Scientia Iranica, Vol. 15, No. 2, pp 170–174

SCIENTIA
IRANICA

GLR Detector for Coded Signals in Noise and Interference

A.A. Tadaion*, M. Derakhtian[1], M.M. Nayebi[2] and M.R. Aref[2]

In this paper, the detection of a coded signal in additive white Gaussian noise and the interference is studied, where there is no knowledge about the correlation between the received symbols and about the noise and interference parameters. The Maximum Likelihood (ML) estimates of the unknown parameters are found, they are substituted in the probability density functions of the observation and the Generalized Likelihood Ratio (GLR) detector is derived. This detector can also be used for the activity detection of a signal in unknown Inter-Symbol Interference (ISI). In this case, the interference is modeled as the unknown correlation between the received symbols. Simulation examples are performed to evaluate the performance of the proposed detector.

INTRODUCTION

Detection methods for signals with correlation between the received symbols in noise and interference are of special importance in the implementation of a communication system. Signal activity detection is an important stage in real communication systems, since the performance of the succeeding stages, such as demodulation and decoding, depends on correct knowledge of the signal activity. The signal symbols are either independent or correlated with some known/unknwon properties. The correlation between the received symbols in a transmission system may be because of the coding on the transmitted information or due to the Inter-Symbol Interference (ISI) imposed on the signal, as the signal goes through the channel. Some attempts are made to propose efficient detectors, in terms of performance and computational complexity, for such a detection problem in a variety of applications, including spectrum management and surveillance, signal confirmation and some other intelligence gathering activities, interference identification, modulation classification and, also, in electronic warfare and threat analysis (e.g., see [1-9] and the references therein).

The traditional method of detecting the presence of a signal is the energy detector, in which the energy level of the received signal is compared with a predetermined threshold, to detect whether a signal is active or not [3]. However, such a method is susceptible to the a priori knowledge of the noise variance and the interference [1]. It must be noted that most of the existing methods for signal presence detection assume that the signal, or some of its parameters, and/or the noise parameters are known, while, in practice, they may not be available to the receiver. In addition, mostly, the presence detection problem has not been studied in cases where the received symbols are correlated. The authors have proposed Generalized Likelihood Ratio (GLR) detectors for the activity detection of a Phase Shift Keying (PSK) signal with unknown amplitude and phase in unknown white Gaussian noise [10].

In this paper, the signal presence detection in Additive White Gaussian Noise (AWGN) is studied, where the received symbols are correlated. The GLR detector for this problem is derived. It is noted that the well-known problem of the activity detection of a signal in noise and ISI can also be formulated and solved by the proposed detector. Therefore, the recent problem is formulated and the proposed GLR detector is applied to solve it.

The remainder of the paper is organized as follows. First, the signal presence detection problem is formulated as a binary hypothesis test. Then, the Maximum Likelihood (ML) estimates of the unknown parameters are founded and, by substituting them in the Likelihood Ratio (LR), the GLR detector is derived. After that, the presence detection of a signal in ISI is studied and, assuming that the channel is

*. *Corresponding Author, Department of Electrical Engineering, Yazd University, Yazd, I.R. Iran.*

1. *Department of Electrical Engineering, Shiraz University, Shiraz, I.R. Iran.*

2. *Department of Electrical Engineering, Sharif University of Technology, Tehran, I.R. Iran.*

unknown, the GLR solution is proposed. No assumption is made of the signal in this derivation and some simulation examples and their results are reported to evaluate the performance of the presented detector. Finally, the paper is concluded.

PROBLEM FORMULATION

The desired problem is modeled as a binary hypothesis testing problem. In hypothesis $\mathcal{H}_0$, the samples of the white Gaussian noise that are independent are received. In this hypothesis, no assumption of receiving the signal is made. The white Gaussian noise samples are of the zero mean and unknown variance, σ^2. In hypothesis $\mathcal{H}_1$, the signal is received in AWGN. The transmitted symbols are assumed to be chosen from a symmetric constellation, such as a PSK family or some QAM (Quadrature Amplitude Modulation), such as 16QAM. The assumption is that the received symbols are correlated. For instance, the transmitter uses a coding that is unknown for the receiver and, so one can assume that the received symbols are the samples of a Gaussian process with zero mean and unknown covariance matrix R. In this formulation, ISI is not assumed, the receiver noise is additive white Gaussian and independent of the signal. The hypothesis testing problem is, then, as follows:

