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An Inverse Problem Method for Gas Temperature Estimation in Partially Filled Rotating Cylinders

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The objective of this article is to study gas temperature estimation in a partially filled rotating cylinder. From the measured temperatures on the shell, an inverse analysis is presented for estimating the gas temperature in an arbitrary cross-section of the aforementioned system. A finite-volume method is employed to solve the direct problem. By minimizing the objective function, a hybrid effective algorithm, which contains a local optimization algorithm, is adopted to estimate the unknown parameter. The measured data are simulated by adding random errors to the exact solution. The effects of measurement errors on the accuracy of the inverse analysis are investigated. Two optimization algorithms are used in determination of the gas temperature. The conjugate gradient method is found to be better than the Levenberg-Marquardt method, since the former produces more accurate results for the same measurement errors. A good agreement between the exact value and the estimated result has been observed for both algorithms.

INTRODUCTION

In recent years, much effort has been devoted to study inverse analysis in many design and manufacturing problems, especially when direct measurement of surface conditions is not possible. The analysis of Inverse Heat Conduction Problems (IHCP) has numerous applications in various branches of science and engineering.

Inverse heat transfer problems, in contrast to direct heat transfer problems, belong to a set of problems that become unstable under small changes in the initial data. In fact, well and ill-posed problems of mathematical physics are distinguished and difficulties associated with the solution of IHTP should also be recognized. Mathematically, inverse heat transfer problems belong to a class called ill-posed [1-7], whereas standard heat transfer problems are well-posed.

Conventionally, the inverse problem includes two phases: The process of analysis and the process of optimization. In the analysis process, the unknown parameters or functions are assumed and the results of the problem are solved directly through either analytical methods [8], such as Laplace transform [9], or numerical methods, such as finite difference and finite element methods [10-14] and dynamic programming [15]. In the optimization process, an optimizer, such as in the Levenberge-Marquardt method, conjugated gradient method and steepest descent method [16,17], is used to guide the exploring points systematically to search for a new set of guess parameters. These new data could be substituted for unknown parameters in the analysis process.

Several numerical and experimental studies have been performed for the estimation of unknown point heat sources and boundary conditions in cylindrical coordinate systems using IHCPs [18-22]. The investigation of thermal behavior in hollow cylinders employing an inverse methodology has also been reported [23-25].

A literature survey of the available work, points out the lack of investigation of thermal behavior in a rotating cylindrical container using an inverse methodology.

A partially filled rotating cylinder is a fundamental physical concept that can be used in mathematical modeling of many industrial processes, such as drying, heating (cooling), calcining, separating, mixing, reducing, roasting, sterilizing, food making and metallurgical processes and in the sintering of a variety of materials [26-36].

In this study, a parameter estimation of quasi-steady conduction-convection heat problems has been undertaken using a numerical inverse analysis solver based on finite volume formulation in cylindrical coor-

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dinates. The gas temperature in an arbitrary cross-section of a rotating cylinder has been determined using an inverse algorithm. The effectiveness of the inverse algorithm was assessed using a sensitivity study. For assurance as to the effectiveness of the developed algorithm, different experimental data were examined. To discuss the correlation between measuring locations and the accuracy of the results, two types of measuring location were considered. Two inverse heat techniques were used for hot gas estimation in an arbitrary cross-section of an infinite long rotating cylinder. For damping instabilities in the inverse analysis procedure, iterative parameter estimation techniques were considered.

PROBLEM FORMULATION

The considered partially filled rotating cylinder is presented, schematically, in Figure 1. The geometrical properties are 4.5 m for the internal radius, 6 m for the external radius and 2.25 m for the bed depth. The rotational speed of the hollow cylinder is 1.5 rad/s and the used material properties in the direct model are given in Table 1.

GOVERNING EQUATION

In the cylinder, the bed (plug flow) region rotates as a rigid body about the hollow cylinder axis. A cylindrical coordinate system is used in this region, as well as for the wall itself. Energy conservation for any cylindrical control volume in the plug flow region and the wall (refractory lining), as shown in Figure 2, requires the following:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k_i r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k_i\frac{\partial T}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(k_i\frac{\partial T}{\partial z}\right)=\rho_i Cp_i u_\theta\frac{\partial T}{r\partial\theta},\quad i=w,p, \tag{1}$$

where;

$$u_\theta = r\omega.$$

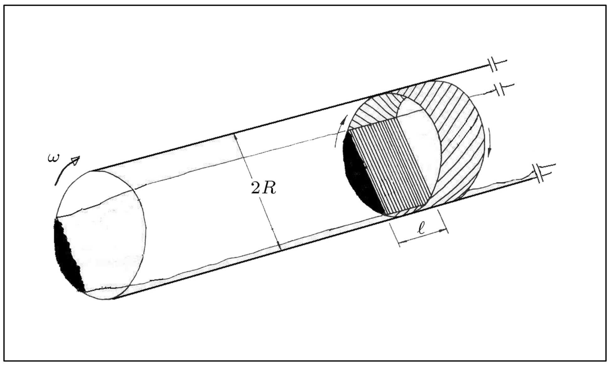


Figure 1. 3-D schematic presentation of partially filled rotating cylinder.

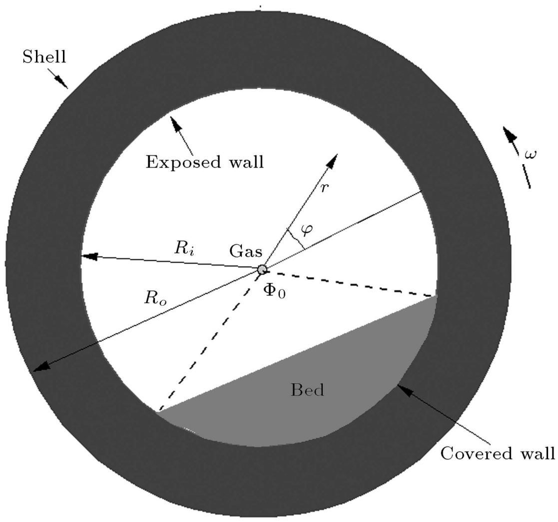


Figure 2. Two-dimensional cross-section model.

Table 1. Relevant physical properties.

