Transaction B: Mechanical Engineering
Vol. 16, No. 2, pp. 140–148

SCIENTIA IRANICA

Spectator Model in D Meson Decays

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Abstract. *In this research, the effective Hamiltonian theory is described and applied to the calculation of current-current* ($Q_{1,2}$) *and QCD penguin* ($Q_{3,\cdots,6}$) *decay rates. The channels of charm quark decay in the quark levels are:* $c \to du\bar{d}$, $c \to du\bar{s}$, $c \to su\bar{d}$ *and* $c \to su\bar{s}$ *where the channel* $c \to su\bar{d}$ *is dominant. The total decay rates of the hadronic of charm quark in the effective Hamiltonian theory are calculated. The decay rates of D meson decays according to Spectator Quark Model (SQM) are investigated for the calculation of D meson decays. It is intended to make the transition from decay rates at the quark level to D meson decay rates for two body hadronic decays,* $D \to h_1 h_2$. *By means of that, the modes of nonleptonic* $D \to PV$, $D \to PP$, $D \to VV$ *decays where* V *and* P *are light vector with* $J^P = 0^-$ *and pseudoscalar with* $J^P = 1^-$ *mesons are analyzed, respectively. So, the total decay rates of the hadronic of charm quark in the effective Hamiltonian theory, according to Colour Favoured (C-F) and Colour Suppressed (C-S) are obtained. Then the amplitude of the Colour Favoured and Colour Suppressed (F-S) processes are added and their decay rates are obtained. Using the spectator model, the branching ratio of some D meson decays are derived as well.*

Keywords: *Effective Hamilton; c quark; D meson; Spectator model; Hadronic; Colour favoured; Colour suppressed.*

INTRODUCTION: EFFECTIVE HAMILTONIAN THEORY

As a weak decay in the presence of strong interaction, D meson decays require special techniques. The main tool to calculate such D meson decays is the effective Hamiltonian theory. It is a two step program, starting with an Operator Product Expansion (OPE) followed by a Renormalization Group Equation (RGE) analysis. The necessary machinery has been developed over the past years. The derivation starts as follows: If the kinematics of the decay are of the kind where the masses of the internal particle, M_i, are much larger than the external momenta, P, $M_i^2 >> p^2$, then, the heavy particle can be integrated out. This concept takes concrete form with the functional integral formalism. It means that the heavy particles are removed as dynamical degrees of freedom from the theory. Hence, their fields no longer appear in the effective Lagrangian. Their residual effect lies in the generated effective vertices. In this way, an effective low energy theory can be constructed from a full theory like the standard model. A well known example is the four-Fermi interaction, where the W-boson propagator is made local for $M_W^2 >> q^2$ (q denotes the momentum transfer through the W):

$$-i(g_{\mu\nu})/(q^2 - M_W^2) \to ig_{\mu\nu}[(1/M_W^2) + (q^2/M_W^4) + \cdots], \quad (1)$$

where the ellipsis denotes terms of a higher order in $1/M_W$.

Apart from the t quark the basic framework for weak decays quarks is the effective field theory relevant for scales M_W, M_Z, $M_t >> \mu$ [1]. This framework, as seen above, brings in local operators, which govern "effectively" the transition in question.

It is well known that the decay amplitude is the product of two different parts the phases of which are made of a weak (Cabbibo-Kobayashi-Maskawa) and a strong (final state interaction) contribution. The weak contributions to the phases change sign when going to the CP-conjugate process while the strong ones do not. Indeed, the simplest effective Hamiltonian without QCD effects ($c \to su\bar{s}$) is:

$$H_{\rm eff}^0 = 2\sqrt{2} G_F V_{sc} V_{su}^* Q_1, \quad (2)$$

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Received 13 January 2007; received in revised form 10 November 2007; accepted 12 April 2008

where G_F is the Fermi constant, V_{ij} are the relevant CKM factors and:

$$Q_1 = (\bar{s}_\alpha c_\alpha)_{V-A}(\bar{u}_\beta s_\beta)_{V-A}, \tag{3}$$

is a $(V-A)$. $(V-A)$ is the current-current local operator.

This simple tree amplitude introduces a new operator, Q_2, and is modified by the QCD effect to:

$$H_{\text{eff}} = 2\sqrt{2}G_F V_{sc}V_{su}^*(C_1Q_1 + C_2Q_2), \tag{4}$$

$$Q_2 = (\bar{s}_\beta c_\alpha)_{V-A}(\bar{u}_\alpha s_\beta)_{V-A}, \tag{5}$$

where C_1 and C_2 are Wilson coefficients. The situation in the standard model is, however, more complicated because of the presence of additional interactions in particular penguins, which effectively generate new operators. These are, in particular, the gluon, photon and Z^0-boson exchanges and penguin c quark contributions, as seen before.

Consequently, the relevant effective Hamiltonian for D-meson decays generally involves several operators, Q_i, with various colour and Dirac structures, which are different from Q_1. The operators can be grouped into two categories [2]: $i = 1, 2$-current-current operators; $i = 3, \cdots, 6$-gluonic penguin operators. Moreover, each operator is multiplied by a calculable Wilson coefficient, $C_i(\mu)$:

$$H_{\text{eff}} = 2\sqrt{2}G_F \sum_{i=1}^{6} d_i(\mu)Q_i(\mu), \tag{6}$$

where scale μ is discussed below, $d_i(\mu) = V_{\text{CKM}}C_i(\mu)$ and V_{CKM} denotes the relevant CKM factors that are:

$$d_{1,2} = V_{ic}V_{jk}^*C_{1,2},$$

$$d_{3,\cdots,6} = -V_{tc}V_{tk}^*C_{3,\cdots,6}. \tag{7}$$

EFFECTIVE HAMILTONIAN DECAY RATES

The effective $\Delta C = 1$ Hamiltonian at scale $\mu = O(m_c)$ for tree plus penguin term is given by:

$$\begin{aligned} H_{\text{eff}}^{\Delta C=1} = 2\sqrt{2}G_F\{&[d_{1s}(\mu)Q_1^s(\mu) + d_{2s}(\mu)Q_2^s(\mu)] \\ &+ [d_{1d}(\mu)Q_1^d(\mu) + d_{2d}(\mu)Q_2^d(\mu)] \\ &- \sum_{i=3}^{6} d_i(\mu)Q_i(\mu)\}. \end{aligned} \tag{8}$$

Here, $d_{1,\cdots,6}$ are defined by Equation 7, $d_{1,2s,d} = d_{1,2}(i = j = s, d)$ and index k refers to d or s quarks.

