Interrelationship among Permeability, Porosity, Specific Surface Area and Irreducible Fluid Saturation

George V. Chilingarian[1]

The correlation between porosity ϕ and permeability k has been studied both theoretically and experimentally for a long time. Finally a breakthrough has been achieved. On adding two other critical variables, i.e., irreducible fluid saturation S_{wr} and specific surface area s_s, the coefficient of correlation R between porosity and permeability was raised to 0.997 for several carbonate reservoirs, many microfractured.

As an example, for Vuktyl'skiy gas condensate reservoir in the former USSR:

$$\log k = 0.9532 - 2.7880 \times 10^{-2} S_{wr} - 5.5597 \times 10^{-4} s_s + 1.3309 \times 10^{-1} \phi + 1.1707 \times 10^{-5} S_{wr} \times s_s ,$$

where k is in mD, ϕ is in percent, s_s is in cm^2/cm^3 of pore volume and $R = 0.997$. Similar correlations should be determined for the Asmari limestones of Iran.

INTRODUCTION

Permeability, which characterizes the ability of rocks to allow the movement of fluids contained in their pores, is one of the most important parameters describing the porous media. Normally, in order to measure the permeability, the sample must have a simple geometric shape (e.g., cylinder or cube) and certain dimensions. On the other hand, measurements of porosity, pore-size distribution and specific surface area do not require special geometric dimensions. The correlations among permeability and other easier-to-measure parameters, therefore, have been studied theoretically and experimentally [1]. In practice, the most often reported correlation is that between the permeability and porosity [2,3]. The coefficient of correlation for the porosity-permeability relationship varies from sample to sample, with a better correlation if the porosity used in the calculation is measured when a core contains the irreducible fluid. Porosity does not reflect the number and width of fractures, the pore sizes and topological structure, whereas the specific surface area does. Thus, it appears advisable to relate permeability simultaneously to porosity, specific surface area, irreducible water/oil saturation, grain size/pore size/throat size distribution, tortuosity, etc.

In this study, the author examined the interrelationship among permeability k, porosity ϕ, specific surface area s_s, and residual water saturation S_{wr} for carbonate reservoirs in the

1. Department of Petroleum Eng., University of Southern California, L.A., CA, USA, 90089-1211.

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former USSR [1]. Parameters most influencing the permeability were included, whereas unnecessary ones were not used.

FUNCTION FORM

The following form of mathematical relation was used:

$$\log k = a_0 + a_1 S_{wr} + a_2 s_s + a_3\phi + a_4 S_{wr} \times s_s + a_5 S_{wr} \times \phi + a_6 s_s \times \phi. \quad (1)$$

The above equation contains three linear terms and three cross-product terms. This equation was used because of its simplicity; it can be linearized as follows:

$$Y = \sum_{j=0}^{m} a_j X_j, \quad (2)$$

where $Y = \log k$, x_j's are the corresponding terms following a_j in Equation 1 and $m = 6$.

The constants $a_0, a_1, \ldots, a_m$ are estimated by a standard linear regression technique [4,5] i.e., by minimizing:

$$Q(a) = \sum_{i=1}^{N} \left(\log k_i - \sum_{j=0}^{m} a_j X_{ij} \right)^2, \quad (3)$$

where Q is the squared difference between the measured and calculated values.

CASE STUDIES

Data used in this study represent four carbonate reservoir areas in the former USSR. The first one examined is Vuktyl'skiy gas-condensate deposit (Table 1). Lithology varies from dolomites (porous, porous-cavernous, finely-cavernous, crystalline, slightly calcareous, crystalline dense, etc.) to true limestones

Table 1. Petrophysical properties of carbonate rocks of Vuktyl'skiy gas condensate deposit (see [6], Table 37).

S_{wr}, %	s_s, cm^2/cm^3	ϕ^*, %	$S_{wr}s_s$	$S_{wr}\phi$	$s_s\phi$	k (mD)
16	2156	7	34496	112	15092	3.6
28	7070	8	197960	224	56560	0.4
19	3878	8	73682	152	31024	1.6
25	2058	10	51450	250	20580	11.5
12	1827	12	21924	144	21924	26.0
12	1113	12	13356	144	13356	76.0
9	1421	13	12789	117	18473	58.0
9	1428	13	12852	117	18564	63.0
28	2030	13	56840	364	26390	28.0
5	945	14	4725	70	13230	138.0
9	3668	14	33012	126	51352	9.5
7	644	14	4508	98	9016	294.0
3	854	15	2562	45	12810	208.0
16	2142	15	34272	240	32130	36.0
8	1001	15	8008	120	15015	167.0
4	532	18	2128	72	9576	1011.0
4	441	20	1764	80	8820	1910.0

* "Effective" (intercommunicating) porosity as used in USA.

(dolomitized, with very fine caverns, and micro-grained).

