Non-Equilibrium Effects in Steam Turbines

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This paper gives an overall review of progress made in studies of problems associated with the behavior of wet steam in turbines. Brief summaries of nucleation theory and equations governing two-dimensional two-phase flows of steam are given and a cascade tunnel specially designed to study these problems and some of the results obtained are briefly described.

INTRODUCTION

The importance of supersaturation in the separation of liquid phase from steam undergoing expansion has been recognized since the early decades of the century. Nevertheless, conventional procedures adopted for predicting the behavior of steam in the two phase state are based on the use of steam tables. These procedures assume that the fluid remains in thermodynamic equilibrium throughout, becomes wet immediately as the state path crosses the saturation line and totally disregard the behavior of the liquid. For these assumptions to have any validity, the changes must occur very slowly. In practical systems, for example in a flowing vapor undergoing expansion, the changes in the fluid condition are accomplished in finite time and condensation must occur at a rate to match them. For this purpose, a driving potential is necessary and the system will have to depart from thermodynamic equilibrium. Indeed it is often the case that the rate of phase change will differ drastically from that required to match the flow changes and the departure of the system from equilibrium is correspondingly greater. The state of progress in the study of these problems as relevant to steam turbines is briefly reviewed in this paper. It must, however, be emphasized that these problems are not particularly limited to steam and will arise in all fluids. Their discussion in relation to steam turbines is a consequence of the research interests of the author.

A more realistic approach to dealing with two phase flows involving substantial departures from equilibrium would be to consider wet vapor as consisting of a large number of very small droplets suspended in dry parent vapor. The droplets will be generally carried by the vapor and interact with it. After any disruption of the system, the vapor is affected by the changes directly but the droplets have to respond as the result of interaction with the vapor. For a given wetness fraction, the smaller the droplets, the better is the interphase contact. Clearly, the response of the mixture to a change depends on the number and the size of the droplets present. To understand the behavior of the system the interphase interaction should be taken into account.

In the particular case of the flow of steam

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in turbines, in general the fluid is initially superheated but in due course becomes wet and some of the stages must operate on a two-phase mixture. The performance of the wet stages is affected by the interaction between the phases and the detailed behavior of the liquid. However, before considering their application to flow in these machines, the processes of droplet formation and behavior will be briefly reviewed.

FORMATION OF DROPLETS

Measure of Supersaturation

The state of the vapor is specified by its pressure p and temperature T_G. To take a concrete example, say that $p = 1$ bar and $T_G = 80°\text{C}$. The saturation temperature corresponding to p is denoted by $T_s(p)$ which in the present case is 99.6°C. The saturation pressure corresponding to T_G denoted by $p_s(T_G)$ is 0.4736 bar. Comparison of p with $p_s(T_G)$ or $T_s(p)$ with T_G indicates that the vapor is not in an equilibrium state. Its condition is said to be supercooled or supersaturated. The departure from equilibrium can be quantified either by the ratio $p/p_s(T_G) = 2.1115$, which is termed the supersaturation ratio, or by $T_s(p) - T_G = 19.6°\text{C}$, which is the supercooling.

Thermodynamic Aspects

At low pressures, the equilibrium vapor pressure over a curved surface $P_s(T, r)$ is related to its radius of curvature by the Kelvin-Helmholtz relation.

$$\ln \frac{p_s(T, r^*)}{p_s(T)} = \frac{2\sigma}{r^* \rho_L RT}, \tag{1}$$

where ρ denotes the density, σ the surface tension, R the gas constant for the vapor, suffix L refers to the liquid phase and r^* is the radius of curvature of the surface, it will be seen that, for a given temperature, the size of the droplets and the logarithmic supersaturation ratio are inversely related. Thus for given vapor conditions p and T_G, droplets with a radius of r^* will be in unstable equilibrium with the surroundings. Droplets with a larger radius need a lower supersaturation and will grow under the prevailing conditions while those with a smaller radius will find the surrounding supersaturation insufficient and tend to evaporate.

The existence of the critical radius is a barrier to the condensation of a pure vapor removed from contact with external surfaces. In order to condense, the molecules must form a droplet of radius r^*, which is against their natural tendency. Thus, when the state path of an expanding vapor crosses the saturation line, the vapor will initially supercool. The only route to the formation of supercritical droplets is through chance collisions within the body of the vapor.

Homogeneous Nucleation Theory

Formation of critical droplets within the body of a pure supercooled vapor is termed homogeneous nucleation. This is in contrast to heterogeneous nucleation which refers to condensation on foreign particles present in the vapor. The first theory to describe homogeneous nucleation was proposed in 1926 and the subject has been undergoing continuous development since. It was reasoned that, even in a superheated vapor, clusters of molecules are continuously formed and disrupted by molecular collisions. At any one instant the number of clusters n_g containing g molecules present in the vapor would follow the Boltzmann distribution law,

$$n_g = n_1 e^{-\Delta G/kT}, \tag{2}$$

where n_1 is the number of single molecules in the system and k is the Boltzmann constant. The rate at which a cluster would capture molecules, C_g, can be calculated from the kinetic theory, i.e.:

$$C_g = \frac{q\rho\bar{c}}{4m}, \tag{3}$$

where $\bar{c}$ is the mean molecular velocity, m the mass of one molecule and q the condensation coefficient denoting the fraction of molecules colliding with the surface which condense on it. It was then argued that, for a critical cluster, the probability of decay and growth would be equal and the nucleation rate would be half the product $C_{g^*} n_{g^*}$.