$$\begin{cases} \mathcal{H}_0 : \mathbf{r}[k] = \mathbf{u}[k], & \text{the signal is absent,} \\ \mathcal{H}_1 : \mathbf{r}[k] = \mathbf{v}[k], & \text{the signal is present,} \end{cases}$$

$$k = 0, \cdots, K-1, \tag{1}$$

where $\mathbf{r}[k] = [r_0[k], \cdots, r_{N-1}[k]]^T$ is the baseband representation of the received signal vector in the kth time interval (that can be anywhere in the received sequence), $\mathbf{u}[k] = [u_0[k], \cdots, u_{N-1}[k]]^T \sim \mathcal{N}(\mathbf{0}, \sigma^2 I_N)$, in hypothesis $\mathcal{H}_0$, is the complex white Gaussian noise with zero mean and unknown variance σ^2. In hypothesis $\mathcal{H}_1$, $\mathbf{v}[k] = [v_0[k], \cdots, v_{N-1}[k]]^T \sim \mathcal{N}(\mathbf{0}, R)$ represents the correlated signal symbols plus AWGN. Since the modulated signal samples are correlated, due to the coding applied to them, the signal plus noise in hypothesis $\mathcal{H}_1$ is modeled as a Gaussian process with zero mean and covariance matrix R. In addition, the observation data is considered as some N-blocks, which is necessary for the estimation of the covariance matrix, R (see e.g. [11]).

GENERALIZED LIKELIHOOD RATIO DETECTOR FOR CODED SIGNALS

According to the Neyman-Pearson criteria, the optimal test is obtained by constructing the LR and comparing it with a threshold. This threshold is adjusted, such that the Probability of False Alarm, (P_{fa}), be less than a predetermined value. In composite hypothesis testing problems, where one or both of the hypotheses contain unknown parameters, if the constructed test maximizes the Probability of Detection (P_{d}) for all values of the unknown parameters, the detector is Uniformly Most Powerful (UMP) test. However, the UMP test does not exist for all composite hypothesis testing problems [12]. Therefore, the UMP tests are looked for within the class of invariant tests or unbiased tests, namely, UMP Invariant (UMPI) or UMP Unbiased (UMPU) tests, respectively. The other well-known test is the GLR test, which mostly performs close to optimum [10]. In this detector, the ML estimates of the unknown parameters are substituted in the likelihood ratio and the resulting likelihood ratio is compared with a threshold [13].

In the following, to derive the GLR detector for Equation 1, the ML estimates of the unknown parameters, σ^2 and R are substituted, under each hypothesis in the probability density functions (pdfs) of the observation under each hypothesis and the LR is constructed. The pdf of the observation signals under the hypotheses, $\mathcal{H}_0$ and $\mathcal{H}_1$, are as follows:

$$f(\mathbf{r}[0], \cdots, \mathbf{r}[K-1]; \mathcal{H}_0) = \frac{1}{(2\pi\sigma)^{2KN}} \exp\left\{-\frac{1}{\sigma^2}\sum_{k=0}^{K-1} \mathbf{r}^H[k]\mathbf{r}[k]\right\}, \tag{2a}$$

$$f(\mathbf{r}[0], \cdots, \mathbf{r}[K-1]; \mathcal{H}_1) = \frac{1}{((2\pi)^{2N}|R|)^K} \exp\left\{-\sum_{k=0}^{K-1} \mathbf{r}^H[k]R^{-1}\mathbf{r}[k]\right\}, \tag{2b}$$

where $(.)^H$ is the conjugate transpose and $|.|$ is the determinant of the matrix. In the following, the ML estimates of the unknown parameters, σ^2 and R, will be found. Taking the natural logarithm of both sides of Equation 2b, one has:

$$\ln f(\mathbf{r}[0], \cdots, \mathbf{r}[K-1]; \mathcal{H}_0) = -KN\ln(4\pi^2\sigma^2) - \frac{1}{\sigma^2}\sum_{k=0}^{K-1} \mathbf{r}^H[k]\mathbf{r}[k], \tag{3}$$

and by taking the derivation of the recent equation, with respect to σ^2, the ML estimate of the σ^2 will be as follows:

$$\widehat{\sigma^2} = \frac{1}{KN}\sum_{k=0}^{K-1} \mathbf{r}^H[k]\mathbf{r}[k]. \tag{4}$$