The partial decay rate in the c rest frame is (see Appendix A):

$$\begin{aligned} d^2\Gamma_{Q_1,\cdots,Q_6}/dp_i dp_k = (G_F^2/\pi^3)p_ip_kE_{\bar{j}}\{&\alpha_1(p_i.p_k/E_iE_k) \\ &+ \alpha_2(p_i.p_{\bar{j}}/E_iE_{\bar{j}}) \\ &+ \alpha_3(m_km_{\bar{j}}/E_kE_{\bar{j}})\}. \end{aligned} \tag{9}$$

After the change of variable to x and y, the decay rate is given by:

$$d^2\Gamma_{Q_1,\cdots,Q_6}/dxdy = \Gamma_{0c}I_{ps}^{EH}. \tag{10}$$

SPECTATOR MODEL

In the spectator model [3], the spectator quark is given a non-zero momentum, having in this work a Gaussian distribution represented by a free (but adjustable) parameter, Λ:

$$P(|p_s|^2) = (1/\pi^{3/2}\Lambda^3)e^{-(p_s^2/\Lambda^2)}. \tag{11}$$

The probability distribution of a three momentum for spectator quarks is given by:

$$dP(p_s) = P(|p_s|^2)d^3p_s = P(|p_s|^2)d\Omega p_s^2 dp_s. \tag{12}$$

And $P(|p_s|^2)$ is normalized according to:

$$4\pi\int_0^\infty P(|p_s|^2)p_s^2dp_s = 1. \tag{13}$$

Here, however, a tentative spectator model is considered based upon the idea of duality between quark and hadron physics at the high energies of c quark and D meson decays. The decays at the quark level, even including the penguins, are basically short distance processes. In our proposed spectator quark model, the long distance hadronization is largely a matter of incoherently assigning regions of the final quark phase space to the different mesonic systems. For example, consider a D meson $c\bar{u}$ or $c\bar{d}$ to be a heavy stationary c quark accompanied by a light spectator constituent antiquark, which has a spherically symmetric normalized momentum distribution, $P(|p_s|^2)d^3p_s$. The total meson decay rate through a particular mode is then assumed to be:

$$\Gamma_{\text{total}} = \int \frac{d^2\Gamma}{dp_idp_k}P(|p_s|^2)d^3p_sdp_idp_k, \tag{14}$$

which is equal to the initiating decay rate. Ignoring, for the moment, any constraints due to quark colour, it is supposed the spectator antiquark, q_s, combines with the quark, q ($q = q_i$ or q_k), to form the meson system.

For example, if $q = q_i$, a mass, $M_{q_i q_s}$, is assigned to the system such that:

$$M_{is}^2 = (p_i + p_s).(p_i + p_s) = m_i^2 + m_s^2 + 2(E_i E_s - p_i p_s \cos\theta_{is}). \quad (15)$$

Constraining p_i and p_s to have mass M_{is}, it can be inferred from Equation 14 that:

$$\frac{d\Gamma}{dM_{is}} = 2M_{is}\int \frac{d^2\Gamma}{dp_i dp_k} P(|p_s|^2)\delta(M_{is}^2 - m_i^2 - m_s^2 - 2(E_i E_s - p_i p_s \cos\theta_{is})) \times 2\pi p_s^2 dp_s d(\cos\theta_{is}) dp_i dp_k. \quad (16)$$

Hence:

$$\frac{d\Gamma}{dM_{is}} = 2\pi M_{is}\int \frac{p_s dp_s}{p_i}\frac{d^2\Gamma}{dp_i dp_k} P(|p_s|^2) dp_i dp_k, \quad (17)$$

where the integration region is restricted by the condition $|\cos\theta_{is}| \leq 1$. We also assign a mass, $M_{k\bar{j}}$, to the second quark-antiquark system such that:

$$M_{k\bar{j}}^2 = (p_k + p_{\bar{j}}).(p_k + p_{\bar{j}}) = m_k^2 + m_{\bar{j}}^2 + 2(E_k E_{\bar{j}} - p_k p_{\bar{j}} \cos\theta_{k\bar{j}}), \quad (18)$$

or:

$$M_{k\bar{j}}^2 = (p_k + p_{\bar{j}}).(p_k + p_{\bar{j}}) = (p_b - p_i).(p_c - p_i) = m_c^2 + m_i^2 - 2m_c E_i. \quad (19)$$

The variable E_i or p_i determines the mass, $M_{k\bar{j}}$. Taking this mass to be the independent variable, we have:

$$\frac{d^2\Gamma}{dM_{is}dM_{k\bar{j}}} = \frac{2\pi M_{is} M_{k\bar{j}}}{m_c}\int \frac{E_i p_s}{p_i^2}\frac{d^2\Gamma}{dp_i dp_k} P(|p_s|^2) dp_s dp_k. \quad (20)$$

The integration range is restricted by $|\cos\theta_{kj}| \leq 1$,

$$\cos\theta_{k\bar{j}} = (m_k^2 + m_{\bar{j}}^2 - M_{k\bar{j}}^2 + 2E_k E_{\bar{j}})/2p_k p_{\bar{j}}. \quad (21)$$

This mode of quark and antiquark combination process is called (C-F) (Colour Favoured). Finally, by integration, the partial decay rates, $\Gamma(M_{is}, M_{k\bar{j}})$, are computed into quark systems with masses less than M_{is} and $M_{k\bar{j}}$. With suitable binding, these partial decay rates are equated with corresponding rates into mesons. Of particular interest are the quark antiquark systems forming the lowest mass 0^- and 1^- meson states, as data exists for charmed quark systems that can be used to test the spectator quark model.