Clearly this is a gross oversimplification of the problem, because the molecular clusters lose molecules as well as capture them and the distribution law of clusters should differ between supercooled and superheated conditions. The rate of formation of critical clusters should thus be expressed as:

$$I_{g^*} = \frac{C_{g^*} n_{g^*}}{K}. \tag{4}$$

It may be said that the determination of the correct expression for K has been the object of the subsequent investigations. The search has generally proceeded along two separate approaches, the kinetic and the statistical mechanical. The development by Zeldovitch using the kinetic approach is referred to as the classical theory [1]. A lucid and graphical description of this theory is given by McDonald [2]. The statistical mechanical approach originated in the late 1930's and was aimed at removing many of the uncertainties associated with the classical theory. A comprehensive treatment of the subject is given by Dunning [3]. A variety of hybrid theories have also been developed and aim to improve the results of the classical theory by grafting correction factors derived from a statistical treatment. Despite the continuous effort devoted to resolving the many controversies associated with some aspects of the theory, a definitive treatment has yet to emerge. Nevertheless, the classical treatment, subject to some minor modifications, is in reasonable agreement with experimental observations. It is, therefore, generally adopted in engineering applications. The expression recommended by the author for the calculation of the number of droplets formed per unit volume of vapor is:

$$I_{st} = \frac{q}{1+v}\left(\frac{2\sigma}{\pi m^3}\right)^{\frac{1}{2}} \frac{\rho_s(T_G)}{\rho_L} \exp\left(-\frac{4\pi_r^{*2}\sigma}{3kT}\right), \tag{5}$$

where

$$v = \frac{q\rho_G}{\alpha}\sqrt{\frac{RT_G}{2\pi}}\left(\frac{\ell^2}{RT_G^2} - \frac{\ell}{2T_G}\right), \tag{6}$$

and $\rho_s(T_G)$ is the density of saturated vapor at temperature T_G, ℓ the enthalpy of phase change and α the heat transfer coefficient. The above expression is the result of the classical theory as refined by Courtney [4] and by Kantrowitz [5].

Condensation Coefficient and Energetics of Clusters

Two of the parameters associated with the nucleation rate which are still subject to some uncertainty are the condensation coefficient and the surface tension of small clusters.

With the introduction of the correction due to Kantrowitz, the solutions become insensitive to values of q larger than 0.1. On the other hand, studies of molecular collisions with surfaces indicate that only a small fraction of the reflections correspond with those of elastic collisions with the surface, while over 90% of the reflected molecules behave as if in some way held by the surface and re-emitted in a random direction. This would be consistent with imagining that these molecules had penetrated the surface to emerge after collisions with the molecules forming the interior material of the solid surface. In the parallel case of vapor molecules colliding with a droplet of their own liquid, if it can be defined that the molecules penetrating the surface can be regarded as having condensed, it follows that the condensation coefficient should have a value close to unity.

In the derivation of the nucleation equation, the free energy of the formation of an embryo is described in terms of the macroscopic surface tension of a flat film. As expressed by Equation 5, the exponential term contains $r^{*2}\sigma$. Using Equation 1 to eliminate r^* shows that the exponential term is proportional to σ^3. Thus the question naturally arises as to whether the specific surface free energy of small clusters is affected by their small size, as small changes in the value of σ will alter the nucleation rate drastically.

A large number of investigations into the energetics of small clusters have been reported in the literature. In terms of the present summary, a valuable contribution to the subject

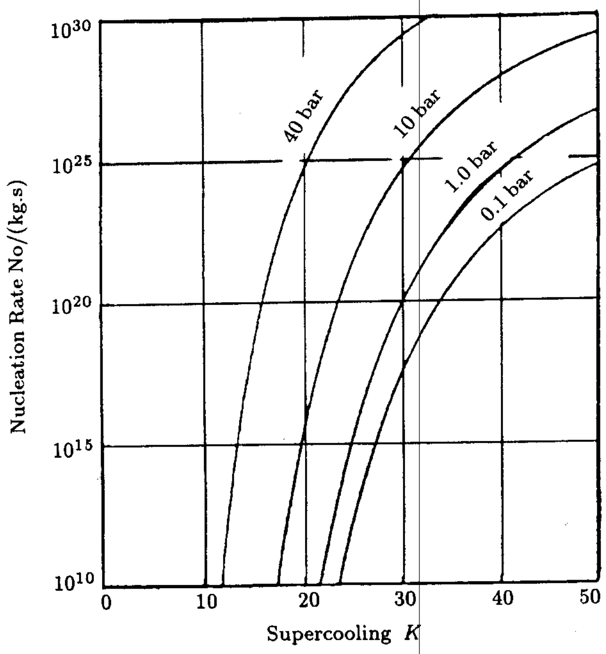

Figure 1. Variations of nucleation rate with supercooling.

is that by Plummer and Hale [6]. In this, calculations of bond energies within ensembles of water molecules were carried out. The resulting free energies of clusters is not a smoothly varying function of the number of molecules in the cluster, but a mean line drawn through the results agrees well with the value, based on regarding the surface tension to be that of a flat surface.

Combination of equations describing droplet formation and growth with standard gas dynamic conservation equations allows the paths of condensing, one-dimensional flows to be predicted theoretically. In flows in converging-diverging nozzles, best all round agreement between theoretical and experimental results can be obtained by taking the nucleation rate to be in accordance with the classical theory subject to corrections due to Courtney and Kantrowitz regarding the specific surface free energy of small clusters to be the same as that for a flat surface and the condensation coefficient as unity. Variations of nucleation rate with supercooling at constant pressure based on the above models are shown in Figure 1.

The presence of foreign particles in the fluid is a complicating factor, as they will provide surface for the condensation of vapor. Condensation on these surfaces will occur at very little supercooling but the rate of separation of liquid will depend on the number of particles present. If the number of foreign particles is relatively small or the expansion fast, the rate of condensation will not be sufficient to maintain the system in equilibrium and the vapor will supercool. Then, if the supercooling increases sufficiently, the vapor will nucleate homogeneously and the liquid phase present will consist of two families of droplets.

