Decentralized Adaptive Control of a Nuclear Reactor

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The adaptive decentralized control of large scale systems (LSS) using local state feedback is investigated. The composite system is formed by the interconnection of linear time-invariant multi-input subsystems with unknown parameters. A control approach using only local state feedback is developed for the regulation and tracking problems. Sufficient conditions that guarantee desired stability properties under certain structural perturbations are defined in the form of algebraic criteria. The control scheme is applied to the reactor core and primary heat transport loop of a nuclear reactor. Simulation results are presented.

INTRODUCTION

Many large scale systems (LSS) are characterized by the multiplicity of inputs and states, as well as by the fact that the overall system has several control stations, each being responsible for the operation of a portion of the system. This situation arising in a control system design is refered to as decentralization [1].

Due to the physical configuration and/or high dimensionality of such systems, a centralized control is neither economically feasible nor even necessary. Therefore, in many applications of feedback control theory to LSS's, some degree of restriction is assumed to prevail on the transfer of information. In some cases, a total decentralization is assumed, that is, every local control input is obtained from the local system variables and possibly external inputs. In others, an intermediate restriction on the information is possible such that interconnecting variables among subsystems are available to the local controller.

Thus, the decentralized control approach attempts to avoid difficulties in data gathering, storage requirements, computer program debuggings, and geographical separation of system components. Decentralized state feedback is also an effective way to handle uncertainty in complex systems [2], in which, usually, high feedback gains are set to meet the worst case constraints.

A considerable improvement in the gain allocation as well as in the performance of the closed-loop system can be achieved using adaptive decentralized feedback. The local gains are adaptively adjusted to the levels necessary to neutralize the interconnections and, at the same time, drive the subsystems with unknown parameters to the relaxed operating point, in the case of regulation, or toward the performance of the locally chosen reference models.

In recent years, decentralized control approaches combined with adaptive schemes have appeared in the literature [3-5], especially on the control of robot manipulators [6]. The adaptive decentralized state feedback design used in [3] deals with the case of single input subsystems and the knowledge of the local control gain vectors. On the other hand, the design approach

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proposed in [4] treats the multi-input subsystem case but requires knowledge of the local control gain matrices as well as the subsystem interconnection matrices.

In this paper we use the results of the reference adaptive control (MRAC) of linear systems via state feedback [7] to obtain an adaptive decentralized control for the case of subsystems with multiple inputs and unknown parameters and interconnections. Sufficient conditions for decentralized adaptive regulation in the form of algebraic criteria are established which guarantee the asymptotic stability under certain structural perturbations. In the case of tracking, the decentralized adaptive control scheme guarantees boundedness of all the closed-loop system signals and the convergence of the state error to a residual set.

Finally, this control structure is applied to the model of a liquid metal cooled nuclear reactor [8] (LMR) operating as a baseline power plant. The subsystems considered are the reactor core and the primary heat transport loop. Simulation results are also included.

LARGE SCALE SYSTEM

Consider a multivariable linear time-invariant system which is described as an interconnection of N subsystems and is represented by

$$\dot{x}_i = A_i x_i + B_i u_i + g_i(x) \quad i = 1, \ldots, N, \tag{1}$$

$$g_i(x) = \sum_{j=1}^{N} A_{ij} x_j, \tag{2}$$

which for the i-th subsystem $x_i \in R^{n_i}$ is the state vector, $u_i \in R^{l_i}$ is the control vector, and $g_i(x) \in R^{n_i}$ is the interaction vector from the other subsystems. The parameters A_i, B_i, and A_{ij} are unknown constant matrices and all the pairs (A_i, B_i) are completely controllable. The composite system is described as

$$x = Ax + Bu + Hx, \tag{3}$$

where $x = [x_1^T \ldots x_N^T]^T$ is the overall system state vector, $u = [u_1^T \ldots u_N^T]^T$ is the corresponding control vector, the matrix

$$H = \begin{bmatrix} 0 & A_{12} & \cdots & A_{1N} \\ A_{21} & 0 & & \vdots \\ \vdots & & \cdot & \vdots \\ A_{N1} & \cdots & \cdots & 0 \end{bmatrix} \tag{4}$$

is the interconnection matrix, and $A = diag(A_i)$ and $B = diag(B_i)$.

The decentralized adaptive control problem is to design a set of N local adaptive controllers u_i such that the states of the composite system (Equation 3) are regulated to zero or track the state trajectories of a given reference model formed by the locally defined reference system models. Each subsystem is controlled independently on the basis of its own performance criterion and locally provided information, that is, there is no sharing of information among the local controllers.

