158	0.225
QG1	0.926	0.875	0.875	0.869	1.127	0.879
QG5	-0.536	-0.457	-0.455	-0.447	-0.682	-0.484
PL1	0.00	0.00	0.00	0.00	0.000	0.000
PL2	0.60	0.60	0.60	0.60	0.585	0.594
PL3	0.45	0.45	0.45	0.45	0.450	0.450
PL4	0.40	0.40	0.40	0.40	0.400	0.400
PL5	0.20	0.20	0.20	0.20	0.200	0.200
QL1	0.00	0.00	0.00	0.00	0.000	0.000
QL2	0.10	0.10	0.10	0.10	0.097	0.099
QL3	0.15	0.15	0.15	0.15	0.150	0.150
QL4	0.05	0.05	0.05	0.05	0.050	0.050
QL5	0.10	0.10	0.10	0.10	0.100	0.100
V1	1.060	1.060	1.051	1.051	1.054	1.053
V2	0.969	0.969	0.959	0.960	0.950	0.962
V3	0.984	0.983	0.974	0.975	0.967	0.976
V4	0.980	0.980	0.971	0.971	0.963	0.973
V5	1.000	1.000	0.991	0.992	0.980	0.993
$\delta 1$	0.000	0.000	0.000	0.000	0.000	0.000
$\delta 2$	-11.569	-12.022	-12.209	-12.234	-11.854	-11.992
$\delta 3$	-6.035	-6.363	-6.460	-6.447	-6.346	-6.392
$\delta 4$	-6.553	-6.925	-7.029	-7.015	-6.917	-6.954
$\delta 5$	-1.748	-2.319	-2.329	-2.281	-2.333	-2.330
Frequency	50.000	49.731	49.964	49.757	49.959	49.983
$\phi 1$	-1.714	-1.475	-1.516	-1.560	-1.277	-1.452
$\phi 2$	-8.200	-8.122	-8.258	-8.359	-7.809	-8.032
Performance Index	–	–	1.3164	1.1542	3.7877	1.3594
No. of Iterations	–	–	1	1	1	1
Emerg. Con.****	–	Frequency	–	–	–	–

* C.G.R. = Crisp Generation Reallocation
* F.G.R. = Fuzzy Generation Reallocation
*** LMCB = Load Model Considered for Bus
**** Emerg. Con. = Emergency Condition

p.u. and 0.3 p.u., respectively. The details are described in the following parts.

Example 1:

To illustrate the application of the method, it is assumed that there is 33 percent (0.2 p.u.) generation loss at bus 5 of the system. The predisturbance results as well as post-disturbance conditions are shown by Cases 1 and 2 of Table 1, respectively. As shown, the system frequency has been violated from its respective limit. The frequency drop is 0.2688 Hz (more than the maximum permissible limit of 0.2 Hz). Case 3 demonstrates that the system is transformed to a normal state following only generation reallocation by crisp

Table 2. The simulation results of Example 2.