Property	Charge	Wall
k (W/m/°K)	0.692	0.4
ρ (kg/m^3)	1680	1334
C_p (J/kg/°K)	1298	1100-1300

In order to predict temperature distribution in the considered long rotating cylinder, the following assumptions were made:

1. Due to high length-to-diameter ratio effects in the system, the conduction heat transfer in the longitudinal direction is negligible, i.e. $\frac{\partial T_i}{\partial z}=0.0$,
2. The charge behavior at the cross-section is as in rolling bed mode,
3. The hydrodynamic behavior of the charge at the bed is as plug flow,
4. Both the hollow cylinder wall and the bed are taken to be relatively gray [37-40],
5. The flowing gas (freeboard gas) is taken to be relatively gray,
6. There are no radial temperature gradients in the freeboard gas, so that either phase could be characterized by a unique temperature at a given cross-section of the hollow cylinder.

Applying appropriate simplifications to the physical model, the governing partial differential equation (Equation 1) reduces as follows [28]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k_i r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k_i\frac{\partial T}{\partial \theta}\right)=\rho_i Cp_i \omega\frac{\partial T}{\partial\theta},\quad i=w,p. \tag{2}$$

According to Figure 2, the following boundary conditions could be used:

a. At the exposed inner wall for $\Phi_0 \leq \theta \leq 2\pi - \Phi_0$:

$$r = R_i,$$

$$\Phi_0 \leq \theta \leq 2\pi - \Phi_0 :$$

$$-k\frac{\partial T}{\partial r} = (h_{C_{g\to ew}} + h_{R_{g,p\to ew}})(T_g - \overline{T}_{ew}). \quad (3)$$

b. At the charge surface (exposed solid surface) for $2\pi - \Phi_0 \leq \theta \leq \Phi_0$;

$$r = R(\theta),$$

$$2\pi - \Phi_0 \leq \theta \leq \Phi_0 :$$

$$-k\frac{\partial T}{\partial r} = (h_{C_{g\to p}} + h_{R_{g,ew\to p}})(T_g - \overline{T}_p). \quad (4)$$

The value of T_g in Equations 3 and 4 should be obtained by inverse analysis.

c. Initial conditions for θ;

$$R_i - Tic \leq r \leq R_o :$$

$$T(r, \theta = 0) = T(r, \theta = 2\pi)$$
$$\frac{\partial T(r,\theta=0)}{\partial r} = \frac{\partial T(r,\theta=2\pi)}{\partial r}. \quad (5)$$

The radiative heat transfer coefficients, i.e. $h_{R_{g,p\to ew}}, h_{R_{g,w\to p}}$, can be calculated from a multi-zone radiation model [40,41].

The convective heat transfer coefficient between the flowing hot gas and the exposed wall can be determined from the following correlation [32,40,42,43]:

$$h_{C_{g\to ew}} = 0.036\frac{k_g}{D}\mathrm{Re}^{0.8}\mathrm{Pr}^{0.33}\left(\frac{D}{L}\right)^{0.055}. \quad (6)$$

The convective heat transfer coefficient from the flowing hot gas to the bed (plug flow) can be determined from the following correlation [28,32,40]:

$$h_{C_{g\to p}} = 0.4G_g^{0.62}. \quad (7)$$

d. At the outer hollow cylinder surface (shell) for $0 \leq \theta \leq 2\pi$;

$$r = R_o,$$

$$0 \leq \theta \leq 2\pi :$$

$$-k\frac{\partial T}{\partial r} = (h_{C_{sh\to\infty}} + h_{R_{sh\to\infty}})(\overline{T}_{sh} - T_\infty). \quad (8)$$

The radiative heat transfer coefficient at the outer shell, $h_{R_{sh\to\infty}}$, could be evaluated using:

$$h_{R_{sh\to\infty}} = \frac{\sigma\varepsilon_{sh}(\overline{T}_{sh}^4 - T_\infty^4)}{(\overline{T}_{sh} - T_\infty)}. \quad (9)$$

The natural convection, ($\mathrm{Re}_D^2 < Gr_D$), heat transfer between the shell surface of the cylinder and the surroundings could be evaluated by [44]:

$$\mathrm{Nu} = h_{sh}\frac{D}{k_\infty} = 0.6 + \frac{0.386Ra^{1/6}}{[1+(0.559/\mathrm{Pr})^{9/16}]^{8/27}};$$

$$10^{-4} < \mathrm{Ra} < 10^{12}, \quad (10)$$

where:

$$\mathrm{Ra} = \mathrm{Pr}\ Gr_D. \quad (11)$$

INVERSE ANALYSIS CONCEPT

As stated in the introduction, in practical problems, it may be difficult to obtain some of the unknown parameters and functions through direct measurement, as the desired positions of the sensors cannot be accessed. However, they can be obtained through inverse heat conduction analyses. Inverse analysis is a discipline that provides tools for the efficient use of the sensor values of measured temperature data in estimation of the structure of the mathematical models and, also, unknown parameters, functions and the initial and boundary conditions appearing in mathematical models [1-14]. For the parameter or function estimation, error is defined by the difference between the measured and estimated or computed temperatures ($e = T^e - T^c$). Minimization of this error is the main goal of inverse analysis. One of the minimization strategies is to apply the standard least squares method.

The solution of this inverse problem for the estimation of N unknown parameters, P_j, $j = 1, \cdots, N$, is based on minimization of the ordinary least squares norm, given by [24]:

$$S(P) = [T^e - T^c(P)][T^e - T^c(P)]^T. \quad (12)$$

The estimated temperature, $T_i^c(P) = T(P, x_i)$, is obtained from the solution of the direct problem at the measurement location, x_i, which corresponds with the experimental temperature, T^e. To minimize the least squares norm given by Equation 12, the derivative of $S(\mathbf{P})$, with respect to each of the unknown parameters, $\mathbf{P} = [p_1, p_2, \cdots, p_n]$, should be equated to zero. Such a necessary condition for the minimization of $S(\mathbf{P})$ can be represented in matrix notation by equating the gradient of $S(\mathbf{P})$, with respect to the vector of parameter $\mathbf{P}$ to zero, that is:

$$\nabla S(\mathbf{P}) = 2\left[-\frac{\partial T^{c^T}(P)}{\partial P}\right][T^e - T^c(P)] = 0. \qquad (13)$$

Sensitivity Analysis

The solution of inverse problems and the accuracy of the estimated parameters and functions are directly related to the sensitivity of the measurements to the errors in input data, due to the location of the sensors and frequency of oscillations. The sensitivity matrix is determined by differentiating Equation 12, with respect to the vector of the unknown parameters or functions as follows:

$$\mathbf{J}(P) = \frac{\partial T^{c^T}(P)}{\partial P}. \qquad (14)$$

The elements of the sensitivity matrix are called sensitivity coefficients. A sensitivity coefficient is very important because it indicates the magnitude of a change in the temperature, due to perturbations in the value of parameters or functions.