All six terms (see Equation 1) were used to calculate the regression equation, and the value of 0.9967 for the correlation coefficient R indicates that a strong relation exists between permeability and other petrophysical properties (Table 2). The porosity used here is the "intercommunicating" porosity measured when the pores do not contain irreducible fluids, which usually gives a poor correlation between permeability and porosity alone [2]. This is because dead-end pore spaces, which hold part of the irreducible fluid, contribute little to the fluid flow. However, because irreducible water saturation is included in the equation, the dead-end pore spaces have been indirectly considered. Specific surface area, together with the irreducible water saturation, reflects the pore/grain size of the rock sample, which can significantly result in totally different values of permeability for two rock samples which may have the same values of porosity. Specific surface area also takes into account the presence of fractures, which greatly increase the permeability with only a minor effect on porosity (porosity due to the fractures alone $\leq 1\%$).

Next, the contribution of each term was examined and unnecessary ones, having negligible effect on permeability, were removed. This is achieved by using the F test. In Equation 1, each term has a corresponding F-value. The larger the F-value, the stronger the corresponding term affects the permeability. The terms with F-values smaller than a given critical value are statistically not important in the equation. Only one term with the least F-value, however, should be eliminated each time, after which the constants a_j's must be recalculated. This process is repeated until all the terms have F-values higher than the given critical value. Then, one can state that, statistically, the remaining terms affect the permeability.

The constant vector (regression coefficient), a, is recalculated as shown in Table 3. After eliminating two insignificant terms, the matching is as good as the previous one, because the coefficient of correlation remains unchanged. The final correlation equation is as follows:

$$\log k = 0.9532 - 2.7880 \times 10^{-2} S_{wr} - 5.5597 \times 10^{-4} s_s + 1.3309 \times 10^{-1} \phi + 1.1707 \times 10^{-5} S_{wr}(s_s), \quad (R = 0.997). \tag{4}$$

The results are plotted in Figure 1. In this figure the effects of specific surface area (per unit of pore volume) and irreducible water saturation, S_{wr}, are considered in the data fitting, showing both the measured and fitted data for

Table 2. Regression coefficient a_j's and corresponding ratios of variances F_j's. Vuktyl'skiy gas condensate deposit.

j	**fitted coefficient a_j**	**term**	F_j
0	8.6323×10^{-1}	constant	4.21
1	-2.3047×10^{-2}	S_{wr}	0.53
2	-5.3394×10^{-4}	s_s	3.60
3	1.3847×10^{-1}	ϕ	33.48
4	1.0987×10^{-5}	$S_{wr}s_s$	5.58
5	-2.6844×10^{-4}	$S_{wr}\phi$	0.01
6	-1.3178×10^{-6}	$s_s\phi$	0.01
		total	195.17

$F_{0.25}(6,10) = 1.58, F_{0.25}(1,10) = 1.49, R = 0.9967$

Table 3. Regression coefficient a_j's and corresponding ratios of variances F_j's. Vuktyl'skiy gas condensate deposit.

j	**fitted coefficient a_j**	**term**	F_j
0	9.5325×10^{-2}	constant	22.99
1	-2.7880×10^{-2}	S_{wr}	26.46
2	-5.5967×10^{-4}	s_s	139.83
3	1.3309×10^{-1}	ϕ	164.93
4	1.1709×10^{-5}	$S_{wr}s_s$	40.76
		total	331.49

$F_{0.01}(4,12) = 5.41, F_{0.01}(1,12) = 9.33, R = 0.9967$

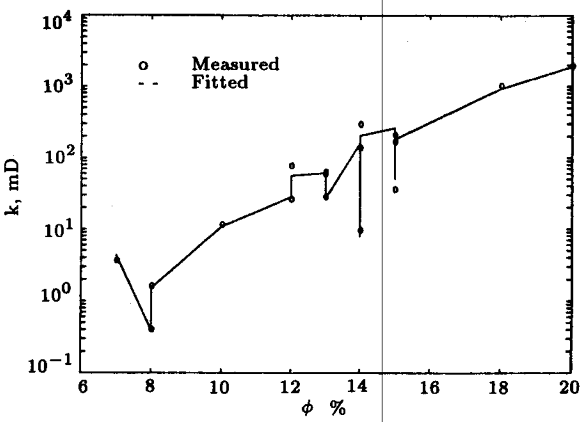

Figure 1. Plots of permeability, k, versus intercommunicating (open) porosity, ϕ_o.

