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Review Article

Sensitivity Based Health Monitoring of Structures with Static Response

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A sensitivity based parameter identification method is presented to detect the damage of existing structures, using applied sets of static forces at one subset of degrees of freedom and measured displacements at a subset of degrees of freedom that may overlap completely, partially, or not at all. The algorithm follows an output error approach, which minimizes the deviation between a measured and a theoretical displacement, in lieu of the commonly used force error function. An iterative scheme is developed utilizing a first order Taylor series expansion to linearize the associated non linear problem. The algorithm automatically adjusts the structural element stiffness parameters, in order to improve the comparison between a measured and a theoretical response in an optimal way. The measured input required in the present study is artificially generated. The effect that a noisy displacement measurement has on an identification procedure is also studied. A procedure is also identified, in order to select the limited number of DOF required to perform successful parameter identification, reducing the impact of measurement errors on the identified parameters. The algorithm is elucidated by a numerical example on frame structures.

INTRODUCTION

Structural damage may be viewed as any deviation in a structure's original geometric or material properties that may cause undesirable stresses, displacements or vibrations, leading to the weakening of the structure, which adversely affects the current or future performance of the system. This weakening and, also, deviations may be due to cracks, loose bolts, broken welds, corrosion and fatigue etc. In all cases, damage can severely affect the safety and serviceability of the structure. Hence, an early detection of any damage is paramount. Visual inspection has always been the most commonly used method for detecting damage in a structure. But, the increased size and complexity of today's structures reduce the efficiency of visual inspections. There are various methods of testing to assess the strength of the materials before construction, but these strengths may reduce, due to various factors in the proper structure. Different non-destructive test methods are available to assess the material strengths of the structure without damaging it. However, all these techniques normally assess the damage qualitatively, but cannot quantify the damage scenarios.

The need for quantitative damage detection methods that can be applied to complex structures has led to extensive research, in the recent past, on parameter identification, using test data. A detailed review on system identification and damage detection may be found in Doebling et al. [1]. System identification techniques, based on the dynamic data, have been developed extensively, compared to that of static data. But, the static responses are more locally sensitive than the frequency in structural damage detection, as recognised by Jenkins et al. [2]. It is also well noted that, in the dynamic method, large amounts of data need to be implemented, accompanied by expensive dynamic instrumentation, since it needs mass, stiffness and damping properties. The static damage identification method is usually simpler than the dynamic method, as it involves stiffness properties only. Moreover, the equipment in static testing is comparatively cheaper and deformation and strain can be obtained rapidly and economically. Therefore, these methods are attracting much attention in the field of civil engineering. However, with simple

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static data, the damage in stiffness can be identified up to a certain level of accuracy. If the primary goal of the identification is to find element stiffness degradation, this can be reasonably achieved by static parameter identification, which is inherently easy and simpler than dynamic methods of parameter identification.

Information on damage detection, based on static data (both displacement and strain), is reported in an isolated manner. Various approaches to damage detection are found, using static displacement data, i.e. sensitivity based update methods [3,4] with incomplete measurement [5], a heuristic method to select a limited number of degrees of freedom [6], force and displacement error minimization [7,8], an adaptive parameter-grouping scheme [9] and a regularization technique [10] etc. Structural parameter identification, utilizing elemental strain measurements, is also found [11-14]. Sanayei and Onipede [15] presented analytical parameter identification, providing the structure's present load carrying capacity, and applied it to the plate structure [16]. Sanayei et al. [6] presented the results of experiments on a small-scale steel frame model in order to explain "the displacement equation error function", "the displacement output error function" and "the strain output error function" methods of structural parameter estimation. Mahnken and Kuhl [17] proposed a parameter identification algorithm, following gradient enhanced damage models, in which the non-uniform distributions of state variables, such as stresses, strains and damage variables, were taken into account. Identification algorithms, using static data, together with frequencies, are also studied [18-20]. Chou and Ghaboussi [21] proposed identification as an optimization problem, using output error and equation error as objective functions under a static load. Papadimitriou [22] presented a methodology for designing optimal sensor configurations and excitation characteristics. Oliver and Vidal [23] adopted the minimization of the gradient of the gap between test and simulation applied to a simple case of the bending of the elastic-viscoplastic beam. Recent studies [24] are found in determining damage properties from full field displacement measurements. Robert-Nicoud et al. [25] employed a static response for model calibration, identifying the cause of the structural behaviour, such as the support condition and the material properties etc. Candidate models whose response reasonably matches the measured static data are identified. A mixed integer nonlinear least-squares problem, for identifying the damage in truss structures, is presented by Araki and Miyagi [26]. The algorithm is applied separately to the static and the modal data.