Using the definition of sensitivity from Equation 14, the sensitivity coefficient distribution can be determined by differentiating governing Equation 2, with respect to the vector of the unknown parameters or functions, P, as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k_i r\frac{\partial J}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k_i\frac{\partial J}{\partial \theta}\right) = \rho_i C p_i \omega \frac{\partial J}{\partial \theta}, \quad i = w, p. \qquad (15)$$

The initial and boundary conditions read as follows:

$r = R_{\text{in}}$,

$\Phi_0 \leq \theta \leq 2\pi - \Phi_0$:

$$-k\frac{\partial J}{\partial r} = (h_{C_{g\to ew}} + h_{R_{g,p\to ew}}), \qquad (16)$$

$r = R(\theta)$,

$2\pi - \Phi_0 \leq \theta \leq \Phi_0$:

$$-k\frac{\partial J}{\partial r} = (h_{C_{g\to p}} + h_{R_{g,ew\to p}}), \qquad (17)$$

$Ri - Tic \leq r \leq Ro$:

$J(r, \theta = 0) = J(r, 2\pi)$,

$$\frac{\partial J(r, \theta = 0)}{\partial r} = \frac{\partial J(r, \theta = 2\pi)}{\partial r}. \qquad (18)$$

Inverse Analysis Methods

In inverse analysis methods, unknown quantities can be estimated by minimizing the least squares, Equation 12. These methods, based on estimation of an unknown quantity, can be divided into two groups; parameter estimation and function estimation. There are several parameter estimation methods available for inverse analysis to treat the ill-posed nature of inverse problems, e.g., the least squares method modified by the addition of a regularization term, and the use of a conjugate gradient method, where the regularization is inherently built into the iterative procedure [1,2,5,6]. There are some powerful techniques for solving inverse heat transfer problems. These techniques are as follows [24]:

1. Levenberg-Marquard method for parameter estimation,
2. Conjugate Gradient method for parameter estimation,
3. Conjugate Gradient method with an adjoint problem for parameter estimation,
4. Conjugate Gradient method with an adjoint problem for function estimation.

The parameter estimation approaches are, in general, nonlinear and the number of the unknown parameters is strongly limited [7]. However, in the parameter estimation methods, there is a priori information regarding the functional forms of the unknown parameters and their locations. It is also generally assumed that the number of these parameters/functions and their coefficients are known.

However, for cases with no priori information of the functional forms, one must use the function estimation methods of the inverse analysis, such as the adjoint conjugate gradient method and the function specification method. The solution of function estimation problems is highly sensitive to noise in the measured data. Even if data were measured exactly, the inverse function estimation problems would be difficult. However, for function estimation, if some information is available on the functional form of the unknown function, the inverse problem of estimating this function is reduced to estimating a finite number of parameters.

The Levenberg-Marquardt and the conjugate gradient methods, based on minimization of the objective function, are the two iterative algorithms used in this study for parameter estimations. These inverse heat conduction methods for estimating unknown parameters and functions are conducted for certain sequential items, i.e. solution of the direct problem, solution of the inverse problem and a convergence criterion. These methods can be suitably arranged in iterative procedures. In this research, both Levenberg-Marquardt and Conjugate Gradient methods are used for gas temperature estimation.

Levenberg-Marquardt Method

In this parameter estimation method, for the minimization of Equation 12, its gradient, with respect to the vector of the unknown parameters, must be equated to zero.

Using the definition of the sensitivity matrix, Equation 13 could be written as:

$$-2\mathbf{J}^T(P)[T^e - T^c(P)] = 0. \tag{19}$$

The solution of Equation 19 for nonlinear estimation problems requires an iterative procedure, which is obtained by linearizing the vector of the estimated temperatures, $T_i^c(\mathbf{P})$, with a Taylor series expansion around the current solution, $\mathbf{P}^k$, at iteration k. Such a linearization is given by:

$$T^c(P) = T^c(P^k) + \mathbf{J}^k(P - P^k). \tag{20}$$

Equation 20 is substituted into Equation 19 and the resulting expression is rearranged to yield the following iterative procedure, in order to obtain the vector of the unknown parameters, $\mathbf{P}$. Inverse problems are generally very ill conditioned, especially near the initial guess used for the unknown parameters. Levenberg-Marquardt alleviates such difficulties by utilizing an iterative procedure in the following form [24,25]:

$$P^{k+1} = P^k + [(\mathbf{J}^k)^T\mathbf{J}^k + \mu^k\mathbf{\Omega}^k]^{-1}(\mathbf{J}^k)^T[T^e - T^c(P^k)], \tag{21}$$

where $\mathbf{\Omega}^k$ is a diagonal matrix used for alleviating the ill-conditioned problem and μ^k is a positive scalar called the damping factor.

Conjugate Gradient Method

The conjugate gradient method is a powerful iterative parameter or function-estimation algorithm for the solution of linear and nonlinear inverse problems. In this method, the following principle relations are considered [24,25].

In the conjugate gradient method, the minimization procedure is performed by using the following iteration:

$$P^{k+1} = P^k - \beta^k d^k. \tag{22}$$

At each iteration step, a suitable step size, β^k, is taken along the direction of descent in order to minimize the objective function.