It is also possible that the spectator antiquark combines with the quark q_k for which we get:

$$\frac{d^2\Gamma}{dM_{ks}dM_{i\bar{j}}} = \frac{2\pi M_{ks} M_{i\bar{j}}}{M_c}\int \frac{E_k p_s}{p_k^2}\frac{d^2\Gamma}{dp_i dp_k} P(|p_s|^2) dp_s dp_i. \quad (22)$$

This process Colour Suppressed (C-S). In some meson decays, for example, $D^+(c\bar{d}) \rightarrow (d\bar{d}) + (\bar{d}u)$ is initiated by the quark decay, $c \rightarrow du\bar{d}$, and the spectator, $\bar{d}$, could have combined with the d or the u. In this case, results are shown if the processes produce incoherence or assume coherence. In sum, the decay rates of B mesons for process (C-F) and process (C-S) are:

$$\Gamma_{(\text{C-F})} = \int_{m_{\min\,is}}^{m_{\text{cut}\;is}}\int_{m_{\min\,k\bar{j}}}^{m_{\text{cut}\;k\bar{j}}} \frac{d^2\Gamma}{dM_{is}dM_{k\bar{j}}} dM_{is} dM_{k\bar{j}},$$

$$\Gamma_{(\text{C-S})} = \int_{m_{\min\,ks}}^{m_{\text{cut}\;ks}}\int_{m_{\min\,i\bar{j}}}^{m_{\text{cut}\;i\bar{j}}} \frac{d^2\Gamma}{dM_{ks}dM_{i\bar{j}}} dM_{ks} dM_{i\bar{j}}. \quad (23)$$

where $m_{\min\,is} = (m_{q_i} + m_{q_s})$, $m_{\text{cut}\;is} = M_{q_i q_s}$ and so on. Turning now to the colour factors, what may be regarded as two extreme possibilities are examined. In the first, here called model (A), Equations 9, 20 and 22 are taken at face value, that is, no attempt is made to follow the flow of colour and it is assumed that all colour flow is looked after by the gluon fields in the meson system. In the second, here called model (B), consider the possibility that the lowest mass meson states are only formed if the quark-antiquark pairs are in a colour singlet state. That is, it is assumed that the colour distribution caused by a quark-antiquark pair in an octet state will result in more complex meson systems than the lowest mass 0^- and 1^- states.

Projecting out the colour singlet states results only in a modification of coefficients α_1, α_2 and α_3 of Equation A3. Physical hadrons are singlets in colour space, and are termed colourless or white. Colour symmetry is absolute and weak quark currents are, as hadrons, white. This means, for example, that $\bar{q}_s q_i$ is, in fact, a sum of three terms:

$$q_s\bar{q}_i = q_{s\alpha}(h)\bar{q}_{i\alpha}(h) = q_s(1)\bar{q}_i(\bar{1}) + q_s(2)\bar{q}_i(\bar{2}) + q_s(3)\bar{q}_i(\bar{3}), \quad (24)$$

where it suffices for 1, 2 and 3 to stand for yellow, blue and red, respectively. The spectator is a colour singlet with the c quark. Hence, the colour factors for Q_1,

Q_2Q_3, Q_4, Q_5 and Q_6 are given by:

$$Q_1, Q_4, Q_6 \rightarrow \text{process (C-F)} \equiv \sqrt{3},$$

$$\text{process (C-S)} \equiv 1/\sqrt{3},$$

$$Q_2, Q_3, Q_5 \rightarrow \text{process (C-F)} \equiv 1/\sqrt{3},$$

$$\text{process (C-S)} \equiv \sqrt{3}.$$

Consequently, factors α_1, α_2 and α_3 of Equation A3 are different for process (C-F) and process (C-S) of model (B). For process (C-F), represented by Equation 27, they become:

$$\alpha_1 = 3\left|d_1 + (d_2/3) + (d_3/3) + d_4\right|^2,$$

$$\alpha_2 = 3\left|(d_5/3) + d_6\right|^2. \tag{25}$$

While for process (C-S), represented by Equation 58, they are:

$$\alpha_1 = 3\left|(d_1/3) + d_2 + d_3 + (d_4/3)\right|^2,$$

$$\alpha_2 = 3\left|d_5 + (d_6/3)\right|^2. \tag{26}$$

EFFECTIVE HAMILTONIAN SPECTATOR MODEL

We try to calculate the spectator model for the general case, which we call the effective Hamiltonian spectator model. According to Equation 10, the total decay rates for current-current plus penguin operators in the Effective Hamiltonian are given by:

$$\Gamma_{Q_1,\cdots,Q_6} = \Gamma_{0c} I_{ps}^{EH}. \tag{27}$$

The differential decay rates for the two boson system in the spectator quark model for current-current plus penguin operators in the Effective Hamiltonian are given by (see Appendix B):

$$\frac{d^2\Gamma_{Q_1,\cdots,Q_6}}{d(q_{si}/M_c)d(q_{k\bar{j}}/M_c)} = \Gamma_{0c}\frac{8q_{si}q_{k\bar{j}}}{\sqrt{\pi}M_c}\frac{\beta^2}{\Lambda}\frac{\sqrt{(2m_i/M_c)^2+x^2}}{x^2} \times \int_0^1 dy \int_0^1 dz \zeta^{\text{eff}}_{ps(q,z)} z e^{-\beta^2 z^2}. \tag{28}$$