In order to find the ML estimate of R, by taking the natural logarithm of both sides of Equation 2b and since one has:

$$\sum_{k=0}^{K-1} \mathbf{r}^H[k] R^{-1} \mathbf{r}[k] = \text{trace}\left(R^{-1} \sum_{k=0}^{K-1} \mathbf{r}[k]\mathbf{r}[k]^H \right),$$

the following is reached:

$$\begin{aligned} &\ln f(\mathbf{r}[0], \cdots, \mathbf{r}[K-1]; \mathcal{H}_1) \\ &\quad = -2NK \ln(2\pi) - K \ln(|R|) \\ &\quad - \text{trace}\left(R^{-1} \sum_{k=0}^{K-1} \mathbf{r}[k]\mathbf{r}[k]^H \right). \end{aligned} \tag{5}$$

If one takes the derivation of the recent equation with respect to R, from the identities $(\partial/\partial R) \ln(|R|) = R^{-1}$ and:

$$\begin{aligned} &\frac{\partial}{\partial R}\text{trace}\left(R^{-1} \sum_{k=0}^{K-1} \mathbf{r}[k]\mathbf{r}[k]^H \right) \\ &\quad = R^{-1} \sum_{k=0}^{K-1} \mathbf{r}[k]\mathbf{r}[k]^H R^{-1}, \end{aligned}$$

one obtains:

$$\hat{R} = \frac{1}{K} \sum_{k=0}^{K-1} \mathbf{r}[k]\mathbf{r}^H[k]. \tag{6}$$

Substituting Equaion 4 in Equation 2a and Equation 6 in Equation 2b, respectively, the likelihood ratio is as follows:

$$\begin{aligned} &\frac{f(\mathbf{r}[0], \cdots, \mathbf{r}[K-1]; \mathcal{H}_1)|_{(6)}}{f(\mathbf{r}[0], \cdots, \mathbf{r}[K-1]; \mathcal{H}_0)|_{(4)}} = \frac{\left(\widehat{\sigma^{2N}}\right)^K e^{-N}}{|\hat{R}|^K e^{-NK}} \\ &= \left(\frac{\left(\frac{1}{NK} \sum_{k=0}^{K-1} \mathbf{r}^H[k]\mathbf{r}[k] \right)^N}{\frac{1}{K} \left| \sum_{k=0}^{K-1} \mathbf{r}[k]\mathbf{r}^H[k] \right|} \right)^K e^{N(K-1)}, \end{aligned} \tag{7}$$

where $a|_{(.)}$ means substituting the results of equations numbered by (.) in the expression of a. After removing constants and, since x^{KN} is an increasing function of x, comparing the above likelihood ratio with the threshold, T^{KN}, is equivalent to comparing:

$$\frac{\frac{1}{NK} \sum_{k=0}^{K-1} \mathbf{r}^H[k]\mathbf{r}[k]}{\left(\frac{1}{K} \left| \sum_{k=0}^{K-1} \mathbf{r}[k]\mathbf{r}^H[k] \right| \right)^{\frac{1}{N}}},$$

with threshold T. By dropping the constants (that can be absorbed in the threshold) and taking the Nth root, the GLR test rejects $\mathcal{H}_0$, if:

$$L_{\text{GLR}}(\mathbf{r}) = \frac{\text{trace}(S)}{|S|^{\frac{1}{N}}} > \eta_{\text{GLR}}, \tag{8}$$

where $S = \sum_{k=0}^{K-1} \mathbf{r}[k]\mathbf{r}^H[k]$ and trace(.) is the trace of the matrix. Therefore, in order to determine if the received signal does contain a coded signal or is only noise, one can construct the matrix, S, and compare the ratio of Equation 8 with a threshold. The threshold, η_{GLR}, in this detector is obtained, based on the maximum allowable value of the probability of false alarm, P_{fa}; i.e. the threshold is selected, such that the P_{fa} does not exceed a pre-determined value. It must be noted that the detection threshold is independent of the SNR and can be obtained by a Monte-Carlo simulation.

The model used in this problem can also be used for the detection of a signal that is passed through an ISI channel. In the following section, the corresponding detector is proposed.