The direction of the descent is obtained as a linear combination of the negative gradient direction at the current iteration, with the direction of descent of the previous iteration, which is as follows:

$$d^k = \nabla S(P^k) + \gamma^k d^{k-1}, \tag{23}$$

where:

$$\gamma^k = \frac{\sum\limits_{j=1}^{N}\{[\nabla S(P^k)]_j[\nabla S(P^k) - \nabla S(P^{k-1})]_j\}}{\sum\limits_{j=1}^{N}[\nabla S(P^k)]_j^2},$$

$$\gamma^0 = 0. \tag{24}$$

The gradient direction vector is determined as:

$$\nabla S(P^k) = -2(\mathbf{J}^k)^T[T^e - T^c(P^k)]. \tag{25}$$

Temperature vector, $T(P^k - \beta^k d^k)$, can be linearized with a Taylor series expansion and, then, the minimization, with respect to β^k, is performed to yield the following expression for the search step size:

$$\beta^k = \frac{\sum\limits_{j=1}^{N}[\mathbf{J}^k d^k]^T[T^c(P^k) - T^e]}{\sum\limits_{j=1}^{N}\{[\mathbf{J}^k]^T d^k\}^2}. \tag{26}$$

Convergence Criterion

The above-mentioned iterative algorithms need a criterion to stop the iterative procedure of the solution. The following condition is used for terminating the iterative process:

$$S(P^{k+1}) < \varepsilon_1,$$

$$\left\|(\mathbf{J}^k)^T[Y - T(P^k)]\right\| < \varepsilon_2,$$

$$\left\|P^{k+1} - P^k\right\| < \varepsilon_3. \tag{27}$$

The solution of inverse problems using an iterative algorithm may become well posed if the convergence criterion is used to stop the iterative procedure.

NUMERICAL ANALYSIS

Direct Method Algorithm

The finite volume-method is employed to solve the bed-wall governing equation for the cross-section of the partially filled rotating cylinder. Rectangular grids with cylindrical control volumes are used in both the plug flow (charge) and the wall regions. The grid independency study, as shown in Figure 3, on the mesh in the solution domain, indicates that the 131 × 45 grids in θ and r directions, respectively, are sufficient to produce grid independent results. The generated mesh for the computational domain is presented in Figure 4.

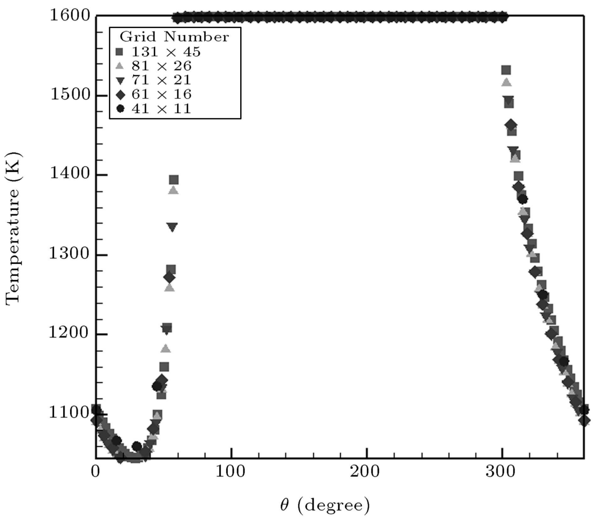


Figure 3. The effect of different grid sizes on radial temperature distribution.

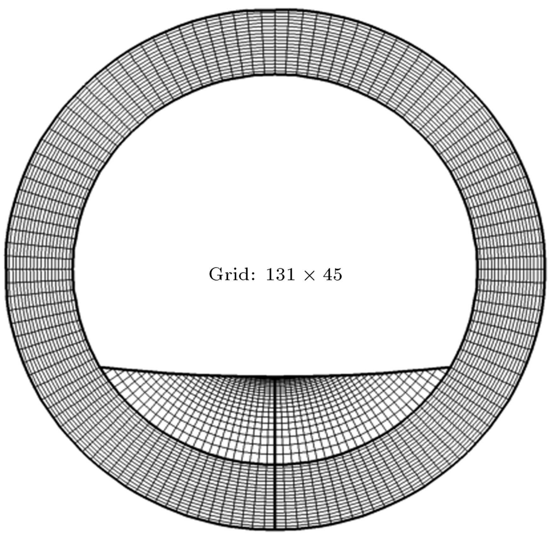


Figure 4. Computational grid for partially filled rotating cylinder (cross section).

Inverse Method Algorithm

A quasi-steady-state heat conduction-convection inverse problem was developed for the partially filled rotating cylinder. As shown in Figure 5, the gas tem perature in the arbitrary cross-section, T_{gas}, could be estimated using the inverse problem method. For estimation of the unknown parameter, the objective function, $S(\mathbf{P}^k)$, should be optimized. This objective function is produced, based on the measured and computational temperature differences at the shell surface of the rotating hollow cylinder. Due to employment of the L.M.M. algorithm, Equations 20 or 21 should be computed iteratively. To perform the iteration, according to C.G.M, the direction of the descent vector, d^k, in Equation 23 and the step size, β^k, in Equation 26

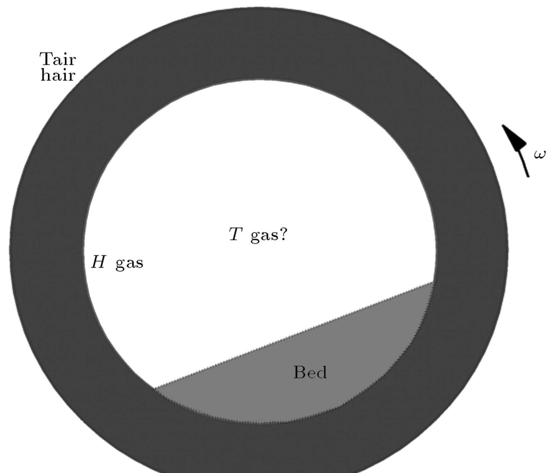


Figure 5. Layout of cross-section model with thermocouple arrangement shown.

require computation. For both optimization algorithms, the iteration producer is continued until one of the criteria introduced in Equation 27 is satisfied.

RESULTS

The properly developed inverse algorithm should lead to the correct value of the parameter in question. The effectiveness of the inverse algorithm can be assessed using the sensitivity study. To assess the effectiveness of the developed algorithm, different experimental data with 0%, 1% and 5% error were examined. To discuss the correlation between the measuring locations and the accuracy of the results, two types of measuring location were considered. The first location is the entire perimeter of the shell surface of the rotating cylinder's cross-section and the second location is the bedside zone of the shell. For each location, three sets of thermocouple arrangement, as shown in Figure 5, were considered. Both Levenberg-Marquardt and Conjugate-Gradient optimization algorithms were applied for each thermocouple arrangement.