Now, we can integrate over the two mass cuts (two boson systems), and obtain the hadronic decay rates, as follows:

$$\Gamma'_{Q_1,\cdots,Q_6} = \int_{\text{min}}^{m_{\text{cut}}}\int_{\text{min}'}^{m'_{\text{cut}}} \frac{d^2\Gamma_{Q_1,\cdots,Q_6}}{d(q_{si}/M_c)d(q_{k\bar{j}}/M_c)} dm_{\text{cut}} dm'_{\text{cut}},$$

$$= \Gamma_{0c}\int_{\text{min}}^{m_{\text{cut}}}\int_{\text{min}'}^{m'_{\text{cut}}} \frac{8q_{si}q_{k\bar{j}}}{\sqrt{\pi}M_c}\frac{\beta^2}{\Lambda}\frac{\sqrt{(2m_i/M_c)^2+x^2}}{x^2}$$

$$\int_0^1 dy \int_0^1 dz \zeta^{\text{eff}}_{ps(q,z)} z e^{-\beta^2 z^2} dm_{\text{cut}} dm'_{\text{cut}}. \tag{29}$$

DECAY RATES OF PROCESSES C-F PLUS C-S ($F+S$) OF EFFECTIVE HAMILTONIAN

Now, we want to calculate the decay rates of the effective Hamiltonian ($Q_1, \cdots, Q_6$) for $F+S$ at quark-level and the spectator model. The effective Hamiltonian for $F+S$, is given by:

$$H_{\text{eff}}^{A+B} = H_{\text{eff}}^{b\rightarrow ik\bar{j}} + H_{\text{eff}}^{b\rightarrow i\bar{j}k}, \tag{30}$$

where $H_{\text{eff}}^{c\rightarrow ik\bar{j}}$ is defined by Equation 8 and we can obtain $H_{\text{eff}}^{c\rightarrow i\bar{j}k}$ (see Appendix C). The decay rates of current-current plus penguin for $F+S$ are given by (see Appendix C):

$$d^2\Gamma_{EH}^{F+S}/dxdy = \Gamma_{0c} I_{pc}^{F+S}. \tag{31}$$

NUMERICAL RESULTS

As an example of the use of the formalism above, we use the standard Particle Data Group [4] parameterization of the CKM matrix with the central values,

$$\theta_{12} = 0.221, \qquad \theta_{13} = 0.0035, \qquad \theta_{23} = 0.041,$$

and choose the CKM phase, δ_{13}, to be $\pi/2$. Following Ali and Greub [2], internal quark masses are treated in tree-level loops with the values (GeV) $m_b = 4.88$, $m_s = 0.2$, $m_d = 0.01$, $m_u = 0.005$, $m_c = 1.5$, $m_e = 0.0005$, $m_\mu = 0.1$, $m_\tau = 1.777$ and $m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0$.

Following G. Buccella [5], the effective Wilson coefficients, C_i^{eff}, is chosen for the various $c \rightarrow q$ transitions.

a) The total decay rate and branching ratios hadronic modes, according to the effective Hamiltonian theory (see Equation 10), are shown in Table 1. We see that mode $c \rightarrow su\bar{d}$ is dominant. The total c-quark decay rate of the effective Hamiltonian is given by:

$$\Gamma_{\text{total}}^{EH}(c \rightarrow \text{ anything})$$

$$= \Gamma(c\rightarrow s \text{ anything}) + \Gamma(c\rightarrow d \text{ anything})$$

$$= 9.261 \times 10^{-13} + 0.606 \times 10^{-13} \text{ GeV}$$

$$= 9.867 \times 10^{-13} \text{ GeV}.$$

Table 1. Decay rates (Γ) and Branching Ratio (BR) of effective Hamiltonian of c-quark.

Process	$\Gamma_{EH} \times 10^{-15}$	$\text{BR}_{EH} \times 10^{-3}$
$c \rightarrow du\bar{d}$	31.689	32.12
$c \rightarrow du\bar{s}$	1.0785	1.093
$c \rightarrow su\bar{d}$	409.44	414.95
$c \rightarrow su\bar{s}$	23.836	24.157

b) Now the mean lives of the charm quark (D meson) are obtained theoretically and compared with the experimental mean life of $D^{\pm}$, D^0 and D_s^+, so:

$$\text{Mean life}\ {}^{EH}_{\text{theory}}(D) = \hbar/\Gamma^{EH}_{\text{total}}$$
$$= 1.067 \times 10^{-12}\ \text{sec}, \qquad (32)$$

and:

$$\text{Mean life}\ _{\text{exp}}(D^+) = (1.040 \pm 0.007) \times 10^{-12}\ \text{sec},$$
$$\text{Mean life}\ _{\text{exp}}(D^0) = (0.410 \pm 0.001) \times 10^{-12}\ \text{sec},$$
$$\text{Mean life}\ _{\text{exp}}(D_s^+) = (0.461 \pm 0.015) \times 10^{-12}\ \text{sec}. \qquad (33)$$

Also, we can compare the branching ratio of the semileptonic theoretically and experimentally, so:

$$\text{BR}(c \to e^+\ \text{anything})_{\text{theory}} = \text{BR}(c \to se^+\nu_e)$$
$$+ \text{BR}(c \to de^+\nu_e) = 147.96 \times 10^{-3}$$
$$+ 8.702 \times 10^{-3} = 15.67 \times 10^{-2}, \qquad (34)$$

and:

$$BR_{\text{exp}}(D^+ \to e^+\ \text{anything}) = (17.2 \pm 1.9) \times 10^{-2},$$
$$BR_{\text{exp}}(D^0 \to e^+\ \text{anything}) = (6.87 \pm 0.8) \times 10^{-2},$$
$$BR_{\text{exp}}(D_s^+ \to e^+\ \text{anything}) < 20 \times 10^{-2}. \qquad (35)$$