The i-th stable local reference model is given by

$$\dot{x}_{m_i} = A_{m_i} x_{m_i} + B_{m_i} r_i \quad i = 1, \ldots, N, \tag{5}$$

where $x_{m_i} \in R^{n_i}$ is the reference state vector, $r_i \in R^{l_i}$ is a piecewise continuous uniformly bounded reference input vector, and A_{m_i} is a stable matrix. It is furthur assumed that an $(l_i \times n_i)$ matrix K_i^* and an $(l_i \times l_i)$ matrix L_i^* exist, such that

$$A_i + B_i K_i^* = A_{m_i}, \tag{6}$$

$$B_i L_i^* = B_{m_i}, \tag{7}$$

which are known as the matching conditions between the plant and the reference model [7]. The composite reference system is described by

$$\dot{x}_m = A_m x_m + B_m r, \tag{8}$$

where

$$x_m = [x_{m_1}^T \dots x_{m_N}^T]^T,$$

$$r = [r_1^T \dots r_N^T]^T,$$

$$A_m = diag(A_{m_i}), \text{ and}$$

$$B_m = diag(B_{m_i}).$$

In the case of regulation, that is, $r = 0$ and $x_m = 0$, the objective is to find local adaptive control inputs u_i to drive the states of the plant to the origin. For the tracking problem, the local controllers are determined such that the error $e = x - x_m$ between the plant and the reference model, as well as all the signals in the closed-loop system remain bounded. Due to the interconnections $g_i(x)$, $i = 1, \dots, N$, it is not possible to ensure $\lim_{n \to \infty} e(t) = 0$ for all bounded reference input vectors r. However, we can achieve convergence of the state error e to some bounded residual set. The solution to the regulation and tracking problems is covered in the next section.

DECENTRALIZED ADAPTIVE CONTROL

The problem of determining the local control inputs u_i, $i = 1, \dots, N$, such that the state of the plant is driven to zero or tracks some desired trajectory, while the same stability of the closed-loop system is maintained, is presented next.

Regulation

In the regulation problem, the reference input r, as well as the state of the reference model x_m, are set to zero, and the local controllers are chosen as

$$u_i(t) = L_i(t) K_i(t) x_i(t), \tag{9}$$

where the feedback gain matrix $K_i(t)$ and the feedforward matrix $L_i(t)$ are adjusted according to the adaptive laws

$$\dot{K}_i = -(B_{m_i}^T P_i x_i) x_i^T \Gamma_i, \tag{10}$$

$$\dot{L}_i = -L_i (B_{m_i}^T P_i x_i)(K_i x_i)^T L_i^T \Lambda_i L_i, \tag{11}$$

where

$$T_i = T_i^T > O,$$

$$\Lambda_i = \Lambda_i^T > O,$$

$$P_i = P_i^T > O,$$

and P_i satisfies the Lyapunov equation

$$A_{m_i}^T P_i A_{m_i} = -Q_i,$$

$$Q_i = Q_i^T > O. \tag{12}$$

The closed-loop subsystems are described by

$$\dot{x}_i = [A_{m_i} + B_i(L_i K_i - L_i^* K_i^*)] x_i + \sum_{j=1}^{N} A_{ij} x_j. \tag{13}$$

The closed-loop decoupled subsystems have the property that if $K_i(t) \equiv K_i^*$ and $L_i(t) \equiv L_i^*$, then the plant together with the controller is identical to the reference model, hence, the closed loop system stability is guaranteed. As shown below, the adaptive laws in Equations 10 and 11 assure boundedness of the parameters and convergence of the state errors to zero for the decoupled subsystems only when the initial parameter values $K_i(t_o)$ and $L_i(t_o)$ lie in the vicinity of the desired values K_i^* and L_i^*.

The presence of interconnections among subsystems can change the stability properties of the decoupled subsystems, and thus, it is necessary to obtain sufficient structural conditions to guarantee the stability of the overall system. These conditions are given by the following theorem.

