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Unaxisymmetric Heat Transfer in the Axisymmetric Stagnation-Point Flow of a Viscous Fluid on a Cylinder with Simultaneous Axial and Rotational Movement Along with Transpiration

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The unaxisymmetric heat transfer of an unsteady viscous flow, in the vicinity of an axisymmetric stagnation-point of an infinite circular cylinder, with simultaneous axial and rotational movement, along with transpiration, U_o, is investigated, when the angular velocity, axial velocity and wall temperature or wall heat flux vary arbitrarily with time. The impinging free stream is steady and with a strain rate of $\bar{k}$. An exact solution of the Navier-Stokes equations and energy equation is derived in this problem. A reduction of these equations is obtained by the use of appropriate transformations for the most general case, when the transpiration rate is also time-dependent. However, results are presented only for uniform values of this quantity. The general self-similar solution of unsteady unaxisymmetric heat transfer is obtained, in which unaxisymmetry is due to the sinusoidal variation of the temperature, with respect to the surface position of the rotating cylinder, and unsteadiness is because of the sinusoidal variation of the temperature of each point of the cylinder surface, with respect to time and, also the rotation of the cylinder. All the solutions are presented for Reynolds numbers, $\mathrm{Re} = \bar{k}a^2/2\nu$, ranging from 0.1 to 10.0 for different values of Prandtl number and selected values of the dimensionless transpiration rate, $S = U_0/\bar{k}a$, where a is the cylinder radius and ν is the kinematic viscosity of the fluid. The local coefficient of heat transfer (Nusselt number) is found to be independent of time and place, though the cylinder wall temperature or wall heat flux are both functions of time and place.

INTRODUCTION

The problem of finding the exact solutions of Navier-Stokes equations is a very difficult task. This is primarily due to the fact that these equations are nonlinear. An exact solution of these equations, governing the problem of a two-dimensional stagnation flow against a flat plate, has been given by Hiemenz [1]. Howarth [2] and Davey [3] presented results for asymmetric cases of stagnation flow against a flat plate. The first exact solution to the problem of an axisymmetric stagnation flow on an infinite circular cylinder was obtained by Wang [4]. Gorla [5-9], in a series of papers, studied steady and unsteady flows and heat transfer over a circular cylinder in the vicinity of the stagnation-point for the cases of constant axial movement and the special case of the axial harmonic motion of a non-rotating cylinder. In more recent years, Cunning, Davis, and Weidman [10] have considered the stagnation flow problem on a rotating circular cylinder with a constant angular velocity. They have also included the effects of suction and blowing in their study. Takhar, Chamkha and Nath [11] have investigated the unsteady viscous flow in the vicinity of an axisymmetric stagnation point of an infinite circular cylinder, for the particular case when both the axial velocity of the cylinder and the free stream velocity vary, inversely, as a linear function of time. The study considered by Saleh and Rahimi [12] presents the axisymmetric stagnation-point flow and heat transfer of a viscous fluid on a moving cylinder, with time-dependent axial velocity and uniform transpiration. The effects of axial and rotational movements, simultaneously, and unaxisymmetric thermal

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loading in an axisymmetric stagnation point flow on a cylinder have not yet been investigated. These studies are perhaps of interest in cooling and centrifugal processes in industry, calcinations of cement and in the accelerating phases of rocket motors, in which axial and rotational movements are simultaneously present etc. Other different types of motion of a cylinder have applications, for example, in steady state cooling processes, start up and stopping stages of centrifugal processes and in sinusoidal blenders in industry.

In the present analysis, the problem of unaxisymmetric heat transfer in the axisymmetric stagnation-point flow of a viscous fluid on a cylinder with simultaneous axial and rotational movement along with transpiration is considered. An exact solution of the Navier-stokes equations and the energy equation is obtained. The general, self-similar solution of unsteady unaxisymmetric heat transfer is obtained, in which unaxisymmetry is because of the sinusoidal variation of temperature, with respect to the surface position of a rotating cylinder and in which unsteadiness is because of the sinusoidal variation of the temperature of each point of the cylinder surface, with respect to the time and also rotation of the cylinder. The solutions are presented for different values of Reynolds and Prandtl number and the velocity component of the flow and selected values of the dimensionless temperature function. Particular cases of these results are compared with the existing results of Gorla [6,8,9].

PROBLEM FORMULATION

The unaxisymmetric heat transfer of the unsteady incompressible flow of a viscous fluid in the neighborhood of the axisymmetric stagnation-point of an infinite circular cylinder with simultaneous axial and rotational movement with uniform normal transpiration, U_o, at its surface is considered, where $U_o > 0$ corresponds to suction into the cylinder, although the formulation of the problem is for the more general case of a time-dependent transpiration rate. The flow configuration is shown in Figure 1 in cylindrical coordinates (r, ϕ, z) with corresponding velocity components (u, v, w). The cylinder has a simultaneous time-dependent rotation and axial movement and the wall temperature or the wall heat flux is also a function of time. A radial external flow of strain rate $\overline{k}$ impinges on the cylinder of radius, a, centered at $r = 0$. The unsteady Navier-Stokes and energy equations in the cylindrical polar coordinates governing the axisymmetric flow and unaxisymmetric heat transfer are given by:

Mass:

$$\frac{\partial}{\partial r}(ru) + r\frac{\partial w}{\partial z} = 0. \tag{1}$$

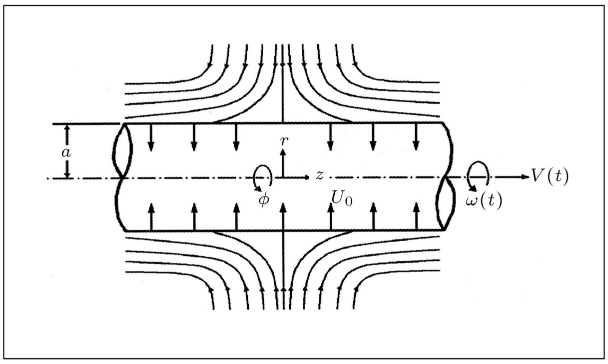

Figure 1. Schematic diagram of a cylinder with simultaneous axial and rotational movement under radial stagnation flow in the fixed cylindrical coordinates system $(r, \phi z)$.

Momentum:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} - \frac{v^2}{r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right), \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{uv}{r} + w\frac{\partial v}{\partial z} = \nu\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2}\right), \tag{3}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right). \tag{4}$$

Energy:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + \frac{v}{r}\frac{\partial T}{\partial \phi} + w\frac{\partial T}{\partial z} = \overline{\alpha}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}\right), \tag{5}$$

where P, ρ, v and $\overline{\alpha}$ are the fluid pressure, density, kinematic viscosity and thermal diffusivity. Because of the infinite length of the cylinder, the variations of flow quantities are not a function of z. The boundary conditions for the velocity field are as follows:

$r = a$:

$$u = -U_o(t), \qquad v = a.\omega(t), \qquad w = V(t), \tag{6}$$

$r \to \infty$:

$$\frac{\partial u}{\partial r} = -\overline{k}, \qquad \lim it_{r\to\infty} rv = 0, \qquad w = 2\overline{k}z. \tag{7}$$

Here, Relation 6 are transpiration and no-slip boundary conditions on the cylinder wall, where $U_o(t)$ is the transpiration rate, $\omega(t)$ is the angular velocity and $V(t)$ the axial velocity of the cylinder. Relations 7 show that the viscous flow solution approaches, in a manner analogous to the Hiemens flow, the potential stagnation field, as $r \to \infty$ [10]. The presence of the stagnation flow allows the condition of zero circulation at infinity to be imposed on the swirl velocity. This form of boundary condition is only from the mathematical point of view, however, from a physical viewpoint, the flow can usually be considered inviscid, when r is far enough away from the cylinder surface ($r >> a$).