DROPLET VAPOR INTERACTION

Exchange Processes

Once formed, the droplets interact with the surrounding vapor and exchange momentum, heat and mass with it. These processes are very similar in nature but in terms of their consequences are more conveniently divided into mechanical, thermodynamic and aerodynamic. The term "mechanical effects" is used to describe the drag, deposition and other consequences of the movement of the droplets, while the thermodynamic/aerodynamic consequences result from the heat and mass exchange processes which have to be considered simultaneously. The actual rates of exchange in these processes depend on the relative magnitude of diameter of the droplets d and the mean free path of the molecules $\bar{\ell}$. The ratio $\bar{\ell}/d$ is termed the Knudsen number and denoted by Kn. Usually, the water droplets formed in steam flowing in turbines grow from small to large in comparison with the mean free path.

Movement of Water in Turbines

Droplets formed by nucleation are generally much smaller than 1μm in diameter and only a small difference in velocity is sufficient to cause them to accelerate with the main flow. Consequently, they follow the streamlines very closely. On the other hand, droplets larger than 10-20 μm are very massive in comparison and develop considerable velocity differences with the main stream.

A typical configuration of flow over a row of turbine blades as calculated by Ghassemi [7] is shown in Figure 2. There is change in both the flow velocity and direction in passing over the row. As already discussed, the finer droplets follow the streamlines closely but the larger droplets depart from them. The limiting path of droplets of $1.4\mu m$ diameter is shown by the full line in the figure. Droplets entering the passage below this line will pass through but those entering above it will deposit on the blade pressure surface.

The mechanism described above is termed "inertial deposition". Droplets can also be carried to the surface by molecular and turbulent diffusion as well as by eddy impaction.

Droplets deposited on blades form films and rivulets on the surface which are driven towards the trailing edge by the shearing action of the main stream. In the case of the rotor blades, the surface water is also driven radially by the rotation of the blades. The liquid film reaching the trailing edge is re-entrained into the flow in the form of coarse droplets. Particularly in the case of droplets leaving the stator blades, the distance between the succeeding rows of blades is insufficient to allow appreciable acceleration of the entrained liquid. Consequently, on reaching the succeeding row, the entrained droplets will be travelling in the wrong direction relative to it and will strike the leading edge causing braking and erosion.

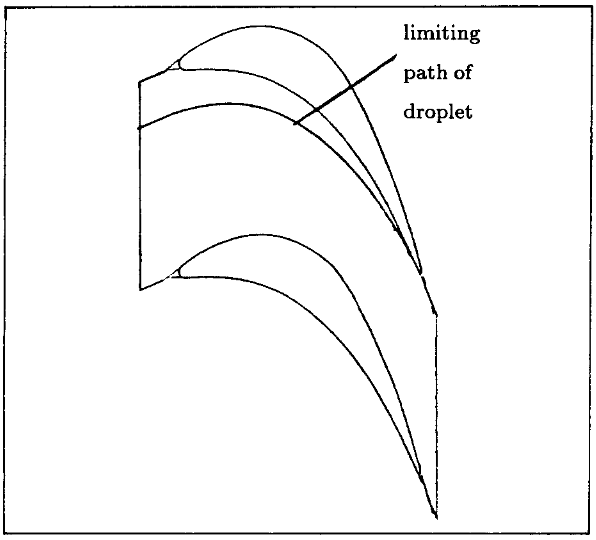

Figure 2. Flow over typical blade passage.

The individual phenomena involved in the above chain of events occur in many other situations and have been studied widely in the literature [8,9].

In studies of droplet behavior in turbines, to measure the size of the small water droplets, the technique receiving widest development is light extinction and the measurements in operating turbines carried out by Walters and Skingley [10] are the most extensive. Nevertheless, there is still much uncertainty about the size and the distribution of these droplets in turbines. In addition, because of the complexity of these machines, no direct measurement of the deposition rates in operating turbines has been attempted. However, it is generally assumed that droplets depositing on blade surfaces will subsequently reappear as coarse water and the proportion of the theoretical moisture existing in the form of large droplets is taken as a measure of the deposition rate. In turbines fitted with suction slots, measurement of the water extracted provides another measure of the deposition rate.

Some experimental measurements of deposition rates of small particles in cascades of turbine blading have also been reported [11, 12]. In these studies the agreement between theoretical estimates and direct measurements is good. In the case of flow in turbines, the use of idealized methods of estimating the diameter of the fine droplets yield values with substantially lower deposition rates than are observed [13]. The suggested mechanisms proposed to explain the difference all concern the departures between the real flow and fluid conditions in contrast to the idealized assumptions in the theoretical models. Adopting measured droplet sizes as their basis, Young, Yau and Walters [14] have estimated deposition rates which are in good agreement with direct observations.

Thermodynamic and Aerodynamic Consequences

The most tangible mechanical consequence of the presence of liquid phase in steam is erosion of blading, but, in practice, the effect on the output of the turbine is relatively small. The greater influence of the two-phase effects on the output of a turbine is through the thermodynamic and aerodynamic consequences of the interphase heat and mass transfer. The thermodynamic consequences are the irreversibilities associated with the internal heat transfer within the fluid and the aerodynamic consequences result from the effect of heat release on the behavior of the vapor. The equations describing interphase heat and mass transfer are given in Appendix 1.

To predict the behavior of nucleating and wet steam flows, the equations describing droplet behavior have to be combined with the standard gas dynamic conservation equations and treated as a set. In the case of wet steam formed by nucleation, the droplets are generally very small and interphase slip can be neglected with little loss of accuracy. A brief outline of the equations and the arrangement for treating two-dimensional two-phase flows is given in Appendix 2. Some cases of condensation in high speed flows studied in the literature will now be briefly considered.