Theorem 1:

Let

$$q_i = \min \lambda(Q_i), \tag{14}$$

$$a_{ij} = \| P_i A_{ij} \|, \tag{15}$$

where $\lambda(Q_i)$ is the set of eigenvalues of Q_i. These eigenvalues are all positive real from the symmetric positiveness condition on Q_i. If there exist constants $\delta_i > 0$, $i = 1, \ldots, N$, such that the $N \times N$ matrix S with elements

$$S_{ij} = \begin{cases} \delta_i q_i & i = j \\ -(\delta_i a_{ij} + \delta_j a_{ji}) & i \neq j, \end{cases} \tag{16}$$

is positive definite, then the controller parameters $K_i(t)$, $L_i(t)$ are bounded and $\lim \| x(t) \| = 0$ as $t \to \infty$ when the initial conditions of $K_i(t)$ and $L_i(t)$ lie in some bounded neighborhood of their matching condition values K_i^* and L_i^*.

Proof:

Consider the positive definite function

$$V = \sum_{i=1}^{N} \delta_i [V_{i_1} + V_{i_2} + V_{i_3}], \tag{17}$$

where

$$V_{i_1} = x_i^T P_i x_i, \tag{18}$$

$$V_{i_2} = tr[(K_i - K_i^*)\Gamma_i^{-1}(K_i - K_i^*)^T], \tag{19}$$

$$V_{i_3} = tr[\Psi_i \Lambda_i^{-1} \Psi_i^T], \tag{20}$$

with

$$\Psi_i = L_i^{*^{-1}} - L_i^{-1}, \tag{21}$$

$$\dot{\Psi}_i = -(B_{m_i}^T P_i x_i)(K_i x_i)^T L_i^T \Lambda_i. \tag{22}$$

Using the properties of the trace operation

$$\begin{aligned} &tr(A) = tr(A^T), \\ &tr(A + B) = tr(A) + tr(B), \\ &tr(Ayx^T) = x^T Ay , \end{aligned} \tag{23}$$

the matching conditions of Equations 6 and 7, and the Lyapunov Equation 12, the time derivatives of V_{i_1}, V_{i_2}, and V_{i_3} along the solutions of Equations 13, 10, and 22 respectively, are given by

$$\begin{aligned} \dot{V}_{i_1} &= -x_i^T Q_i x_i + 2x_i^T P_i B_i (L_i K_i - L_i^* K_i^*) x_i \\ &\quad + 2x_i^T P_i \sum_{j=1}^{N} A_{ij} x_j, \\ \dot{V}_{i_2} &= -2x_i^T P_i B_i (L_i^* K_i - L_i^* K_i^*) x_i, \\ \dot{V}_{i_3} &= -2x_i^T P_i B_i (L_i - L_i^*)(K_i x_i). \end{aligned} \tag{24}$$

Thus, the time derivative of V can be expressed as

$$\dot{V} = \sum_{i=1}^{N} \delta_i \left[-x_i^T Q_i x_i + 2x_i^T P_i \sum_{j=1}^{N} A_{ij} x_j \right]. \tag{25}$$

For the symmetric positive definite matrix Q_i, we have

$$q_i \| x_i \|^2 \leq x_i^T Q_i x_i \leq \max \lambda(Q_i) \| x_i \|^2, \tag{26}$$

with q_i as defined in Equation 14. Clearly, in the absence of interconnections, the existence of the Lyapunov function V in Equation 17 assures global stability in the $\{x, K, \Psi\}$ space, with $K = diag(K_i), \Psi = diag(\Psi_i)$. However, since our interest is in the parameter errors $\tilde{L}_i \equiv L_i - L_i^*$ and not $\Psi_i = L_i^{*^{-1}} - L_i^{-1}$, then only uniform stability is implied in the $\{x, K, \tilde{L}\}$ space. The function V in Equation 17 is not radially unbounded in this latter space.

Considering now the interconnections among subsystems, let us use Equations 14, 15, and 26 to obtain the following inequality for $\dot{V}$:

$$\dot{V} \leq -\sum_{i=1}^{N} \delta_i \left[q_i \| x_i \|^2 - \| x_i \| \sum_{j=1}^{N} 2a_{ij} \| x_i \| \right] = -\bar{x}^T S_a \bar{x}, \tag{27}$$

where

$$\bar{x} = [\| x_1 \| \ldots \| x_N \|]^T \tag{28}$$

$$S_a = \begin{bmatrix} \delta_1 q_1 & -2\delta_1 a_{12} & \cdots & -2\delta_1 a_{1N} \\ -2\delta_2 a_{21} & \delta_2 q_2 & & \vdots \\ \vdots & & \cdot & \\ -2\delta_N a_{N1} & \cdots & & \delta_N q_N \end{bmatrix} \tag{29}$$