For the temperature field, one has:

$r = a:$

i) $T = T_w(\phi, T)$ for the defined wall temperature,

ii) $\dfrac{\partial T}{\partial r} = -\dfrac{q_w(\phi, t)}{k}$ for the defined wall heat flux,

$$r \to \infty: \qquad T \to T_\infty, \tag{8}$$

and the two boundary conditions with respect to ϕ are:

$$T(r,0,t) = T(r,2\pi,t),$$

$$\frac{\partial T}{\partial \phi}(r,0,t) = \frac{\partial T}{\partial \phi}(r,2\pi,t), \tag{9}$$

where k is the thermal conductivity of the fluid, T_∞ is a constant and $T_w(\phi,t)$ and $q_w(\phi,t)$ are temperature and heat flux at the wall cylinder, respectively.

A reduction of the Navier-Stokes equations is obtained by applying the following transformations:

$$u = -\bar{k}\frac{a}{\sqrt{\eta}}f(\eta,\tau), \qquad v = 2\bar{k}\frac{a}{\sqrt{\eta}}G(\eta,\tau),$$

$$w = 2\bar{k}f'(\eta,\tau)z + H(\eta,\tau), \qquad P = \rho\bar{k}^2 a^2 p, \tag{10}$$

where $\tau = 2\bar{k}t$ and $\eta = (r/a)^2$ are dimensionless time and radial variables and the prime denotes differentiation with respect to η. Transformations in Equations 10 satisfy Equation 1 automatically and their insertion into Equations 2 to 4 yields a coupled system of differential equations in terms of $f(\eta,\tau)$, $G(\eta,\tau)$ and $H(\eta,\tau)$ and an expression for the pressure:

$$\eta f''' + f'' + \mathrm{Re}\left[1 - (f')^2 + ff'' - \frac{\partial f'}{\partial \tau}\right] = 0, \tag{11}$$

$$\eta G'' + \mathrm{Re}\left[fG' - \frac{\partial G}{\partial \tau}\right] = 0, \tag{12}$$

$$\eta H'' + H' + \mathrm{Re}\left[fH' - f'H - \frac{\partial H}{\partial \tau}\right] = 0, \tag{13}$$

$$p - p_0 = -\left[\frac{f^2}{2\eta} + \frac{1}{\mathrm{Re}}f' + 2\left(\frac{z}{a}\right)^2 - \frac{1}{2K^2}\int_0^\eta \frac{G^2(\xi)}{\xi^2}d\xi\right]. \tag{14}$$

In these equations, the prime indicates differentiation, with respect to η, and $\mathrm{Re} = \frac{\bar{k}a^2}{2\nu}$ is the Reynolds number. From Conditions 6 and 7, the boundary conditions for Equations 11 to 13 are as follows:

$\eta = 1:$

$$f = S(\tau), \qquad f' = 0, \qquad G = \Omega(\tau), \qquad H = V(\tau),$$

$\eta \to \infty:$

$$f' = 1, \qquad G = 0, \qquad H = 0, \tag{15}$$

in which, $S(\tau) = \frac{U_0(\tau)}{\bar{k}a}$ and $\Omega(\tau) = \frac{\omega(\tau)}{2\bar{k}}$ are the dimensionless wall transpiration rate and dimensionless angular velocity of the cylinder, respectively.

To transform the energy equation into a non-dimensional form, for the case of a defined wall temperature, one introduces:

$$\Theta(\eta,\phi,\tau) = \frac{T(\eta,\phi,\tau) - T_\infty}{T_w(\phi,\tau) - T_\infty}. \tag{16}$$

Making use of Equations 10 and 16, the energy equation may be written as:

$$\eta\Theta'' + \Theta' + \frac{1}{4\eta}\left(\frac{\partial^2\Theta}{\partial\phi^2}\right) + 2\frac{\partial T_w/\partial\phi}{(T_w - T_\infty)}\frac{\partial\Theta}{\partial\phi} + \frac{\partial^2 T_w/\partial\phi^2}{(T_w - T_\infty)}\Theta + \mathrm{Re.Pr}\left\{\left(f\Theta' - \frac{\partial\Theta}{\partial\tau} - \frac{\partial T_w/\partial\tau}{(T_w - T_\infty)}\Theta\right) - \frac{G}{\eta}\left(\frac{\partial\Theta}{\partial\phi} + \frac{\partial T_w/\partial\phi}{(T_w - T_\infty)}\Theta\right)\right\} = 0, \tag{17}$$

where $\mathrm{Pr} = \frac{\nu}{\alpha}$ is the Prandtl number and the boundary conditions are:

$$\Theta(1,\phi,\tau) = 1, \qquad \Theta(\infty,\phi,\tau) = 0, \tag{18}$$

$$\Theta(\eta,0,\tau) = \Theta(\eta,2\pi,\tau),$$

$$\frac{\partial\Theta}{\partial\phi}(\eta,0,\tau) = \frac{\partial\Theta}{\partial\phi}(\eta,2\pi,\tau). \tag{19}$$

For the case of a defined wall heat flux, one introduces:

$$\Theta(\eta,\phi,\tau)=\frac{T(\eta,\phi,\tau)-T_\infty}{aq_w(\phi,\tau)/2k}. \quad (20)$$

Now, making use of Equations 10 and 20, the energy equation can be written as:

$$\eta\Theta''+\Theta'+\frac{1}{4\eta}\left(\frac{\partial^2\Theta}{\partial\phi^2}+2\frac{\partial q_w/\partial\phi}{q_w}\frac{\partial\Theta}{\partial\phi}+\frac{\partial^2 q_w/\partial\phi^2}{q_w}\Theta\right)$$
$$+\mathrm{Re.Pr}\left\{\left(f\Theta'-\frac{\partial\Theta}{\partial\tau}-\frac{\partial q_w/\partial\tau}{q_w}\Theta\right)\right.$$
$$\left.-\frac{G}{\eta}\left(\frac{\partial\Theta}{\partial\phi}+\frac{\partial q_w/\partial\phi}{q_w}\Theta\right)\right\}=0, \quad (21)$$

with the boundary conditions as follows:

$$\Theta'(1,\phi,\tau)=-1, \qquad \Theta(\infty,\phi,\tau)=0, \quad (22)$$

$$\Theta(\eta,0,\tau)=\Theta(\eta,2\pi,\tau),$$

$$\frac{\partial\Theta}{\partial\phi}(\eta,0,\tau)=\frac{\partial\Theta}{\partial\phi}(\eta,2\pi,\tau). \quad (23)$$

SEMI-SIMILAR SOLUTIONS

Semi-similar Equations 11 to 13, 17 and 21 for different time-dependent functions for $S(\tau)$, $\Omega(\tau)$, $V(\tau)$, $T_w(\phi,\tau)$ and $q_w(\phi,\tau)$ with different choices of Re and Pr numbers, can be solved numerically. In this paper, only self-similar solutions are presented, which appear in the next section.

