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Particle Swarm Optimization Method for Optimal Reactive Power Procurement Considering Voltage Stability

B. Mozafari[*,2,3], T. Amraee[1,2,3], A.M. Ranjbar[2,3] and M. Mirjafari[3]

This paper presents and utilizes an Improved Particle Swarm Optimization algorithm (IPSO) for reactive power management in restructured power systems. Reactive power procurement is modeled as a Security Constraint Optimal Power Flow (SCOPF), which incorporates a voltage stability problem. This is a major concern in power system control and operation. The model attempts to minimize the cost of reactive power procurement and energy losses as a main objective, while the technical criteria and voltage stability margin, especially, are treated as soft constraints. From a mathematical point of view, the reactive power market can be expressed as a nonlinear non-convex optimization problem with multi-local minima. In most cases, the non-convexity results in a failure of the mathematical-based optimization algorithm to find the global optimum. Thus, the PSO, a powerful heuristic searching algorithm, is developed and implemented to find the global optimum of the reactive power market objective function. The feasibility of the methodology (IPSO) is tested over an IEEE30 bus system, while the obtained simulation results are compared with the gradient-based approach, using General Algebraic Modeling System (GAMS) software, the original PSO and another evolutionary programming called a Genetic Algorithm (GA). The results demonstrate that the IPSO can converge to better feasible solutions with less iteration and can be successfully used for reactive power scheduling in deregulation environments.

INTRODUCTION

Voltage stability constrained reactive power dispatching in deregulated power networks is a difficult task facing an Independent System Operator (ISO) that is mandated to provide equitable ancillary services. In a vertically integrated power system, reactive power facilities are operated under a monopoly by the system operator to meet technical requirement, such as improving the voltage profile and reducing the loss of transmission lines or, even, increasing the voltage stability margin to avoid instability, due to load perturbation or equipment failure. However, there is not a clear competition mechanism for providing reactive power in this structure. With the advent of restructuring, generation, transmission and distribution are separated into distinct entities in terms of ownership and management. Thus, the various competitive market structure models are designed and developed for the pricing of electricity and, also, associated ancillary services [1,2]. In these structures, new concerns are introduced into the power system operation to handle both technical and economical issues efficiently. Optimal reactive power procurement and voltage control are the most important issues in restructured power systems. Usually, the strategies for dealing with reactive power control are in accordance with electricity market strategies. In recent years, various approaches have been proposed for the optimal distribution of re-

*. *Corresponding Author, Science and Research Branch, Islamic Azad University, Tehran, I.R. Iran.*
1. *Science and Research Branch, Islamic Azad University, Tehran, I.R. Iran.*
2. *Department of Energy and Environment, Niroo Research Institute, Tehran, I.R. Iran.*
3. *Department of Electrical Engineering, Sharif University of Technology, Tehran, I.R. Iran.*

active power among providers in power pool electricity markets. In one approach, active and reactive power markets are cleared together, at the same time. For example, Xie et al. [3] apply a mathematical method to minimize both active and reactive generation costs while, in [4,5], the reactive powers generated by static compensators are also treated as ancillary services and their associated costs are considered correspondingly. In the second approach, the reactive power market is executed independently of the energy market. In this perspective, a method is established that aims to dispatch efficient reactive power at a minimum cost, while real power transactions are assumed to be fixed [6]. As a matter of fact, that part of the reactive power, which is used for improving various technical features of the power system, is inherently assumed as an ancillary service. This means that generators should provide some parts of the reactive power in a mandatory manner to satisfy grid code requirements.

Generally, an optimal reactive power procurement problem can be usually formulated as a nonlinear programming (NLP) problem. The final purpose is to optimize the objective function, while satisfying sets of constraints. Usually, equality constraints are power flow equations and the inequality constraints are the limits on control variables, voltages, active/reactive power generation and line flows. The mathematically-based optimization algorithms are very efficient in handling linear convex problems. However, the studies, which have been performed on reactive power and voltage control optimization, show that the reactive power market is usually a non-convex NLP with more than one feasible region where the optimal solution might be found at any point within any such region. Hence, the mathematical algorithms are very sensitive to initial conditions or starting points [7]. The stochastic behavior implemented in the most population-based evolutionary algorithms makes them good candidates for obtaining the optimal solution. A survey regarding application of intelligent techniques for reactive power/voltage control in the power system is presented in [8]. Recently, Kennedy and Eberhart have developed a new powerful evolutionary computation technique, called the PSO, which comprises a very simple concept. This paradigm can be implemented in a few lines of computer code. It requires only primitive mathematical operators and is computationally inexpensive in terms of both memory and speed requirements [9].