Since

$$\bar{x}^T S_a \bar{x} = \bar{x}^T \frac{(S_a^T + S_a)}{2} \bar{x} = \bar{x}^T S \bar{x},$$

then, Equation 27 can be expressed as

$$\dot{V} \leq -\bar{x}^T S \bar{x}, \tag{30}$$

where

$$S = \begin{cases} \delta_i q_i & i = j \\ -(\delta_i a_{ij} + \delta_j a_{ji}) & i \neq j \end{cases}, \tag{31}$$

If S is positive definite, then $\dot{V}$ is negative semidefinite. Again, the solutions $x_i(t)$, $K_i(t)$, and $L_i(t)$ to Equations 13, 10, and 11 respectively, are bounded when $K_i(t_o)$ and $L_i(t_o)$ are sufficiently close to their desired values K_i^* and L_i^*.

Using Barbalat's lemma [11] we know from Equation 30 and the boundedness of $x(t)$ that $\ddot{V}(t)$ is bounded. Hence $\dot{V}(t)$ is uniformly continuous. Since $V(t)$ is a nonincreasing function of time and is bounded from below, it converges to a finite value V_∞. Thus, $\lim_{t\to\infty} \int_0^t \dot{V} dt = V_\infty - V_o < \infty$ and hence, $\lim \dot{V}(t) = 0$ as $t \to \infty$, that is, $\lim \| x(t) \| = 0$ as time progresses.

Tracking

For the tracking problem, the local control inputs are determined by

$$u_i(t) = L_i(t) K_i(t) x_i(t) + L_i(t) r_i(t), \tag{32}$$

with the adaptive laws for the feedback and feedforward gain matrices K_i and L_i given by

$$\dot{K}_i = -(B_{m_i}^T P_i e_i) x_i^T \Gamma_i - \sigma_i K_i \Gamma_i, \tag{33}$$

$$\dot{L}_i = -L_i (B_{m_i}^T P_i e_i)(K_i x_i + r_i)^T L_i^T \Lambda_i L_i, \tag{34}$$

where

$$e_i = x_i - x_{m_i}, \tag{35}$$

and σ_i is a design positive scalar parameter.

Using the matching conditions in Equations 6 and 7, the state error dynamics are obtained as

$$\dot{e}_i = A_{m_i} e_i + B_i (L_i K_i - L_i^* K_i^*) x_i + B_i (L_i - L_i^*) r_i + \sum_{j=1}^{N} A_{ij} x_j, \tag{36}$$

A persistent input due to the interconnections acts as a disturbance in the overall error equation and therefore the solution $e(t)$ may not converge to, or may not even possess, an equilibrium. The following theorem establishes sufficient conditions for boundedness and convergence of the state error to a residual set.

Theorem 2:

If the matrix S defined by Equation 16 is positive definite and the initial parameter values $K_i(t_o)$ and $L_i(t_o)$ are sufficiently close to their desired values K_i^* and L_i^* respectively, then the solution to Equations 33, 34, and 36 is ultimately bounded. Furthermore, there exist finite non-negative constants T and c_o such that for all $t \geq T$ the solution $e_i(t)$ is inside the set

$$D_o = \left\{e, K_i : \left[\frac{\lambda_s}{2} \parallel e \parallel^2 + \sum_{i=1}^{N} \delta_i \sigma_i \parallel K_i - K_i^* \parallel^2\right] \leq (1+c_o)d_o\right\}, \tag{37}$$

where

$$d_o = \frac{b_o^2}{2\lambda_s} \parallel x_m \parallel^2 + \sum_{i=1}^{N} \delta_i \sigma_i \parallel K_i^* \parallel^2, \tag{38}$$

$\lambda_s = \min \lambda(S)$, and b_o is a positive constant.

Proof:

Consider the positive definite function

$$V = \sum_{i=1}^{N} \delta_i [V_{i_1} + V_{i_2} + V_{i_3}], \tag{39}$$

where

$$V_{i_1} = e_i^T P_i e_i, \tag{40}$$

$$V_{i_2} = tr[(K_i - K_i^*)\Gamma_i^{-1}(K_i - K_i^*)^T], \tag{41}$$

$$V_{i_3} = tr[\Psi_i \Lambda_i^{-1} \Psi_i^T], \tag{42}$$

with matrix Ψ_i and its dynamic equation given by

$$\Psi_i = L_i^{*^{-1}} - L_i^{-1}, \tag{43}$$

$$\dot{\Psi}_i = -(B_{m_i}^T P_i e_i)(K_i x_i + r_i)^T L_i^T \Lambda_i, \tag{44}$$

with P_i satisfying Equation 12.