It is observed that the theoretical and experimental results are close.

c) In the spectator quark model, the value $\Lambda = 0.6$ GeV [6] is used. For the maximum mass of the quark-antiquark systems, (m_{cut}), a value midway between the lowest mass 1^- state and the next most massive meson is taken. Thus, we take, for ($s\overline{u}$) or ($s\overline{d}$), $m_{\text{cut}(u\overline{d})} = 0.877$ GeV between $\rho(0.770)$ and $a_0(0.984)$; for ($u\overline{u}$) and ($d\overline{d}$), $m_{cut(u\overline{u})} = m_{\text{cut}(d\overline{d})} = 0.870$ GeV between $\omega(0.782)$ and $\eta'(0.958)$.

For example, it is possible to calculate the Branching Ratios of mode $c \to du\overline{d}$ in the tree-level and in the effective Hamiltonian spectator model. The mode, $c \to du\overline{d}$, is for decays $D^+ \to \pi^0\pi^+$, $D^+ \to \eta\pi^+$, $D^+ \to \rho^0\pi^+$, $D^+ \to \omega\pi^+$, $D^+ \to \pi^0\rho^+$, $D^+ \to \eta\rho^+$, $D^+ \to \rho^0\rho^+$ and $D^+ \to \omega\rho^+$. In this case, we have got a two-boson system and, therefore, two masses of cut for the boson system. Masses of cut for a two boson system, $m_{\text{cut1}} = 0.870/M_c$ and $m_{\text{cut2}} = 0.877/M_c$ are chosen. Theoretically, the branching ratio of the effective Hamiltonian spectator model is given by:

$$\text{BR}_{EH}(c \to du\overline{d})$$
$$= \Gamma_{EH}(c \to du\overline{d})_{\text{cut1,cut2}}/\Gamma_{\text{total}\ EH}(c \to \text{anything})$$
$$= 1.2502 \times 10^{-14}/9.867 \times 10^{-13}$$
$$= 1.2671 \times 10^{-2}.$$

The masses of some mesons and the masses of cut are shown in Table 2. The results are presented in Table 3 and compared, where data is available, with the sum of the branching ratios into mesons with masses less than the above cutoff masses. Also, all the experimental and theoretical D meson decays in the spectator model are classified and given in Table 3.

d) The decay rates of c quark for $F + S$ (shown in Table 4), and the total decay rates of $F + S$ are given by:

$$(c \to du\overline{d}) \quad D^+ \to (\pi^0, \eta, \rho^0, \omega), (\pi^+, \rho^+),$$
$$\text{BR}^{F+S}_{EH} = 2.1023 \times 10^{-2},$$
$$(c \to su\overline{d}) \quad D^+ \to (\pi^+, \rho^+), (\overline{K}^0, \overline{K}^{*+}),$$
$$\text{BR}^{F+S}_{EH} = 51.2871 \times 10^{-2},$$
$$(c \to su\overline{s}) \quad D^+ \to (\eta', \phi), (K^+, K^{*+}),$$
$$\text{BR}^{F+S}_{EH} = 3.8671 \times 10^{-2}.$$

Table 2. The masses of some mesons and the masses of cut for D meson decay processes.

System of Quark	**Particle**	**Mass (GeV)**	**Cutoff Mass (GeV)**
$s\overline{u}$, $s\overline{d}$	K	0.494	1.081
	K^*	0.892	
	K^{**}	1.270	
$u\overline{d}$	π	0.140	0.877
	ρ	0.770	
	a_0	0.984	
$u\overline{u}$, $d\overline{d}$	π^0	0.140	0.870
	η	0.547	
	ρ^0	0.770	
	ω	0.782	
	η'	0.958	
$s\overline{s}$	η'	0.958	1.150
	ϕ	1.020	
	ϕ	1.680	

Table 3. Experimental and theoretical processes of spectator model of branching ratios of hadronic for D meson decays.