The literature indicated three different approaches to static damage detection, i.e., the displacement equation error function, the output error function and the strain output error function. Most of these works involve the formulation of an optimization problem, to minimize error between the analytical and measured quantities in a finite element framework. The major problem in static identification is the lack of information available, compared to the dynamic ones. Moreover, for a particular structure, it is difficult to identify the damage components, contributing nothing, or fairly little, to structural deformation under a certain load case. This may be partially overcome by proper pre-analysis for an actual scheme of loading and measurement.

The focus of the present work is on the sensitivity based damage detection method. Sensitivity based update methods are based on the first order Taylor series, which minimizes an error function of the perturbed matrices. The damage identification algorithm is posed as a determination of the modified parameter vectors. The basic theory is the minimization of an error function typically the force error function [3,4,27]. Sanayei and Onipede [15] used the approach to minimize the applied forces and the forces produced by applying a static measured displacement. They also examined the sensitivity of the noisy measurements [6]. A sensitivity based algorithm is proposed, where the static forces are applied to a set of DOF and the displacements are measured at another DOF. The conventional output error approach is ill-posed, particularly with noisy data [10]. In the present study, the parameter identification algorithm follows the output error approach. But, the error function is taken as the deviation between the measured and the theoretical displacements at selected DOF, instead of the force error function minimization approach. An iterative procedure is applied to adjust the structural element stiffness parameters automatically, in order to improve the comparison between the simulated and the theoretical response in an optimal way. A sensitivity analysis is employed at the heart of this iterative identification procedure for direct identification of the optimal values of the structural parameters at the element level. The number and location of the force application and the displacement measurement points have an immense impact on the error in estimates. A procedure is also identified to select a limited number of DOF to perform successful parameter identification, reducing the impact of measurement errors on the identified parameters. The effect of the noisy displacement measurement on the identification procedure is also studied. This pretest simulation uses an error sensitivity analysis to determine the effect of measurement errors on the estimated parameter. It is seen that the measurement scheme and the level of accuracy in the measured data can drastically affect the accuracy of the identified parameters. A sample problem, on a frame structure, is presented to demonstrate the proposed identification algorithm.

DAMAGE IDENTIFICATION

The static equilibrium equation of a structure in a displacement based finite element framework can be expressed as follows:

$$[K]\{u\} = \{f\}, \tag{1}$$

where $[K]$, $\{f\}$ and $\{u\}$ are the global stiffness matrix, force and displacement vector, respectively. In the parameter identification algorithm, the finite element model, the topology of the structure, the element behaviour and connections details are specified at the outset. There are two sets of Degrees Of Freedom (DOF) in measurement: The applied Force Degrees Of Freedom (FDOF) and the measured Displacement Degrees Of Freedom (DDOF). These two sets of DOF may, or may not, overlap. 'NSF' sets of forces are applied at FDOF, one set at a time, and 'NSF' sets of displacements are measured at DDOF, corresponding to each applied force set. Note that each set of forces should be neither equal to any other set nor a linear combination of the previous sets of applied forces. These sets of applied forces and measured displacements are concatenated horizontally into a force matrix, $[F]$, and a displacement matrix, $[U]$ as:

$$\begin{bmatrix} f_1^1 & f_1^2 & \cdots f_1^{\mathrm{NSF}} \\ f_2^1 & f_2^2 & \cdots f_2^{\mathrm{NSF}} \\ . & . & \cdots . \\ . & . & \cdots . \\ f_{ndof}^1 & f_{ndof}^2 & \cdots f_{ndof}^{\mathrm{NSF}} \end{bmatrix} = [K] \begin{bmatrix} u_1^1 & u_1^2 & \cdots u_1^{\mathrm{NSF}} \\ u_2^1 & u_2^2 & \cdots u_2^{\mathrm{NSF}} \\ . & . & \cdots . \\ . & . & \cdots . \\ u_{ndof}^1 & u_{ndof}^2 & \cdots u_{ndof}^{\mathrm{NSF}} \end{bmatrix},$$

$$\text{i.e.} \qquad [F] = [K][U]. \tag{2}$$

In fact, not all displacements in $[U]$ needed to be measured. Therefore, Equation 2 is partitioned into $[u_a]$ and $[u_b]$ i.e. the measured and unmeasured displacements, respectively as below:

$$\begin{bmatrix} f_a \\ f_b \end{bmatrix} = \begin{bmatrix} K_{aa} & K_{ab} \\ K_{ba} & K_{bb} \end{bmatrix} \begin{bmatrix} u_a \\ u_b \end{bmatrix}. \tag{3}$$