CONDENSATION IN ONE-DIMENSIONAL FLOWS

The convergent-divergent nozzle offers a relatively simple means of investigating high speed condensing flows and has been used for this purpose in numerous studies. A typical arrangement used for the study of nucleating flow of steam is shown in Figure 3. The passage shape is shown as Figure 3a, while the axial pressure distribution is given as Figure 3b and the path of the expansion on the enthalpy-entropy diagram as Figure 3c. In addition, a pressure distribution curve for completely superheated expansion from the same initial pressure is given as the dotted line in Figure 3b. Steam enters the nozzle in a superheated state. This is designated as point A on the enthalpy-entropy diagram. In the initial part of the expansion, the steam is superheated but the state path crosses the saturation line at point C.

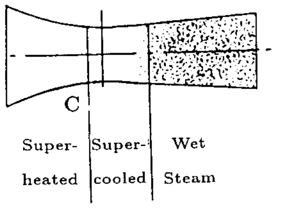

Figure 3a. Passage shape

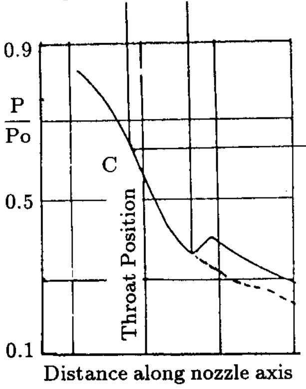

Figure 3b. Axial pressure distribution

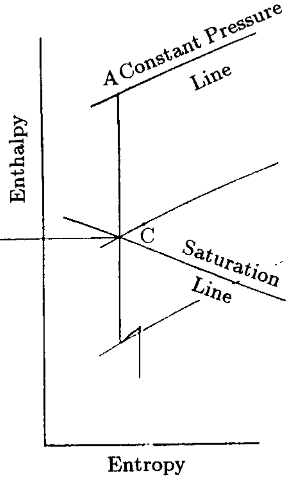

Figure 3c. Path of expansion

Figure 3. General features of nucleating flows in convergent-divergent nozzles.

Comparing the pressure traverses for the superheated and nucleating tests, there will be no difference between the two curves for a while after the state path has crossed the saturation line in the nucleating test. This is because, for the reasons already given, the rate at which steam can produce critical droplets within itself is totally insufficient to bring it to equilibrium initially and the fluid behaves as if the saturation line did not exist. Hence the two pressure traverses remain almost identical for some distance. To all intents and purposes, supercooled steam behaves as a dry fluid having equation of state and values of specific heats and isentropic index very similar to superheated steam. However, this state of affairs does not last for ever. When steam is expanded isentropically, its temperature drops faster than the saturation temperature corresponding to its pressure. Thus, in the case of supercooled steam, its supercooling increases with further expansion. Increases in supercooling decrease the size of the critical droplets and increase the nucleation rate. In due course a point is reached where the number of droplets present within the vapor is sufficient to affect its behavior and the two distribution curves begin to diverge.

At low and moderate pressures and starting with steam which is in an equilibrium state, the amount of expansion necessary to cause substantial nucleation accelerates it to supersonic velocities. Consequently, the point of departure between corresponding dry and nucleating expansions occurs in the divergent part of the nozzle. Once the number of nuclei in the flow has become significant, their growth restores the flow to near equilibrium. The release to the supersonic flow of latent heat associated with the growth of the droplets has a tendency to retard the flow and raise its pressure. Consequently, the wet pressure distribution exhibits a characteristic knee shape in the zone of rapid condensation. But this characteristic only applies to supersonic flows. The addition of heat to a sub-sonic flow will accelerate it, causing the pressure to drop. The position of maximum supercooling is termed the Wilson point.

In the earlier stages of the development of the subject, the measurements consisted mainly of axial pressure distributions but following interest in the behavior of wet steam in turbines, a substantial number of measurements of droplet sizes have also been reported.

As already indicated, comparisons between theoretical solutions and experimental observations of nucleating flows in convergent-divergent nozzles allowed the effect of the proposed refinements of the nucleation theory to be validated against direct measurements. In general, because the effects of friction, the condensation coefficient and variations in surface tension occur side by side, the axial pressure distributions could be matched by more than one combination of these variables. With the availability of droplet measurements, the choice has been narrowed. Typical comparisons between the droplet measurements by Petr and theoretical solutions has been reproduced from [15] in Figure 4. In the theoretical results presented, the nucleation rate has been taken to be in accordance with the classical theory, subject to the corrections due to Courtney and Kantrowitz. It will be seen that the solutions based on taking the condensation coefficient to have a value of unity and the specific surface tension of small droplets to be the same as that for a flat surface give best all round agreement with the measurements. The same choice of variable was therefore used by the author in subsequent studies. Furthermore, because of its relative simplicity the one-dimensional treatment developed offered a valuable means of investigating the general features of nucleating and two-phase flows.