	Case 1	**Case 2**	**Case 3**	**Case 4**	**Case 5**	**Case 6**
State	Prefault	Postfault	Postfault	Postfault	Postfault	Postfault
Method	–	–	C.LSGR*	F.LSGR**	C.LSGR	F.LSGR
LMCB*3**	No	No	No	No	Yes	Yes
PG1	1.325	2.006	1.773	1.884	1.760	1.798
PG5	0.400	0.421	0.415	0.340	0.388	0.386
QG1	0.926	0.942	0.649	0.945	0.746	0.794
QG5	-0.536	0.172	0.235	0.002	0.119	0.110
PL1	0.00	0.00	0.000	0.000	0.000	0.000
PL2	0.60	0.60	0.423	0.441	0.432	0.455
PL3	0.45	0.45	0.450	0.450	0.404	0.406
PL4	0.40	1.00	0.999	1.000	1.000	1.000
PL5	0.20	0.20	0.200	0.200	0.200	0.200
QL1	0.00	0.00	0.000	0.000	0.000	0.000
QL2	0.10	0.10	0.071	0.073	0.072	0.076
QL3	0.15	0.15	0.150	0.150	0.135	0.135
QL4	0.05	0.45	0.449	0.450	0.450	0.450
QL5	0.10	0.10	0.100	0.100	0.100	0.100
V1	1.060	1.060	1.059	1.051	1.061	1.049
V2	0.969	0.951	0.976	0.952	0.975	0.957
V3	0.984	0.943	0.961	0.939	0.962	0.945
V4	0.980	0.931	0.950	0.927	0.951	0.933
V5	1.000	1.000	1.014	0.990	1.011	0.996
δ1	0.000	0.000	0.000	0.000	0.000	0.000
δ2	-11.569	-19.363	-12.929	-13.894	-12.726	-13.865
δ3	-6.035	-10.935	-9.056	-9.480	-8.715	-9.207
δ4	-6.553	-11.959	-9.840	-10.334	-9.550	-10.096
δ5	-1.748	-2.858	-2.786	-2.880	-2.711	-2.809
Frequency	50.00	49.870	49.997	49.848	49.979	49.776
ϕ1	-1.714	-6.511	-4.578	-4.765	-4.207	-4.572
ϕ2	-8.200	-15.788	-8.758	-9.586	-8.607	-9.667
Performance Index	–	–	178.905	172.813	175.425	172.532
No. of Iterations	–	–	3	1	2	1
Emerg. Con.****	–	$\left\{ V3, V4;\ \delta_2 - \delta_5;\ \phi_2 \right.$	–	–	–	–

* C.LSGR = Crisp Load Shedding and Generation Reallocation

** F.LSGR = Fuzzy Load Shedding and Generation Reallocation

*** LMCB = Load Model Considered for Bus

**** Emerg. Con. = Emergency Condition

LP method (after one iteration). The phase angle of the two phase-shifting transformers are 1.516° and −8.258°. Meanwhile, the calculated performance index is 1.3164. To eliminate emergency state of Case 2, fuzzy LP method has been also tried. The results are shown by Case 4 (Table 1). After only one iteration, the emergency condition has been eliminated. The performance index is 1.1542, computed with the same crisp coefficients as before. Since the performance index in the generation reallocation problem indicates the deviations of the generators production from the nominal state, as a result, the reduction of this value based on the fuzzy optimization represents a more acceptable solution compared with the crisp solution.

The same disturbance is reapplied, but now with the load of bus 2 considered to be dependent on voltage and frequency (Equations 14 and 15) with the following coefficients [13]:

$$k_{p_2}=0.03,\ p_{p_2}=0.2,\ p_{c_2}=0.3,\ p_{z_2}=0.5$$
$$k_{q_2}=0.00,\ q_{p_2}=0.2,\ q_{c_2}=0.3,\ q_{z_2}=0.5\ . \quad (42)$$

The results of the generation reallocation solution, with crisp and fuzzy methods, are shown by Cases 5 and 6 of Table 1, respectively. In both cases, the system is transformed to normal sate after one iteration. The performance index by fuzzy method is 1.3594 which is less than the value by crisp approach (3.7877).

Example 2:

The associated load of bus 4 is increased by an active power of 0.6 p.u. (Power Factor = 0.83). The normal predisturbance results are shown by Case 1 of Table 2. Case 2 represents the severe emergency condition after the disturbance. The voltage magnitude of buses 3 and 4, the phase angle difference of line 2 ($\delta_2-\delta_5$) and finally the phase angle of the phase-shifting transformer 2 have been violated from their respective limits. The crisp and fuzzy LSGR optimization results are shown by Cases 3 and 4. The crisp solution transforms the system to normal state in three iterations where the performance index is 178.905. The fuzzy solution converges in one iteration with a lower performance index of 172.813. Thus, using the fuzzy optimization method, the number of iterations of the LP solutions, based on the algorithm flow chart (Figure 1), and the value of the performance index are reduced. This shows that the fuzzy LP solution is more appropriate than the crisp solution. Also, comparison of Cases 3 and 4 indicates that the total active and reactive load power (which can be supplied) of the crisp solution are 2.072 p.u. and 0.77 p.u., but these figures for the fuzzy solution consist of 2.091 p.u. and 0.773 p.u. . Thus, similarly, the load curtailment values in the fuzzy LP method are less than the crisp LP method.