The matrix of unmeasured displacements, $[u_b]$, is condensed out, following static condensation, and Equation 3 reduces to the following:

$$[f_a^*] = [K_{aa}^*]\,[u_a]\,, \tag{4}$$

where:

$$[f_a^*] = \left[[f_a] - [K_{ab}][K_{bb}]^{-1}[f_b]\right],$$

and:

$$[K_{aa}^*] = \left[[K_{aa}] - [K_{ab}][K_{bb}]^{-1}[K_{ba}]\right]. \tag{5}$$

The matrices $[f_a]$ and $[f_b]$ are normally known in a typical test program. The analytical stiffness matrices, $[K_{aa}]$, $[K_{ab}]$, $[K_{bb}]$ and $[K_{ba}]$, are functions of the structural parameters defined in $\{h\}^T = \{h_1 \quad h_2 \quad \cdots \quad h_{\mathrm{NUP}}\}$. If the stiffness parameter changes, the measured displacement, $[\overline{u_a}]$, will vary from the displacement, as obtained from Equation 4. It is to be noted here that Equation 5 is a nonlinear function of the stiffness parameter, involving the inversion of $[K_{bb}]$. In order to identify the stiffness parameters, an error matrix is defined as $[E(h)]$ of size NMD (Number of Measured Dofs,) x NSF, each element of which, e_{aij}, is:

$$e_{aij}(h) = u_{aij}(h) - \overline{u}_{aij}, \tag{6}$$

where $i = 1, 2, \cdots$, NMD and $j = 1, 2, \cdots$, NSF. If the stiffness parameters, $\{h\}$, are the original values with no damage, then, $[E(h)]$ will be zero, otherwise it will be non-zero. To adjust the parameters, $\{h\}$, in the displacement vector, a 'zero' approximation (initial value) for the vector, h, as h_o and the use of the first order Taylor expansion of u_{aij}, around h_o, yields:

$$u_{aij}(h) = u_{aij}(h_o) + \sum_{k=1}^{\mathrm{NUP}} \frac{\partial u_{aij}}{\partial h_k} \Delta h_k + \cdots. \tag{7}$$

The column elements of $\{u_a(h)\}$ and $\{\overline{u}_a\}$ are placed vertically, one after the other sequentially to form a vector of size NM (i.e. NMD x NSF). Similar to $\{u_a(h)\}$ and $\{\overline{u}_a\}$, the elements of $[E(h)]$ are assembled into an error vector, $\{E(h)\}$, of size NM by 1. A scalar performance error function may be defined as:

$$J(h) = \{E(h)\}^T\{E(h)\}. \tag{8}$$

Now, the stiffness parameters are obtained by minimizing $J(h)$, with respect to the unknown parameters, $\{h\}$, i.e:

$$\frac{\partial}{\partial\{h\}} J(h) = 0,$$

$$\text{i.e.} \qquad \frac{\partial\left(\{E(h)\}^T\{E(h)\}\right)}{\partial\{h\}} = 0,$$

$$\text{i.e.} \qquad \left(\{u_a(h)\} - \{\overline{u}_a\}\right) \frac{\partial\{u_a(h)\}}{\partial\{h\}} = 0. \tag{9}$$

Substituting Equation 7 in Equation 9 finally yields (details are elaborated in the Appendix):

$$[S(h)]^T[S(h)]\{\Delta h\}=-[S(h)]^T\left\{\{u_a(h_0)\}-\overline{\{u_a\}}\right\}. \tag{10}$$