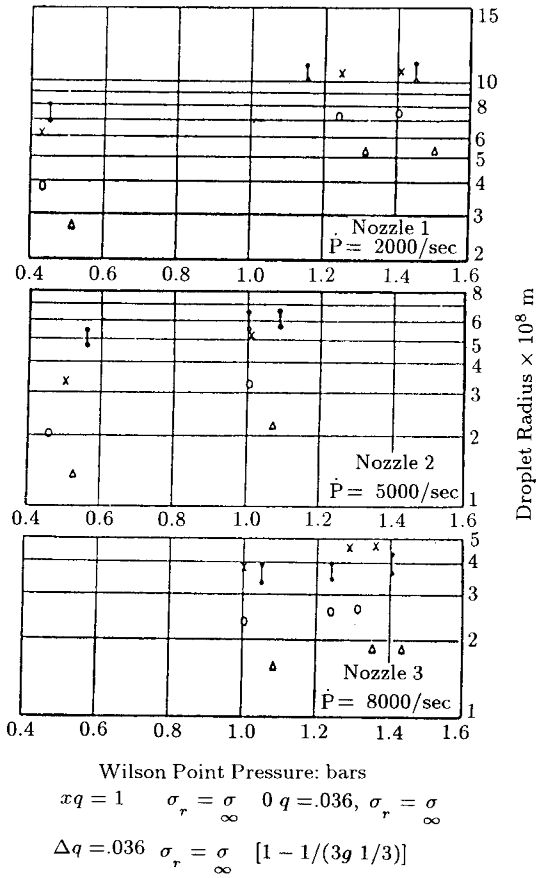

Figure 4. Measured comparison between calculated and measured droplet radii.

The condensation of vapor has three separate consequences for the flow. These are the change in the volume of the vapor condensing into liquid, the internal transfer of heat and the release of the latent heat to the flow.

Because of the large difference between the specific volumes of the liquid and the vapor, condensation reduces the mass flow rate of the vapor content. However, the effect of this change on the overall behavior of the flow is small.

As already discussed, the latent heat of phase change is initially given up to the droplets but has then to be transferred back to the vapor. To achieve heat transfer, the liquid temperature has to be above that of the vapor and the transfer of heat across a temperature difference is irreversible. This process is accompanied by a gain in entropy and loss of output which is generally referred to as "thermodynamic loss". The magnitude of this loss is larger the greater the temperature difference between the droplets and the vapor. The highest difference is associated with the first reversion of the flow when the fluid attains its highest supercooling. There are, however, smaller losses experienced after each rapid expansion of the flow as the fluid departs from, and then recovers, thermodynamic equilibrium.

A less expected but very important consequence of phase change is the effect of the heat released by condensation on the behavior of the vapor. Substantial departures from equilibrium are usually experienced when the vapor is accelerated rapidly. In the example of flow in convergent-divergent nozzles considered, the pressure rise in the zone of rapid condensation is one such consequence, as it can affect the stability of the boundary layer. Indeed, if the zone of rapid condensation occurs just downstream of the throat, it is possible that the heat released by condensation cannot be sustained by the flow and will cause it to become unstable. A further consequence is its effect on the sonic velocity in the fluid and choking of the flow.

CONDENSATION IN TWO-AND THREE-DIMENSIONAL FLOWS

Theoretical Treatments

The problems resulting from supercooling and phase change occur alongside many other complexities associated with flow in turbines. Although a large number of measurements of the performance of operating steam turbines have been reported, without the support of satisfactory theoretical models or a wealth of background cascade measurements, it is difficult to investigate the influence of any one parameter from them. The general case of flow in a turbomachine is three-dimensional and, in principle, the equations describing two-phase flows in these machines are capable of solution by numerical techniques. The earlier studies of nucleating flows were concerned with one-dimensional problems. At the present state of progress, only the treatment of such flows in two-dimensional fields has received significant attention, but the techniques are capable of extension to three-dimensional flows.

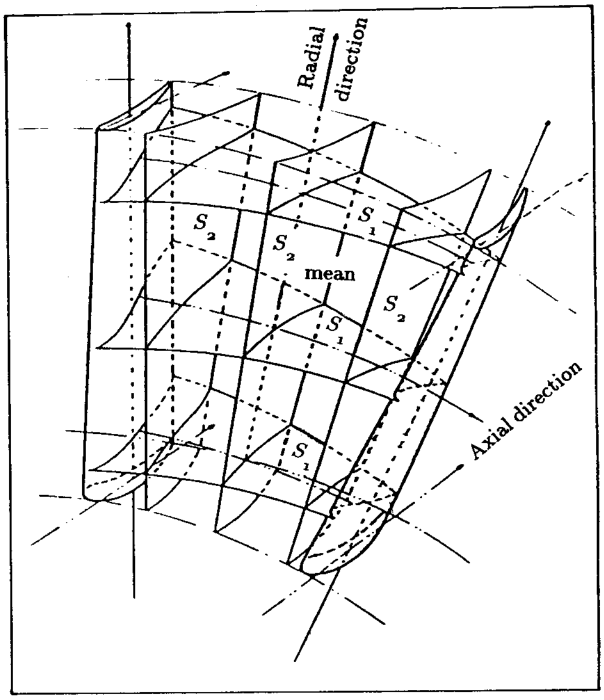

Figure 5. Blade-to-blade and hub-to-tip surfaces in a blade row.

It was shown by Wu [16] that the solution of the three-dimensional flows in a turbomachine can be taken as the combination of solutions in two families of two-dimensional surfaces. As shown in Figure 5, one family consists of cylindrical surfaces co-axial with the machine which intersect the blades at right angles, referred to as the S_1, or blade-to-blade planes. The other family is meridional surfaces containing the axis of the machine, referred to as the S_2, or hub-to-tip planes. Solutions of two-phase flows in both surfaces have been attempted [17, 18, 19, 20]. The next stage in the development of these treatments is validation against experimental observations, but direct measurements suitable for this purpose are scarce.