SELF-SIMILAR SOLUTIONS

Before presenting the self-similar change of variables, it is noted that, for $S(\tau)$ = constant, none of the boundary conditions of Equation 11 are a function of time and by assuming steady-state initial conditions for this equation one has:

$$\tau=0\rightarrow\partial f'/\partial\tau=0.$$

Therefore, in this case, Equation 11 is reduced to the following form:

$$\eta f'''+f''+\mathrm{Re}[1-(f')^2+ff'']=0. \quad (24)$$

Steady-state solutions are obtained by solving this equation. Since none of the boundary conditions on f is time-dependent, then this function does not change with respect to time, and the result of steady-state solution ($\tau=0$) is the same as the solution for all the later times ($\tau>0$). Thus, $f(\eta,\tau)=f(\eta)$ and, consequently, Equation 11 can be reduced to Equation 24.

Semi-similar Equations 12 and 13, as in [12], are reduced to exact differential equations, with the following separation of variables:

$$G(\eta,\tau)=g(\eta)\Omega(\tau),$$

$$\Omega(\tau)=b_1\exp[(\alpha_1+i\beta_1)\tau], \quad (25)$$

$$H(\eta,\tau)=h(\eta)V(\tau),$$

$$V(\tau)=b_2\exp[(\alpha_2+i\beta_2)\tau], \quad (26)$$

$$\eta g''+\mathrm{Re}[fg'-\alpha_1 g-i\beta_1 g]=0, \quad (27)$$

$$\eta h''+h'+\mathrm{Re}[fh'-f'h-\alpha_2 h-i\beta_2 h]=0, \quad (28)$$

in which $i=\sqrt{-1}$ and b, α and β are constants.

The boundary conditions for the Equations 24, 27 and 28 according to Conditions 15 are:

$$\eta=1:$$

$$f=S, \qquad f'=0, \qquad g=1, \qquad h=1,$$

$$\eta\rightarrow\infty:$$

$$f'=1, \qquad g=0, \qquad h=0. \quad (29)$$

To reduce the energy equation to a self-similar form, the following separation of the variable is chosen:

$$\Theta(\eta,\phi,\tau)=\theta(\eta)Q(\phi,\tau). \quad (30)$$

For the boundary Conditions 18, 19, 22 and 23, to admit this separation of the variable, the following conditions must be satisfied:

$$\eta=1:$$

$$T=T_\infty\rightarrow\Theta=1=\theta(1)Q(\phi,\tau)$$

$$\rightarrow\theta(1)=1,\ Q(\phi,\tau)=1, \quad (31)$$

$$\eta\rightarrow\infty:$$

$$T=T_\infty\rightarrow\Theta=0=\theta(\infty)Q(\phi,\tau)$$

$$\rightarrow\theta(\infty)=0 \text{ for a defined wall temperature,}$$

$$\eta=1:$$

$$q=q_\infty\rightarrow\Theta'=-1=\theta'(1)Q(\phi,\tau)$$

$$\rightarrow\theta'(1)=-1, \quad Q(\phi,\tau)=1, \quad (32)$$

$$\eta\rightarrow\infty:$$

$$T=T_\infty\rightarrow\Theta=0=\theta(\infty)Q(\phi,\tau)$$

$$\rightarrow\theta(\infty)=0 \text{ for a defined wall heat flux.}$$

Substituting this result into energy Equations 17 and 21 gives:

$$\eta\theta'' + (1.0 + \mathrm{Re.Pr.}f)\theta' + \left(\frac{1}{4\eta}\frac{\frac{\partial^2 T_w}{\partial\phi^2}}{T_w - T_\infty} - \mathrm{Re.Pr}\left(\Omega\frac{g}{\eta}\frac{\frac{\partial T_w}{\partial\phi}}{T_w - T_\infty} + \frac{\frac{\partial T_w}{\partial\tau}}{T_w - T_\infty}\right)\right)\theta = 0,$$

for a defined wall temperature, (33)

$$\eta\theta'' + (1.0 + \mathrm{Re.Pr.}f)\theta' + \left(\frac{1}{4\eta}\frac{\frac{\partial^2 q_w}{\partial\phi^2}}{q_w} - \mathrm{Re.Pr}\left(\Omega\frac{g}{\eta}\frac{\frac{\partial q_w}{\partial\phi}}{q_w} + \frac{\frac{\partial q_w}{\partial\tau}}{q_w}\right)\right)\theta = 0,$$

for a defined wall heat flux. (34)

In order for Equations 33 and 34 to be self-similar, none of the terms should be a function of dimensionless time and angle ϕ. Therefore, one must have:

1. For the case of a defined wall temperature:

$$T_w(\phi,\tau) - T_\infty = C\exp\{\delta\tau + n(\Omega\tau + \phi)\}, \tag{35}$$

which gives:

$$\frac{\frac{\partial T_w}{\partial\phi}}{T_w - T_\infty} = in, \qquad \frac{\frac{\partial^2 T_w}{\partial\phi^2}}{T_w - T_\infty} = -n^2,$$

$$\frac{\frac{\partial T_w}{\partial\tau}}{T_w - T_\infty} = i(\delta + n\Omega), \tag{35a}$$

2. For the case of a defined wall heat flux:

$$q_w(\phi,\tau) = C\exp\{\delta\tau + n(\Omega\tau + \phi)\}, \tag{36}$$

which gives:

$$\frac{\frac{\partial q_w}{\partial\phi}}{q_w} = in, \qquad \frac{\frac{\partial^2 q_w}{\partial\phi^2}}{q_w} = -n^2,$$

$$\frac{\frac{\partial q_w}{\partial\tau}}{q_w} = i(\delta + n\Omega). \tag{36a}$$

For the quantities in Equations 35a and 36a to be constant, the angular velocity of the cylinder, Ω, must be constant. This means that the cylinder must rotate with a constant angular velocity. Taking the above relations into consideration, the temperature distribution function and heat flux of the cylinder wall have the following changes, with respect to δ, n, Ω and ϕ parameters:

- ϕ: Temperature on the cylinder wall changes as a cosine function,
- Ω: The rotational speed of the cylinder is constant and, thus the position of the surface temperature alternates because of it,
- n: The number of sinusoidal changes on the surface of the cylinder,
- δ: The sinusoidal changes of temperature, with respect to time, on each point of the cylinder,

Finally, substituting the above relations into Equations 33 and 34, the following self-similar equation is obtained for both cases of defined wall temperature and wall heat flux:

$$\eta\theta'' + (1.0 + \mathrm{Re.Pr}f)\theta' - \left[\frac{n^2}{4\eta} + i\mathrm{Re.Pr}\left\{n\Omega\left(\frac{g}{\eta} + 1.0\right) + \delta\right\}\right]\theta = 0. \tag{37}$$

The boundary conditions for this equation are as follows:

$$\eta = 1, \qquad \theta = 1, \qquad \eta \to \infty, \qquad \theta = 0,$$

for defined wall temperature, (38)

$$\eta = 1, \qquad \theta' = 1, \qquad \eta \to \infty, \qquad \theta = 0,$$

for defined wall heat flux. (39)

Equations 24, 27 and 28, along with the boundary Conditions 29, can be solved by using the fourth-order Runge-Kutta method of numerical integration, along with a shooting method.

To solve Equation 37, the following two cases are considered.