Using the trace operation properties in Equation 23, the matching conditions in Equations 16 and 17, and the Lyapunov Equation 12, the time derivatives of V_{i_1}, V_{i_2}, and V_{i_3}, along the solutions of Equations 36, 33, and 44 respectively, are given by

$$\dot{V}_{i_1} = -e_i^T Q_i e_i + 2e_i^T P_i B_i (L_i K_i - L_i^* K_i^*) x_i + 2e_i^T P_i B_i (L_i - L_i^*) r_i + 2e_i^T P_i \sum_{j=1}^{N} A_{ij} x_j,$$

$$\dot{V}_{i_2} = -2e_i^T P_i B_i (L_i^* K_i - L_i^* K_i^*) x_i - 2tr[\sigma_i (K_i - K_i^*) K_i^T],$$

$$\dot{V}_{i_3} = -2e_i^T P_i B_i (L_i - L_i^*)(K_i x_i + r_i).$$

The time derivative of V can then be expressed as

$$\dot{V} = \sum_{i=1}^{N} \delta_i \left[-e_i^T Q_i e_i + 2e_i^T P_i \sum_{j=1}^{N} A_{ij}(e_j + x_{m_j}) - 2tr[\sigma_i (K_i - K_i^*) K_i^T]\right]. \tag{45}$$

Let us now define the following vectors

$$\bar{e} = [\parallel e_1 \parallel \dots \parallel e_N \parallel]^T, \tag{46}$$

$$\bar{x}_m = [\parallel x_{m_1} \parallel \dots \parallel x_{m_N} \parallel]^T, \tag{47}$$

now, proceeding as in the regulation problem to obtain the structural condition on the composite system, and using the inequality

$$-2tr[(K_i - K_i^*)K_i^T] \leq - \parallel K_i - K_i^* \parallel^2 + \parallel K_i^* \parallel^2, \tag{48}$$

the following inequality is obtained for $\dot{V}$

$$\dot{V} \leq -\bar{e}^T S\bar{e} + b_o \|e\|\|x_m\| - \sum_{i=1}^{N} \delta_i \sigma_i \|K_i - K_i^*\|^2 + \sum_{i=1}^{N} \delta_i \sigma_i \| K_i^* \|^2, \tag{49}$$

where b_o is a finite constant which depends on the norms of P_i, A_{ij}, and δ_i, i=1, ..., N.

Noting that

$$\begin{aligned} &-\bar{e}^T S\bar{e} + b_o \| e \|\| x_m \| \\ &\leq -\lambda_s \| e \|^2 + b_o \| e \| \| x_m \| \\ &+ \left[\sqrt{\frac{\lambda_s}{2}} \| e \| - \frac{b_o \| x_m \|}{\sqrt{2\lambda_s}} \right]^2 \\ &= -\frac{\lambda_s}{2} \| e \|^2 + \frac{b_o^2}{2\lambda_s} \| x_m \|^2, \end{aligned} \tag{50}$$

where $\lambda_s = \min \lambda(S)$, we can now express Equation 49 as

$$\dot{V} \leq -\frac{\lambda_s}{2} \| e \|^2 - \sum_{i=1}^{N} \delta_i \sigma_i \| K_i - K_i^* \|^2 + \frac{b_o^2}{2\lambda_s} \| x_m \|^2 + \sum_{i=1}^{N} \delta_i \sigma_i \| K_i^* \|^2 . \tag{51}$$

In view of Equations 39 and 51, the solutions $e(t)$, $K_i(t)$, $L_i(t)$ are uniformly ultimately bounded [9] for all initial conditions $K_i(t_o)$, $L_i(t_o)$ lying in some bounded neighborhood about their matching condition values K_i^*, L_i^*. Further, since $\dot{V} < \varepsilon(c_o) < 0$ for e, K_i outside D_o with $c_o \geq 0$, then for some finite constants T and c_o, the solution $e(t)$, $K_i(t)$ remains inside D_o for all $t > T$.

The use of σ_i is found to be useful in obtaining sufficient conditions for boundedness in the presence of unmodeled interactions. In the absence of such interactions, that is, when each subsystem is decoupled, the design parameters $\sigma_i > 0$ cause nonzero state errors. This is a trade-off between boundedness of all signals in the presence of subsystem's interactions and the loss of exact convergence of the errors to the origin in their absence.

