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Micro Resonator Nonlinear Dynamics Considering Intrinsic Properties

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Abstract. *One of the most important phenomena to affect the motion behaviour of Micro Resonators is their thermal dependency. This has recently received the attention of researchers widely. A thermal phenomenon has two main effects, the first is damping, due to internal friction, and the second is softening, due to Young's modulus-temperature relationship. In this research work, some theoretical and experimental reported results are used to make a proper model, including thermal phenomena. Two Lorentzian functions are used to describe the restoring and damping forces caused by thermal phenomena. In order to emphasize the thermal effects, a nonlinear model of the MEMS, considering capacitor nonlinearity and mid-plane stretching, has been used. The responses of the system are developed by employing a multiple time scale perturbation method on a non-dimensionalized form of the equations. Frequency response, resonance frequency and peak amplitude are examined by varying the dynamic parameters of the modelled system. Finally, Fuzzy Generalized Cell Mapping (FGCM) is introduced and applied to the Micro Resonator's dynamical system behaviour. It is then concluded as to how the model uncertainties and different initial conditions can affect the working domain of the system and/or make it pull in instabilities. At the end, it can be seen that FGCM is a useful method for monitoring the working regions of Micro Resonators, while varying system parameters.*

Keywords: *Micro resonator; Thermal effects; Nonlinear dynamics; Fuzzy generalized cell mapping.*

INTRODUCTION

A Micro Electro Mechanical System (MEMS) is the integration of mechanical elements, sensors, actuators and electronics on a common silicon substrate through micro fabrication technology. While the electronics are fabricated using Integrated Circuit (IC) process sequences, the micromechanical components are fabricated using compatible "micromachining" processes that selectively etch away parts of the silicon wafer or add new structural layers to form the mechanical and electromechanical devices. MEMS is an enabling technology allowing the development of smart products, augmenting the computational ability of microelectronics, with the perception and control capabilities of microsensors and microactuators, and expanding the space of possible designs and applications.

In recent years, MEMS [1] has created a startling revolution in today's technology. It is now possible to produce micro accelerometers [2], micro gyroscopes [3], RF-MEMS filters [4] and resonant sensors [5], which are dimensionally less than a millimeter on each side. They also have high sensitivity and resolution characteristics and low power consumption capabilities in providing digital output data characteristics. However, the manufacturing of mechanical parts in micro scales is difficult; the understanding, identification and control of these physical systems are essential [1,6]. Such effects as fluidity [7] and electromagnetic thermal [8,9] and mechanical systems are also an essential problem. Some of these physical systems are directed to energy dissipation in the system. There are many dissipation mechanisms that contribute to lowering the quality factor. Difficulties have arisen in the process of creating high quality factor (Q) MEMS or NEMS, where the Qs are found to be less than expected in MEMS scales and which need more investigation on fundamental loss expected mechanisms [3]. There are many research explanations of the observed behavior, including support losses [10], bulk defects, losses associated with electrical contacts, surface effects, squeeze film effects [7] and thermal effects. Temperature dependent properties of the microbeam materials play a significant role in

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the design and application of microsystems, utilizing a microbeam or a micro cantilever resonator [8,9,11]. Most of the micro resonators must work at resonance frequencies, like sensors or actuators. A typical micro resonator is built by attaching a microplate to the tip of a long micro cantilever, or to the middle of a microbeam. The microplate is in use as the moving electrode of a variable capacitor, whose other electrode is fixed to the frame of the MEMS device. The approach of this paper is to investigate the mathematical modeling of thermal effects and nonlinearities caused by capacitor and midplane stretching in the dynamic behavior and sensitivity analysis of micro resonators. The effects of thermal phenomena are modeled as an increase in damping [12] and a decrease in stiffness rates [13], both as Lorentzian functions of excitation frequency [14]. The steady state response frequency-amplitude dependency of such system will be derived using the multiple time scale perturbation method by considering the nonlinearity of the actuated force [15,16] and midplane stretching [5,17,18]. The developed analytical equations define the frequency response of the system close to resonance frequency, which can be utilized to explain the dynamics of the system, as well as resonant frequency and peak amplitude. Finally, Fuzzy Generalized Cell Mapping [16,19,20] will be used, and it will be shown how an alteration in the damping factor and initial condition can influence the pull-in instabilities.