Simple Case of Steady-State Unaxisymmetric Heat Transfer

Unsteadiness is because of the temperature boundary conditions (cylinder wall temperature and wall heat flux functions) being a function of time. To remove this unsteadiness, all the coefficients, including δ and Ω, must be zero. Based on this, Equation 37 and the cylinder wall temperature and wall heat flux are simplified as follows:

$$\eta\theta'' + (1.0 + \mathrm{Re.Pr}f)\theta' - \frac{n^2}{4\eta}\theta = 0, \tag{40}$$

$$T_w(\phi) - T_\infty = C\exp(in\phi) = C[\cos(n\phi) + i\sin(n\phi)]$$
$$= A\cos(n\phi), \quad (41)$$

$$q_w(\phi) = C\exp(in\phi) = C[\cos(n\phi) + i\sin(n\phi)]$$
$$= A\cos(n\phi). \quad (42)$$

If $n = 0$, the very simple cases of constant wall temperature and constant heat flux of a cylinder are obtained, which were solved by Gorla [9]. Self-similar Equation 40, along with the boundary Conditions 38 and 39 and knowing the $f(\eta)$ function, was solved, using the fourth-order Runge-Kutta method of numerical integration, along with a shooting method for different values of n, Reynolds number and Prandtl number.

More General Case of Unsteady Unaxisymmetric Heat Transfer

The constant rotational speed, Ω, of the cylinder and the coefficient, δ, presenting the time-dependent variation of temperature of each point of the cylinder surface, are the terms causing unsteadiness. In this case, Equation 37 and boundary Conditions 38 and 39 must be considered with no changes. Considering the dimensionless temperature as:

$$\theta(\eta) = \theta_1(\eta) + i\theta_2(\eta), \quad (43)$$

gives:

$$\begin{cases} \eta\theta_1'' + (1.0 + \text{Re.Pr.}f)\theta_1' - \frac{n^2}{4\eta}\theta_1 \\ \quad +\text{Re.Pr}\left[n\Omega\left(\frac{g}{\eta} + 1.0\right) + \delta\right]\theta_2 = 0 \\ \\ \eta\theta_2'' + (1.0 + \text{Re.Pr.}f)\theta_2' - \frac{n^2}{4\eta} \\ \quad -\text{Re.Pr}\left[n\Omega\left(\frac{g}{\eta} + 1.0\right) + \delta\right]\theta_1 = 0 \end{cases} \quad (44)$$

The boundary conditions become:

$$\eta = 1: \quad \theta = 1 \rightarrow \theta_1 = 1, \quad \theta_2 = 0,$$

$$\eta \rightarrow \infty: \quad \theta = 0 \rightarrow \theta_1 = \theta_2 = 0,$$

for a defined wall temperature case, (45)

$$\eta = 1: \quad \theta' = -1 \rightarrow \theta_1' = -1, \quad \theta_2' = 0,$$

$$\eta \rightarrow \infty: \quad \theta = 0 \rightarrow \theta_1 = \theta_2 = 0,$$

for a defined wall heat flux. (46)

Considering Relations 35a and 36a, $n = 0$, $\delta = 0$, $\Omega = 0$ correspond to the very simple case of a cylinder with a constant wall temperature and a constant wall heat flux obtained by Gorla [9] for the first time. $n = 0$, $\delta \neq 0$ and $\Omega \neq 0$ correspond to the axial axisymmetric heat transfer discussed in [12], in which wall temperature or wall heat flux changes harmonically with the time. $n \neq 0$, $\delta = 0$ and $\Omega = 0$ correspond to a steady unaxisymmetric heat transfer case, in which the temperature variation and heat flux variation on the cylinder wall is considered sinusoidal, as in previous section. $n \neq 0$, $\delta \neq 0$ and $\Omega = 0$ correspond to an unsteady unaxisymmetric heat transfer, in which the unaxisymmetry is because of the sinusoidal variation of the temperature, with respect to the surface position of the rotating cylinder, and the unsteadiness is because of the sinusoidal variation of the temperature of each point of the cylinder surface, with respect to time. $n \neq 0$, $\delta = 0$ and $\Omega \neq 0$ correspond to an unsteady unaxisymmetric heat transfer, in which the unaxisymmetry is because of the sinusoidal variation of the temperature with respect to the surface position of the rotating cylinder, and its unsteadiness is because of the constant rotation of the cylinder and the displacement of temperature of each point of the cylinder surface with respect to time. $n \neq 0$, $\delta \neq 0$ and $\Omega \neq 0$ correspond to the most general case of an unsteady unaxisymmetric heat transfer, in which the unaxisymmetry is because of the sinusoidal variation of the temperature with respect to the surface position of the rotating cylinder, and the unsteadiness is because of the sinusoidal variation of temperature of each point of the cylinder surface, with respect to time and, also the rotation of the cylinder.

The coupled system of Equations 44, along with the boundary Conditions 45 and 46, have been solved by using the fourth-order Runge-Kutta method of numerical integration, along with a shooting method [13], for known values of $f(\eta)$ and $g(\eta)$ functions and different values of n, δ, Ω, Reynolds number and Prandtl number. The results are presented in later sections.

HEAT TRANSFER COEFFICIENT

The local heat transfer coefficient and the rate of heat transfer for the case of a defined wall temperature are given by the following:

$$h = \frac{q_w}{T_w - T_\infty} = \frac{-k\left(\frac{\partial T}{\partial r}\right)_{r=a}}{T_w - T_\infty} = -\frac{2k}{a}\Theta'(1, \phi, \tau),$$

for a semi-similar case,

$$h = h_r + ih_i = -\frac{2k}{a}[\theta_1'(1) + i\theta_2'(1)],$$

for a self-similar case. (47)

Or, in terms of Nusselt number, $\mathrm{Nu} = \frac{ha}{2k}$ gives:

$$\mathrm{Nu} = -\Theta'(1, \phi, \tau) \qquad \text{for a semi-similar case,} \qquad (48)$$

$$\mathrm{Nu} = \mathrm{Nu}_r + i\mathrm{Nu}_i = -[\theta_1'(1) + i\theta_2'(1)],$$

for a self-similar case. (49)

And, finally, the heat flux through the cylinder wall is:

$$q_w = -\frac{2k}{a}\Theta'(1, \phi, \tau).(T_w - T_\infty),$$

for a semi-similar case, (50)

$$q_w = -\frac{2k}{a}C\exp[i\{\delta\tau + n(\Omega\tau + \phi)\}][\theta_1'(1) + i\theta_2'(1)],$$

for a self-similar case. (51)

And, for a defined wall heat flux case:

$$h = \frac{q_w}{T_w - T_\infty} = \frac{2k}{a}\frac{1}{\Theta(1, \phi, \tau)},$$

for a semi-similar case, (52)

$$h = \frac{2k}{a}\left(\frac{1}{\theta_1(1) + i\theta_2(1)}\right) = \frac{2k}{a}\left(\frac{\theta_1(1) - i\theta_2(1)}{\theta_1^2(1) + \theta_2^2(1)}\right),$$

for a self-similar case. (53)

And, in terms of Nusselt number,

$$\mathrm{Nu} = \frac{1}{\Theta(1, \phi, \tau)}, \qquad \text{for a semi-similar case,} \qquad (54)$$

$$\mathrm{Nu} = \frac{ha}{2k} = \left(\frac{1}{\theta_1^2(1) + \theta_2^2(1)}\right) = \left(\frac{\theta_1(1) - i\theta_2(1)}{\theta_1^2(1) + \theta_2^2(1)}\right),$$

for a self-similar case. (55)

And, finally, the temperature distribution is:

$$T_w - T_\infty = \frac{a}{2k}\Theta(1, \phi, \tau).q_w,$$

for a semi-similar case, (56)

$$T_w - T_\infty = \frac{a}{2k}C.\exp[i\{\delta\tau + n(\Omega\tau + \phi)\}][\theta_1(1) + i\theta_2(1)],$$

for a self-similar case. (57)

From Relations 47 and 53, it is clearly seen that for self-similar cases of unsteady unaxisymmetric heat transfer, the local heat transfer coefficient (Nusselt number) is neither a function of time nor place, contrary to the fact that the cylinder wall temperature and wall heat flux are both functions of time and place.