1-	Decay of c quark		
2-	Decay of D meson, process (C-F)		
3-	Decay of D meson, process (C-S)		
4-	Experimental branching ratio, process (C-F)		
5-	Total experimental branching ratio, process (C-F)		
6-	Experimental branching ratio, process (C-S)		
7-	Total experimental branching ratio, process (B)		
8-	$m_{\text{cut1}} \times M_c$ GeV, $m_{\text{cut2}} \times M_c$ GeV, process (C-F)		
9-	$m_{\text{cut1}} \times M_c$ GeV, $m_{\text{cut2}} \times M_c$ GeV, process (C-S)		
10-	Effective Hamiltonian branching ratio, model (A), process (C-F)		
11-	Effective Hamiltonian branching ratio, model (A), process (C-S)		
1-	$c \to du\overline{d}$		
2-	$D^+ \to (\pi^0, \eta, \rho^0, \omega)$, (π^+, ρ^+), $D^0 \to (\pi^-, \rho^-)$, (π^+, ρ^+), $D_s^+ \to (K^0, K^{*0})$, (π^+, ρ^+)		
3-	$D^+ \to (\pi^0, \eta, \rho^0, \omega)$, (π^+, ρ^+), $D^0 \to (\pi^0, \eta, \rho^0, \omega)$, $(\pi^0, \eta, \rho^0, \omega)$, $D_s^+ \to (\pi^0, \eta, \rho^0, \omega)$, (K^+, K^{*+})		
4-	$\pi^0\pi^+, (2.5 \pm 0.7)E-3$	$\pi^-\pi^+, (1.25 \pm 0.11)E-3$	$K^0\pi^+, < 8.0E-3$
	$\rho^0\pi^+, < 1.4E-3$		
	$K^*(892)^0\pi^+, (6.5 \pm 2.8)E-3$		
	$\eta\pi^+, (7.5 \pm 2.5)E-3$		
	$\omega\pi^+, < 7.0E-3$		
	$\eta\rho^+, 1.2E-2$		
5-	$< (3.04 \pm 0.25)E-2$	$(1.25 \pm 0.11)E-3$	$(14.5 \pm 2.8)E-3$
6-	$\pi^0\pi^+, (2.5 \pm 0.7)E-3$	$\pi^0\pi^0, (8.4 \pm 2.2)E-4$	$K^+\rho^0, < 2.9E-3$
	$\rho^0\pi^+, < 1.4E-3$		
	$\eta\pi^+, (7.5 \pm 2.5)E-3$		
	$\omega\pi^+, < 7.0E-3$		
	$\eta\rho^+, 1.2E-2$		
7-	$< (3.04 \pm 0.25)E-2$	$(8.4 \pm 2.2)E-4$	$< 2.9E-3$
8-	0.870, 0.877	0.877, 0.877	1.081, 0.877
9-	0.870, 0.877	0.870, 0.870	0.870, 1.081
10-	$1.2671E-2(F+S\ 2.1023\ E-2)$	$1.2834E-2$	$1.2723E-2$
11-	$1.2931E-2$	$1.2543E-2$	$1.2398E-2$
1-	$c \to su\overline{s}$		
2-	$D^+ \to (\overline{K}^0, \overline{K}^{*0})$, $(K^+\ K^{*+})$,	$D^0 \to (K^-, K^{*-}), (K^+, K^{*+})$,	$D_s^+ \to (\eta', \phi), (K^+, K^{*+})$
3-	$D^+ \to (\eta', \phi), (\pi^+, \rho^+)$,	$D^0 \to (\eta', \phi), (\pi^0, \eta, \rho^0, \omega)$,	$D_s^+ \to (\eta', \phi), (K^+, K^{*+})$
4-	$\overline{K}^0K^+, (7.2 \pm 1.2)E-3$	$K^+K^-, (4.33 \pm 0.27)E-3$	$\phi K^+, < 5.0E-4$
	$\overline{K}^*(892)^0K^+, (4.2 \pm 0.5)E-3$	$K^*(892)^+K^-, (3.5 \pm 0.8)E-3$	
	$\overline{K}^0K^*(892)^+, (3.0 \pm 1.4)E-2$	$K^+K^*(892)^-, (1.8 \pm 1.0)E-3$	
	$\overline{K}^*(892)^0K^*(892)^+, (2.6 \pm 1.1)E-2$		
5-	$(6.74 \pm 2.62)E-2$	$(9.63 \pm 2.07)E-3$	$< 5.0E-4$
6-	$\eta'(958)\pi^*, < 9.0E-3$	$\phi\pi^0, < 1.4E-3$	$\phi K^+, < 5.0E-4$
	$\eta'(958)\rho^+, < 1.5E-2$	$\phi\eta, < 2.8E-3$	
	$\phi\pi^+, (6.1 \pm 0.6)E-3$	$\phi\omega, < 2.1E-3$	
	$\phi\rho^+, < 1.5E-2$	$\phi\rho^0, (1.07 \pm 0.29)E-3$	
7-	$(4.51 \pm 0.1)E-2$	$(7.37 \pm 0.29)E-3$	$< 5.0E-4$
8-	1.081, 1.081	1.081, 1.081	1.510, 1.081
9-	1.150, 0.877	1.150, 0.870	1.150, 1.081
10-	$1.9543\ E-2$	$1.9378\ E-2$	$1.9859\ E-2\ (F+S3.8671E-2)$
11-	$1.9213\ E-2$	$1.8975\ E-2$	$1.9453\ E-2$

Table 4. Decay rates and branching ratio of $F+S$ of effective Hamiltonian.

Process	$\Gamma_{HE} \times 10^{-15}$	$\mathrm{BR}_{EH} \times 10^{-2}$
$c \to du\overline{d}$	35.611	31.262
$c \to du\overline{s}$	1.4608	1.2824
$c \to su\overline{d}$	554.45	486.74
$c \to su\overline{s}$	26.927	23.638

CONCLUSIONS

In the present paper, the effective Hamiltonian theory and spectator quark model for c quark are used and the hadronic decays of D mesons are calculated. In this model, decays of the channel hadronic decays of D mesons are added. For colour favoured and colour suppressed, channel $c \to du\overline{d}$ (e.g. $D^+ \to \pi^0\pi^+$) is considered and theoretical values very close to experimental ones are achieved. Finally, cases have been shown in which the theoretical values are better than the amplitude of all the decay rates calculated. Here, the total decay rates of the hadronic of charm quark in the effective Hamiltonian are obtained according to colour favoured and colour suppressed, and then the amplitude of colour favoured and colour suppressed processes are added to them to obtain their decay rates. Also, using the spectator model, the branching ratio of some D meson decays was obtained.

According to Tables 1 and 4, it is possible to compare the decay rates of processes $c \to du\overline{d}$, $c \to du\overline{s}$ and $c \to su\overline{d}$, $c \to su\overline{s}$. The channels of $c \to du\overline{s}$ and $c \to su\overline{d}$ have tree-level decay rates and the channels of $c \to du\overline{d}$ and $c \to su\overline{s}$ have tree-level, plus Penguin, decay rates. According to Tables 1 and 4, we saw that the decay rates of channels $c \to du\overline{d}$ and $c \to su\overline{s}$ of colour favoured plus colour suppressed were more than the decay rates of the effective Hamiltonian. Hence, the branching ratio of colour favoured plus colour suppressed was less than the effective Hamiltonian. When we put the values of channels $c \to du\overline{d}$ and $c \to su\overline{s}$ into the theoretical model, it is observed that the theoretical and experimental values are close.