Sensitivity Analysis

Sensitivity coefficients are, in some sense, the key to the success of the estimation procedures of the parameter and verify the influence of the measured parameters (measured temperatures). The sensitivity variations for gas temperature estimation are computed in four different layers of the rotating cylinder wall and are presented in Figure 6. As seen from this result, the sensitivity coefficient for gas temperature estimation at the inner wall is better, with respect to the outer wall (shell), for all thermocouple arrangements. In addition, the sensitivity coefficient in the bedside zone of the cross-section (at inner or outer walls) is considerable

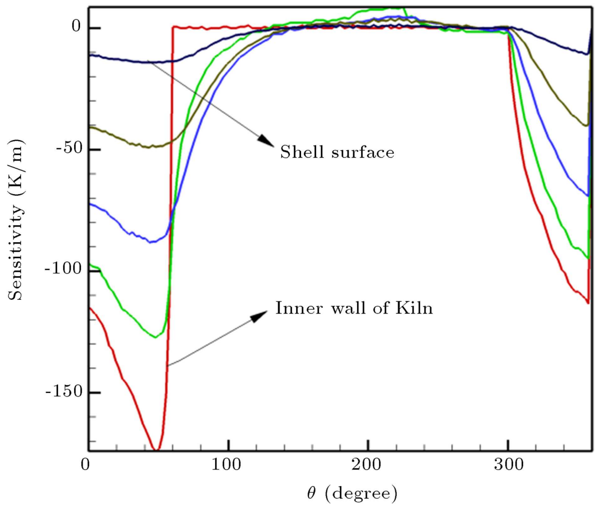


Figure 6. Sensitivity study for gas temperature estimation in different wall layers of cross section.

with respect to other zones. Therefore, it is expected that computational analysis based on covered wall actual temperature data, converges faster compared with computational analysis based on other isoclines experimental data.

Experimental Data

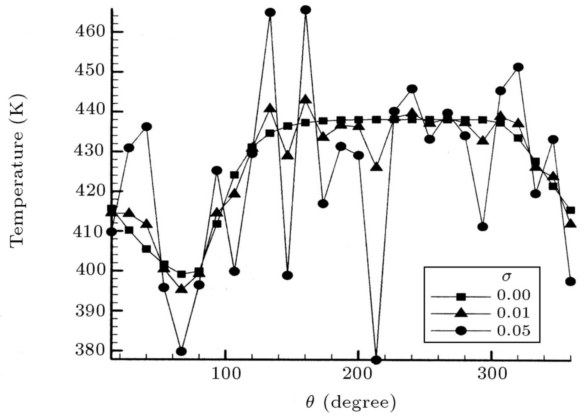
In order to simulate the measured temperature, T_{measure}, (by imaginary thermocouples) realistic noises are introduced to contaminate the theoretical temperature, T_{exact}, computed from the solution of the direct problem. The results involving the random measurement errors, the normally distributed uncorrelated errors with non-zero mean and constant standard deviation were assumed. Thus, the measured temperature, T_{measure}, could be expressed as follows:

$$T_{\text{measure}} = T_{\text{exact}}(1 + \lambda\sigma), \tag{28}$$

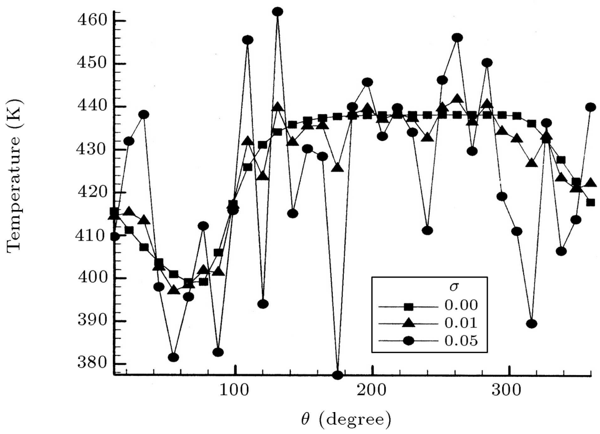
where T_{exact} is the solution of the direct problem with the exact gas temperature, 1600°K, σ is the standard deviation of the measurement and λ is the random variable, which is within $-2.576 \leq \lambda \leq 2.576$ for 99.43% confidence bound [16,22-25]. The produced measuring data for shell and bedside zones of a cylinder with 0%, 1% and 5% errors are shown in Figures 7 and 8, respectively.

Numerical Test Case 1

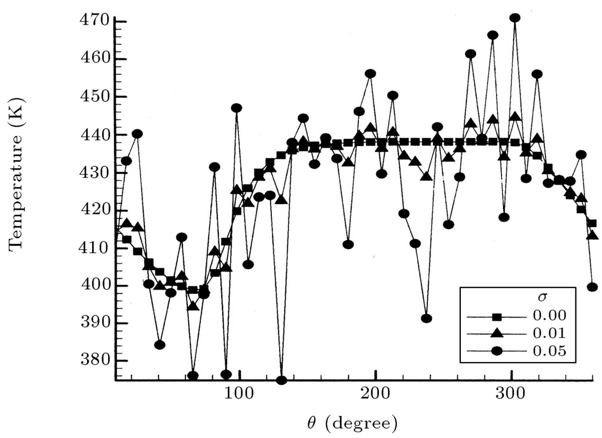
As shown in Figure 5, 27, 33 and 44 thermocouples, in turn, are arranged at the perimeter shell surface at an arbitrary cross-section of the rotating cylinder. The results of this inverse study for C.G.M and L.M.M algorithms are shown in Figures 9 and 10, respectively. The options (a), (b) and (c) are related to the 26,



(a)



(b)



(c)

Figure 7. Experimental temperature with different error for (a) 27, (b) 33 and (c) 44 thermocouples arrangement on the shell.