The same disturbance is reapplied, but now with load of bus 3 considered to be dependent on voltage and frequency (Equations 14 and 15) with the same coefficients as in Example 1. The results of LSGR solution, with crisp and fuzzy methods, are shown by Cases 5 and 6 of Table 2, respectively. The performance indices are 175.425 and 172.532, respectively. As shown through Cases 5 and 6 of Table 2, the number of iterations for the fuzzy LP solution compared with the crisp LP method are reduced.

CONCLUSION

A mathematical formulation for the optimum load shedding and generation reallocation problem using fuzzy linear programming has been presented. The fuzzy LP is a proper alternative in performing the traditional optimal LSGR procedure, a method to control the power system during emergency conditions. At the same time, the fuzzy LP application to LSGR problem provides practical adjustments to the operation of a real power system.

In the power system case studies, the simulation results show that the fuzzy solution method for the LSGR problem is more flexible than the crisp solution method. Additionally, the performance index of fuzzy solution is less than its respective value of the crisp solution.

Also, the load curtailment values in a fuzzy environment are less than the corresponding crisp case. More important, the fuzzy based approach to the solution of the LSGR problem accommodates more realistic models to characterize the behavior of practical power systems operations.

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NOMENCLATURE

The general quantities used are defined below.

F	system frequency.
NB	number of system buses.
NG	number of generators.
NL	number of transmission lines.
NT	number of tap-changing transformers.
NP	number of phase-shifting transformers.
PG, QG	active and reactive power generation.
PL, QL	load active and reactive powers.
P, Q	active and reactive power injections.
$PG_{\text{set}}, QG_{\text{set}}$	setting of active and reactive power of a generator.
$PL_{\text{set}}, QL_{\text{set}}$	setting of active and reactive power of a load.
P_R	rated output of a generator.
R	rated regulation of a generator.
V_i, δ_i	voltage magnitude and angle at bus i.
V_{LB}	load base voltage value.
t	tap value of a transformer.
a_i, b_i, c_i, d_i	coefficients of the objective function.
p_z, q_z	coefficients of constant impedance load.
p_c, q_c	coefficients of constant current load.
p_p, q_p	coefficients of constant power load.
k_p, k_q	coefficients of frequency dependent part of the load.
min	minimum value of a variable.
max	maximum value of a variable.
ϕ	angle value of a phase-shifting transformer.
$\Psi_{ij}^{\max}$	maximum phase angle difference between buses i and j.
Y_{ij}, θ_{ij}	an element of the admittance matrix Y.
Δ	deviation operator.

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APPENDIX A

For LP applications, the nonlinear terms in Equation 1 may be piecewise linearized, as shown for example in Figure A.1. In order to apply LP, each component of the objective function ($\alpha_i \cdot \Delta S_i^2$) and its corresponding variable (ΔS_i) are represented by four simple linear functions with four variables ($\Delta S_i'$, $\Delta S_i''$, $\Delta S_i'''$, $\Delta S_i''''$). Thus, $\alpha_i \cdot \Delta S_i^2$ is approximated by the expression:

$$\rho_{i_1} \cdot \Delta S_i' + \rho_{i_2} \cdot \Delta S_i'' + \rho_{i_3} \cdot \Delta S_i''' + \rho_{i_4} \cdot \Delta S_i'''' \ , \tag{A.1}$$

as:

$$\Delta S_i = \Delta S_i''' + \Delta S_i'''' - \Delta S_i' - \Delta S_i'' \ , \tag{A.2}$$