The PSO has also been applied to some optimization problems in the power systems. In [10,11], the PSO is implemented for well-known economic dispatch problems. Yoshida et al. [12] have presented a two-step procedure for handling voltage stability criteria in the reactive power optimization problem. In the first stage, the PSO is used to minimize transmission power losses, while, in the second stage, the maximum loading parameter of the system is evaluated using a CPFLOW method [13] for each best solution found by the PSO. The first solution that satisfies the minimum requirements of the system operator is selected as the final best solution. However, the PSO has not been implemented for power market scheduling, especially the reactive power market constrained by voltage stability criteria.

In this paper, the PSO is modified to solve the reactive power market, which incorporates the voltage stability criterion. This model conforms to the power pool electricity market. The objective function includes the costs associated with energy losses, as well as reactive power generation costs, while the voltage stability criterion is implemented into equality constraints. The feasibility of the proposed method is demonstrated and compared with the original PSO, the GA and results obtained from the GAMS software.

PSO APPROACH

Original Algorithm

The particle swarm optimization algorithm is a new evolutionary computation technique motivated by simulation of the social behavior of a folk or a group of people [9]. Each individual refers to a particle and presents a candidate solution to the optimization problem. Two different versions of the PSO algorithm exist, which can be implemented and tested, namely, the gbest PSO and the lbest PSO [14]. In the gbest topology, all members move toward the best particle in the population, however, in the lbest topology, each swarm aims at the best particle in its surrounding neighborhood. In the original version, neighborhood size is constant and neighbors do not change during a run. The impact of randomly created neighborhood topologies on PSO performance has been investigated in [15]. A modified particle swarm optimizer is presented in [16], which reduces a number of setting parameters, such as maximum and minimum velocity ($V_{\max}, V_{\min}$), associated with the original algorithm. In the following, the application of the PSO method for a general form of optimization problem is described. Let the optimization problem be written, as follows:

$$\text{Minimize } F(X)$$

$$\text{s.t}$$

$$G(X) = 0,$$

$$H(X) < 0,$$

$$X^{\min} \leq X \leq X^{\max}. \tag{1}$$

In this formulation, the vector, X, contains control variables. PSO can be applied to an optimization problem through the following steps.

Step 1

Using penalty factors, Equation 1 can be transformed into the form of the augmented objective function without any constraints. In such a case, the control variable limits, ($X^{\min} \leq X \leq X^{\max}$), define the feasible solution region, and initial population is generated in such a way as to fulfill these conditions. Moreover, a random velocity vector is assigned to each particle. These velocity vectors are generated according to the lower and upper bounds of control variables. Let X and V indicate the particle position and its corresponding velocity in a d-dimensional search region, respectively. Then, the ith particle can be represented as $X_i = (x_{i1}, x_{i2}, \cdots, x_{id})$ and its flight speed can be expressed as $V_i = (v_{i1}, v_{i2}, \cdots, v_{id})$.

Step 2

The augmented objective function is used to evaluate the fitness of each individual in the population. The obtained value for each particle is compared with the previous one, which is stored in the agent memory. If a decrease in value is noted (for the minimization problem), the new explored point is substituted with the prior position. This value suggests the self-experience of a particle and can be written in the form of $P_{\text{best } i} = (P_{\text{best } i1}, P_{\text{best } i2}, \cdots, P_{\text{best } id})$ for the ith agent.

Step 3

In the lbest version, the initial generation is categorized into different groups where the particle's neighbors are specified. One methodology to determine neighbors is presented in [15]. Implementing the gbest version requires no additional operation. In this case, all individuals are considered to be a particle's neighbors.

Step 4

Referring to the evaluated fitness value of each particle obtained in Step 2, the best agent in each group can be distinguished. This value is shown by $L_{\text{best } h}$, where h denotes the number of the group. In the gbest version, there is one neighborhood, which contains all population members and, hence, $L_{\text{best } h}$ is replaced with G_{best}, accordingly. The $L_{\text{best } h}$ or G_{best} store the best experience attained by members of the hth neighborhood or the best experience achieved by all members of the population in every epoch.

The modified velocity and position of each particle can be calculated, using the current velocity, random weighted distance from the $P_{\text{best}.i,h}$ and random weighted distance from the $L_{\text{best}.h}(G_{\text{best}})$, by the following formulas. The process is shown plainly in Figure 1.

