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# Noncoherent Integration after MTI Filters

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In this paper, the $N_e$ approximate method for calculation of integration loss after moving target indication (MTI) filters of radars is generalized to MTI filters with nonuniform pulse repetition frequency (PRF). Then, an exact method for analyzing the radar signal processor (MTI filter, squarer and integrator) is presented.

## INTRODUCTION

To increase the probability of detection and reduce the false alarm rate, it is necessary to improve the signal to interference ratio. Integration has an important role in increasing the signal to noise ratio [1-4] and is usually used in radar systems. A simplified block diagram of a common radar signal processor is depicted in Figure 1.

In coherent (predetection) integration, the phase relation between pulses is preserved so that integrating over $N_i$ pulses (where $N_i$ is the number of pulses integrated) results in $N_i$ times increase in signal to noise ratio. However, in noncoherent (postdetection) integration which is used in Figure 1, there will be some loss due to phase relation deterioration [1-4], thus the effective number of integrated pulses ($N_e$) will be less than $N_i$. In addition, MTI filters cause the noise to become colored (correlated) and, hence, result in some additional loss in integration [5-10]. In this paper, work concerning loss of integration on uncorrelated and correlated (clutter) noises after MTI filters are reviewed. Then, the discussion is generalized to nonuniform PRF, with the aid of general model of MTI filters presented in [11].

Because of the approximate nature of the $N_e$ method (which is used for integration loss calculations in this discussion), another method will also be presented for exact analysis of complete MTI processors.

## INTEGRATION AFTER MTI FILTERS WITH UNIFORM PRF

For signal detection, the variation of output average

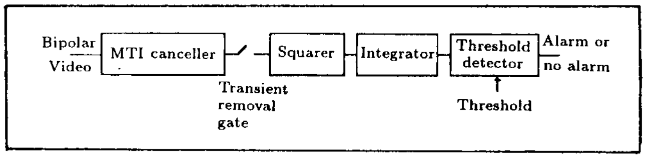


**Figure 1.** Simple block diagram of signal processing part of a radar.

due to signal with respect to noise power must be recognized. Therefore, $N_e$, which describes the effective number of integrated pulses, is defined as follows [5]:

$$N_e = \frac{(\sigma^2/m^2)_{\text{in}}}{(\sigma^2/m^2)_{\text{out}}}, \tag{1}$$

where $\sigma$ and $m$ are standard deviation and mean deviation (of the signal at the input or the output of the integrator), respectively.

Presence of an MTI filter results in noise correlation and, consequently, a decrease in the effective number of integrated pulses ($N_e$). It has been shown in [6,7] that using $N_e$ in well known curves of detection probability, $P_d$, (versus $S/N$ and false alarm probability, $P_{fa}$) leads to a good approximation for the required $S/N$ to achieve the given $P_d$ and $P_{fa}$, which differ from the exact results obtained by computer simulation with 0.3 - 0.6 dB.

First assume that the integrator is an FIR filter (tapped-delay-line integrator) which adds the total $N_i$ pulses coming from the MTI filter (after passing the square law detector) with equal weights. For this situation [5,8]:

$$N_e = \frac{N_i^2}{N_i + 2\sum_{k=1}^{N_i-1}(N_i - k)\rho_y^2(kT)}, \tag{2}$$

where $\rho_y(\tau) = \lambda_y(\tau)/\lambda_y(0)$ is the normalized autocorrelation function of MTI filter output and $T$ the

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radar pulse repetition period (1/PRF). The following approximate formula can also be used:

$$N_e = \frac{1}{\int_{-T_i}^{T_i} \frac{T_i-|\tau|}{T_i^2}\rho_y^2(\tau)d\tau}, \tag{3}$$

which leads to a satisfying approximation for small values of $\sigma_c T(\sigma_c T < 0.4)$ [2], where $\sigma_c$ is the standard deviation of the clutter spectrum and $T_i$ in Equation 3 is the total integration time, i.e.,

$$T_i = N_i T. \tag{4}$$

In an MTI filter:

$$y(nT) = \sum_{l=0}^{N-1} a_l x[(n-l)T], \tag{5}$$

where $a_l$'s are MTI filter coefficients and $N$ is the number of coefficients. Therefore, the autocorrelation function of an MTI filter can be written in terms of the input autocorrelation function as:

$$\begin{aligned}\lambda_y(kT) &= \overline{y(nT)y(nT+kT)}\\ &= \sum_{l=0}^{N-1}\sum_{m=0}^{N-1} a_l a_m \overline{x[(n-l)T]x[(n+k-m)T]}\\ &= \sum_{l=0}^{N-1}\sum_{m=0}^{N-1} a_l a_m \rho_x[(l-m+k)T]\\ &= d_0\rho_x(kT)\\ &\quad+\sum_{i=1}^{N-1} d_i\{\rho_x[(k+i)T]+\rho_x[(k-i)T]\},\end{aligned} \tag{6}$$

where,

$$d_i = \sum_{l=0}^{N-i-1} a_l a_{l+i}, \quad i=0,1,\dots,N-1, \tag{7}$$

therefore [6]:

$$\begin{aligned}\rho_y(kT) = \frac{\lambda_y(kT)}{\lambda_y(0)} &= \frac{\sum_{l=0}^{N-1}\sum_{m=0}^{N-1} a_l a_m \rho_x[(l-m+k)T]}{\sum_{l=0}^{N-1}\sum_{m=0}^{N-1} a_l a_m \rho_x[(l-m)T]}\\ &= \frac{d_0\rho_x(kT)+\sum_{i=1}^{N-1} d_i\{\rho_x[(k+i)T]+\rho_x[(k-i)T]\}}{d_0+2\sum_{i=1}^{N-1} d_i\rho_x(iT)}\end{aligned} \tag{8}$$

If the input to the filter is white noise, then:

$$\rho_x[(i-j)T] = \delta(i-j) = \begin{cases}1 & \text{if } i=j\\ 0 & \text{otherwise}\end{cases},$$

so:

$$\lambda_y(0) = d_0 = \sum_{i=0}^{N-1} a_i^2, \tag{9}$$

and

$$\rho_y(kT) = \sum_{i=0}^{N-1} \frac{d_i}{d_0}\delta(i-k), \quad k>0. \tag{10}$$

Substituting this into Equation 2 results in:

$$N_e = \frac{N_i^2}{N_i + 2\sum_{k=1}^{N_i-1}(N_i-k)(\frac{d_i}{d_0})^2}, \tag{11}$$

for example, for a double canceller, Equation 7 results in:

$$d_0 = 6, \quad d_1 = -4, \quad d_2 = 1$$

and Equation 11 yields:

$$\begin{aligned}N_e &= \frac{N_i^2}{N_i + 2[(N_i-1)(-4/6)^2 + (N_i-2)(1/6)^2]}\\ &= \frac{18N_i^2}{35N_i - 18},\end{aligned}$$

which for large $N_i$, approaches $\frac{18}{35}N_i$, leading to a loss of $10\log[(35/18)^{1/2}] = 1.4$ dB. According to Kretschmer [9], for a large $N_i$ and an $n$th order binomial filter:

$$\frac{N_e}{N_i} = \left[\frac{(1)(3)(5)\cdots(m-1)}{(2)(4)(6)\cdots(m)}\right]\times \left[\frac{(m+2)(m+4)\cdots(2m)}{(m+1)(m+3)\dots(2m+1)}\right], \tag{12}$$

where:

$$m = 2n. \tag{13}$$

If the input to the filter is clutter, assuming a Gaussian spectrum for clutter:

$$\rho_x(kT) = \rho_c(\tau)\big|_{\tau=kT} = \exp(-2\pi^2\sigma_c^2k^2T^2), \tag{14}$$

$\rho_y(kT)$ can be derived by substituting this into Equation 8 and then $N_e$ can be calculated using Equation 2.

Figures 2 to 4 show $N_e$ versus $\sigma_c T$ for various values of $N_i$ for the cases of first, second and third order binomial filters and also in a case where the MTI filter is absent (but a square law detector is present).