Vuktyl'skiy gas-condensate deposit. Similarly determined, for carbonate reservoir rocks of Central Asia (see Table 4), the correlation equation is:

$$\log k = 3.8690 - 1.0536 \times 10^{-1} S_{wr} - 4.1979 \times 10^{-4} s_s + 6.5363 \times 10^{-6} S_{wr}(s_s) + 2.8324 S_{wr}(\phi), \quad (R = 0.985). \tag{5}$$

For carbonate reservoirs of Kuybyshev Along-Volga Region (see Table 5) the correlation equation is:

$$\log k = 2.1085 - 5.0777 \times 10^{-2} S_{wr} - 4.3785 \times 10^{-4} s_s + 7.9959 \times 10^{-2} \phi + 7.6326 \times 10^{-6} S_{wr}(s_s), \quad (R = 0.988). \tag{6}$$

The correlation equation for the carbonate reservoir rocks of Orenburg Deposit (see Table 6) is as follows:

$$\log k = 3.4351 - 2.0443 \times 10^{-1} S_{wr} + 9.5086 \times 10^{-6} S_{wr}(s_s) + 8.0217 \times 10^{-3} S_{wr}(\phi) - 2.3892 \times 10^{-5} s_s(\phi), \quad (R = 0.981). \tag{7}$$

Table 4. Petrophysical properties of carbonate reservoirs (both limestones and dolomites) of Central Asia (see [6], Table 46).

S_{wr}, %	s_s, cm^2/cm^3	ϕ, %	$S_{wr}s_s$	$S_{wr}\phi$	$s_s\phi$	k (mD)
32	3465	13	110880	416	45045	9.6
16	658	14	10528	224	9212	302.0
10	231	14	2310	140	3234	2400.0
25	2324	15	58100	375	34860	29.0
30	1750	15	52500	450	26250	55.0
30	3311	16	99330	480	52976	18.0
25	1169	16	29225	400	18704	151.0
63	13860	17	873180	1071	235620	1.3
22	1316	17	28952	374	22372	129.0
58	12880	19	747040	1102	244720	2.1
43	6650	19	285950	817	126350	7.4
37	8358	20	309246	740	167160	5.6
46	6300	22	289800	1012	138600	14.0
27	3542	22	95634	594	77924	44.0
23	1848	22	42504	506	40656	150.0
22	840	23	18480	506	19320	883.0

Table 5. Petrophysical properties of carbonate reservoirs of the Kuybyshev Along-Volga Region (see [6], Table 37). Lithology is mainly limestone.

S_{wr}, %	s_s, cm^2/cm^3	ϕ, %	$S_{wr}s_s$	$S_{wr}\phi$	$s_s\phi$	k (mD)
58	7875	10.1	456750	585.8	79537.5	0.8
7	427	13.0	2989	91.0	5551.0	577.0
22	2016	14.8	44352	325.6	29836.8	39.0
21	1890	17.5	39690	367.5	33075.0	74.0
16	1330	17.6	21280	281.6	23408.0	151.0
41	5285	18.1	216685	742.1	95658.5	10.4
52	10850	19.4	564200	1008.8	210490.0	3.0
15	1309	19.4	19635	291.0	25394.6	209.0
16	1253	19.7	20048	315.2	24684.1	239.0
17	700	20.5	11900	348.5	14350.0	860.0
42	9520	21.0	399840	882.0	199920.0	5.0
10	518	21.0	5180	210.0	10878.0	1671.0
52	7210	23.4	374920	1216.8	168714.0	11.9
32	5432	23.7	173824	758.4	128738.4	22.0
18	2310	23.7	41580	426.6	54747.0	123.0
14	749	24.0	10486	336.0	17976.0	1207.0

Table 6. Petrophysical properties of carbonate reservoirs of Orenburg Deposit, (see [6], Table 24).

S_{wr}, %	s_s, cm^2/cm^3	ϕ, %	$S_{wr}s_s$	$S_{wr}\phi$	$s_s\phi$	k (mD)
20	2632	9.2	52640	184.0	24214.4	5.50
42	10164	10.3	426888	432.6	104689.2	0.52
31	5810	11.0	180110	341.0	63910.0	1.93
19	2471	12.0	46949	228.0	29652.0	14.00
33	7000	12.4	231000	409.2	86800.0	1.89
39	9296	13.0	362544	507.0	120848.0	1.24
15	2212	13.0	33180	195.0	28756.0	22.00
27	5551	13.2	149877	356.4	73273.2	3.65
9	770	13.4	6930	120.6	10138.0	198.00
25	2866	13.5	71650	337.5	38691.0	3.40
16	1183	13.8	18928	220.8	16325.4	92.00
25	2285	14.5	57125	362.5	33132.5	5.34
17	3451	15.7	58667	266.9	54180.7	16.00
20	2261	17.0	45220	340.0	38437.0	39.00
10	616	17.0	6160	170.0	10472.0	590.00
18	1274	18.2	22932	327.6	23186.8	181.00
12	1050	20.0	12600	240.0	21000.0	258.00

As one can see, in general, there is a good correlation between permeability and other petrophysical properties. For different lithologies, however, each term weighs differently. The complex pore structures may contribute to such diversity. Care, therefore, should be taken when using such empirical correlations because many factors, e.g, mineralogy and diagenesis can greatly affect the pore configuration.

CONCLUSIONS

The use of multi-variable linear regression analysis enabled the author to demonstrate experimentally that the permeability can accurately be estimated from the other petrophysical properties. It was shown that, besides porosity, specific surface area and irreducible water saturation are important factors for such permeability estimations. Such experimental relations should be established for other reservoir rocks.

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