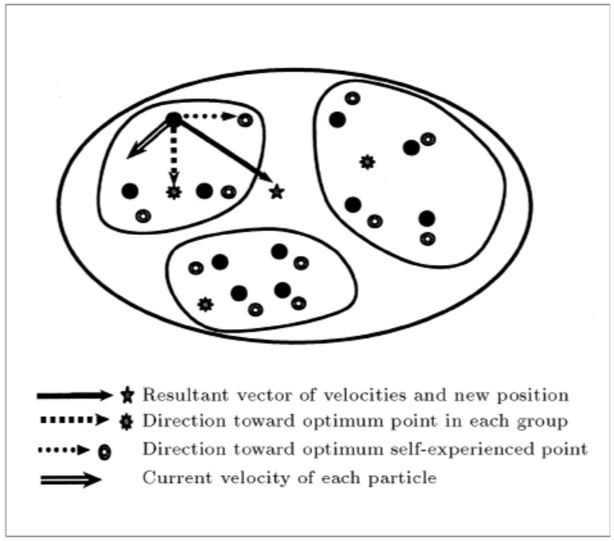

Figure 1. Concept of particle's movement by PSO algorithm.

$$V_{i,h}^{(t+1)} = w \times V_{i,h}^{(t)} + c_1 \times \text{rand}_1(.) \times (P_{\text{best } i,h} \quad X_i^{(t)}) + c_2.\text{rand}_2(.) \times (L_{\text{best } h} \quad X_{i,h}^{(t)}), \tag{2}$$

$$X_{i,h}^{(t+1)} = X_{i,h}^{(t)} + V_{i,h}^{(t+1)}, \tag{3}$$

where:

$i = 1, 2, \cdots, m$	points to the ith particle,
m	the number of particles,
t	a counter of iterations,
w	an inertia weight,
c_1, c_2	the acceleration constants,
$\text{rand}_1(.), \text{rand}_2(.)$	uniform random generator function in range of [0,1].

Equations 2 and 3 are used for implementing the PSO lbest version. These equations are also applicable for the gbest version substituting $h = 1$ and G_{best} instead of $L_{\text{best } h}$ into Equation 2.

The acceleration constants, c_1 and c_2, represent the weighting of the stochastic acceleration terms that pull each particle toward the P_{best} and L_{best} positions. Low values allow particles to roam far from the target regions before being tugged back. On the other hand, the high values result in abrupt movement towards, or, backwards, away from the target regions. Therefore, according to previous experiences, these factors are set to 2.0 during all simulations. The PSO algorithm is very sensitive to the value of inertia weight, w, presented in Equation 2. The w adjusts the PSO dynamic behavior during searching procedures, where $w = 1$ guaranties the PSO to converge by iterations. High values of w put particles to fly over the local minima, however, low values allow particles to intensify searching local areas. Generally, the w decreases linearly from about 0.9 to 0.4 during the run, according to the following equation:

$$w = w_{\max} \quad \frac{w_{\max} \quad w_{\min}}{\mathrm{iter}_{\max}} \times \ \mathrm{iter},$$

where $W_{\max}$ is 0.9, $W_{\min}$ is 0.4 and $\mathrm{iter}_{\max}$ is maximum iteration number.

Improved Algorithm (IPSO)

PSO movement's instructions can be effectively applied to all members except for the best agent position, namely, $L_{\text{best } h}$, where this position is identical to its current position and its best previous experience. This causes the best agent moves only based on its weighted previous velocity. This becomes serious when $w \leq 1$ tends to zero velocity after a number of iterations. Thus, the probability of local minima escaping may decrease in this situation. To solve this deficiency, in this section, a method, which randomly selects the best position for each particle, is presented.

In Step 4 of the PSO original algorithm, particles that exist in neighborhood, h, move toward $L_{\text{best } h}$, which can be modified, as follows.

Let the initial population be divided into N neighborhoods, where $h = 1, 2, \cdots, N$ and all $L_{\text{best } h}$ are available at this stage. For each individual, L_{best} is assigned, where this value is selected among $L_{\text{best } h}$s, based on one of the random selection procedures, such as the Roulette Wheel selection scheme. In this approach, all candidates are assigned weights based on their fitness values and then a random selection is used to determine the social leaders. Accordingly, this provides a situation, in which particles in one neighborhood move toward other groups and increase overlap among the flock. The aim of this process is to maintain population diversity.