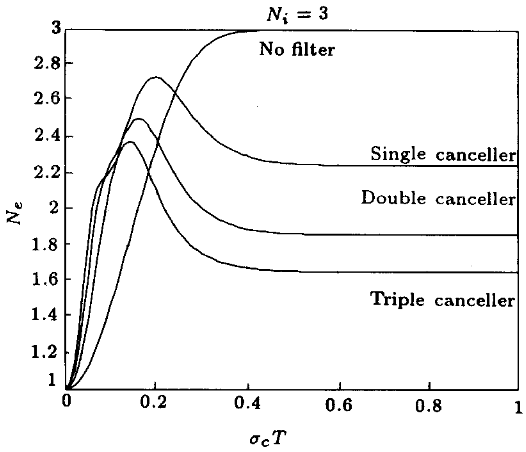


**Figure 2.** Effective number of integrated samples of correlated noise after the MTI filter for $N_i = 3$.

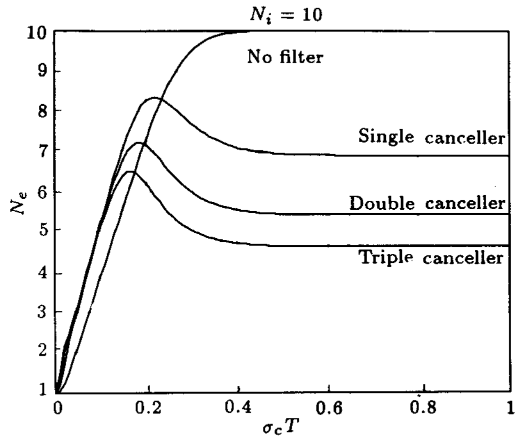


**Figure 3.** Effective number of integrated samples of correlated noise after the MTI filter for $N_i = 10$.

When $\sigma_c T \to \infty$, $N_e$ approaches the value obtained from Equation 11.

To consider clutter and noise simultaneously, it is sufficient to substitute $\rho_x(\tau)$ in Equation 8 by:

$$\rho_x(\tau) = \frac{\delta(\tau) + (C/N)_{\text{in}}\rho_c(\tau)}{1 + (C/N)_{\text{in}}} = \frac{\rho_c(\tau) + (N/C)_{\text{in}}\delta(\tau)}{1 + (N/C)_{\text{in}}}, \quad (15)$$

where $(C/N)_{\text{in}}$ is the clutter to noise ratio at the filter input. In this case, Equation 8 leads to [7]:

$$\rho_y(kT) = \frac{(N/C)_i d_k U_{k,N-1} + d_0\rho_c(kT)}{d_0[1 + (N/C)_i] + 2\sum_{i=1}^{N-1} d_i\rho_c(iT)} + \frac{\sum_{i=1}^{N-1} d_i\{\rho_c[(k+i)T] + \rho_c[(k-i)T]\}}{d_0[1 + (N/C)_i] + 2\sum_{i=1}^{N-1} d_i\rho_c(iT)}, \quad (16)$$

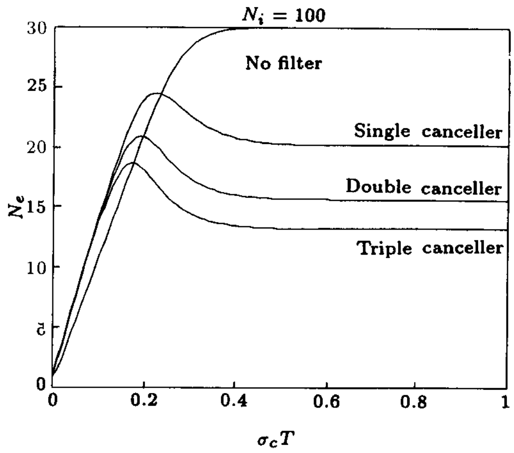


**Figure 4.** Effective number of integrated samples of correlated noise after the MTI filter for $N_i = 100$.

where:

$$U_{k,l} = \begin{cases} 1 & \text{if } |k| \le l \\ 0 & \text{otherwise} \end{cases}. \quad (17)$$

Up to now, it has been assumed that the integrator adds the input pulses with equal weights, but if the pulses are added together with $b_i$ weights [10]:

$$N_e = \frac{(\sum_{i=1}^{N_i} b_i)^2}{\sum_{i=1}^{N_i}\sum_{j=1}^{N_i} b_i b_j \rho_{y_{ij}}^2}, \quad (18)$$

where:

$$\rho_{y_{ij}} = \rho_y(\tau)\big|_{\tau=T(i-j)}, \quad (19)$$

and $\rho_y(\tau)$ is the normalized autocorrelation function of the MTI filter output and can be substituted from Equation 8. Therefore:

$$N_e = \frac{(\sum_{i=1}^{N_i} b_i)^2 \left\{\sum_{l=0}^{N-1}\sum_{m=0}^{N-1} a_l a_m \rho_x[(l-m)T]\right\}^2}{\sum_{i=1}^{N_i}\sum_{i=1}^{N_j} b_i b_j \left\{\sum_{l=0}^{N-1}\sum_{m=0}^{N-1} a_l a_m \rho_x[(l-m+i-j)T]\right\}^2}$$

$$= \frac{(\sum_{i=1}^{N_i} b_i)^2 [d_0 + 2\sum_{l=1}^{N-1} d_l \rho_x(lT)]^2}{\sum_{i=1}^{N_i}\sum_{i=1}^{N_i} b_i b_j (d_0 \rho_x[(i-j)T] + \sum_{l=1}^{N=1} d_l \{\rho_x[(i-j+l)T] + \rho_x[(i-j-l)T]\})^2} \quad (20)$$