If the Numbers of Unknown Parameters (NUP) are greater than the Number of Independent Measurements (NIM), a unique solution of Equation 11 does not exist. If NUP is equal to NIM, a direct inversion solves Equation 10 as follows:

$$\{\Delta h\}=-[S(h)]^{-1}\{u(h_0)-\{u\}\}. \tag{11}$$

When NUP is less than NIM, then, $[S(h)]$ will not be a square matrix. The method of least squares can be utilized to compute the unknown parameters for each iteration, i.e:

$$\{\Delta h\}=-\left[[S(h)]^T[S(h)]\right]^{-1}[S(h)]^T\{u(h_0)-\{u\}\}. \tag{12}$$

Equation 12 can be set up as an iterative procedure for parameter identification, i.e:

$$\{h\}^{i+1}=\{h\}^i+\{\Delta h\}. \tag{13}$$

The next iteration starts with Equation 4, where the updated stiffness matrix is used (obtained from the updated parameter through Equation 13). Various criteria may be used for the convergence of the algorithm, i.e. changes in the error matrix, $E(h)$; parameters, $\{\Delta h\}$, or relative changes in the parameters, compared to their initial values, $\{\Delta h\}/\{h_i\}$. The tolerance limits are set to control the desired accuracy in the identified parameters.

In order to identify a unique set of parameters from a given set of measurements, NIM must be greater than or equal to the number of unknown parameters, otherwise there may exist an infinite number of values of parameters that satisfy the measurements. The number of independent measurements is the total number of measurements less the number of redundant measurements, due to symmetry in the measurements. The number of independent measurements for single load cases is NIM $=(m_1+m_2)(m_2+m_3)-\frac{1}{2}m_2(m_2-1)$, where m_1 is the measured displacement DOF only (no forces applied), m_2 is the measured displacement and applied force DOF and m_3 is the applied force DOF only (no displacements measured). For the special case of a complete overlap between FDOF and DDOF, NIM $=\frac{1}{2}m_2(m_2+1)$.

EFFECT OF MEASUREMENT ERROR

In an actual measurement setup, no matter how accurate the measurements are, some error is bound to occur. The performance of the identification algorithm in the presence of errors must be investigated, in order to study any potential problems. In particular, even if the algorithm converges, the magnitude of the errors in the input measurements and resulting errors in the identified parameters may not be comparable. It is, therefore, essential to estimate the relationship between the errors in the measurements and the errors in the identified parameters, prior to any testing program. It may, then, only be possible to establish limits on measurement errors compatible with the accuracy requirements in the parameter identification.

The errors associated with different measuring devices can have different error distributions. Typical error distributions that are used to simulate test data are uniform and normal probability density functions (PDFs). A uniform PDF represents a banded type of error, with an equal probability of occurrence throughout the predefined limits. A normal PDF represents an error behavior that is not banded, but has a higher probability of occurrence closer to the actual values. Measurement errors may be added to the simulated measurements, either as proportional errors or absolute errors. Proportional errors generate the largest error at the maximum value of the measurements, but absolute errors are added to the simulated measurements, regardless of their magnitudes. Both types of error and their error distributions seem to be insufficient for modelling various measurement errors in the field. However, any combination of error types and distributions can be used to study the error behavior of the identification algorithm. For the presented study, zero-mean uniform random numbers are generated and scaled by the corresponding percentage measurement error and, then, added absolutely to the displacement measurements. In order to obtain $[u_a]$, a set of random numbers are generated with zero-mean between -1 to +1, for each observation. 'NSF' sets of random numbers are then placed column-wise in a matrix, $[R_u]$. The percentage error of each displacement measurement point is used to scale each element of $[R_u]$. All of the NMD percentage errors are placed in the diagonal matrix, $[{}^\backslash E_{u_\backslash}]$. The correct displacements are used as the mean value, i.e. $[u_{a0}]$. The simulated measured displacements are obtained as follows:

$$[u_a]=[u_{a0}]+[{}^\backslash E_{u_\backslash}][R_u]. \tag{14}$$

Once the simulated displacements with error are obtained, iterative structural parameter identification is performed following the previous section. The procedure is repeated several times to compute the required statistical properties of the identified parameters. The sample size of the simulation should be large enough for statistically stable results. The fluctuations of the maximum output error have been studied against a sample size. When the fluctuation is very small, it is taken as the simulation numbers for the error sensitivity study. The mean of the parameter estimates is as follows:

$$\{\bar{h}\} = \frac{1}{\text{NOBS}} \sum_{i=1}^{\text{NOBS}} \{h\}_i, \tag{15}$$

where NOBS is the total number of simulations. The bias of the parameter estimates in percentage of the damaged values is as follows:

$$\text{BIAS}(\{h\}) = \frac{\{\bar{h}\} - \{h_d\}}{\{h_d\}}(100). \tag{16}$$

In the above, h_d is the actual value of the parameters. The Grand mean Percentage Error (GPE) is defined as the mean of the bias of the parameters in Equation 16, as follows:

$$\text{GPE} = \frac{1}{\text{NUP}} \sum_{j=1}^{\text{NUP}} |\text{BIAS}(h_j)|. \tag{17}$$