33 and 44 thermocouples, respectively. These figures illustrate that the estimated results have excellent approximations with the exact value of the gas temperature when the measurement error is not considered to be $\sigma = 0.0$. In addition, the estimated results

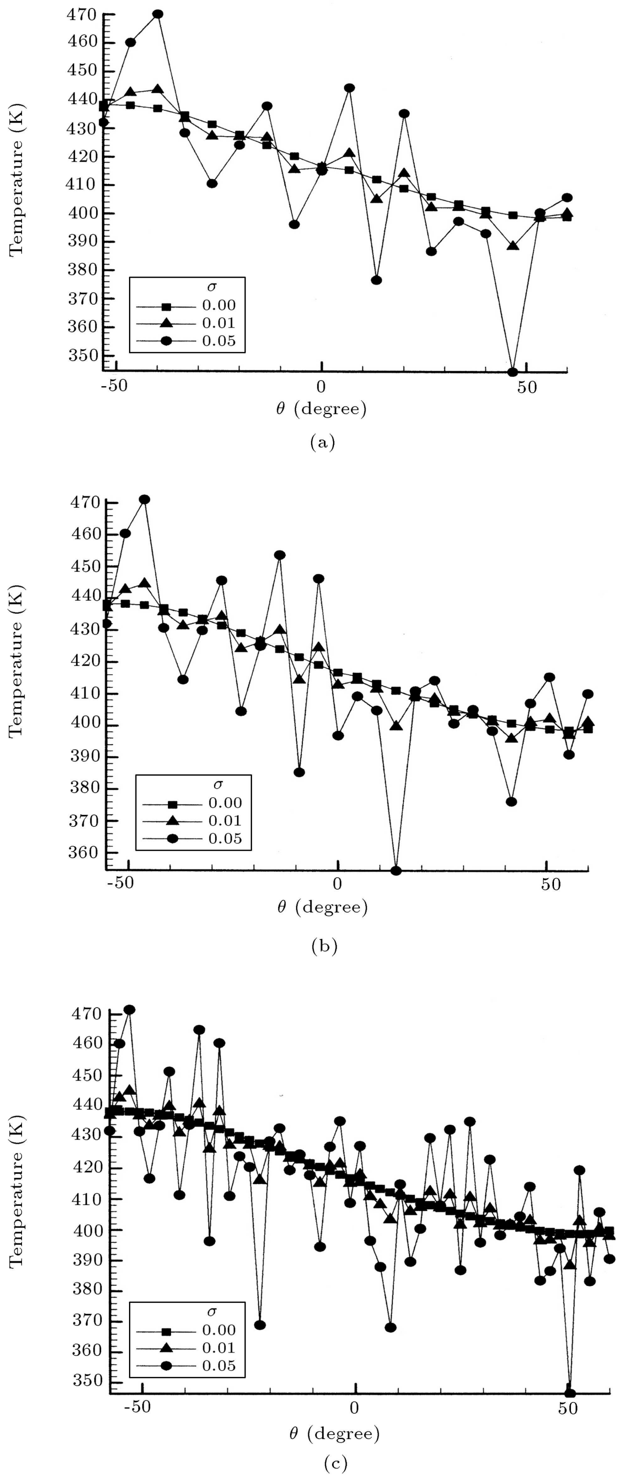


Figure 8. Experimental temperature with different error for (a) 18, (b) 26 and (c) 51 thermocouples arrangement on the bedside zone.

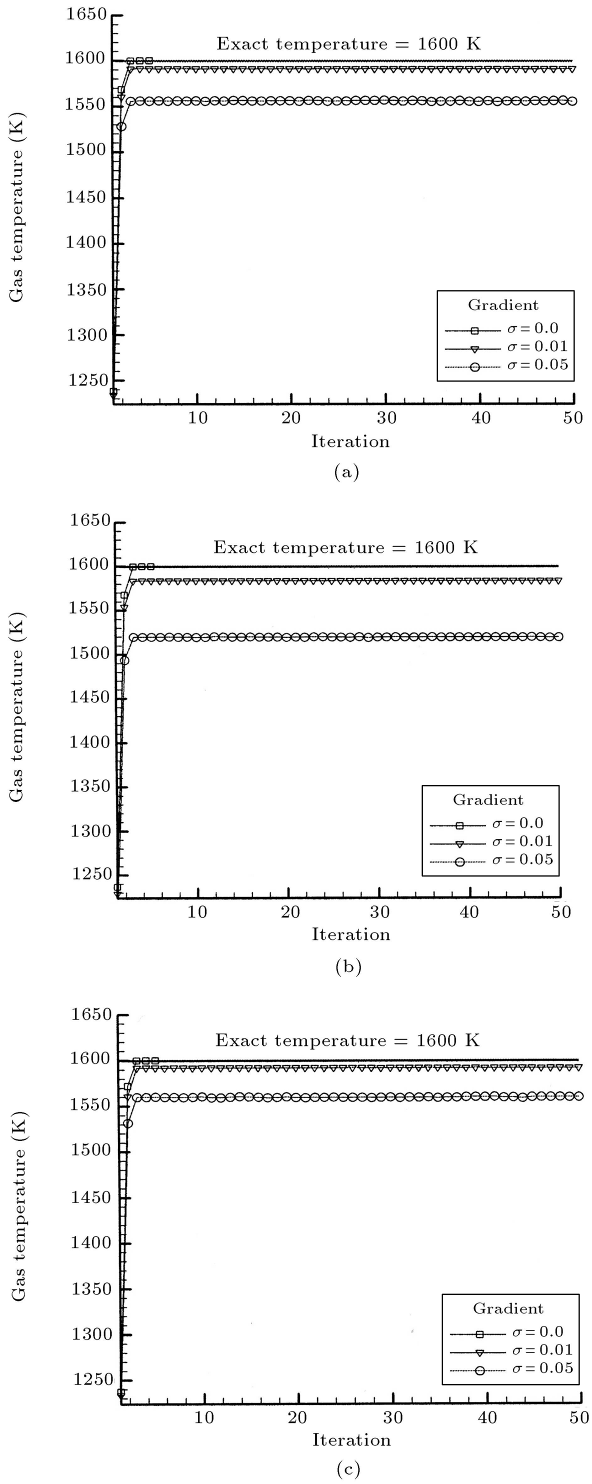


Figure 9. Gas temperature estimation for (a) 27, (b) 33 and (c) 44 thermocouples on the shell based on C.G.M. algorithm.

are in good agreement with the aforementioned exact value, even when the measurement error, $\sigma = 0.01$, is considered. These results show that the accuracy of the gas temperature increases as the number of thermocouples are increased to an optimum value. For example, as shown in Figures 9 and 10, it is obvious that an optimum value for the thermocouples arrangement is 33. Also, the results obtained by using C.G.M are better than the results obtained by using L.M.M. regarding accuracy and CPU time.