## GENERALIZATION OF Ne METHOD TO MTI FILTERS WITH NONUNIFORM PRF

Since the general model presented in [11] covers all various types of MTI filters, the $N_e$ method for this general model is employed. In this case, in place of Equation 5:

$$y_i = y(t_{(i-1)s+N-1}) = \sum_{k=0}^{N-1} a_{i,k} x(t_{(i-1)s+N-1-k}), \tag{21}$$

where $x$ is the filter input and $y$ its output, $a_{i,k}$ the $k$th coefficient of $i$th filter. Furthermore, it is assumed that each time the filter slides "$s$" pulses forward [11].

From Equation 21 it can be concluded that:

$$\lambda_{y_{i,j}} = \overline{y_i y_j}$$

$$= \sum_{k=0}^{N-1}\sum_{l=0}^{N-1} a_{i,k} a_{j,l} \overline{x(t_{(i-1)s+N-1-k}) x(t_{(i-1)s+N-1-l})},$$

$$= \sum_{k=0}^{N-1}\sum_{l=0}^{N-1} a_{i,k} a_{j,l} \rho_x (t_{i,N-1-k} - t_{j,N-1-l}), \tag{22}$$

where:

$$t_{i,j} = t_{(i-1)s+j}.$$

Equation 21 can also be written as:

$$y_i = \sum_{k=0}^{N-1} w_{i,k} x(t_{(i-1)s+k}), \tag{23}$$

where:

$$w_{i,k} = a_{i,N-1-k}. \tag{24}$$

Therefore:

$$\lambda_{y_{i,j}} = \sum_{k=0}^{N-1}\sum_{l=0}^{N-1} w_{i,k} w_{j,l} \rho_x (t_{i,k} - t_{j,l}). \tag{25}$$

Note that in Equations 22 and 25, since the filter is time-variant, $\lambda_{y_{i,j}}$ is not a function of the time difference only. For example, $\lambda_{y_{i,i}}$ is not equal for various values of $i$, therefore, the definition of Equation 1 is not directly applicable because $(\sigma^2/m^2)_{\text{in}}$ is not constant and changes from pulse to pulse. Consequently, $N_e$ is defined as:

$$N_e = \frac{[(\sigma^2/m^2)_{\text{in}}]_{\text{ave}}}{(\sigma^2/m^2)_{\text{out}}} \tag{26}$$

and as in Equation 18:

$$N_e = \frac{(\sum_{i=1}^{N_i} b_i)^2}{\sum_{i=1}^{N_i}\sum_{j=1}^{N_i} b_i b_j \rho^2_{y_{i,j}}} \tag{27}$$

and now $\rho_{y_{i,j}}$ is defined as:

$$\rho_{y_{i,j}} = \frac{\lambda_{y_{i,j}}}{\lambda_{y_{i,i}}}. \tag{28}$$

is so because MTI filter in its general form [11] is a time-variant system that experiences $I$ distinct variations where $I$ is a function of $s$ and the number of employed periods. Thus, the integrator experiences $I$ situations for its input.

Equations 28 and 25 result in:

$$\rho_{y_{i,j}} = \frac{\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} w_{i,k} w_{j,l} \rho_x (t_{i,k} - t_{j,l})}{(1/I)\sum_{i=1}^{I}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} w_{i,k} w_{i,l} \rho_x (t_{i,k} - t_{i,l})} \tag{29}$$