The Grand Standard Deviation percentage error (GSD) is defined as the standard deviation of the bias of parameters in Equation 16, i.e:

$$\text{GSD} = \sqrt{\frac{1}{\text{NUP}} \sum_{j=1}^{\text{NUP}} (|\text{BIAS}(h_j)| - \text{GPE})^2}. \tag{18}$$

Prior to actual testing, the input/output error relationship can be used to determine the accuracy limits on the measurements, in order to achieve an acceptable error level in the identified parameters. One of the most important aspects of parameter identification is the selection of force application locations (FDOF) and displacement measurement locations (DDOF). The number and location of FDOF and DDOF can have an immense impact on the error in the parameter estimates. For a given percentage error in DDOF, the error in the parameter estimates can, potentially, vary from small to medium to large and to extremely large identification errors, thus totally overshadowing the identified parameters. So, it is essential to measure a subset of DOF that is not highly error sensitive. The uncertainty associated with the set of measurements as quantified here can be used for each case to choose the most suitable force application locations and displacement measurement locations.

NUMERICAL STUDY

A two-dimensional frame structure, as shown in Figure 1, is used to demonstrate the parameter identification algorithm. Damage is introduced by reducing the stiffness parameters of the elements. The FEM is utilized to obtain the simulated measured response at one subset of degrees of freedom, using the damaged values of the parameters.

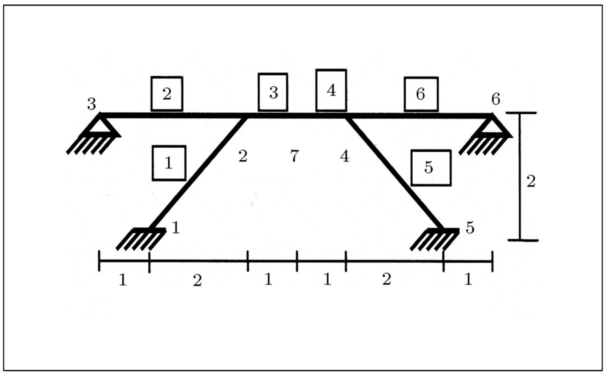

Figure 1. Two-dimensional frame model.

The algorithm starts with these simulated measurements, in place of measured data (as no experiments are done in the present study) and the undamaged values of the parameters as the initial value. Finally, it is examined whether or not the values of parameters converged to the pre-introduced damaged values. Since the frame structure is composed of elements capable of bending, as well as axial deformation, two parameters, i.e. the cross sectional area, A, and the moment of inertia, I, are considered here for identification. The modulus of elasticity of all the elements is assumed as unity. The undamaged and damaged values of the parameters are given in Table 1.

These unknown parameters are numbered as 1 to 12, corresponding to elements 1 to 6 (e.g., parameters 3 and 4 are the area and the moment of inertia of element

Table 1. Undamaged and damaged values of the parameters.

Element	PU	Initial Undamaged Values h_{ud}	Final Damaged Values h_d
1	A1	2	1
	I1	0.667	0.0833
2	A2	3	2
	I2	2.25	0.667
3	A3	3	2
	I3	2.25	0.667
4	A4	3	2
	I4	2.25	0.667
5	A5	2	1
	I5	0.667	0.0833
6	A6	3	2
	I6	2.25	0.667

Table 2. Parameter identification of frame model.

Case	FDOF	DDOF	NUP	PU	NIM	Iteration	Results
1	4-6, 9-12,18-21	4-6, 9-12, 18-21	12	1-12	66	2	Converged
2	4, 5, 10, 11, 19	4, 5,10,11,20	12	1-12	19	4	Converged
3	19, 20	4, 5, 10, 11, 19, 20, 21	12	1-12	13	5	Converged

number 2). In each case, a unit force is applied to FDOF and a set of displacements is measured at the selected DDOF. Three different cases are presented in Table 2. In all cases, all the 12 parameters are assumed to be unknown. Case 1 is an example of the complete overlap of FDOF and DDOF. Cases 2 and 3 are examples of partial overlap.