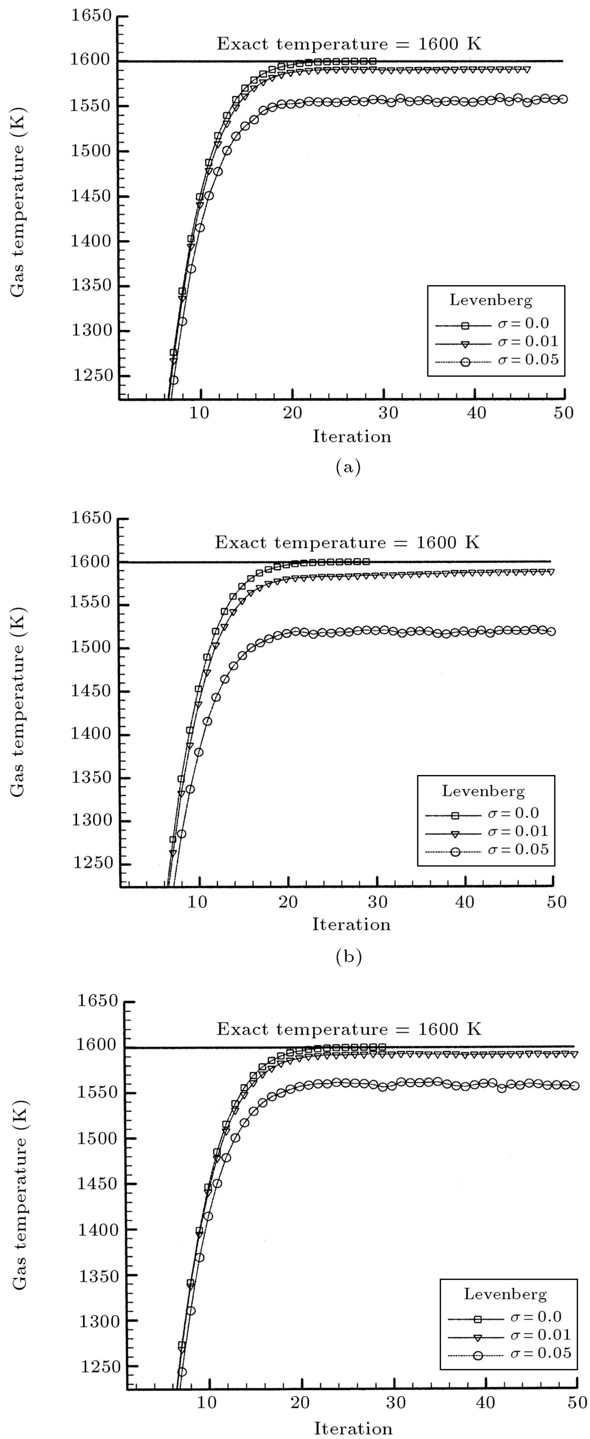


Figure 10. Gas temperature estimation for (a) 27, (b) 33 and (c) 44 thermocouples on the shell based on L.M.M. algorithm.

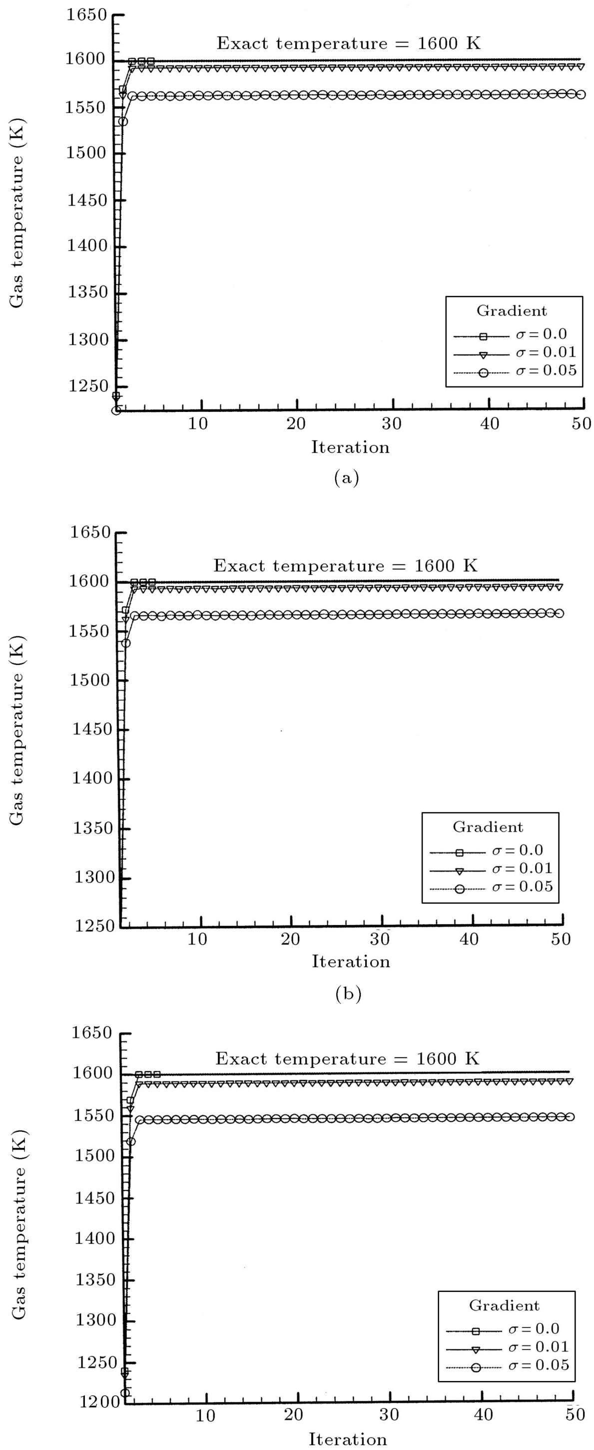


Figure 11. Gas temperature estimation for (a) 18, (b) 26 and (c) 51 thermocouples on the bedside zone based on C.G.M. algorithm.