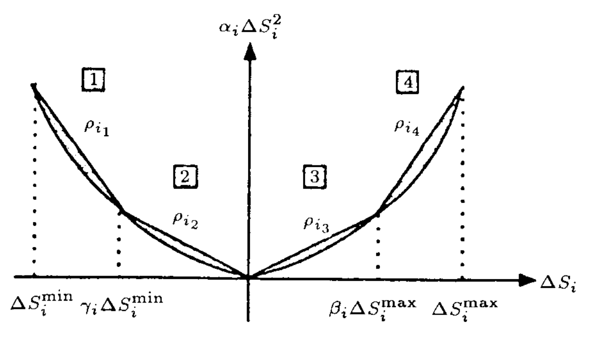

Figure A.1. Linearization of quadratic objective function.

and:

$$S_i^{\min} - S_i^\circ \leq \Delta S_i \leq S_i^{\max} - S_i^\circ \ , \qquad (A.3)$$

then, the constraints in terms of new variables become:

$$0 \leq \Delta S_i' \leq (1-\gamma_i)\cdot|\Delta S_i^{\min}| \ , \qquad (A.4)$$

$$0 \leq \Delta S_i'' \leq \gamma_i\cdot|\Delta S_i^{\min}| \ , \qquad (A.5)$$

$$0 \leq \Delta S_i''' \leq \beta_i\cdot\Delta S_i^{\max} \ , \qquad (A.6)$$

$$0 \leq \Delta S_i'''' \leq (1-\beta_i)\cdot\Delta S_i^{\max} \qquad (A.7)$$

A more clear picture of these relations is shown in Figure A.1 . It should be mentioned that in Figure A.1, ΔS_i is an indication for ΔPG_i, ΔQG_i, ΔPL_i, ΔQL_i variables, and α_i denotes either of a_i, b_i, c_i, d_i coefficients, while ρ_{i1} through ρ_{i4} denote the absolute value of each portion slope. γ_i and β_i are taken so that the dashed area is minimum. It is easy to show that for this to be the case, $\gamma_i = \beta_i = 0.5$.

APPENDIX B

The membership function of the fuzzy number $\tilde{\alpha}_i$ (defined by the decision maker) can be considered as the one shown in Figure B.1.

With the account that in Figure A.1 $\gamma_i = \beta_i = 0.5$, then the membership functions of $\tilde{\rho}_{i3}$ and $\tilde{\rho}_{i4}$ (that are the right piece lines slope of Figure A.1) can be calculated as (see Figure B.2):

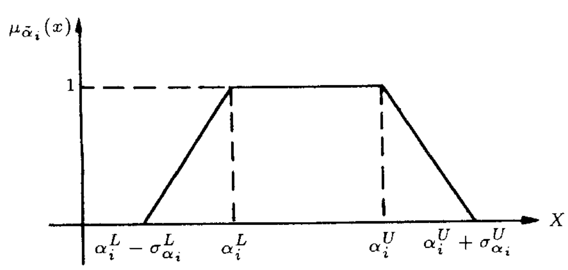

Figure B.1. The membership function of $\tilde{\alpha}_i$.

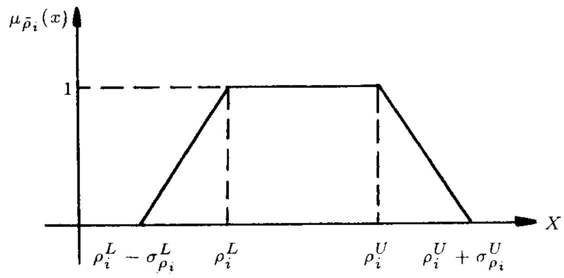

Figure B.2. The membership function of $\tilde{\rho}_i$.