For $s = 1$ (moving window) and $t_i = iT$ (uniform PRF), Equations 27 and 29 reduce to Equation 20. If, in the above equation, the nonequal occurance of filters due to limitation of $N_p$ (total input pulses of MTI) is to be considered [11], then:

$$N_i = 1 + IP(\frac{N_p - N}{s}), \tag{30}$$

where $IP(.)$ denotes the integer part. This means that $N_p - N_i$ pulses are discarded by the transient removal gate of Figure 1 and

$$\rho_{y_{i,j}} = \frac{\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} w_{i,k} w_{j,l} \rho_x (t_{i,k} - t_{j,l})}{[1/(Ik_1 + k_2)]\sum_{i=1}^{I}[k_1 + U(k_2 - i)]\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} w_{i,k} w_{i,l} \rho_x (t_{i,k} - t_{i,l})}, \tag{31}$$

where $k_1$ is the repetition number of all filters and $k_2$ the number of filters repeated once more ($k_1 + 1$ times). Figure 5 compares the value of $N_e$ for fixed and moving window single canceller ($N = 2$). For both cases, $N_p = 6$ has been assumed, therefore, according to Equation 30, $N_i$ will equal 5 for the moving window ($s = 1$) and 3 for the fixed window ($s = N = 2$). It is observed that for the fixed window, $N_e$ approaches 3

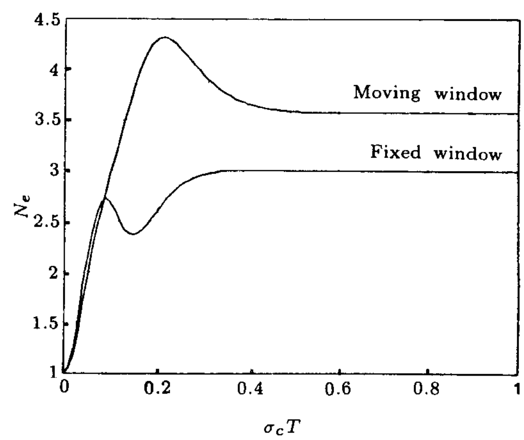


**Figure 5.** Comparison of $N_e$ for fixed and moving window single cancellers assuming $N_p = 6$.

(i.e., $N_i$) as $\sigma_c$ increases. This is due to the fact that in fixed window filters, the output pulses are constructed from different batches of input pulses and, thus, the output noise is not correlated. However, in moving window filters, $N_e$ approaches $\frac{2}{3}N_i$, approximately (as can be predicted by Equation 12). It is understood that although moving window filters cause some degree of correlation, they usually have larger $N_e$ because of their larger $N_i$ (for the same $N_p$).

## EXACT METHOD FOR NONCOHERENT INTEGRATOR AND MTI FILTER ANALYSIS

As previously mentioned, the $N_e$ method is only an approximate model for calculation of the required input signal to interference ratio to achieve the given $P_d$ and $P_{fa}$. This is because of two reasons, first is the fact that the integration loss (due to the MTI filter before the integrator) is not completely independent of the input signal to interference ratio, second is that the integrator is actually a lowpass filter and although a portion of signal energy (even if it has a Doppler) is transfered to dc due to square-law detector, there will be a frequency dependency for the entire integrator and square-law detector set.

The problem of analyzing the set of two linear filters (such as IF amplifiers and video or audio amplifiers) with a square-law detector between them has been of interest for a long time [10,12]. Castella [13] presented a method for calculating the signal to interference ratio at the output of a set of integrators and square-law detectors in terms of input signal to interference and clutter to noise ratios and integrator coefficients. However, he did not consider the presence of the MTI filter.

A simplified block diagram of a radar signal processor has been shown in Figure 1. In practice, two channels (in-phase and quadrature) are used to solve the blind phase problem. If the received signal is $A\text{Cos}[(\omega_0 + \omega_d)t + \theta]$, where $\omega_d$ is the Doppler angular frequency, it will be synchronously detected with $\text{Cos}\omega_0 t$ at in-phase channel of radar receiver and samples of $A\text{Cos}(\omega_d t + \theta)$ will be obtained. For a uniform PRF radar, if the MTI filter coefficients are shown with $w_k$ and their number is $N$, this will lead to [14]:

$$y_i = A\sum_{k=0}^{N-1} w_k \text{Cos}(k\omega_d T + \theta) = A[H_c(\omega_d)\text{Cos}\theta - H_s(\omega_d)\text{Sin}\theta], \tag{32}$$

where:

$$H_c(\omega_d) = \sum_{k=0}^{N-1} w_k \text{Cos}(k\omega_d T) \tag{33}$$

and:

$$H_s(\omega_d) = \sum_{k=0}^{N-1} w_k \text{Sin}(k\omega_d T). \tag{34}$$

$H_M(\omega_d)$ and $\alpha(\omega_d)$ can be defined as:

$$H_M(\omega_d) = (|H_c(\omega_d)|^2 + |H_s(\omega_d)|^2)^{1/2}, \tag{35}$$

$$\alpha(\omega_d) = \text{tg}^{-1}\left[\frac{H_s(\omega_d)}{H_c(\omega_d)}\right]. \tag{36}$$