In Table 2, all cases lebelled 'converged' implied that the algorithm identifies the damage values of the unknown parameters accurately. In Case 1, forces are applied and displacements are measured at all FDOF and DDOF. When all DOF are measured, there are 66 independent measurements and only 12 unknown parameters. The rapid convergence is achieved, due to the extra number of measurements. Since no errors are introduced in the measurements, there are no errors in the identified parameters. In Case 2, there is a partial overlap between the measured DOF. In this case, there are 12 unknown parameters and 19 independent measurements converged in four iterations. Case 3 has two applied forces and seven measured displacements. This induces 13 independent measurements, one more than the 12 required for identification purposes, and convergence is achieved in five iterations.

The identification results presented so far are based on simulated displacement data having no error. An error sensitivity analysis is also performed, as described in the following section, for the selection of an error tolerant subset of DOF. The results of such two simulation cases are presented here. In the first case, there are 2 FDOF and 7 DDOF for measurement and, in the second case, 5 FDOF and 5 DDOF. For each case, a Monte Carlo experiment is performed assuming a uniform distribution of errors in the measured response. The Grand mean Percentage Error (GPE) of the parameters and the Grand Standard Deviation error (GSD) of the parameters are presented in Table 3.

From the table, it can be observed that Case B has the smallest identification error and that Case A has the larger identification error. In this example, Case B is taken for further investigation of the statistical properties of the parameters. As for small GPE and GSD, the error in the identified parameter will be small. Different percentages of error have been introduced in Case B to observe the effect of errors in the identified parameters. The sample size of the simulation is taken as 1000, based on observations of the statistical fluctuations of the maximum output error against the sample size. This is done for the worst case (i.e. 4% error case) and stability will be much better for lower values of random error. The results of the simulations are shown in Figure 2. The maximum and minimum values of the parameters for all simulations are realistic, i.e. they are neither negative nor abnormally high. It is clear from the identified parameters, that when the magnitude of the input error is very small, the errors in the identified parameter are identical to the input error. However, if errors increase, the output error propagates. It is also experienced that, for large input error, the algorithm diverges. Thus, the proposed algorithm works well when the error in input measurement is comparatively small. For larger errors, modifications may be needed in the proposed algorithm.

Table 3. Identification error percentage for extreme cases.

Case	Measured FDOF	Measured DDOF	GPE	GSD
A	19,20	4,5,10,11,19,20, 21	11.65	23.64
B	4,5,10,11,19	4,5,10,11,20	1.04	1.96

CONCLUSIONS

The parameter identification methods presented here used a limited number of applied forces at some DOF and measured static displacements at other DOF. The algorithm developed is capable of detecting large

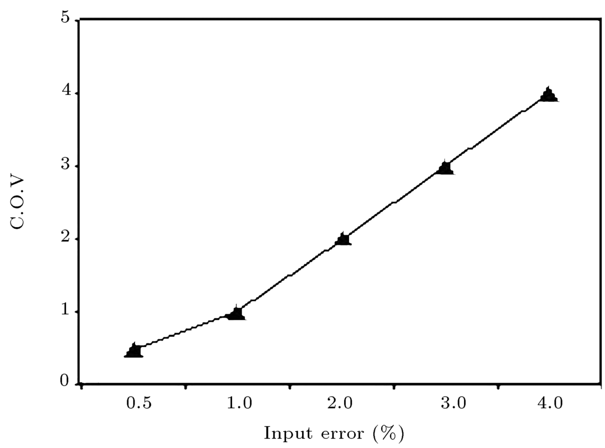

Figure 2. Maximum C.O.V. in identified parameters with varying error in measured input data.