FORMULATION OF THE REACTIVE POWER MARKET PROBLEM

In vertically integrated power systems, a single entity (provider) performs all of the basic functions of production, transportation and delivery. It is also responsible for developing the power system in each section. Transmission lines provide single path connecting generation sources to consumption areas. The static voltage security assessment is strictly dependent on power system network operation. From a reactive power point of view, in vertically integrated utilities, the system operator makes an effort to dispatch reactive power resources in order to maintain the security of the system under normal and, also, under various loading conditions. The reactive reserve can be determined doing different ($N \quad 1$) contingency analyses, according to the priority list. In such a system, the main objective is the fulfillment of some technical requirements, such as improving the voltage profile, minimizing real power losses, increasing the available transmission capability of the network etc. [7,8]. At the same time, there is less attention paid to the reactive power revenue for each supplier.

However, in restructured power systems, financial interests introduce new dimensions to an open market system, where the cost and contribution of different reactive power facilities should be evaluated more precisely than before. The methodology presented here considers the predefined loadability limit or voltage stability margin as a soft constraint. Considering the voltage stability constraint in the reactive power market modeling provides independent system operator with insurance against entering the power system into unstable conditions, which may happen due to load changes. In this paper, there is no intention of dealing with reactive power reserve procurement and, consequently, no contingency analysis will be carried out into the presented model and simulation. Thus, a voltage stability constrained reactive power market can be generally formulated, as follows.

Reactive Market Objective Function

$$\begin{aligned}
\text{Minimize } & f(\underline{Q}_g, \underline{Q}_g^{vsm}, \underline{Q}_{sh}, \underline{X}) \\
&= \sum_{i=1}^{N_g} C_{gqi}(Q_{gi}) + \sum_{i=1}^{N_g} A_{gqi}^{vsm}.C_{gqi}(Q_{gi}^{vsm}) \\
&+ \sum_{i=1}^{N_{sh}} C_{shi}(Q_{shi}) + 0.5.MCP.\sum_{i,j}(G_{i,j} \\
&\times (V_i^2 + V_j^2 \quad 2V_iV_j\cos(\theta_i \quad \theta_j))), \qquad (4)
\end{aligned}$$

where:

i, j:	index for buses,
$\underline{Q}_g$:	generators VAr output at normal condition,
$\underline{Q}_g^{vsm}$:	generators VAr output when loads are increased,
A_{gqi}^{vsm}:	the ratio between the cost of reactive energy and reactive power reserve,
N_g:	number of generators,
$\underline{Q}_{sh}$:	static VAr compensators output,
N_{sh}:	number of static compensators,
V:	voltage at bus in per unit,
$G_{i,j}=\mathrm{real}(Y_{i,j})$:	conductance of line $i \quad j$,
θ:	angle associated with Y bus,

Y	admittance matrix of power system,
$\underline{X} = [\underline{\theta}, \underline{V}]$:	state vector,
MCP	Market Clearing Price.

The objective function consists of the total payments of the reactive power procurement under normal and increasing load conditions and the payment associated with real power losses of transmission lines. In this model, the reactive power reserve necessary to protect the system against voltage instability is procured, simultaneously, with the reactive energy. In this structure, it is assumed that the slack generator provides network losses at a MCP price, which has been previously settled in the electricity market. The cost of providing real power loss into the objective function of reactive power procurement is considered, since this term can inherently prevent the transportation of reactive power from long distance areas. The traveling of reactive power over long transmission lines can increase both active and reactive power losses. Since the defined objective function can minimize the cost of active and reactive power losses, ISO can assure the adequate local provision of reactive power. As a result, it seems to be a good objective for reactive power control in deregulated power systems.

Constraints on the Power System and Resources

1. Power flow equations under normal condition:

$$P^{\circ}_{gi} \quad P^{\circ}_{di} = \sum_j V_i Y_{i,j} V_j \cos(\theta_{i,j} + \delta_j \quad \delta_j), \qquad (5)$$

$$Q^{\circ}_{gi} + Q^{\circ}_{shi} \quad \tan(\varphi^{\circ}_{di}).P^{\circ}_{di} = \sum_j V_i Y_{i,j} V_j \sin(\theta_{i,j} + \delta_j \quad \delta_j), \qquad (6)$$

where:

$P^{\circ}_{gi}, P^{\circ}_{di}$:	denote generation and consumption of active powers,
φ°_{di}:	denotes power angle of consumption loads.