DECENTRALIZED ADAPTIVE CONTROL OF A NUCLEAR REACTOR

The control design approach developed in the previous section is now applied to the system formed by the reactor core and primary heat transport loop models of one module of a multimodular nuclear power plant [8] with liquid metal as the primary coolant (LMR).

LMR Reactor System

The LMR reactor is considered a large scale system formed by the interconnection of two subsystems, one being the reactor core, and the other, the primary heat transport loop. The system module is shown in Figure 1. Each module also comprises a recirculating steam generator and a steam drum. All modules are connected to a common steam heater that feeds

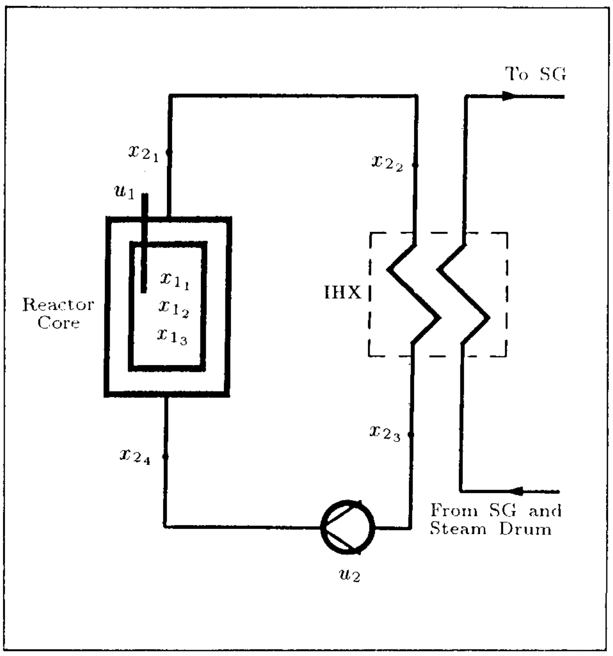

Figure 1. Liquid metal cooled reactor.

the turbine.

During long periods of time, a nuclear power plant is devised as a baseline system providing constant electrical power to the grid. Thus, the reactor is maintained at near full neutron power level which can be achieved with the control rods half-way inserted.

Under these conditions, the dynamic behavior of the plant can be considered linear around the equilibrium and the plant management goal is to regulate the system at the specified operating point.

In our case, the two interconnected subsystems [10] are described by the following equations:

$$\begin{aligned}\dot{x}_1 &= A_1 x_1 + B_1 u_1 + A_{12} x_2,\\ \dot{x}_2 &= A_2 x_2 + B_2 u_2 + A_{21} x_1,\end{aligned} \tag{52}$$

where the nominal parameter values are given by

$$A_1 = \begin{bmatrix} -952.38 & 952.38 & -1008.3 \\ 0.01 & -0.01 & 0 \\ 0.14371 & 0 & -0.49477 \end{bmatrix}, \tag{53}$$

$$B_1 = \begin{bmatrix} 6666.67 \\ 0 \\ 0 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 470.54 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.49477 & 0 & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.66661 & 0 & 0 & 0.27889 \\ 0.022401 & -0.022401 & 0 & 0 \\ 0 & 0.069336 & -0.17961 & 0 \\ 0 & 0 & 0.022401 & -0.022401 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} -0.12512 \\ 0 \\ 0.031108 \\ 0 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 0 & 0 & 0.38772 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The state variable and control inputs give the deviations from their equilibrium values and represent the following physical variables:

x_{1_1}: Normalized neutron power.

x_{1_2}: Normalized neutron precursor concentration.

x_{1_3}: Fuel to reference temperature ratio.

x_{2_1}: Core coolant outlet to reference temperature ratio.

x_{2_2}: IHX primary inlet to reference temperature ratio.

x_{2_3}: IHX primary outlet to reference temperature ratio.

x_{2_4}: Core inlet to reference temperature ratio.

u_1: Normalized control rod position.

u_2: Primary pump fractional flow.