Now it can be written that:

$$y_i = A\{H_M(\omega_d)\text{Cos}[\theta + \alpha(\omega_d)]\}. \tag{37}$$

At quadrature channel, the input is synchronously detected by $\text{Sin}\omega_0 t$ and similarly:

$$y_q = A\{H_M(\omega_d)\text{Sin}[\theta + \alpha(\omega_d)]\}. \tag{38}$$

Therefore:

$$y_T = |y_i|^2 + |y_q|^2 = A^2|H_M(\omega_d)|^2. \tag{39}$$

Consequently, the filter total power gain is described with the following equation and is independent of input phase:

$$P(\omega_d) = \frac{y_T}{A^2} = |H_M(\omega_d)|^2 = |\sum_{k=0}^{N-1} w_k \text{Cos}k\omega_d T|^2 + |\sum_{k=0}^{N-1} w_k \text{Sin}k\omega_d T|^2 = |\sum_{k=0}^{N-1} w_k e^{jk\omega_d T}|^2. \tag{40}$$

The above equations, however, are for uniform PRF (with moving window) but the MTI filter in its general form, as previously stated, experiences $I$ distinct variations. Therefore, if $P_l(\omega_d)$ is used for power gain of the $l$th filter (i.e., $l$th variation):

$$P_l(\omega) = |H_M(\omega)|^2 = |\sum_{k=0}^{N-1} w_{l,k} e^{j\omega t_{l,k}}|^2 = |H_{cl}(\omega)|^2 + |H_{sl}(\omega)|^2, \tag{41}$$

where:

$$H_{cl}(\omega) = \sum_{k=0}^{N-1} w_{l,k}\text{Cos}\omega t_{l,k} = H_{Ml}(\omega)\text{Cos}\alpha_l(\omega) \tag{42}$$

and

$$H_{sl}(\omega) = \sum_{k=0}^{N-1} w_{l,k}\text{Sin}\omega t_{l,k} = H_{Ml}(\omega)\text{Sin}\alpha_l(\omega). \tag{43}$$

$w_{l,k}$ in these equations is the $k$th coefficient of the $l$th filter and:

$$t_{l,k} = t_{(l-1)s+k} \ , \ l = 1, 2, ..., I, \ k = 0, 1, ..., N-1 \tag{44}$$

is the time at which the $k$th input pulse to the $l$th filter is received.

Now assume that the input to the in-phase channel of MTI filter is:

$$v_i(t) = (2S)^{1/2}\text{Cos}\omega_d t + x(t) \tag{45}$$

and for quadrature channel:

$$v_q(t) = (2S)^{1/2}\text{Sin}\omega_d t + y(t), \tag{46}$$

where $x(t)$ and $y(t)$ are in-phase and quadrature components of the interference, repectively. Without loss of generality, $x$ and $y$ can be considered normalized such that:

$$\overline{x^2} = \overline{y^2} = 1. \tag{47}$$

With this normalization, $S$ in Equations 45 and 46 will be the $SIR_{in}$ (input signal to interference ratio). If the $l$th filter output is shown by $Y_l$, considering Equations 41-43, the following equation is obtained:

$$Y_l = [(2S)^{1/2}(P_l(\omega_d))^{1/2}\text{Cos}\alpha_l(\omega_d) + \hat{x}_l]^2 + [(2S)^{1/2}(P_l(\omega_d))^{1/2}\text{Sin}\alpha_l(\omega_d) + \hat{y}_l]^2, \tag{48}$$

where $\hat{x}_l$ and $\hat{y}_l$ are in-phase and quadrature components of interference at the filter ouput and:

$$\alpha_l(\omega_d) = \text{tg}^{-1}\left(\frac{H_{sl}(\omega_d)}{H_{cl}(\omega_d)}\right). \tag{49}$$