In Table 3, it is seen that the effective Hamiltonian branching ratio of colour favoured plus colour suppressed is better than the effective Hamiltonian branching ratio of colour favoured or colour suppressed alone. The experimental value of the branching ratio of channel $c \to su\overline{s}$ (e.g. $D_s^+ \to \eta' K^+$, $D_s^+ \to \phi K^{*+}$) is less than $5.0E-4$ and the theoretical value of the branching ratio of colour favoured plus colour suppressed is less than 3.8671 $E-2$. It is observed that the experimental and theoretical values are close. Also, for channel $c \to du\overline{d}$ (e.g. $D^+ \to \pi^0\rho^+$, $D^+ \to \rho^0\pi^+$, $D^+ \to \omega\rho^+$, $D^+ \to \eta\pi^+$), the experimental value of the branching ratio is less than (3.04 ± 0.25) $E-2$ and the theoretical value of the branching ratio of colour favoured plus colour suppressed is less than 2.1023 $E-2$. We see that the experimental and theoretical values are close.

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APPENDIX A

Partial Decay Rate

Squaring spin average term $Q_1, \cdots, Q_6$ is given by:

$$[(\tilde{\sigma}^\mu)(\tilde{\sigma}_\mu)_{LL} + (\tilde{\sigma}^\mu)(\sigma_\mu)_{LR}]^2_{sp-av}$$

$$= \alpha_1(1/16)(1+v_i)(1+v_k)(1+v_j)[1-\cos(\theta_k-\theta_i)]$$

$$+\alpha_2(1/16)(1-v_i)(1+v_k)(1-v_j)[1+\cos(\theta_k-\theta_j)]$$

$$+\alpha_3(1/16)\sqrt{1-v_i^2}(1+v_k)\sqrt{1-v_j^2}[1+\cos(\theta_j-\theta_i)$$

$$-\cos(\theta_k-\theta_j)-\cos(\theta_k-\theta_i)]. \quad \text{(A1)}$$

We must obtain the eight terms of the helicity states of the above equation and then add them up. So:

$$[(\tilde{\sigma}^\mu)(\tilde{\sigma}_\mu)_{LL} + (\tilde{\sigma}^\mu)(\sigma_\mu)_{LR}]^2_{sp-av}$$

$$= (\alpha_1/2)[1 - v_i v_k \cos(\theta_k - \theta_i)]$$

$$+ (\alpha_2/2)[1 + v_k v_j \cos(\theta_j - \theta_k)]$$

$$+ (\alpha_3/2)\sqrt{1-v_i^2}\sqrt{1-v_j^2}. \quad \text{(A2)}$$

After adding all colour combinations, α_1, α_2 and α_3 give:

$$\alpha_1 = |d_1 + d_2 + d_3 + d_4|^2 + 2|d_1 + d_4|^2 + 2|d_2 + d_3|^2,$$

$$\alpha_2 = |d_5 + d_6|^2 + 2|d_5|^2 + 2|d_6|^2,$$

$$\alpha_3 = \mathrm{Re}\{(3d_1 + d_2 + d_3 + 3d_4)d_6^* + (d_1 + 3d_2 + 3d_3 + d_4)d_5^*\}. \quad \text{(A3)}$$

The angle between the particle velocities must be physical, $-1 \le \cos(\theta_k - \theta_i) \le 1$ and $-1 \le \cos(\theta_j - \theta_k) \le 1$. So, we should take variable p_i and p_k, or x and y as:

$$p_i = xM_c/2, \qquad p_k = yM_c/2. \quad \text{(A4)}$$

After the change of variable to x and y, the decay rate is given by:

$$d^2\Gamma_{Q_1,\cdots,Q_6}/dxdy = \Gamma_{0c} I_{ps}^{EH}, \quad \text{(A5)}$$

where:

$$I_{ps}^{EH} = \alpha_1 I_{ps}^1 + \alpha_2 I_{ps}^2 + \alpha_3 I_{ps}^3, \quad \text{(A6)}$$

where:

$$I_{ps}^1 = 6xy.f_{ab}.(1 - h_{abc}),$$

$$I_{ps}^2 = 6xy.f_{bc}.(1 + h_{bca}),$$

$$I_{ps}^3 = 6xy.f_{ac}.h_{xa}.h_{yc}. \quad \text{(A7)}$$

f_{ab}, f_{bc}, f_{ac}, h_{abc} and h_{bca} are defined by:

$$\Gamma_{0c} = G_F^2 M_c^5/192\pi^3,$$

$$f_{ab} = 2 - \sqrt{x^2 + a^2} - \sqrt{y^2 + b^2},$$

$$h_{abc} = \frac{(f_{ab})^2 - (c^2 + x^2 + y^2)}{2\sqrt{x^2 + a^2}\sqrt{y^2 + b^2}},$$

$$f_{bc} = 2 - \sqrt{x^2 + b^2} - \sqrt{y^2 + c^2},$$

$$h_{bca} = \frac{(f_{bc})^2 - (a^2 + x^2 + y^2)}{2\sqrt{x^2 + b^2}\sqrt{y^2 + c^2}},$$

$$f_{ac} = 2 - \sqrt{x^2 + a^2} - \sqrt{y^2 + c^2},$$

$$h_{acb} = \frac{(f_{ac})^2 - (b^2 + x^2 + y^2)}{2\sqrt{x^2 + a^2}\sqrt{y^2 + c^2}},$$

$$h_{xa} = [1 - (x^2/(x^2 + a^2))]^{1/2},$$

$$h_{yc} = [1 - (y^2/(y^2 + c^2))]^{1/2},$$

$$h_{xb} = [1 - (x^2/(x^2 + b^2))]^{1/2},$$

$$h_{yb} = [1 - (y^2/(y^2 + b^2))]^{1/2}. \quad \text{(A8)}$$

Also:

$$h_{xa} = [1 - (x^2/(x^2 + a^2))]^{1/2},$$

$$h_{yc} = [1 - (y^2/(y^2 + c^2))]^{1/2}. \quad \text{(A9)}$$

APPENDIX B

Two Boson System Spectator Model

The differential decay rates for a two boson system in the spectator quark model for current-current plus penguin operators in the effective Hamiltonian was obtained, where:

$$\zeta_{ps(q,z)}^{\text{eff}} = \alpha_1 \zeta_1^{\text{eff}} + \alpha_2 \zeta_2^{\text{eff}} - \alpha_3 \zeta_3^{\text{eff}}. \quad \text{(B1)}$$