Decentralized Adaptive Controller

The stable linear decoupled reference subsystems are described by

$$\begin{aligned}\dot{x}_{m_1} &= A_{m_1} x_{m_1} + B_{m_1} r_1,\\ \dot{x}_{m_2} &= A_{m_2} x_{m_2} + B_{m_2} r_2,\end{aligned} \tag{54}$$

where

$$A_{m_1} = \begin{bmatrix} -6734.4 & -2579.4 & -1086.4 \\ 0.01 & -0.01 & 0 \\ 0.14371 & 0 & -0.49477 \end{bmatrix}, \tag{55}$$

$$A_{m_2} = \begin{bmatrix} -1.0868 & -0.16777 & 0.32498 & 0.40762 \\ 0.022401 & -0.22401 & 0 & 0 \\ 0.10446 & 0.11105 & -0.26041 & -0.032005 \\ 0 & 0 & 0.22401 & -0.22401 \end{bmatrix},$$

$$B_{m_1} = \begin{bmatrix} 7000 \\ 0 \\ 0, \end{bmatrix}$$

$$B_{m_2} = \begin{bmatrix} -0.13138 \\ 0 \\ 0.032664 \\ 0, \end{bmatrix}$$

The pairs (A_i, B_i), (A_{m_i}, B_{m_i}), $i = 1, 2$, are completely controllable and the subsystems satisfy the matching conditions given in Equations 6 and 7. Furthermore, the structural conditions on the subsystem's interconnections, established in Theorem 1, are met, thus assuring that all the closed-loop system signals are bounded and that $\lim \| x(t) \| = 0$ as $t \to \infty$.

The two local controllers u_i, are given by

$$u_i(t) = L_i(t) K_i(t) x_i(t), \qquad (56)$$

with $K_i(t)$ and $L_i(t)$ being adjusted according to the adaptive laws in Equations 10 and 11, respectively.

Identity matrices of proper dimensions for Γ_i and Λ_i were used in the adaptive law equations of the state feedback and control feedforward gain matrices for each subsystem.

The symmetric positive definite matrices P_i, $i = 1, 2$, which are also used in the adaptive laws, Equations 10 and 11, were obtained after solving the corresponding Lyapunov Equation 12. Likewise, the two positive definite matrices Q_i, required by these Lyapunov equations, were set equal to the identity matrices.

The simulation of the closed-loop system was performed assuming a perturbation on the neutron power of 10% above the specified operating value, which results in a deviation of +0.1 for all the plant state variables.

The initial values of the feedback adaptive gain matrices $K_i(t)$, $i = 1, 2$, were set to zero, and those of the adaptive input matrices $L_i(t)$ were set to 80% of their matching condition values.

The results of the simulation are shown in Figures 2 to 6. The neutron power and the core coolant outlet temperature are brought back to their nominal values in about 500 seconds. The adaptive feedback and input gain matrices approach steady state values asymptotically. Likewise, the control inputs converge to the origin as expected.

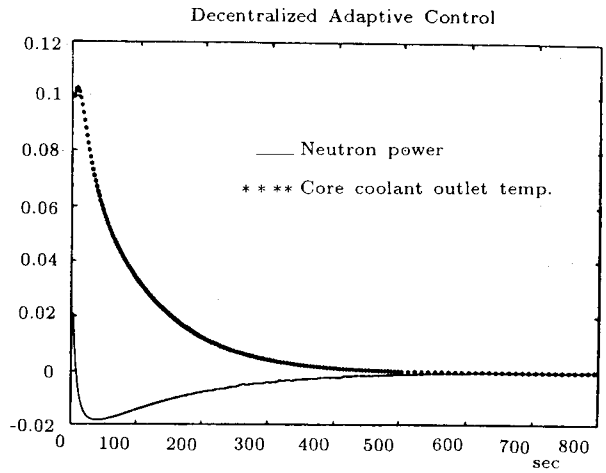

Figure 2. Neutron power and core coolant outlet temperature.

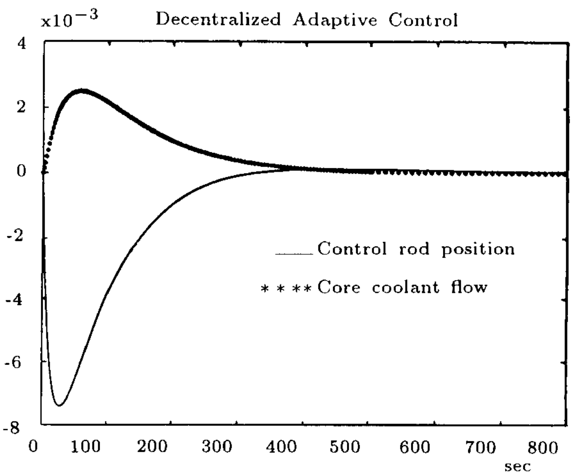

Figure 3. Control rod and core coolant flow inputs.