THERMAL EFFECTS

The thermoelastic behavior of a micromechanism, such as a microbeam, will be identified in two different parts; the first one being thermal damping, which is the energy dissipation mechanism [12,15], and the second one being thermal relaxation, which affects the rigidity of the material [13,15]. Thermoelastic damping is proportional to exiting frequencies; hence, when the principal natural frequency increases while the size of the devices decreases, the thermoelastic damping effects become more significant. Thermal energy dissipation is caused by irreversible heat flow across the thickness of the micro cantilever as it oscillates. For simulating the damping force corresponding to thermal damping, Jazar et al. [12] introduce a frequency dependent force:

$$f_{Td} = C_T L\left(\frac{\omega^2}{\omega_i^2}\right)\dot{z}, \qquad L(x) = \frac{x}{1+x^2}, \tag{1}$$

where c_T defines the thermal damping per unit length of the microbeam, which depends on the geometries and material properties of the microbeam and must be determined experimentally, ω_i is the natural frequency, ω is frequency of excitation, $\dot{z}$ is the velocity in the system of one degree of freedom and L is the Lorentzian function, as defined. Since the warming up of the microbeam material is Lorentzian frequency dependent, the effect of stiffness softening of the microbeam is also a frequency dependent characteristic. So, in this work, a negative softening function to define this behavior is proposed. More specifically, a negative restoring force with stiffness, as a Lorentzian function of excitation frequency [13], determines the drop in the linear rigidity stiffness force, $EI(\partial^4 w/\partial x^4)$.

$$f_{Ts} = -k_T\frac{\omega/\omega_i}{1+(\omega/\omega_i)^2}w = -k_T L\left(\frac{\omega^2}{\omega_i^2}\right)w. \tag{2}$$

The breaking frequency of thermal stiffness softening is also at the fundamental resonance frequency. The softening stiffness coefficient per unit length, k_T, which depends on the geometrical parameters and material properties of the microbeam, must be determined experimentally.

MICRO RESONATOR REDUCED ORDER MODEL CONSIDERING INTRINSIC EFFECTS

The micro resonator, cantilever or clamped-clamped, as depicted in Figure 1, is composed of a beam resonator, a plate underneath in contact with the beam and one (or more) capacitive transducer electrode(s). The unidirectional electrostatic force, f_e, between two electrodes is:

$$f_e = \frac{\varepsilon_0 A(v-v_p)^2}{2(d-w_p)^2}, \tag{3}$$

where $\varepsilon_0 = 8.85\times 10^{-12} As/Vm$ is the permittivity in the vacuum, A is the area of the microplate and w_p is the moving plate variable, which is a function

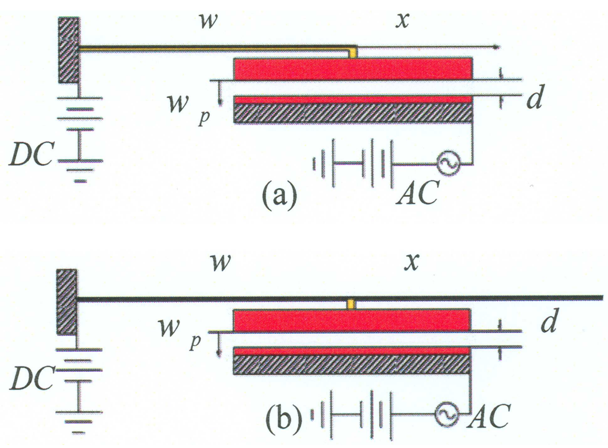


Figure 1. A micro cantilever and a clamped-clamped microbeam model of a MEMS and its voltage connections.

of electrode length and time, identifying the lateral motion of the capacitor free plate attached to the microbeam. But, w is the beam deflection, which is also a function as: $w = w(x,t)$. It is said that, at the connecting point between the microbeam and electrode capacitor: $w = w_p$. The exiting voltage sources are composed of a DC polarization voltage, v_p, and an AC actuating voltage, $v = v_i \sin(\omega t)$ [16]. The governing equation describing lateral vibrations of the microbeam, using the Euler-Bernoulli approach in elastic beams, can be summarized and simplified to the following equation, assuming uniform geometry for the beam.

$$\rho\frac{\partial^2 w}{\partial t^2} + c\frac{\partial w}{\partial t} + EI\frac{\partial^4 w}{\partial x^4} + c_T\frac{\omega/\omega_1}{1+(\omega/\omega_1)^2}\frac{\partial w}{\partial t} - k_T\frac{\omega/\omega_1}{1+(\omega/\omega_1)^2}w = \frac{\varepsilon_0 A(v-v_p)^2}{2(d-w_p)^2} + \left(\hat{N} + \frac{EA_0}{2L}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx\right)\frac{\partial^2 w}{\partial x^2}, \tag{4}$$

where the first term on the left hand side is the inertia force per unit length (ρ is the mass per unit length), the second term on the left side is the damping term, the third term on the left side is the restoring force, due to the elasticity of the microbeam (E is Young's modulus, if the beam isn't narrow, Young's modulus (E) must be replaced by $E/(1-v^2)$, where v is the Poisson ratio), and I is the moment of inertia of the beam cross section. The fourth and fifth terms on the same side are concluded from Equations 1 and 2, the first term on the right hand side is a nonlinearly, w_p, dependent electro capacitor force, due to Equation 3, and the final term on the right side of the above equation is the restoring force, due to the internal tensional force of the microbeam (A_0 is the area of the cross section and L is length of the beam), which generates additional nonlinearity, called intrinsic nonlinearity, in the system. For changing this highly nonlinear equation to a dimensionless form, in order to make it simply solvable, first, the electrostatic force is diminished and, then, this diminished electrostatic force is replaced by the aid of the other dimensionless factors of the resulting equation, as introduced in [12-15]. To make the equation of motion dimensionless, the following variables are identified.