The integration region is restricted by condition $\cos\theta_{is} \le 1$, thus:

$$\zeta_1^{\text{eff}}, \zeta_2^{\text{eff}}, \zeta_3^{\text{eff}} = \begin{cases} \zeta_{1ps}^{\text{eff}}, \zeta_{2ps}^{\text{eff}}, \zeta_{3ps}^{\text{eff}} & \text{if } (f_{si(z)})^2 \le 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{(B2)}$$

where:

$$f_{si(z)} = \Big([(m_i + m_s)/M_c]^2 - (q_{si}/M_c)^2 + (1/M_c)\sqrt{m_s^2 + (\beta\Lambda z)^2} \times \sqrt{(2m_i/M_c)^2 + x^2}\Big)/(\beta\Lambda xz/M_c).$$

Therefore, using Equation A7, the phase space parameters will be defined by:

$$\zeta_{1ps}^{\text{eff}} = 6xy.f_{ab}.(1 - h_{abc}),$$

$$\zeta_{2ps}^{\text{eff}} = 6xy.f_{bc}.(1 + h_{bca}),$$

$$\zeta_{3ps}^{\text{eff}} = 6xy.f_{ac}.h_{xa}.h_{yc}. \quad \text{(B3)}$$

APPENDIX C

Processes C-F Plus C-S ($F+S$)

Adding the amplitudes of two terms for b quark spin 1/2 and -1/2 gives:

$$[(\tilde{\sigma}^{\mu})(\tilde{\sigma}_{\mu})]^{F+S}_{1/2} = A_1[\sin((\theta_k-\theta_j-\theta_i)/2) + \sin((\theta_k+\theta_j-\theta_i)/2)]$$
$$A_2[\sin((\theta_j-\theta_k-\theta_i)/2) + \sin((\theta_k-\theta_j-\theta_i)/2)]$$
$$A_1[\sin((\theta_j-\theta_k-\theta_i)/2) + \sin((\theta_j+\theta_k-\theta_i)/2)]$$
$$A_3[\sin((\theta_k-\theta_j-\theta_i)/2) + \sin((\theta_j-\theta_k-\theta_i)/2)].$$
$$[(\tilde{\sigma}^{\mu})(\tilde{\sigma}_{\mu})]^{F+S}_{-1/2} = A_1[\cos((\theta_k-\theta_j-\theta_i)/2) - \cos((\theta_k+\theta_j-\theta_i)/2)]$$
$$A_2[\cos((\theta_j-\theta_k-\theta_i)/2) + \cos((\theta_k-\theta_j-\theta_i)/2)]$$
$$A_1[\cos((\theta_j-\theta_k-\theta_i)/2) - \cos((\theta_j+\theta_k-\theta_i)/2)]$$
$$A_3[\cos((\theta_k-\theta_j-\theta_i)/2) + \cos((\theta_j-\theta_k-\theta_i)/2)]. \quad \text{(C1)}$$

Here A_1, A_2 and A_3 refer to the colour and the helicity factor,

$$A_1 = (\sqrt{\alpha_1}/4)\sqrt{1+v_i}\sqrt{1+v_k}\sqrt{1+v_j},$$
$$A_2 = (\sqrt{\alpha_2}/4)\sqrt{1-v_i}\sqrt{1+v_k}\sqrt{1-v_j},$$
$$A_3 = (\sqrt{\alpha_2}/4)\sqrt{1-v_i}\sqrt{1-v_k}\sqrt{1+v_j}. \quad \text{(C2)}$$

We must square these terms and, by averaging over the b quark spin 1/2 and -1/2, we have:

$$([(\tilde{\sigma}^{\mu})(\tilde{\sigma}_{\mu})]^{F+S}_{sp-av})^2 = [(3/2)\alpha_1 + \alpha_2 - \alpha_3\sqrt{1-v_i^2}\sqrt{1-v_k^2} - \alpha_1 v_i v_k \cos(\theta_k-\theta_i)]$$
$$- [\alpha_3\sqrt{1-v_i^2}\sqrt{1-v_j^2} + \alpha_1 v_i v_j \cos(\theta_j-\theta_i)]$$
$$+ [\alpha_1\sqrt{1-v_k^2}\sqrt{1-v_j^2} + (1/2)(\alpha_1-2\alpha_2)v_k v_j \cos(\theta_k-\theta_j)], \quad \text{(C3)}$$

where α_1, α_2 and α_3 are defined by Equation A3.

The decay rates of current-current plus penguin for $F+S$ was obtained such that:

$$I^{F+S}_{pc} = I_{1ps} + I_{2ps} + I_{3ps}, \quad \text{(C4)}$$

where:

$$I_{1ps} = 6xy.f_{ab}.[\alpha_1((3/2)-h_{abc}) + \alpha_2 - \alpha_3 h_{xa}h_{yb}],$$
$$I_{2ps} = -6xy.f_{ac}.[\alpha_1 h_{acb} + \alpha_3 h_{xa}h_{yc}],$$
$$I_{3ps} = 6xy.f_{bc}.[(\alpha_1/2)h_{bca} + \alpha_2(h_{xb}h_{yc} - h_{bca})]. \quad \text{(C5)}$$