changes in structural elements correctly, including element failure. The number of linearly independent measurements must be, at least, equal to the number of unknown parameters. This is a necessary condition, but not a sufficient condition for parameter identification. It is observed that, sometimes, for a particular set of FDOF and DDOF, although NIM is equal to NUP, the algorithm does not converge. However, the algorithm converges faster when more independent measurements are made. In order to identify a specific subset of parameters, the measured degrees of freedom should be chosen to facilitate the unique identification of the desired parameters. Though any subset can be chosen for measurement, maintaining NUP<NIM, the algorithms do not converge in all the cases. A pre-test error sensitivity analysis will result in the selection of a set of error tolerant DOF for the measurement that can be used for parameter identification and damage assessment. A better location of measurement can be decided from the values of GPE and GSD. The precision of the measurements can be selected, so that the errors in the identified structural parameters are within the specific limits. For example, in the present study, Case B is taken for further investigation of the statistical properties of the parameters as the GPE and GSD are less compared to case A. However, one should consider the error sensitivity of other cases for a better understanding and it is understood that this aspect needs further consideration in future studies.

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APPENDIX

Maximization of the scalar performance error function $J(h) = \{E(h)\}^T\{E(h)\}$ gives:

$$\frac{\partial(\{E(h)\}^T\{E(h)\})}{\partial\{h\}} = 0,$$

with Equation 6, which becomes:

$$(\{u_a(h)\} - \{\overline{u}_a\})\frac{\partial\{u_a(h)\}}{\partial\{h\}} = 0.$$

Substituting Equation 7 in the above yields:

$$\left[\left(\{u_a(h_0)\}+\sum_{k=1}^{\mathrm{NUP}}\frac{\partial\{u_a\}}{\partial h_k}\Delta h_k\right)-\{\overline{u}_a\}\right]\frac{\partial\{u_a\}}{\partial\{h\}}=0,$$

i.e.:

$$\left[\sum_{k=1}^{\mathrm{NUP}}\frac{\partial\{u_a\}}{\partial h_k}\Delta h_k+(\{u_a(h_0)\}-\{\overline{u}_a\})\right]\frac{\partial\{u_a\}}{\partial\{h\}} = 0, \tag{A1}$$

or:

$$[S(h)]^T\,[[S(h)]\{\Delta h\} + \{\{u_a(h_0)\} - \{\overline{u_a}\}\}] = 0,$$

or:

$$[S(h)]^T[S(h)]\{\Delta h\}=-[S(h)]^T\left\{\{u_a(h_0)\}-\overline{\{u_a\}}\right\}. \tag{A2}$$

The sensitivity matrix, $[S(h)]$, is formed by differentiating Equation 4, with respect to each parameter, where $[f_a^*]$ is the force applied and $[u_a]$ is the displacement measured. The jth column of the sensitivity matrix can be obtained as follows:

$$\left\{\frac{\partial\{f_a^*\}}{\partial h_j}\right\} = [K_{aa}^*]\left\{\frac{\partial\{u_a\}}{\partial h_j} + \left[\frac{\partial[K_{aa}^*]}{\partial h_j}\right]\{\overline{u}_a\}\right\}.$$

Thus:

$$\{S(h_j)\} = \left\{\frac{\partial\{u_a\}}{\partial h_j}\right\} = -[K_{aa}^*]^{-1}\left\{\left[\frac{\partial[K_{aa}^*]}{\partial h_j}\right]\{\overline{u}_a\} - \left\{\frac{\partial\{f_a^*\}}{\partial h_j}\right\}\right\}, \tag{A3}$$

where:

$$\left[\frac{\partial[K_{aa}^*]}{\partial h_j}\right] = \frac{\partial[K_{aa}]}{\partial h_j} - \frac{\partial[K_{ab}]}{\partial h_j}[K_{bb}]^{-1}[K_{ba}] + [K_{ab}][K_{bb}]^{-1}\frac{\partial[K_{bb}]}{\partial h_j}[K_{bb}]^{-1}[K_{ba}] - [K_{ab}][K_{bb}]^{-1}\frac{\partial[K_{ba}]}{\partial h_j},$$

and:

$$\left\{\frac{\partial\{f_a^*\}}{\partial h_j}\right\} = -\frac{\partial[K_{ab}]}{\partial h_j}[K_{bb}]^{-1}\{f_a\} + [K_{ab}][K_{bb}]^{-1}\frac{\partial[K_{bb}]}{\partial h_j}[K_{bb}]^{-1}\{f_b\}.$$

The sensitivity coefficient in Equation A3 is evaluated for $j = 1$ to NUP. Since sets of forces are applied and sets of displacements are measured, the Number of Independent Measurements (NIM) may be different from the Number of Unknown Parameters (NUP).