PRESENTATION OF RESULTS

In this section, the results obtained from solving the self-similar Equation 40, along with boundary Conditions 38 and 39, for different values of Reynolds number, Re, Prandtl number, Pr, and n for known values of function, $f(\eta)$, are presented. A fourth-order Runge-Kutta method along with a shooting method has been used. This equation is a somewhat simpler form of Equation 37, in which the heat transfer is unaxisymmetric. Interesting results, regarding the fluid flow in this problem, have been discussed in detail in [12-14]. Here, merely the heat transfer results are discussed and presented.

Sample profiles of the non-dimensional temperature, $\theta(\eta)$, and temperature gradient, $\theta'(\eta)$, in terms of η are presented in Figure 2, for wall temperature function, $T_w - T_\infty = A\cos(n\phi)$, and for a non-dimensional transpiration rate of $S = -1.0, 0.0, 1.0$. As is clear, these profiles are for $n = 0.0, 1.0, 2.0, 5.0$, where n is the cycle number of the wall temperature cosine function and $n = 0$ expresses the state of constant wall temperature. From Figure 2a, the increase of the suction rate of fluid into the cylinder and the increase of n reduce the depth of diffusion of the temperature into the fluid. From this figure, when $n \to \infty$, the thickness of the thermal boundary layer tends toward zero. From Figure 2b, the increase of n and S cause the increase of the absolute value of the profile of the initial temperature and, therefore, the coefficient of heat transfer increases and, thus, the thickness of the thermal boundary layer decreases.

Sample profiles of the non-dimensional temperature, $\theta(\eta)$, and the temperature gradient, $\theta'(\eta)$, in terms of η, are given in Figure 3 for the wall heat flux function, $q_w = A\cos(n\phi)$, and for the non-dimensional transpiration rate of $S = -1.0, 0.0, 1.0$. Here, again, the value of $n = 0.0, 1.0, 2.0, 5.0$ has been used, when n is the cycle number of the cylinder wall heat flux and $n = 0$ expresses a constant wall heat flux. From Figure 3a, again, the increase of the suction rate of the fluid into the cylinder and the increase of n reduce the cylinder wall temperature and, naturally, the depth of diffusion of the temperature field of the fluid adjacent to the cylinder wall decreases. Also, the distribution of the non-dimensional temperature in the fluid, which is in the form of a cosine function, is going to tend to zero in a faster trend as n and S increase. Here, again, as $n \to \infty$, the thickness of the thermal boundary layer tends to zero. From Figure 3b, as n and S increase, the depth of diffusion of the temperature field decreases and, thus, the thickness of the thermal boundary layer decreases.

Sample profiles of the non-dimensional temper-

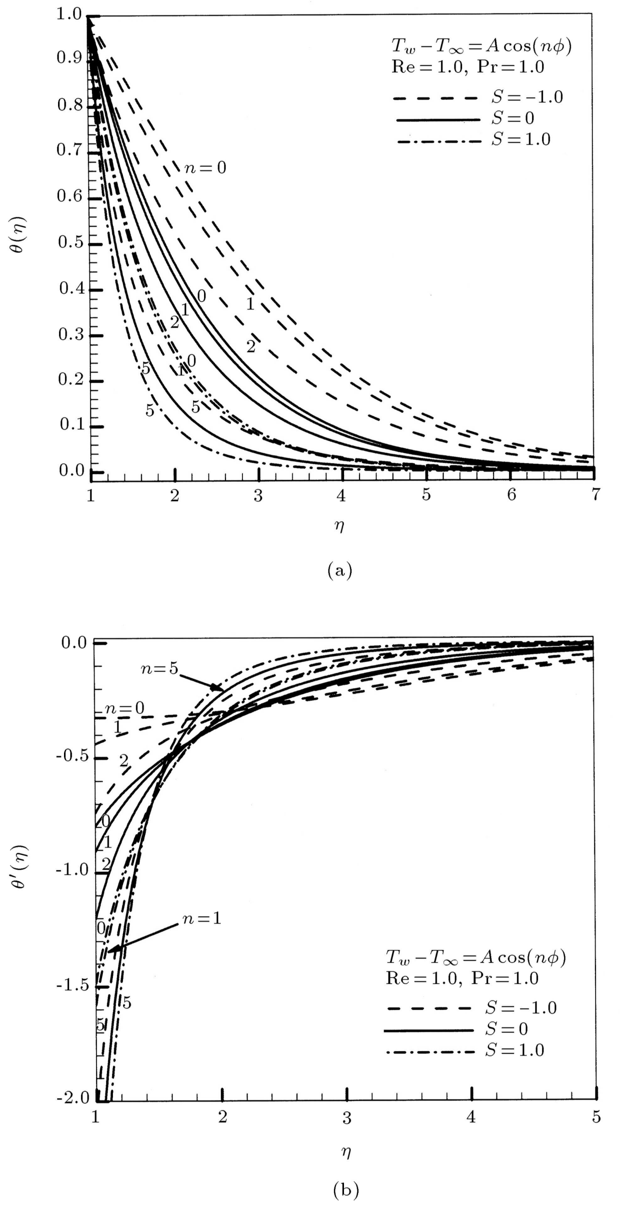

Figure 2. Profiles of (a) $\theta(\eta)$ and (b) $\theta'(\eta)$ for sinusoidal wall temperature function for $S = -1.0, 0.0, 1.0$, Pr = 1, Re = 1 and different values of n.

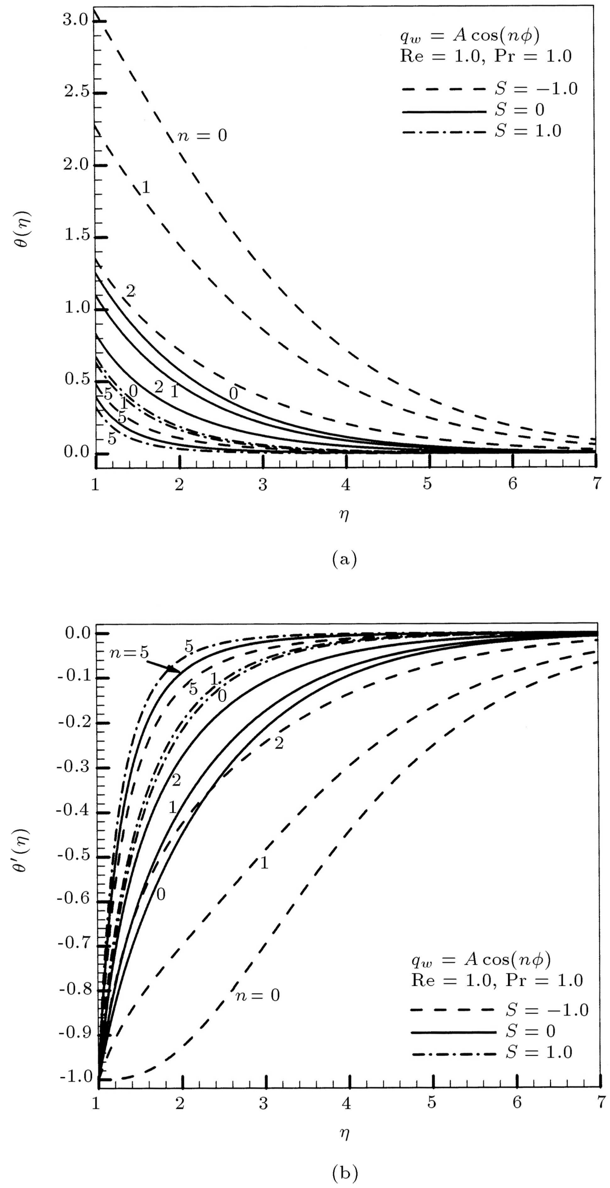

Figure 3. Profiles of (a) $\theta(\eta)$ and (b) $\theta'(\eta)$ for sinusoidal wall heat flux function for $S = -1.0, 0.0, 1.0$, Pr = 1, Re = 1 and different values of n.