GENERALIZED LIKELIHOOD RATIO DETECTOR IN THE PRESENCE OF ISI

In this case, in the null hypothesis, $\mathcal{H}_0$, one only receives noise and, in the other hypothesis, $\mathcal{H}_1$, he receives the signal that is passed through an ISI channel. Assuming that one sample is available at each symbol interval, the received signal is:

$$r_p = \sum_{l=0}^{L-1} h_l s_{p-l} + n_p, \tag{9}$$

where s_p are the received symbols and $h_l, l = 0, \cdots, L-1$ are the unknown channel coefficients of a Finite Impulse Response (FIR) channel. The elements of the covariance matrix of the received signal in hypothesis $\mathcal{H}_1$ are as follows:

$$\begin{aligned} E(r_p r_m^H) &= \sum_{l=0}^{L-1} \sum_{t=0}^{L-1} h_l h_t^H E(s_{p-l} s_{m-t}^H) + \sigma^2 \delta[p-m] \\ &= \sum_{t=0}^{L-1} h_{t+(p-m)} h_t^H + \sigma^2 \delta[p-m]. \end{aligned} \tag{10}$$

For instance, if $L = 3$, the covariance matrix of the received signal is as follows:

$$R = \begin{bmatrix} |h_0|^2+|h_1|^2+|h_2|^2+\sigma^2 & h_0 h_1^H + h_1 h_2^H & h_0 h_2^H \\ h_1 h_0^H + h_2 h_1^H & |h_0|^2+|h_1|^2+|h_2|^2+\sigma^2 & h_0 h_1^H + h_1 h_2^H \\ h_2 h_0^H & h_1 h_0^H + h_2 h_1^H & |h_0|^2+|h_1|^2+|h_2|^2+\sigma^2 \end{bmatrix}. \tag{11}$$

It must be noted that the symbols of the received signal are considered as independent random variables. The hypothesis testing problem will then be:

$$\begin{cases} \mathcal{H}_0 : \mathbf{r}[k] = \mathbf{n}[k], \\ \mathcal{H}_1 : \mathbf{r}[k] = \mathbf{w}[k], \end{cases} \qquad k = 0, \cdots, K-1, \tag{12}$$

where $\mathbf{n}[k] = [n_0[k], \cdots, n_{N-1}[k]]^T \sim \mathcal{N}(\mathbf{0}, \sigma^2 I_N)$ and $\mathbf{w}[k] = [w_0[k], \cdots, w_{N-1}[k]]^T \sim \mathcal{N}(\mathbf{0}, R)$. The assumption is that $\mathbf{w}[k]$ and $\mathbf{w}[m]$ for $0 \leq k \neq m \leq K-1$ are mutually independent. In order to satisfy this condition, one can choose the N-vectors, $\mathbf{r}[k]$, from the received signal, such that they are $N+L$ apart. Note that $\mathbf{r}[k], 1 \leq k \leq K-1$ are used as the secondary data for the estimation of the covariance matrix and, since they can be chosen anywhere in the received signal sequence, they are chosen in such a way that the first components of $\mathbf{r}[k]$ and $\mathbf{r}[k+1]$, i.e. $r_0[k]$ and $r_0[k+1]$ are, at least, $N+L$ apart from each other in the received signal sequence.

Since there is no assumption of the interference, this detector can also detect the coded signals in unknown interference. Obviously, compared to the previous situations, the lower performance in this case is expected, as the number of unknown parameters is increased.

SIMULATION RESULTS

The performance of the proposed detector is evaluated by simulations. In the authors' simulations, the threshold in each test is determined experimentally, as follows: The decision statistics for 10^5 independent trials in the absence of signal were sorted in ascending order and the threshold was chosen as the $\%100 \times P_{\text{fa}}$-percentile of the resulting data. For example, for $P_{\text{fa}} = 0.01$, the threshold is chosen as the $0.01 \times 10^5 = 10^3$th ordered data; i.e., such that $\%100 \times P_{\text{fa}}$ of the decision statistics are above the threshold. This threshold is independent of the unknown parameters. Since the threshold is obtained in the null hypothesis, $\mathcal{H}_0$, and in this hypothesis, if one divides both the numerator and denominator by σ^2, it is as if the received vectors are divided by σ and are of distribution $\mathcal{N}(\mathbf{0}, I_N)$, which is independent of the unknown parameters. Such a detector is called a Constant False Alarm Rate (CFAR) detector. In Figure 1, it is observed that the proposed detector (Equation 8) is CFAR, compared with the energy detector. It is shown that, in contrast to the energy detector, the performance of the proposed GLR detector is constant as the noise variance varies.

Other simulation examples are from the high application problem of the signal detection in ISI. The channel assumed is a channel with the high interference of length 3 and the coefficients $\mathbf{h} = [1, 0.9, 0.8]$. Figure 2 shows the probability of detection versus the signal amplitude for some values of $N = 20, 30$. In these simulations, $P_{\text{fa}} = 0.01$ is assumed and the transmitter uses the Binary PSK (BPSK) modulation. It is observed that, as N increases, the performance of the detector improves. Similar simulations are performed for Quadrature PSK (QPSK) modulations in Figure 3.