$$\begin{cases} \rho_{i_3}^L = \frac{1}{2}\cdot\alpha_i^L\cdot\Delta S_i^{\max} \\ \rho_{i_3}^U = \frac{1}{2}\cdot\alpha_i^U\cdot\Delta S_i^{\max} \\ \sigma_{\rho_{i_3}}^L = \frac{1}{2}\cdot\sigma_{\alpha_i}^L\cdot\Delta S_i^{\max} \\ \sigma_{\rho_{i_3}}^U = \frac{1}{2}\cdot\sigma_{\alpha_i}^U\cdot\Delta S_i^{\max} \end{cases} \qquad (B.1)$$

$$\begin{cases} \rho_{i_4}^L = \frac{3}{2}\cdot\alpha_i^L\cdot\Delta S_i^{\max} \\ \rho_{i_4}^U = \frac{3}{2}\cdot\alpha_i^U\cdot\Delta S_i^{\max} \\ \sigma_{\rho_{i_4}}^L = \frac{3}{2}\cdot\sigma_{\alpha_i}^L\cdot\Delta S_i^{\max} \\ \sigma_{\rho_{i_4}}^U = \frac{3}{2}\cdot\sigma_{\alpha_i}^U\cdot\Delta S_i^{\max} \end{cases} \qquad (B.2)$$

For calculating the membership functions of $\tilde{\rho}_{i1}$ and $\tilde{\rho}_{i2}$, $\Delta S_i^{\max}$ must be changed to the absolute of $\Delta S_i^{\min}$ in Equations B.1 and B.2.

APPENDIX C

Crisp Data

A sample power system (see Figure 4) is considered consisting of two generators, two phase-shifting transformers and five transmission lines. The lines and transformers data are given in Table C.1. The buses data, the minimum (min) and maximum (max) of voltage magnitudes and c_i, d_i coefficients of the objective function (in conjunction with system loads) are shown in Table C.2. The max and min of active and reactive generator powers, and a_i, b_i coefficients of the objective function (in conjunction with generators) are illustrated in Table C.3. The min and max phase angles of

Table C.1. The lines and transformers data.

No.	From Bus	To Bus	Resist.	React.	Suscept.
1	5	3	0.060	0.180	0.000
2	5	2	0.040	0.120	0.000
3	5	4	0.060	0.180	0.040
4	3	4	0.010	0.030	0.020
5	4	2	0.080	0.240	0.050
6	1	5	0.020	0.060	0.060
7	1	3	0.080	0.240	0.050

Table C.2. Bus data and c_i, d_i coefficients.

No.	PG	QG	PL	QL	V	$V^{\min}$	$V^{\max}$	c_i Coeff.	d_i Coeff.
1	0.00	0.00	0.00	0.00	1.06	0.90	1.10	400.0	400.0
2	0.00	0.00	0.60	0.10	1.00	0.95	1.05	400.0	66.7
3	0.00	0.00	0.45	0.15	1.00	0.95	1.05	400.0	133.3
4	0.00	0.00	0.40	0.05	1.00	0.95	1.05	400.0	50.0
5	0.40	0.00	0.20	0.10	1.00	0.95	1.05	400.0	200.0

Table C.3. The limits valve of generators and a_i, b_i coefficients.

No.	$QG^{\min}$	$QG^{\max}$	$PG^{\min}$	$PG^{\max}$	R_i	a_i Coeff.	b_i Coeff.
1	-2.00	2.00	0.20	2.50	0.05	20.00	28.60
5	-2.00	2.00	0.20	1.50	0.05	20.00	14.90

phase shifters are -10 and 10 degrees. Also, the max phase angle difference of lines is considered to be 0.25 radians (14.5 degrees).

Fuzzy Data

The values of the system variable limits (except voltage magnitudes) are described by trapezoidal fuzzy numbers defined as (see Figure 2):

(min value, max value, 10% of min value,
10% of max value) (C.1)

For bus voltage magnitude, the fuzzy numbers are defined by:

(min value, max value, 5% of min value,
5% of max value). (C.2)

The load limits are considered to be crisp. Also, the objective function coefficients are left-right fuzzy numbers (in other words, the min and max values are equal).