ature, $\theta(\eta)$, and the temperature gradient, $\theta'(\eta)$, in terms of η, are depicted in Figure 4 for the wall temperature function, $T_w - T_\infty = A\cos(n\phi)$, and for Prandtl numbers, Pr = 0.5, 1.0, 2.0. Here, the values of $n = 0.0, 1.0, 2.0, 5.0$ have been used. From Figure 4a, the depth of diffusion of the wall temperature adjacent to the cylinder decreases by increasing Prandtl number and value of n, and the variation of the cosine form of the wall temperature is omitted at a lower radial distance. From Figure 4b, the absolute value of the initial slope of the temperature profile increases by increasing the values of $n$and Prandtl number and, therefore, the coefficient of heat transfer increases and the thickness of the thermal boundary layer decreases.

Sample profiles of the non-dimensional temperature, $\theta(\eta)$, and temperature gradient, $\theta'(\eta)$, in terms of η are shown in Figure 5 for the wall heat flux function, $q_w = A\cos(n\phi)$ and for Pr = 0.5, 1.0, 2.0. As can be seen from both of these figures, increasing values of Prandtl number and n causes a reduction in the cylinder wall temperature and its depth of diffusion in the fluid near the cylinder wall, therefore, the thickness of the thermal boundary layer decreases.

The effects of the variation of flow Reynolds number on non-dimensional temperature profiles, $\theta(\eta)$, and temperature gradient, $\theta'(\eta)$, in terms of η for wall temperature and wall heat flux, are presented

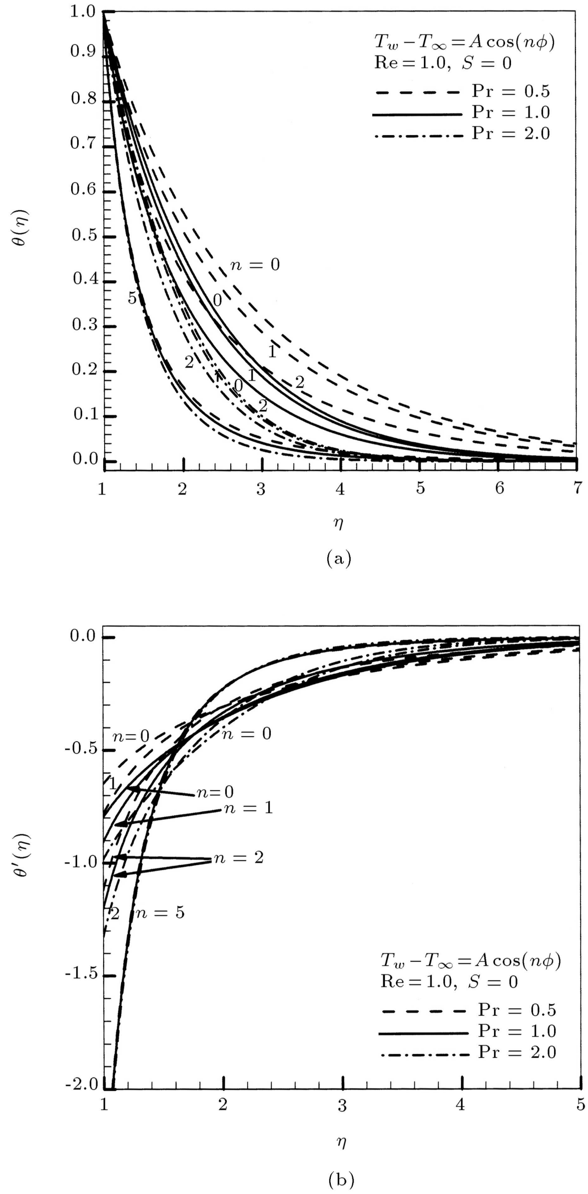

Figure 4. Profiles of (a) $\theta(\eta)$ and (b) $\theta'(\eta)$ for sinusoidal wall temperature function for $S = 0.0$, Pr $= 0.5, 1.0, 2.0$, Re $= 1$ and different values of n.

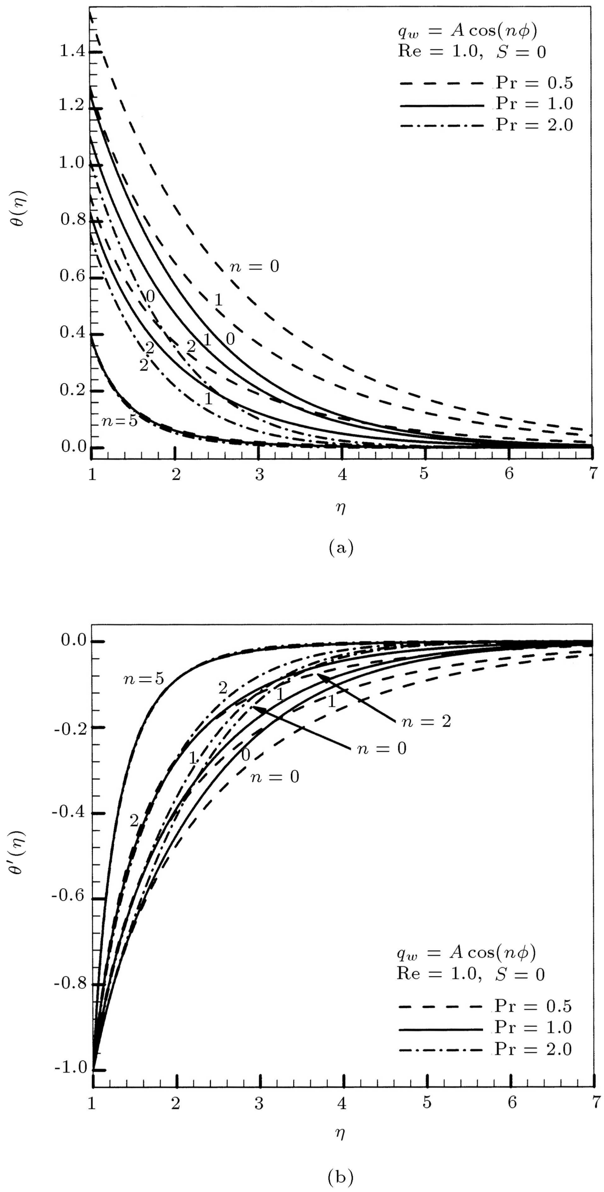

Figure 5. Profiles of (a) $\theta(\eta)$ and (b) $\theta'(\eta)$ for sinusoidal wall heat flux function for $S = 0.0$, Pr $= 0.5, 1.0, 2.0$, Re $= 1$ and different values of n.

in Figures 6 and 7, respectively. These profiles are for Reynolds number Re $= 0.1, 1.0, 10.0$ and selected values of n from zero to five. From these figures, the effect of an increase in Reynolds number is like the effect of an increase of Prandtl number in previous cases, which causes a reduction of fluid temperature adjacent to the cylinder wall and, thus, the variation of a cosine form of the wall temperature tends to zero at a lower radial distance. Also, the increase of Reynolds number increases the absolute value of the initial value of the wall temperature (Figure 6), which on the one hand, increases the coefficient of the heat transfer and on the other hand, decreases the thickness of the thermal boundary layer in the case of a wall heat flux in cosine form (Figure 7).