$$\tau = \omega_1 t, \quad \omega_1 = \frac{n}{L^2}\sqrt{\frac{EI}{\rho}}, \quad z = \frac{x}{L},$$

$$y = \frac{w}{d}, \quad Y = \frac{w_p}{d}, \quad r = \frac{\omega}{\omega_1},$$

$$a_1 = \frac{\varepsilon_0 A L^4}{2n^2 d^3 EI}, \quad a_2 = \frac{cL^2}{n\sqrt{\rho EI}}, \quad a_6 = \frac{c_T L^2}{n\sqrt{\rho EI}},$$

$$a_7 = \frac{k_T L^4}{n^2 EI}, \quad \alpha_1 = \frac{A_0 d^2}{4I}, \quad N = \frac{\hat{N}L^2}{nEI}. \tag{5}$$

Parameter n is a constant, depending on the mode shape of the microbeam. Using these variables, the equation of motion changes to the following dimensionless equation (note that y represents a dimensionless variable, regarding microbeam motion, while Y represents a dimensionless variable, regarding the electrode capacitor):

$$\frac{\partial^2 y}{\partial \tau^2} + a_2\frac{\partial y}{\partial \tau} + \frac{\partial^4 y}{\partial z^4} + a_6\frac{r}{1+r^2}\frac{\partial y}{\partial \tau} - a_7\frac{r}{1+r^2}y = a_1\frac{(v-v_p)^2}{(1-Y)^2} + \left(N + \alpha_1\int_0^1 \left(\frac{\partial y}{\partial z}\right)^2 dx\right)\frac{\partial^2 y}{\partial z^2}. \tag{6}$$

If Equation 6 is considered for the first mode shape, which means the mode number must be $n = 1$, and α_1 must be $\alpha_1 = A_0 d^2/2I$, then, the axial load is considered as a function of the initial axial condition and the axial excitation, as follows:

$$\hat{N} = \frac{EA_0}{L}\delta = \frac{EA_0}{L}(x_0 + x_d\cos(\omega_x t)), \tag{7}$$

where δ, x_0 and x_d are axial deflection, initial stretch and axial excitation amplitude, respectively, and ω_x is the axial excitation frequency. Equation 6 is non-dimensionalized and the following parameters are found:

$$a_8 = \frac{A_0 L x_0}{nI}, \qquad a_9 = \frac{A_0 L x_d}{nI}, \qquad r_x = \frac{\omega_x}{\omega_1}. \tag{8}$$

Separation of the variables solution, $y = Y(\tau).\varphi(z)$, is applied here, where the spatial function, $\varphi(z)$, is called the mode shape function, and must satisfy the boundary conditions. $Y(\tau)$ is the time function and by choosing the first harmonic shape function, this function, $Y(\tau)$, would then represent the maximum deflection of the microbeam, which is the middle deflection for a clamped-clamped end beam and/or a simply supported beam, and also represents the free end deflection for a cantilever beam. A micro cantilever is a microbeam with the following boundary conditions:

$$y(0,\tau) = 0, \qquad \frac{\partial}{\partial z}y(0,\tau) = 0,$$

$$\frac{\partial^2}{\partial z^2}y(1,\tau) = 0, \qquad \frac{\partial^3}{\partial z^3}y(1,\tau) = 0. \tag{9}$$

The first harmonic mode shape satisfying the required boundary conditions is $\varphi(x) = \cos(\pi x/2)$ and the mode shape parameter is $n = \pi^2/4$. At the same time, if a sinusoidal function is used, it can be seen that for a clamped-clamped and a simply supported beam, the mode shape constant (n) equals $4\pi^2$ and π^2, respectively [15]. Then, the required differential equation for the time separated part, $Y(\tau)$, related to a micro cantilever, would be:

$$\ddot{Y} + \left(h + a_6\frac{r}{1+r^2}\right)\dot{Y} + \left(1 - a_7\frac{r}{1+r^2}\right)Y$$
$$= \frac{1}{(1-Y)^2}[(\alpha+\beta) + 2\sqrt{2\alpha\beta}\sin(r\tau) - \beta\cos(r\tau)]$$
$$- a_8 Y - a_9\cos(r_x\tau) - \alpha_1 Y^3, \quad (10)$$