Blow Down Cascade Measurements

Experimental investigations of condensing flows in turbines are hindered by the fact that at low and moderate pressures the limiting supercooling associated with the first break down of supersaturation is high. Consequently, as already discussed, the zones of rapid condensation in steady state tunnels occur in the supersonic part of the flow. This is in contrast to flow in turbines, where, because the shaft extracts energy from the flow, the fluid can supercool sufficiently to nucleate without attaining the speed of sound. Consequently, the turbine nucleating and wet steam conditions cannot be reproduced in steady state tunnels satisfactorily. To produce these conditions realistically requires a supply of supercooled steam. Such a supply can be produced under blow down conditions and a short duration cascade tunnel working on this principle has been constructed at Birmingham University and is the first of its kind [21]. The first results from this facility are becoming available [22]. Brief descriptions of the plant and its principle of operation are given in Appendix 3.

Shadowgraphs of similar superheated and nucleating flows are given in Figure 6a and 6b. The pattern of trailing edge shock waves are clearly evident in the photograph of the superheated test. In the shadowgraph of the nucleating test, the pattern of lines upstream of the cascade extending into the blade passage are formed by water films on the windows. The dark lines downstream of the trailing edge are also formed by water films. The pattern of trailing edge shock waves can also be seen in the shadowgraph. In addition to these, a further dark line can be seen stretching across the passage just downstream of the throat which is absent from the superheated photograph. This line corresponds with the zone of rapid condensation. In the surface pressure measurements, a pressure rise was exhibited at the same location in the nucleating tests which was absent when steam was superheated.

At the time of writing, characteristics of two separate profiles have been investigated using the facility. Somewhat unexpectedly, the zones of rapid condensation occured just downstream of the throat in both families of measurements and the position was insensitive to the inlet supercooling over a wide range.

The measurements have also provided the means of validating theoretical solutions. In the first family of comparisons, although the general features of the flow were predicted reasonably, nevertheless the pressure changes were smeared by the theoretical solutions. It is, however, known that solutions obtained by the time-marching method are grid dependent and, following the introduction of refinements into the algorithms and the grid, the agreement has considerably improved.

It has also been possible to measure the efficiencies of the expansions in the cascade and analyze them further using the theoretical solutions. In general, the magnitude of thermodynamic loss experienced by nucleating flows was in reasonable agreement with the theoretical estimates, but the consequential aerodynamic losses were appreciable in only some of the cases.

The largest difference in efficiency measured between corresponding superheated and nucleating tests was approximately 7%. This figure is substantially below the wetness losses measured by Walters [23] in operating turbines. The difference must be attributed to greater consequential losses experienced in turbine flows. In the example of the solution of the throughflow of nucleating steam in a turbine stage by Bakhtar and Bamkole [20], considerable differences in flow angles were predicted between nucleating and equilibrium solutions. This offers a possible explanation for the existence of high consequential aerody-

Figure 6a. Shadowgraph of superheated flow.

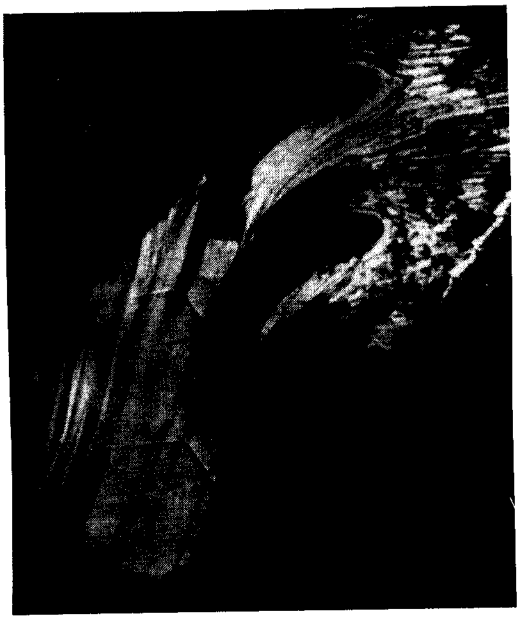

Figure 6b. Shadowgraph of nucleating flow.

namic losses in nucleating turbine stages, which is regarded as avoidable.

CONCLUSION

It has been demonstrated that high speed flows of two-phase fluids are accompanied by substantial departures from thermodynamic equilibrium with major implications for the behavior of the fluid.

The particular problems considered have been associated with the flow of wet steam in turbines, but the problems experienced are not limited to steam and can arise in many other circumstances. Nevertheless, flow in steam turbines alone is of considerable importance because of the wide use of these machines in the generation of electrical power. The losses associated with the flow of wet steam in turbine stages have generally been accepted in the literature as unavoidable. It has, however, been shown that these flows can be reproduced for systematic study and modelled theoretically.

The blow down cascade offers the means of investigating the behavior of different blade profiles. In parallel, two- and three-dimensional flows of nucleating and wet steam are capable of treatment by the time-marching method. The way is therefore open for more efficient designs of nucleating and wet stages of turbines to be developed.

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NOMENCLATURE

B	second virial coefficient
C	rate of capture of molecules by condensation
$\bar{c}$	mean molecular speed
c_L	specific heat of liquid
d	diameter
F	force
G	Gibbs free energy
h	specific enthalpy
I	nucleation rate

K	correction factor
k	Boltzmann's constraint
ℓ	enthalpy of phase change
$\bar{\ell}$	mean free path
m	mass of a molecule
m_r	mass of a droplet of radius r
n	number in the system
N	number of droplets per unit mass
p	pressure
$p_s(T)$	saturation pressure corresponding to T
q	condensation coefficient
r	radius
R	gas constant
s	distance along control volume face
Sn	Schmidt number
T	temperature
$Ts(p)$	saturation temperature corresponding to p
u	component of velocity in X direction
$\bar{U}$	velocity vector
v	component of velocity on Y direction
V	volume
w	wetness fraction
x, y	cartesian coordinates
α	heat transfer coefficient
λ	thermal conductivity
μ	coefficient of viscosity
v	simplifying symbol
ρ	density
$\rho_s(T)$	density of saturated vapor at T
σ	surface tension

Subscripts

g	number of molecules in a cluster
G	vapor
L	liquid
o	stagnation condition
st	steady state

Superscript

$*$	critical

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APPENDIX 1

Interphase Heat and Mass Transfer

Droplets larger than the critical size grow by capturing molecules from the vapor, but they also lose molecules by evaporation because of their thermal energy. Thus, the net gain of mass is the balance between the rates of condensation and evaporation. The enthalpy of condensing molecules is initially given up to the droplets, which raises their temperature. Consequently, in order to grow, the droplets must lose heat to the surroundings.