In Figure 8, variations of the Nusselt number, in terms of the Prandtl number for wall temperature function $T_w(\phi) - T_\infty = A\cos(n\phi)$ and selected values of Reynolds number for (a) $n = 1$, $S = 0$, and (b) $n = 5$, $S = 0$, are shown. It is interesting to note that the value of the Nusselt number is a constant, though the wall temperature or heat flux changes with time and position.

Nusselt number variations, in terms of Prandtl number for wall heat flux function $q_w(\phi) = A\cos(n\phi)$ and selected values of Reynolds number for (a) $n = 1$, $S = 0$, and (b) $n = 2$, $S = 0$, are presented in Figure 9. Again, though the wall temperature

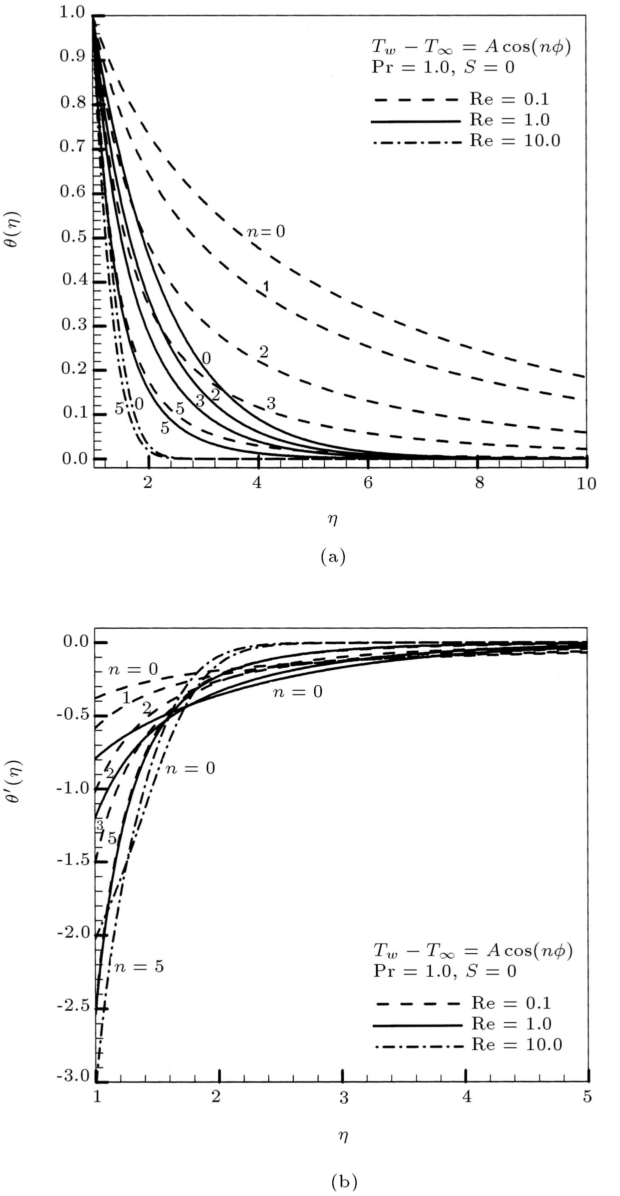

Figure 6. Profiles of (a) $\theta(\eta)$ and (b) $\theta'(\eta)$ for sinusoidal wall temperature function for $S = 0.0$, $\Pr = 1.0$, $\mathrm{Re} = 0.1, 1.0, 10.0$ and different values of n.

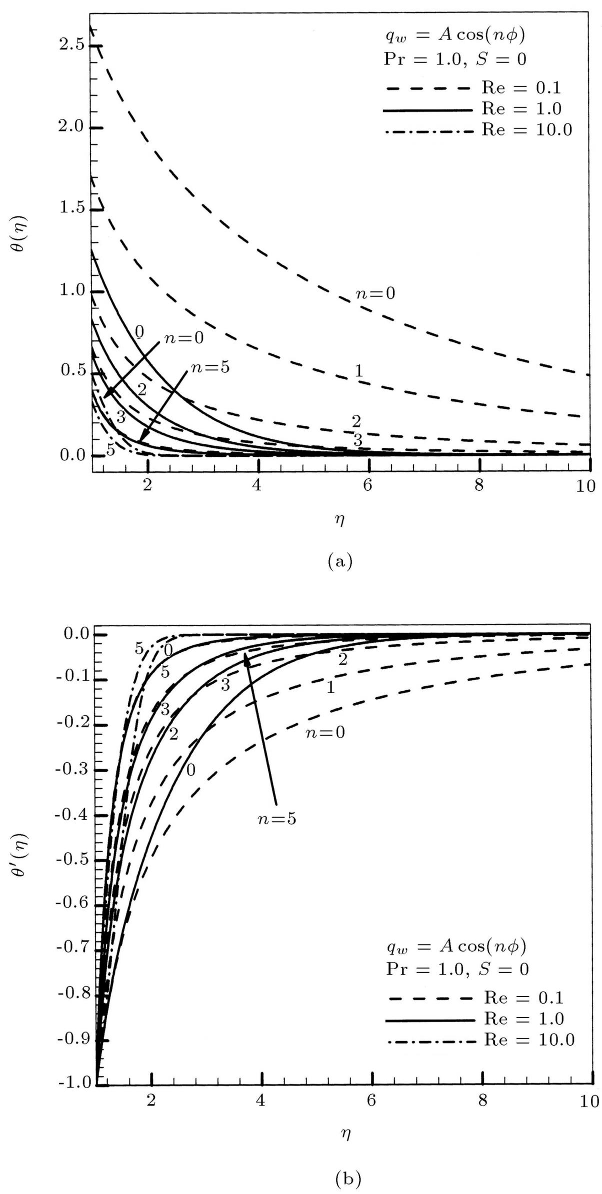

Figure 7. Profiles of (a) $\theta(\eta)$ and (b) $\theta'(\eta)$ for sinusoidal wall heat flux function for $S = 0.0$, $\Pr = 1.0$, $\mathrm{Re} = 0.1, 1.0, 10.0$ and different values of n.

or heat flux changes with respect to time and position the value of the Nusselt number is a constant.

CONCLUSIONS

In this paper, the unaxisymmetric heat transfer of a cylinder for two types of function as an unaxisymmetric wall temperature and an unaxisymmetric wall heat flux in an axisymmetric radial stagnation-point flow, on a cylinder with simultaneous rotational and axial movement, along with transpiration, has been studied. Here, the exact solution has been obtained for the energy equation, for the case of an unaxisymmetric heat transfer for some specific functions for an unaxisymmetric wall temperature distribution and an unaxisymmetric wall heat flux. The general, semi-similar equations have been formulated to present semi-similar solutions for different thermal functions and cylinder movements by use of numerical techniques. The effects of suction and blowing, Reynolds number, Prandtl number and different forms of unaxisymmetric thermal functions on the heat transfer rate have been discussed, for selected cases in a self-similar case. It is interesting to note that, in the case of self-similar solutions, the Nusselt number is a constant value, though wall temperature or wall heat flux change with respect to time and position.