where, $h = a_2$, $\alpha = a_1 v_p^2$, $\sqrt{2\alpha\beta} = a_1 v_p v_i$, $\beta = (a_1/2)\times v_i^2$. The third order expansion (Taylor series) will be used to model the electrostatic force on the micro resonator. Applying the multiple time scales method [21] makes $Y = a(\tau)\cos(r\tau + \gamma(\tau))$, and the following coupled equations yield the following (considering $r_x \approx 2$):

$$a' = \frac{1}{4(1+r^2)} \times (-2(1+r^2)\sqrt{2\beta\alpha}\cos(\sigma\tau - \gamma)$$
$$+ a(-2ra_6 - (1 + r^2(a_9\sin(2\sigma^*\tau - 2\gamma)$$
$$+ 2(h + 3a\sqrt{2\beta\alpha}\cos(\sigma\tau - \gamma)$$
$$+ \beta(1+4a^2)\sin(2\sigma\tau - 2\gamma))))$$
$$+ (a_9\sin(2\sigma^*\tau - 2\gamma)$$
$$+ 2(h + 3a\sqrt{2\beta\alpha}\cos(\sigma\tau - \gamma)$$
$$+ \beta(1+4a^2)\sin(2\sigma\tau - 2\gamma)))), \quad (11a)$$

$$\gamma' = \frac{1}{4(1+r^2)}(-2\sqrt{2\beta\alpha}\sin(\sigma\tau - \gamma)$$
$$+ a(-ra_7 + (1+r^2) + (-18a\sqrt{2\beta\alpha}\sin(\sigma\tau - \gamma)$$
$$+ a_9\cos(2\dot{\sigma}\tau - 2\gamma) + 2(-2(\beta+\alpha)$$
$$+ \beta\cos(2\sigma\tau - 2\gamma)a_8 + (-12(\beta+\alpha)$$
$$+ 8\beta\cos(2\tau\sigma - \gamma) + 3\alpha_1)a^2)))) + 2(-2(\beta+\alpha)$$
$$+ \beta\cos(2\sigma\tau - 2\gamma)a_8 + (-12(\beta+\alpha)$$
$$+ 8\beta\cos(2\tau\sigma - \gamma) + 3\alpha_1)a^2)))), \quad (11b)$$

where $\sigma = r - 1$, $\sigma^* = \frac{(r_x - 2)}{2}$. Assuming a' and $\gamma' - \sigma$ remain zero in the steady state response and $\sigma^* = \sigma$ (the $\gamma - \sigma\tau$ is an argument of the sinusoidal term and must be invariant in time, when $\tau \to \infty$). Eliminating $\gamma(\tau)$ and assuming $0 \leq a \leq 1$ provides a relationship between the parameters of the system to have a periodic steady state response with frequency r, which gives an equation that shows a as a function of β, α, r, h, a_6, a_7, a_8, a_9, α_1. From nonlinear modeling, the amplitude coupled equation is:

$$\frac{1}{4(1+r^2)}(-2(1+r^2)\sqrt{2\beta\alpha}\cos(\gamma)$$
$$+ a(-2ra_6 - (1+r^2)(-a_9\sin(2\gamma)$$
$$+ 2(h + 3a\sqrt{2\beta\alpha}\cos(\gamma)$$
$$- \beta(1+4a^2)\sin(2\gamma))))) = 0, \quad (12a)$$

$$\frac{1}{4(1+r^2)a}(2\sqrt{2\beta\alpha}\sin(\gamma)$$
$$+ a(-ra_7 + (1+r^2)(18a\sqrt{2\beta\alpha}\sin(\gamma)$$
$$+ a_9\cos(2\gamma) + 2(-2(\beta+\alpha) + \beta\cos(2\gamma)a_8$$
$$+ (-12(\beta+\alpha) + 8\beta\cos(2\gamma)$$
$$+ 3\alpha_1)a^2)))) - r + 1 = 0, \quad (12b)$$

which is coupled to the phase angle. The decoupled equation of amplitude is addressed in the Appendix (if the system has no excitation in an axial direction, a_9 can be replaced by zero).

RESULTS OF PARAMETRIC MODEL

Equations 12a and 12b describe the frequency behavior of the micro resonator, indicating that its dynamics are governed by polarization voltage parameter α, alternative excitation voltage parameters β, damping parameter h and the excitation frequency ratio r, as well as the thermal damping and stiffness parameters, a_6 and a_7. The nominal values of a sample micro cantilever, in order to analyze the dynamic behavior of the MEMS, are [12-15]:

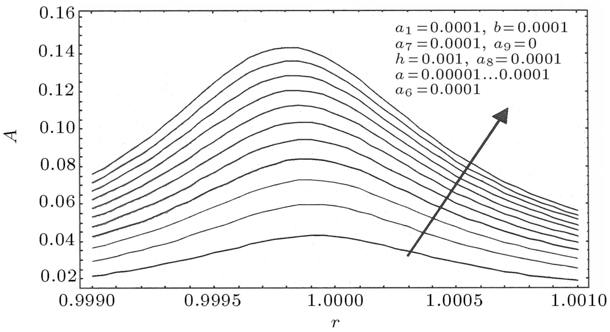
$$m = 1\times 10^{-11}\ \text{kg}, \qquad \alpha = 0.0000553125\ vp^2,$$

$$A = 200\ \mu\text{m} \times 50\ \mu\text{m}, \qquad \beta = 0.00002765625\ v_i^2,$$

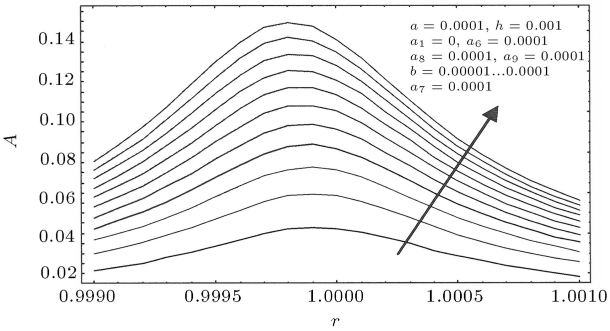
$$k = 1\ \text{N/m}, \qquad d = 2.0\ \mu\text{m},$$

$$c = 1\times 10^{-8}\ \text{Ns/m}.$$

Figures 2a and 2b depict the effect of a variation of the DC and AC voltage for a set of parameters.



(a)



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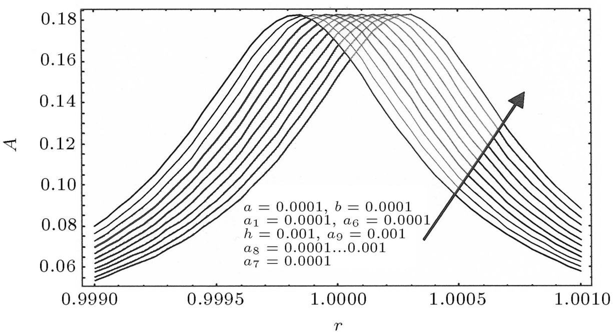
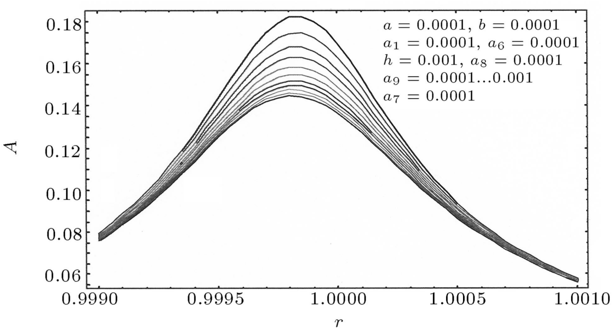


(c)



(d)

Figure 2. Effect of variation of (a) DC voltage, (b) AC voltage, (c) axial load, and (d) axial vibration on frequency response. (Arrows show increasing of the parameter.)

The amplitude of the steady state oscillation increases monotonically by increasing the voltage. Figure 2c shows that the axial load (initial tension) has no effect on the amplitude, but its effects on the resonance frequency are great and the resonance frequency increases smoothly by increasing this load. It can be seen that axial vibration (by considering frequency) increases the amplitude in Figure 2d. In Figure 2, the horizontal axis is frequency, and the vertical axis is amplitude. Increasing the damping ratio diminishes the amplitude of the oscillation, as can be expected. Figure 2 shows that the peak amplitude is not an increasing function of the DC voltage and it increases by increasing α to somewhere, then, it reduces. (Because of that, the intersection can be seen in Figure 3.) Also, A_p (peak value) is a nonlinear increasing function of both polarization and excitation voltages.

Figures 4a and 4b illustrate the thermal effects. Thermal damping diminishes the peak amplitude (Figure 4a) and its effect on the resonance frequency is not so great, but thermal relaxation decreases the resonance frequency and does not affect the system amplitude. In Figure 4c, it is shown that the nonlinearity parameter (α_1) clearly affects the resonance frequency and its increase, but its effect on resonance amplitude is little. It is evident that damping reduces amplitude (Figure 4d), but its effect on resonance frequency is dependent upon other parameters, such as the nonlinearity parameter. For example, if the nonlinearity parameters vanish, then, the damping effect on resonance frequency is dispensable, but if nonlinearity increases, all the parameters that affect amplitude can have an effect on the resonance frequency.