So, the integrator output is equal to:

$$Z = \sum_{l=1}^{N_i} b_l Y_l = \sum_{l=1}^{N_i} b_l[2SP_l(\omega_d) + \hat{x}_l^2 + \hat{y}_l^2 + 2\hat{x}_l(2S)^{1/2}(P_l(\omega_d))^{1/2}\text{Cos}\alpha_l(\omega_d) + 2\hat{y}_l(2S)^{1/2}(P_l(\omega_d))^{1/2}\text{Sin}\alpha_l(\omega_d)] \tag{50}$$

and

$$Z^2 = \sum_{l=1}^{N_i}\sum_{m=1}^{N_i} b_l b_m[2SP_l(\omega_d) + \hat{x}_l^2 + \hat{y}_l^2 + 2\hat{x}_l(2S)^{1/2}(P_l(\omega_d))^{1/2}\text{Cos}\alpha_l(\omega_d) + 2\hat{y}_l(2S)^{1/2}(P_l(\omega_d))^{1/2}\text{Sin}\alpha_l(\omega_d)][2SP_m(\omega_d) + \hat{x}_m^2 + \hat{y}_m^2 + 2\hat{x}_m(2S)^{1/2}(P_m(\omega_d))^{1/2}\text{Cos}\alpha_m(\omega_d) + 2\hat{y}_m(2S)^{1/2}(P_m(\omega_d))^{1/2}\text{Sin}\alpha_m(\omega_d)], \tag{51}$$

therefore:

$$\overline{Z^2} = \overline{Z^2}_{S+1} = \sum_{l=1}^{N_i}\sum_{m=1}^{N_i} b_l b_m[4S^2P_l(\omega_d)P_m(\omega_d) + 4SP_l(\omega_d)\lambda_{mm} + 4SP_m(\omega_d)\lambda_{ll} + 4\lambda_{ll}\lambda_{mm} + 4\lambda_{lm}^2 + 8S(P_l(\omega_d)P_m(\omega_d))^{1/2}\lambda_{lm}\text{Cos}(\alpha_l(\omega_d) - \alpha_m(\omega_d))], \tag{52}$$

where $S+I$ index for $\overline{Z^2}$ implies that both signal and interference have been considered for the above calculation. According to Equation 25, $\lambda_{lm}$ in Equation 52 is equal to:

$$\lambda_{lm} = \overline{y_l y_m} = \sum_{k=0}^{N-1}\sum_{j=0}^{N-1} w_{l,k}w_{m,j}\rho_x(t_{l,k} - t_{m,j}), \tag{53}$$

where $\rho_x(\tau)$ is the normalized autocorrelation function of input interference of the MTI filter. Because of time dependency of the MTI filters, $\lambda_{ll}$ and $\lambda_{mm}$ are not equal (for $l \neq m$) as mentioned previously. In Equation 52:

$$(P_l P_m)^{1/2}\text{Cos}(\alpha_l - \alpha_m) = H_{Ml}H_{Mm}(\text{Cos}\alpha_l\text{Cos}\alpha_m + \text{Sin}\alpha_l\text{Sin}\alpha_m) = H_{cl}H_{cm} + H_{sl}H_{sm} = (\sum_{j=0}^{N-1} w_{l,j}\text{Cos}\omega_d t_{l,j})(\sum_{k=0}^{N-1} w_{m,k}\text{Cos}\omega_d t_{m,k}) + (\sum_{j=0}^{N-1} w_{l,j}\text{Sin}\omega_d t_{l,j})(\sum_{k=0}^{N-1} w_{m,k}\text{Cos}\omega_d t_{m,k}) = \sum_{j=0}^{N-1}\sum_{k=0}^{N-1} w_{l,j}w_{m,k}\text{Cos}\omega_d(t_{l,j} - t_{m,k}) \triangleq \hat{P}_{lm}(\omega_d), \tag{54}$$

consequently:

$$\overline{Z^2}_{S+I} = \sum_{l=1}^{N_i}\sum_{m=1}^{N_i} b_l b_m[4S^2P_lP_m + 4SP_l\lambda_{mm} + 4SP_m\lambda_{ll} + 4\lambda_{ll}\lambda_{mm} + 4\lambda_{lm}^2 + 8S\lambda_{lm}\hat{P}_{lm}]. \tag{55}$$

Eliminating the terms related to signal in Equation 55:

$$\overline{Z^2}_I = 4\sum_{l=1}^{N_i}\sum_{m=1}^{N_i} b_l b_m(\lambda_{ll}\lambda_{mm} + \lambda_{lm}^2), \tag{56}$$

$$\mathrm{SIR_0} = \frac{\overline{Z^2}_{S+I} - \overline{Z^2}_I}{\overline{Z^2}_I} = \frac{\sum_{l=1}^{N_i}\sum_{m=1}^{N_i} b_l b_m [S^2 P_l P_m + S(P_l \lambda_{mm} + P_m \lambda_{ll}) + 2SP_{lm}\lambda_{lm}]}{\sum_{l=1}^{N_i}\sum_{m=1}^{N_i} b_l b_m (\lambda_{ll}\lambda_{mm} + \lambda_{lm}^2)} \tag{57}$$