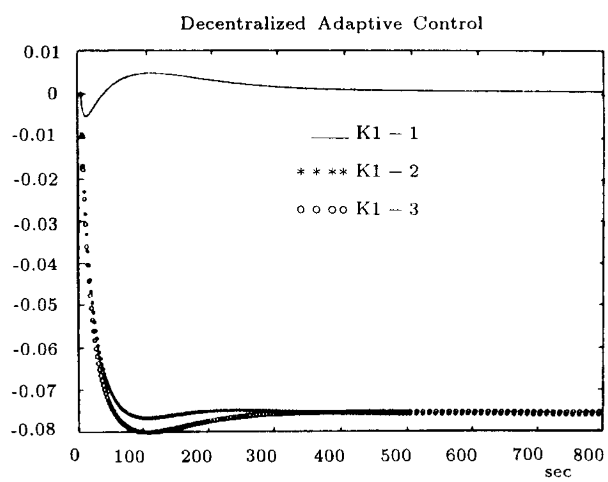

Figure 4. Adaptive feedback gains for subsystem 1.

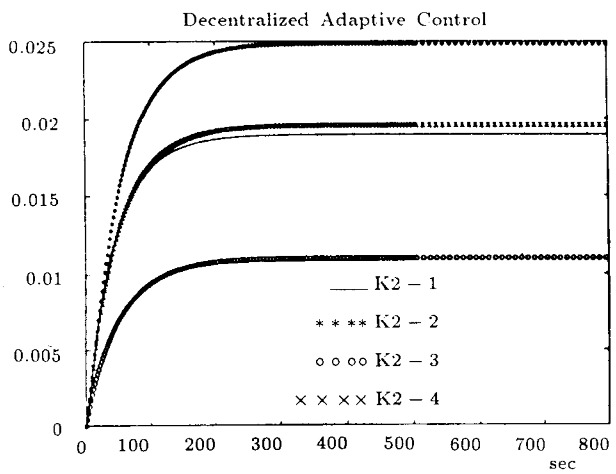

Figure 5. Adaptive feedback gains for subsystem 2.

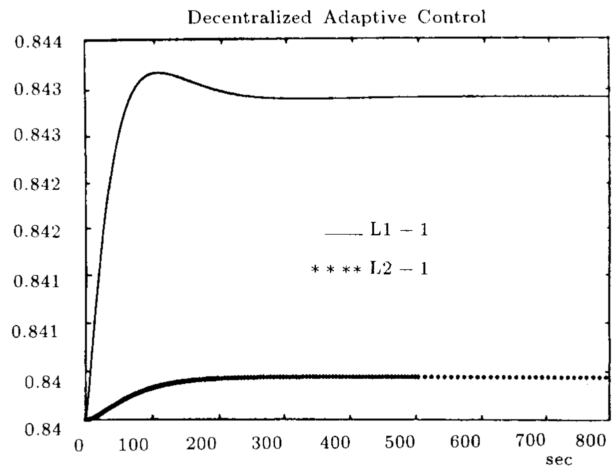

Figure 6. Adaptive gains L1-1 and L2-1.

CONCLUSIONS

A decentralized adaptive control scheme is presented for a class of large scale systems. The system is composed of interconnected linear subsystems with multiple inputs, in which their state and control input matrices as well as the strength of the interconnections are unknown.

The control law for each subsystem, based on local adaptive state feedback, and the adaptive laws of the feedback and feedforward gain matrices, are developed for the stabilization and tracking problems. Sufficient conditions in the form of algebraic constraints are obtained which guarantee asymptotic regulation of the plant states. Moreover, in the case of reference trajectory tracking, the proposed control structure achieves boundedness of all closed loop system signals, as well as convergence of the plant states to a residual set, under certain structural perturbations.

The decentralized adaptive control scheme reduced the number of parameters to be dynamically adjusted through the adaptive laws, compared to the centralized case. In the latter, this number is given by the product of the total number of the system inputs and the sum of the number of the system's states and inputs. For the decentralized case, this figure is computed as the sum over the number of subsystems of the above product applied separately to each subsystem.

Furthermore, the gains of the adaptive controller parameters are generally of lower magnitude than those required in the nonadaptive decentralized control design. The gains are adjusted to the levels necessary to bring the state errors to the origin.

A decentralized adaptive control design is applied to the reactor core and primary heat transport loop subsystems of a liquid metal cooled nuclear reactor. The results of computer simulations for the regulation case are presented. The time responses of the closed loop signals show the stability characteristics of the control scheme and the asymptotic convergence of the state variables to the origin.

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