As illustrated in Figures 5a and 5b, the resonance frequency is a monotonically decreasing function of increasing both polarization and excitation voltages. The behavior of resonance shifting looks linear, with a variation of both voltages. Figure 5c shows the

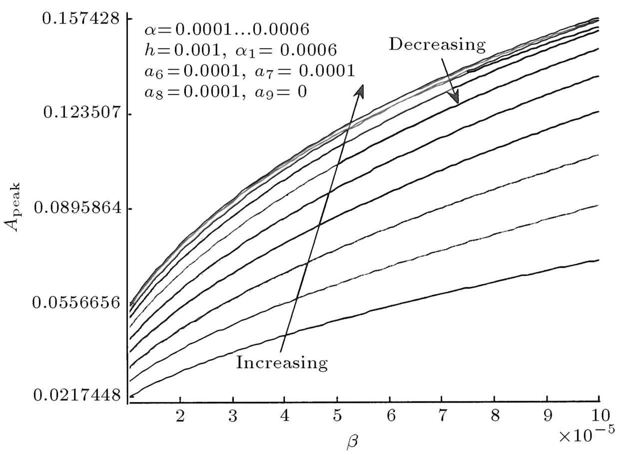


Figure 3. Effect of variation of Ac voltage on peak amplitude.

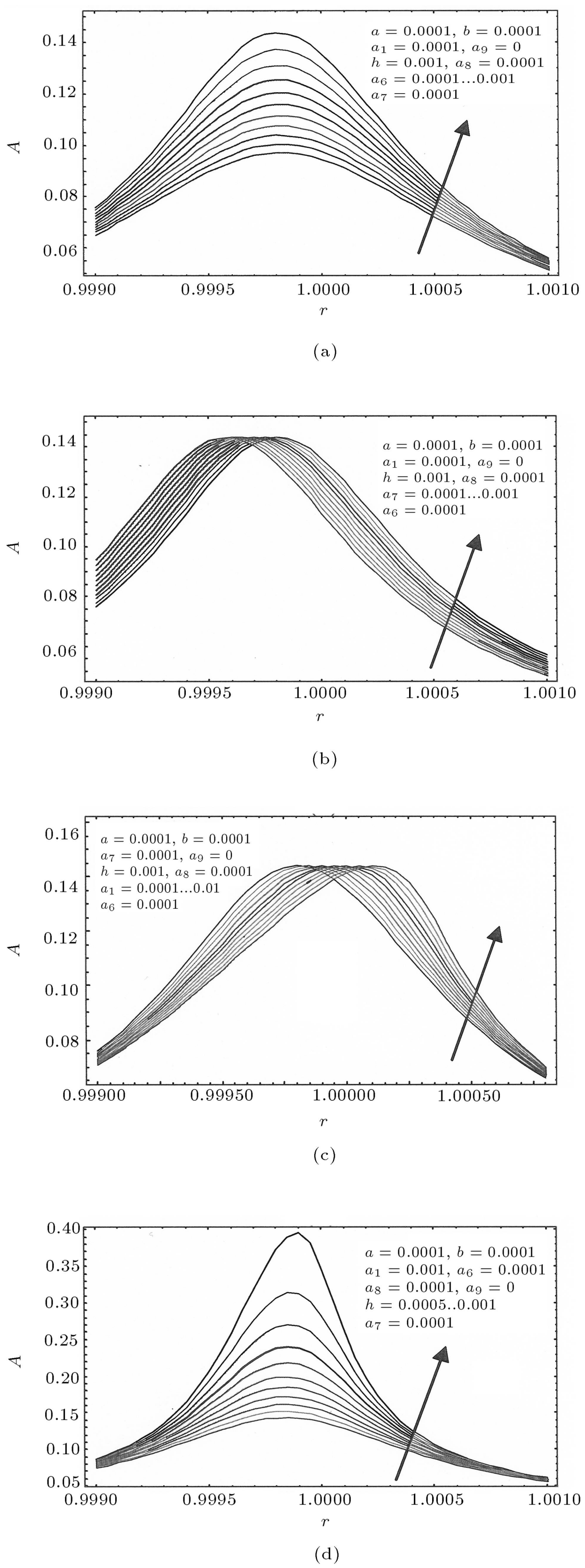


Figure 4. Effects of variation in (a) thermal damping, (b) thermal relaxation, (c) nonlinearity, and (d) damping on frequency response. (Arrows show parameters reduction.)