Equation 57 is interesting because it presents the exact relation between output and input signal to interference ratio for the signal processor. For example, Figure 6 shows the $\mathrm{SIR_0}$ for a moving window double canceller ($s = 1, N = 3$) with (13, 17, 14, 19) periods, assuming $N_p = 10$ (which results in $N_i = 8$ according to Equation 30), $b_i = 1$ (for all $i$), $S = \mathrm{SIR_{in}} = -30$ dB, $(C/N)_{\mathrm{in}} = 40$ dB and a Gaussian clutter with $\sigma_c T_{\mathrm{ave}} = 0.05$. The use of the generalized $N_e$ method (Equations 27 and 29) for the same example results in $N_e = 2.94$, which means an approximate value of $-30+14.6+20\log(2.94)^{1/2} = -10.7$ dB for $SIR_0$ where 14.6 dB is the improvement factor [1] of this MTI filter.

In Equation 57, if the MTI filter is not present and, like Castella, the effect of antenna pattern is to be considered, it is sufficient to set $P_l$ equal to $g_l^2$ where [13]:

$$g_l = \exp\left\{-2.7726\left[\frac{\dot{\theta}_s t_0}{\theta_B} + \left(l - \frac{N_B+1}{2}\right)\frac{\dot{\theta}_s}{\theta_B}\right]^2\right\}. \tag{58}$$

$\dot{\theta}_s$ is the rotation rate of the antenna in degrees per second, $\theta_B$ its 3-dB beamwidth (one way), $t_0$ the time offset of the center of pulse batch from the peak and $N_B = \theta_B/(\dot{\theta}_s T)$ the number of pulses in a dwell time

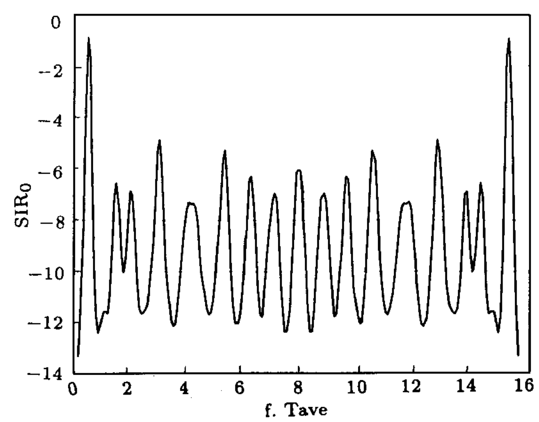


**Figure 6.** Output signal to interference versus Doppler frequency (normalized to average PRF) for an equal weight integrator preceded by a moving window double canceller with (13, 17, 14, 19) periods, assuming $N_p = 10$, $\mathrm{SIR_{in}} = -30$ dB and $(C/N)_{\mathrm{in}} = 40$ dB.

[1]. Substituting $p_l = g_l^2$ and assuming that $N_i = N_B$ and $\lambda_{ll} = 1$ (normalization of the filter power gain), Equation 57 results in:

$$\mathrm{SIR_0} = \frac{\sum_{l=1}^{N_B}\sum_{m=1}^{N_B} b_l b_m [S^2 g_l^2 g_m^2 + S(g_l^2 + g_m^2) + 2S g_l g_m \rho_{lm} \mathrm{Cos}\,\omega_d (t_l - t_m)]}{\sum_{l=1}^{N_B}\sum_{m=1}^{N_B} b_l b_m (1+\rho_{lm}^2)}, \tag{59}$$

as an especial case, which is the same as Castella equation.

## CONCLUSIONS

In this paper, the way to use $N_e$ method for calculating integration loss after an MTI filter with nonuniform PRF is stated and different methods of the PRF variation from this point of view are compared. Then, an exact method for analyzing the entire radar receiver signal processor (consisting of squarer, MTI filter and integrator) is presented. It should be noted that optimization of MTI filters, without taking their effect on the performance of the subsequent circuits into account is not thoroughly correct, because the choice of coefficients and periods of MTI filters affects the integration gain. Using the presented method, it is possible to evaluate the entire set of processor performance due to changes in periods and coefficients of the MTI filter and integrator.

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