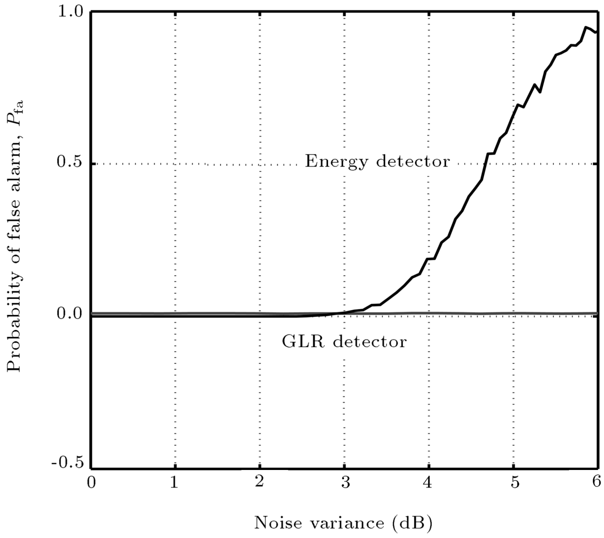

Figure 1. Performance comparison of the proposed GLR and the energy detector in terms of the probability of false alarm versus the noise variance.

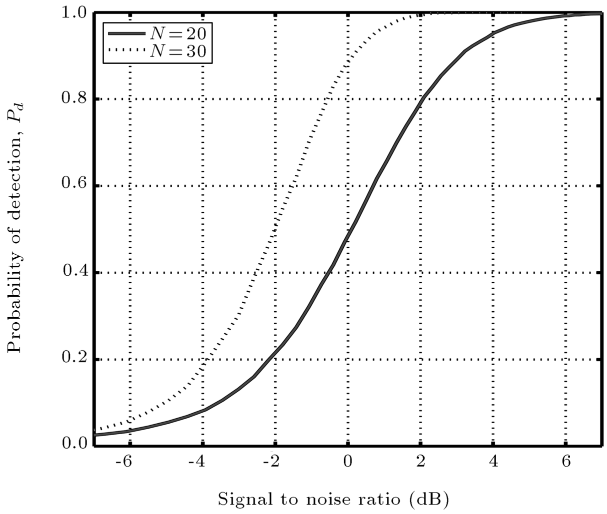

Figure 2. Performance comparison of the proposed GLR for the BPSK signal detection in noise and unknown interference for different values of $N = 20$, 30 and $P_{\text{fa}} = 0.01$.

CONCLUSION

In this paper, the presence detection of a coded signal is studied in cases where there is no knowledge about the correlation between the received symbols. A proper

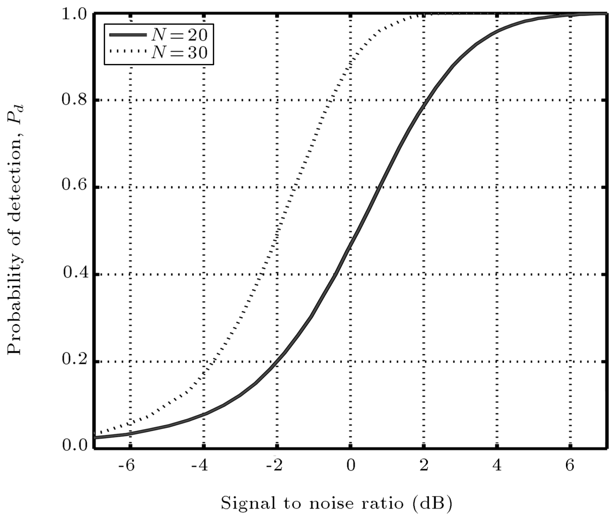

Figure 3. Performance comparison of the proposed GLR for the QPSK signal detection in noise and unknown interference for different values of $N = 20$, 30 and $P_{\text{fa}} = 0.01$.

model is proposed for the problem and the GLR detector is derived by substituting the ML estimates of the unknown parameters in the likelihood ratio. In the authors' assumptions, the noise variance, the received signal amplitude and the coding are unknown. In addition, the signal detection in ISI is described, similar to the previous problem, and the GLR detector is derived. The performance of the proposed detector is evaluated by some simulation examples. It is shown that the authors detector is CFAR, compared with the well-known energy detector.

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