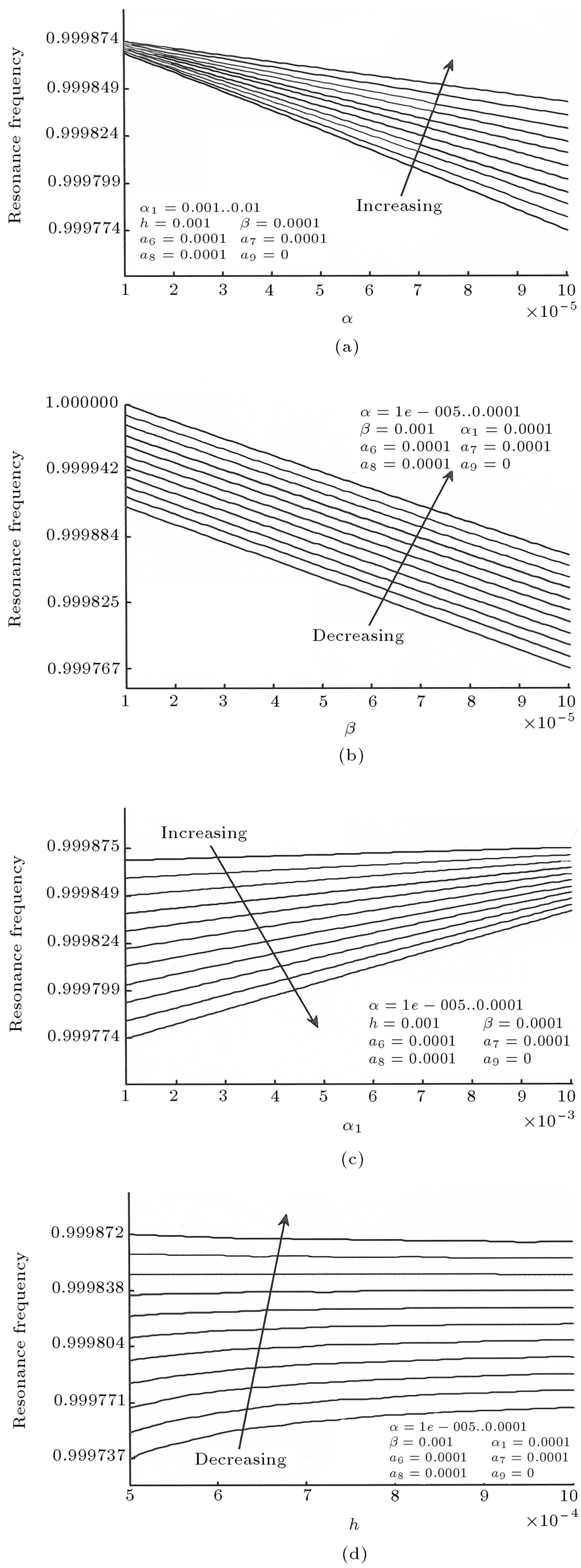


Figure 5. Effects of variation in (a) DC voltage, (b) AC voltage, (c) nonlinearity, and (d) damping on resonance frequency.

effects of a nonlinearity parameter on the resonance frequency. It can be seen that this parameter linearly increases the resonance frequency. The behavior of the resonance frequency is not linear non-monotonic when damping is varied. As shown in Figure 5d, the resonance frequency is ultimately unchanged with a variation of system damping and, as mentioned, the damping effects on the resonance frequency are coupled to the other parameters of the system.

CONSIDERING FUZZINESS OF SYSTEM PARAMETERS

There are some kinds of uncertainties in mechanical systems which are associated with the lack of precise knowledge of the system's parameters and operating conditions. For example, these are originated from the variables in the manufacturing processes. These uncertainties can have significant influence on dynamic responses and the reliability of the system.

The Fuzzy Generalized Cell Mapping (FGCM) is introduced by Hong and Sun [19,20] and, for considering and investigating these uncertainties in multi parameter systems, the FGCM has been expanded in [17] for these systems. In this method, for each uncertain parameter, one probability function is considered, then, the FGCM is applied and the probability of each point in the state space of the system is achieved. The micro resonators quality factor (dissipation parameter) is varied with variation in the system environment and the system's working condition. The FGCM is used for calculating the effects of this variation. The FGCM has been applied to Equation 10 for determining the domain of absorption of the chaotic vibration absorber in the phase plane. The uncertainties from system damping have been considered and the triangular membership function was considered for this analysis (the desired region was divided into 75 × 75 cells and one point was sampled from each cell; the membership functions were divided to 12 segments). Proper identification of the working domain of the system is so important, because it can be shown that loads can be applied to uncertain systems, under which the system is not able to work within the desired domain. Figure 6 shows how and where the pull-in occurs in the system uncertainties.

In Figure 6, the parameters are as follows:

$$\alpha = 10^{-3}, \qquad \beta = 10^{-3}, \qquad r = 0.9999,$$

$$10^{-4} \leq h \leq 2 \times 10^{-1}, \qquad a_8 = 0.001,$$

$$\alpha_1 = 1, \qquad a_6 = a_7 = a_9 = 0,$$

has the Global phase portrait of the micro resonator equation with a fuzzy parameter, with $\mu(h, 2 \times 10^{-4}, 10^{-1}, 10^{-1})$ as the membership function (the triangular membership function, in which the probability in $h = 2 \times 10^{-4}$ is zero and in $h = 10^{-1}$ is one). The membership distribution of fuzzy attractors is depicted as: black dotted points = 1.0, 0.8 < centered zone < 1.0, 0.6 < first inner ring zone < 0.8, 0.4 < second inner ring zone < 0.6, 0.2 < third inner ring zone < 0.4, and 0.0 < outer ring zone < 0.2. Symbol + represents the trajectories, which go out of the interesting domain (pull-in) and symbol * shows a stable cell ($x_1 = Y, x_2 = \dot{Y}$).