The net rate at which a droplet of mass m_r gains mass may be written as:

$$\frac{dmr}{dt} = 4\pi r^2 \rho_L \frac{dr}{dt} = \frac{4\pi_r^2 q}{1 + (0.375\ qSn/Kn)} \sqrt{\frac{R}{2\pi}} \left[\rho_G \sqrt{T_G} - \rho_s(T_L, r)\sqrt{T_L}\right], \tag{7}$$

where $Sc = \mu_G/(\rho_G D)$ is the Schmidt number and D the coefficient of self diffusion.

The heat transfer between a spherical droplet at rest relative to a surrounding medium can be described by the following expression due to Gyarmathy:

$$\frac{\alpha d}{\lambda} = \frac{2}{1 + 3.18\ Kn}, \tag{8}$$

where λ is the thermal conductivity of the vapor. An energy balance for the droplet may be written as:

$$\ell\frac{dm_r}{dt} = 4\pi_r^2\alpha(T_L - T_G) + m_r c_L \frac{d_{T_L}}{dt}, \qquad (9)$$

where c_L is the specific heat of the liquid. The last term in the above expression is small in comparison with the others and can be neglected with little loss of accuracy.

With this assumption, for given vapor conditions and droplet radius, dmr/dt can be eliminated between Equations 7 and 8 and the resulting equation solved for T_L. Physically, this is the quasi-equilibrium temperature attained by the droplet at which it can lose the latent heat released due to its growth. Once T_L is determined, substitution into either Equation 7 or 9 gives the rate of growth of the droplets. The above procedures, though accurate, involve considerable numerical work to implement. For the treatment of flows in two and three dimensional fields, the following approximation due to Gyarmathy offers considerable simplification:

$$[Ts(p) - T_L] = [Ts(p) - T_G](r/r^*). \qquad (10)$$

With the last term on the right hand side of Equation 9 neglected, combination of Equations 9 and 10 provides a relatively straightforward procedure for the calculation of the droplet growth rate and hence the rate of condensation.

APPENDIX 2

Treatment of Two-Dimensional Two-Phase Flows

Governing Equations

Considering the non-steady, two dimensional flow of an inviscid vapor carrying a monodisperse population of spherical droplets, with no interphase slip, the conservation equations of mass, momentum and energy applied to a finite volume element in the x-y rectangular co-ordinate system shown in Figure 7 can be expressed as follows:

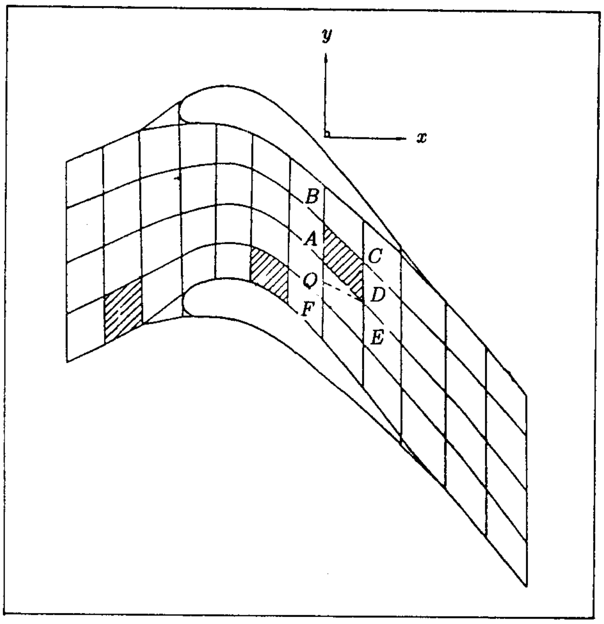

Figure 7. Grid arrangement.

Mass Continuity

$$\Delta t\sum(\rho u d\bar{s}_x + \rho v d\bar{s}_y) = \Delta V\Delta\rho, \qquad (11)$$

where u and v are the velocity components in the x and y directions, ρ is the overall (vapor plus liquid) density of the mixture, ΔV the volume of the element, ds_x and ds_y are the projections of the control volume faces in the x and y directions respectively, Δt is the time increment and the summation is made over the faces of the element.

Momentum in x Direction

$$\Delta t\sum(p + \rho u^2)d\bar{s}_x + \rho uv d\bar{s}_y = \Delta V\Delta(\rho u). \qquad (12)$$

Momentum in y Direction

$$\Delta t\sum\rho uv ds_x + (p + \rho v^2)d\bar{s}_y = \Delta V\Delta(\rho v). \qquad (13)$$

Energy

Since the steady-state solution is of interest, it can be assumed that the flow takes place with constant total enthalpy and the energy equation written as:

$$h_0 = h + \frac{u^2 + v^2}{2}, \qquad (14)$$

where h is the overall (vapor plus liquid) enthalpy of the fluid.

Equation of State for the Vapor Phase

To use mutually consistent thermodynamic properties, it is convenient to adopt a virial equation of state for the vapor phase of the form:

$$p = \rho_G R T_G (1 + B\rho_G), \tag{15}$$

where B is the second virial coefficient.