Numerical Test Case 2

18, 26 and 51 thermocouples, in turn, are arranged at an arbitrary cross-section on the bedside zone of the shell of the rotating cylinder. The results of this test case are presented in Figures 11 and 12. These figures illustrate that the estimated results have excellent approximations with the exact value of the gas temperature when the measurement error is not considered to be $\sigma = 0.0$. Also, the estimated results

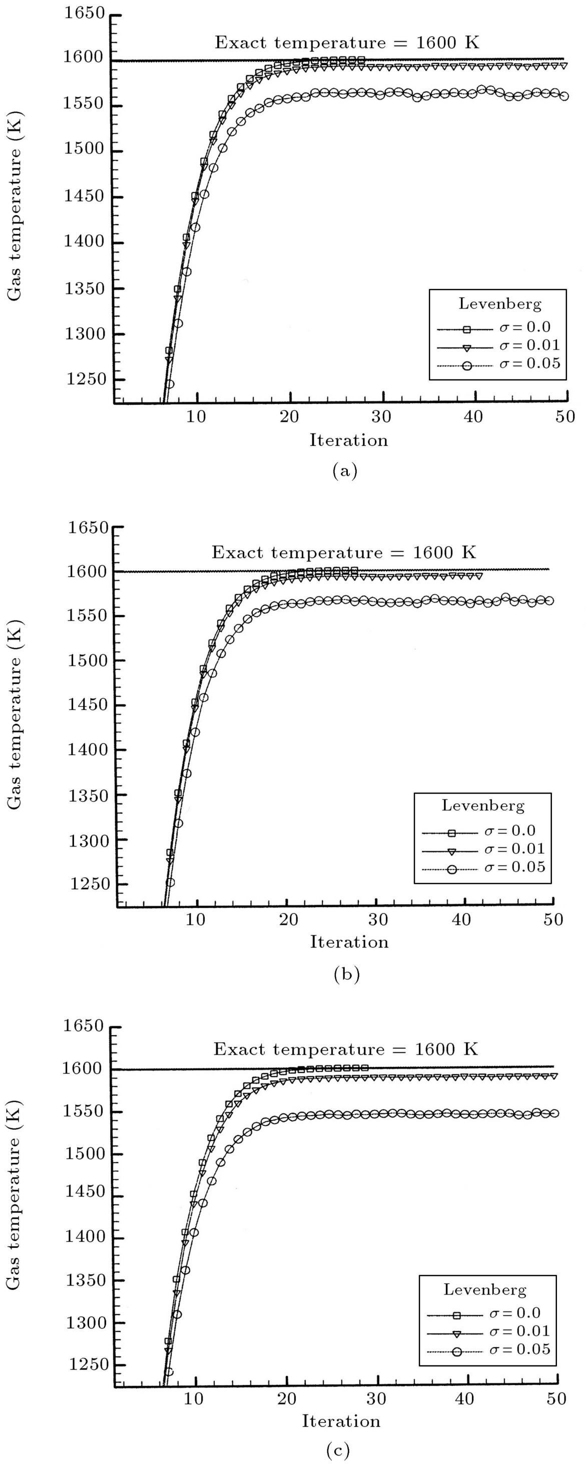


Figure 12. Gas temperature estimation for (a) 18, (b) 26 and (c) 51 thermocouples on the bedside zone based on L.M.M. algorithm.

are in good agreement with the aforementioned exact value, even when the measurement error, $\sigma = 0.01$, is considered. In addition, these results show that the accuracy of the gas temperature increases as the number of thermocouples is increased to an optimum value. As shown in Figures 11 and 12, it is obvious that an optimum value for thermocouple arrangement is 10. Also, the results obtained using C.G.M are better than the results obtained using L.M.M. regarding accuracy and CPU time.

CONCLUSION

In this paper, a method for estimating the hot gas temperature in a partially filled rotating cylinder is developed. This parameter is estimated by an experimental temperature via an inverse method. Two examples were used to show the robustness of the proposed method. The Levenberg-Marquardt method and the conjugate gradient method with a finite volume approach were successfully applied for solution of the inverse problem, in order to determine the gas temperature, by utilizing temperature readings at the shell of the rotating cylinder. The results show that the C.G.M does not require an accurate initial guess of unknown quantities, needs very short CPU time and requires fewer sensors, when compared with the L-MM, in performing the inverse calculations.

From the results, it appears that, by using the proposed method, without measurement error, the exact solution (exact value for gas temperature) can be found, even with only few measurement points. When measurement errors are included, in order to enhance stability and accuracy, temperature data requires more measuring points on the shell surface. Furthermore, fewer measurement points on the bedside of the shell surface, with respect to the whole surface of the shell, may be used to obtain accurate results.

Consequently, the results confirm that the proposed inverse method is effective and efficient for gas temperature estimation in a partially filled rotating cylinder.

NOMENCLATURE

A	area
C_p	specific heat
D	hollow cylinder diameter
d	direction of descent vector
E	emissive power
G	mass flow rate of gas
h	convection coefficient
$\mathbf{J}$	sensitivity matrix
J	radiosity
k	conductivity
M	number of measuring Temperature
N	number of unknowns
Nu	Nusselt number

$\mathbf{P}$	estimated parameter
Pr	Prandtl number
r	polar coordinate
R_i	inner radius of hollow cylinder
R_o	outer radius of hollow cylinder
Re	Reynolds number
Ra	Rayleigh number
S	square root of errors
T	temperature
$\overline{T}$	average temperature
Tic	bed depth
α	absorptivity
β	search step size (descent parameter)
γ	conjugate coefficient
ω	hollow cylinder angular velocity
ε	emissivity
$\varepsilon_1, \varepsilon_2, \varepsilon_3$	tolerance for convergence of the iterative inverse methods
ρ_r	reflectivity
θ	angular coordinate
ρ	density
σ	measurement error
σ_s	Stephan-Boltzman constant
λ	the probability or random value
μ	dynamic viscosity
Φ_0	fill percent
Θ	dimensionless temperature
ΔS	gradient of S

Superscript

c	computational
e	experimental
k	iteration step
T	transpose
$-$	average value

Subscript

a, ∞	ambient
p	plug flow (bed)
cw	covered wall
ew	exposed wall
exact	exact value
g	gas
i	measured parameter index
j	unknown parameter index
measure	measurement value
over	overall
sh	shell
w	wall

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