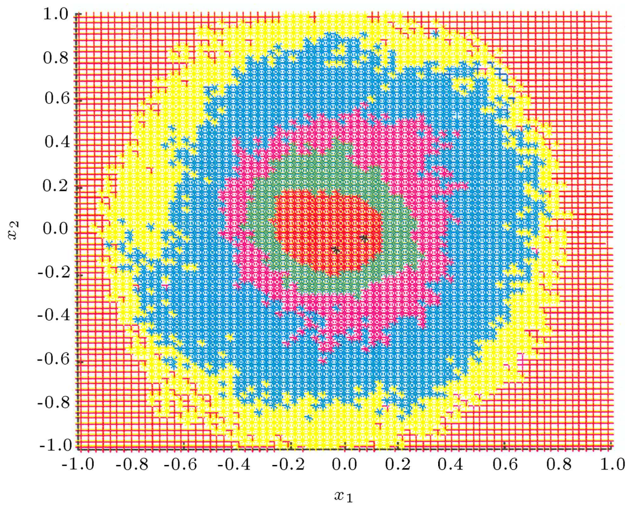


Figure 6. System uncertainties due to pull-in effect.

CONCLUSION

The thermal phenomenon in a micro resonator is modeled by considering the nonlinearities from actuated force and mid-plane stretching in the nondimensionalized equation. The thermal phenomena have been changed to effective forces per unit length of a vibrating microbeam. The thermal properties of a microbeam contribute to the damping system, due to warming up and heat energy dissipation called "thermal damping", and to the restoring system, due to material heat softening called "temperature relaxation".

The highly nonlinear equation governing the system was solved in the first resonance frequency by utilizing the multiple time scales perturbation method. This equation is reduced to two differential equations that can be solved for amplitude, with respect to time, response. The differential equations were solved in a steady state and, then, the frequency responses of the system, while varying the effective dynamic parameters, were plotted and their effects were discussed.

Nonlinearity due to mid-plane stretching affects the system resonance frequency and its increase is because of the variation of damping. It also affects the system resonance frequenc; when it is not considerable or its order is small, then, damping doesn't affect resonance frequency. Temperature relaxation reduces the peak amplitude slightly, while thermal damping has a reduction effect on peak amplitude with a dominant effect. In addition, temperature relaxation shows a significant effect on the shifting of the resonance frequency to lower values. Resonance shifting is a very important phenomenon, especially in resonator-based sensors. It seems to be the most effective source of error in resonance-sensors, which are designed, based on a constant stiffness assumption. Finally, the behavior of the system is investigated by considering the uncertain damping parameters and by showing how and where the pull-in occurs.

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APPENDIX

The decoupled equation of amplitude is:

$$\alpha^2\beta^2(-512(1+r^2)^8X^3\alpha^4\beta^4a_9^2(-2\beta+a_9)^2$$
$$(2\beta+3a_9)^2(-8a^2\beta^3-2\beta(\alpha+2a^2\beta)a_9)$$
$$+2a^2\beta a_9^2+a^2a_9^3)^2+a^4(2\beta+a_9)^8$$
$$(8a^4hr(1+r^2)a_6(2\beta+a_9)^4+4a^4r^2a_6^2(2\beta+a_9)^4$$
$$+(1+r^2)^2(4\beta^2(X^2-2X\beta(\alpha+4a^2\beta)$$
$$+\beta^2(16a^4h^2+\alpha^2-8a^2\alpha\beta))-4\beta(X^2+2X\alpha\beta$$
$$+\beta^2(-32a^4h^2-3\alpha^2+32a^2\alpha\beta))a_9$$
$$+(X^2+2X\beta(11\alpha+8\beta a^2)$$
$$+\beta^2(96a^4h^2+9\alpha^2-176\alpha\beta a^2))a_9^2$$
$$+32a^2\beta(a^2h^2-3\alpha\beta)a_9^3$$
$$+2a^2(2a^2h^2-X-9\alpha\beta)a_9^4))^2)=0, \tag{A1}$$

where X is:

$$X=(-2ra_7a^2(2\beta+a_9)+(1+r^2)(a^2a_9^2$$
$$+2a_8(2\beta+a_9)a^2+2a^2a_9(-2(-1+r+\alpha)$$
$$+3\alpha_1a^2))/(1+r^2)+(\beta(\alpha-4a^2(-2+2r$$
$$+2\alpha+\beta)+12a^4\alpha_1)))/(1+r^2).$$