Two Phase Effects

To deal with this aspect of the problem, the wetness fraction is defined as:

$$w = \frac{4}{3}\pi_r^3 \rho_L N, \tag{16}$$

where r is the mean radius and N the number of droplets per unit mass of the fluid. The number of droplets at the end of a calculating step is the sum of those existing in the fluid at the start of the step and those nucleated over the interval. The mean radius at the end of the step is the root mean square of the radii of the old droplets at the end of the step and that of the newly formed droplets. With the wetness fraction known, the mixture enthalpy and density become:

$$h = (1 - w)h_G + wh_L, \tag{17}$$

$$\rho = \frac{\rho_G}{1 - w}. \tag{18}$$

In combination with the equations describing droplet formation and growth, the above equations are sufficient to describe the flow completely. In the case of steady flows, when the correct values of flow and fluid properties at the calculating points are substituted in the above equations, the change in their values with time becomes zero. But if the estimated values differ from the correct solution, the equations can be used for calculating corrections at each point. Thus, starting with an initial estimate of fluid and flow properties at the calculating points, the above procedure can be repeated until the converged solution is obtained.

Numerical Procedure

With reference to the typical element $ABCD$ in the grid system shown in Figure 7 and considering any of the flow properties ρ, ρu and ρv, its change for the element are obtained from the conservation equations. With the changes in the flow properties of the elements $ABCD$ and $ADEF$ known, its distribution to the calculating points depends on the scheme employed. A common procedure in the time-marching calculations is to assume that changes in flow properties, other than pressure, act downstream. But to improve the stability of the solution, some smoothing is introduced. The value of pressure used for substitution into the momentum equation is calculated from the density at the next downstream point plus a density correction.

To deal with the wetness, the droplet formation and growth rates have to be evaluated for individual fluid packets and hence along the flow streamlines. For this purpose and with reference to Figure 7, the streamline QD through calculating point D is identified. The droplet growth equations are integrated along QD to obtain the wetness terms at D, but the fluid properties thus obtained must be consistent with the remainder of fluid and flow properties calculated from the main conservation equations at D. Thus, when the solutions converge, all the equations are simultaneously satisfied.

APPENDIX 3

General Arrangement and Principle of Operation of Blow-Down Tunnel

The general arrangement of the equipment is shown diagramatically in Figure 8a and a sketch showing the flow passage through the test section as Figure 8b. The receiver is a vessel of 28 m^3 capacity. Valve 1 is a quick-acting valve with a typical opening time of 70 ms and releases the flow to the test section. The test section is essentially a stainless steel fabrication which holds two cover plates 76 mm apart. The blade profiles forming the cascade are mounted

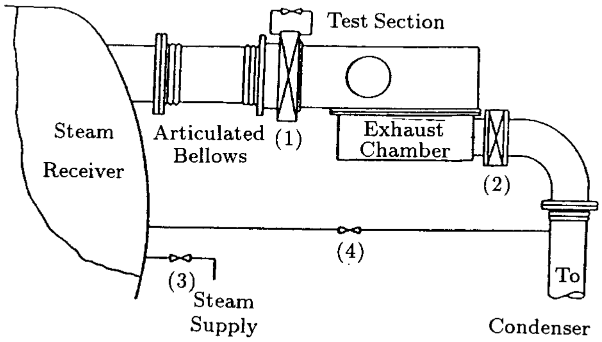

Figure 8a. General arrangement.

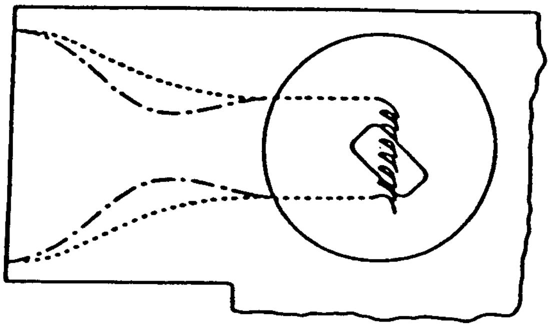

Figure 8b. Flow passage through test **section.**

on circular supporting plates which fit into the test section. The spent steam is discharged via the exhaust chamber to the condenser. The opening of the quick acting valve is followed by transients which decay in 700 ms. Thereafter, a quasi-steady flow is established in the test section which can be studied.

The principle of operation of equipment can be described with reference to the enthalpy-entropy diagram shown in Figure 9. If the receiver is charged with steam at pressure p_1 and then vented to the condenser until the pressure drops to p_2, the steam remaining in the tank will have to expand to expel the fluid which has been lost. In this process the temperature drops faster than the saturation temperature. If the initial and final states, as represented by A and B, are superheated, the fluid will be less superheated at B than at A. Now, to generate supercooled steam if the vessel is charged with saturated steam free from foreign particles as represented by point C, then expansion to p_2 along CD will cause it to supercool without the penalty of giving it kinetic energy. Supercooled steam thus generated is admitted to the test section.

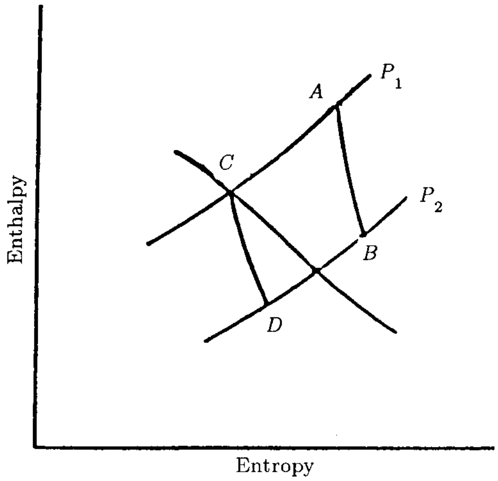

Figure 9. Expansion paths of steam.