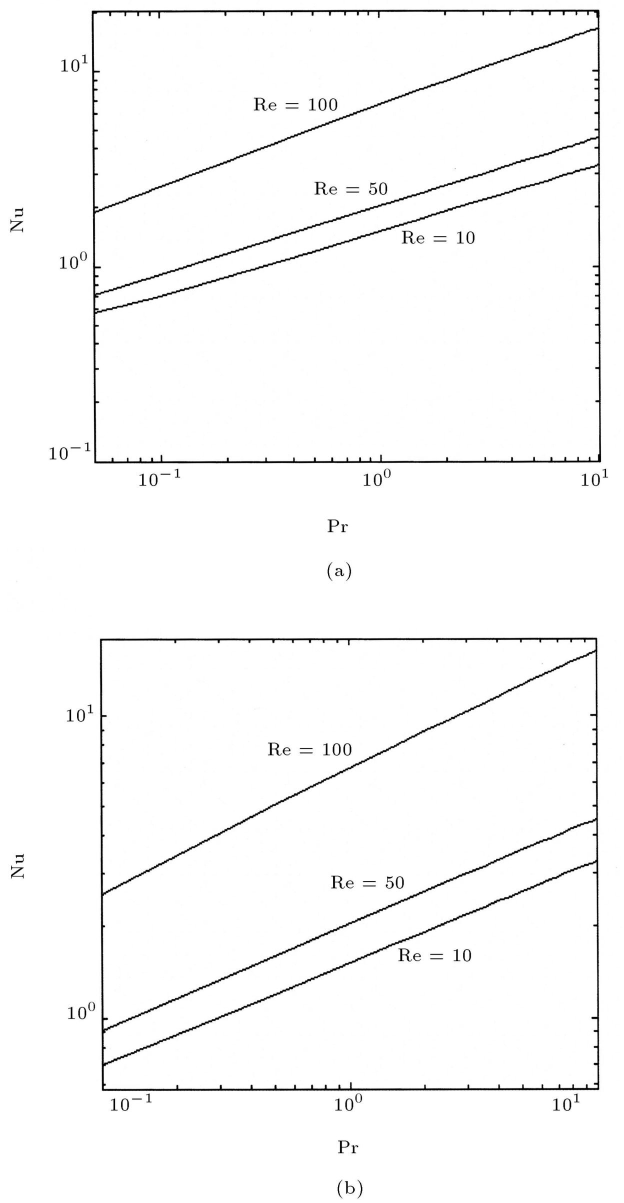

Figure 8. Nusselt number in terms of Pr. number for wall temperature function $T_w(\phi) - T_\infty = A\cos(n\phi)$ and selected values of Reynolds numbers for (a) $n = 1$, $S = 0$ and (b) $n = 5$, $S = 0$.

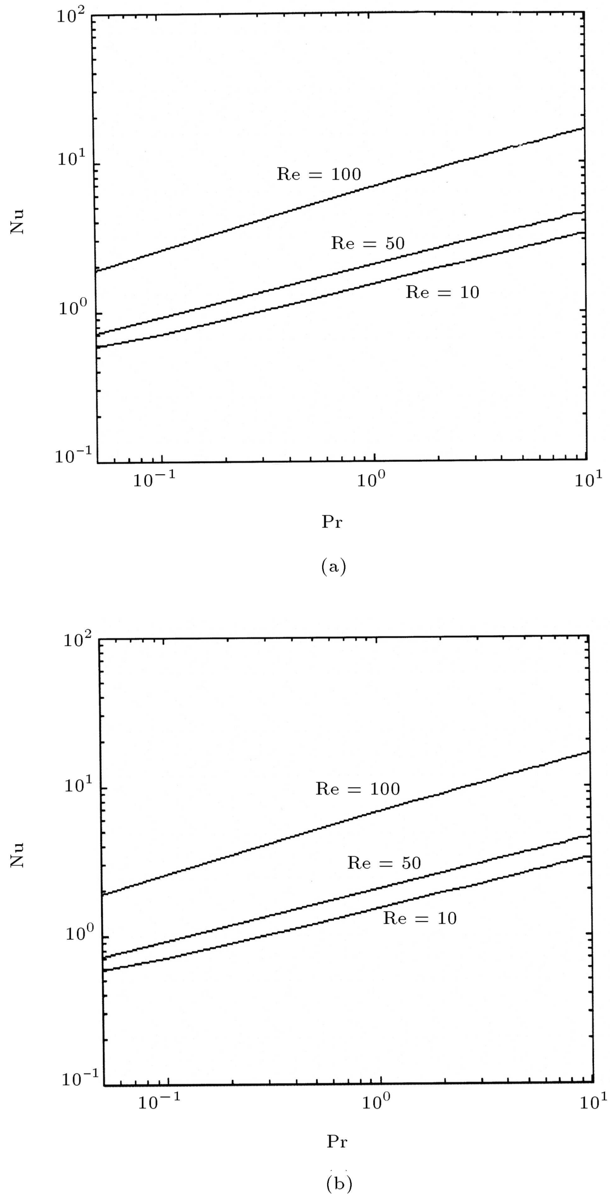

Figure 9. Nusselt number in terms of Pr. number for wall heat flux function $q_w(\phi) = A\cos(n\phi)$ and selected values of Reynolds numbers for (a) $n = 1$, $S = 0$ and (b) $n = 2$, $S = 0$.

NOMENCLATURE

a	cylinder radius (m)
A, B, C	constants
$f(\eta, \tau)$	function related to u-comp. of velocity
$G(\eta, \tau)$	function related to v-comp. of velocity
$g(\eta)$	function, Equation 25
$H(\eta, \tau)$	function related to w-comp. of velocity
h	heat transfer coefficient (w/m^2-°C)
$h(\eta)$	function, Equation 26
i	$\sqrt{-1}$
$\overline{k}$	strain rate (1/sec)
n	number of sinusoidal changes
Nu	Nusselt number
P	fluid pressure (pa)
Pr	Prandtl number
$Q(\phi, \tau)$	function, Equation 30
q_w	wall heat flux (J/kg)
r, ϕ, z	cylindrical coordinates
Re	Reynolds number
S	transpiration rate
t	time (sec)

T	temperature (°C)
T_∞	ambient temperature (°C)
T_w	wall temperature (°C)
u	r-component of velocity (m/sec)
v	ϕ-component of velocity (m/sec)
w	z-component of velocity (m/sec)
U_0	transpiration (m/sec)
$V(t)$	axial velocity of cylinder (m/sec)

Greek

α, β	constants
$\overline{\alpha}$	thermal diffusivity (m^2/hr)
δ	sinusoidal change of temperature
η	dimensionless radial variable
ρ	fluid density (kg/m^3)
ν	kinematic viscosity (m^2/sec)
$\omega(t)$	angular velocity of cylinder (rad/sec)
$\Omega(t)$	dimensionless transpiration
$\theta(\eta)$	function, Equation 30
$\Theta(\eta, \phi, \tau)$	nondimensional temperature
τ	dimensionless time variable

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