2. Power flow equations with fixed predefined voltage stability margin:

$$(1 + k_{gi}.\lambda.\sum_{i=1}^{N_g} P^{\circ}_{gi}).P^{\circ}_{gi} \quad (1 + k_{di}.\lambda.\sum_{i=1}^{N} P^{\circ}_{di}).P^{\circ}_{di} = \sum_j V_i^{vsm} Y_{i,j} V_j^{vsm} \cos(\theta_{i,j} + \delta_{ij}^{vsm}), \qquad (7)$$

$$Q^{vsm}_{gi} + Q^{\circ}_{shi} \quad (1 + k_{di}.\lambda.\sum_{i=1}^{N} P^{\circ}_{di}).\tan(\varphi^{\circ}_{di}).P^{\circ}_{di} = \sum_j V_i^{vsm} Y_{i,j} V_j^{vsm} \sin(\theta_{i,j} + \delta_{ij}^{vsm}). \qquad (8)$$

In the above equations, the subscript "$\circ$" is used to represent the quantities of variables in the base-case, while vsm is used to show the variables under load increasing conditions. In Equations 7 and 8, the percentage of load increasing is modeled by λ. The terms of $\lambda.\sum_{i=1}^{N_g} P_{gi}$ or $\lambda.\sum_{i=1}^{N} P_{di}$ are the total increase of power system generation and load, respectively. k_{gi} is a distribution factor, which defines the direction of generation increase for the ith generator and k_{di} indicates how much of the load increase occurs at bus i. For a given load increase, the sum of distribution factor, k_{gi}, as well as k_{di}, is unity. This means that one has $\sum k_{gi} = 1$ and $\sum k_{di} = 1$. It should be mentioned that numerous directions could be defined and considered for any load increase in the power system using these participation factors. In this paper, the amount of generation/consumption is increased, according to their base-case values. In other words, one can define $k_{gi} = \frac{P^{\circ}_{gi}}{\sum P^{\circ}_{gi}}$ and $k_{di} = \frac{P^{\circ}_{di}}{\sum P^{\circ}_{di}}$;

3. Reactive power-generation limits:

$$Q^{\min}_{gi} \le Q_{gi} \le Q^{\max}_{gi},$$

$$Q^{\min}_{gi} \le Q^{vsm}_{gi} \le Q^{\max}_{gi}, \qquad (9)$$

$$Q^{\min}_{shi} \le Q_{shi} \le Q^{\max}_{shi}. \qquad (10)$$

$Q^{\max}_{gi}$ and $Q^{\min}_{gi}$ are the maximum and minimum reactive power that a generator can provide. These values vary with a change in active power output of a generator. The capability curve of a generator is usually used to demonstrate the relation between its active and reactive power outputs. A typical capability curve of a generator is shown in Figure 2. It is restricted to the maximum stator winding heating limit, which depends on stator current and it is also restricted to the maximum field winding heating limit or the end region heating limit of the rotor when the generator operates in an over or under excitation mode. The feasible operating conditions of a generator can be expressed as a function of its active and reactive operating current point, its terminal and internal setup voltages and its synchronous reactance [17]. However, this paper assumes that generators are allowed to propose their reactive power generation capability, with respect to their active power generation points;

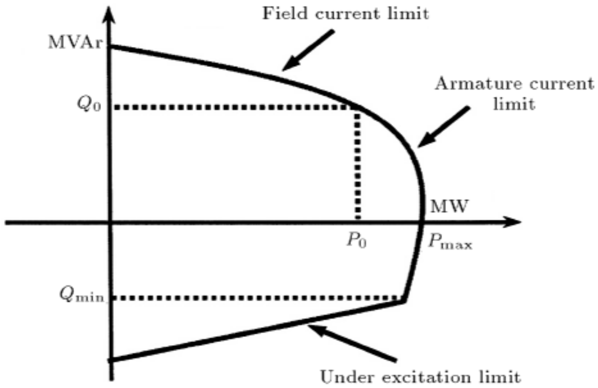

Figure 2. Typical capability curve of a generator.

4. Bus voltage limits:

$$V_i^{\min} \leq V_i \leq V_i^{\max},$$

$$V_i^{\min} \leq V_i^{vsm} \leq V_i^{\max}. \quad (11)$$

$V_i^{vsm} = \text{constant}$, if $i \in$ generator bus.

The value of the voltage stability margin ($\lambda . \sum_{i=1}^{N_g} P_{gi}$ or $\lambda . \sum_{i=1}^{N} P_{di}$) should be predetermined by ISO, who is legally responsible for the security and reliability of the power system. In this paper, there is no intention of calculating the vsm factor, however, a methodology has been presented for modeling voltage stability in the reactive power market. This method is called fixed voltage stability margin formulation. Other useful strategies for handling the voltage stability problem are presented in [18,19].

Reactive Power Production Costs

1. Synchronous generators:
Active power generation decreases the reactive power capability of a generator, as shown in Figure 2. The cost of reactive power production can be modeled using opportunity cost calculation [6]. An approximation for the cost of reactive power production corresponding to the first term of Equation 4, is given in the following equation [5]:

$$C_{gqi}(Q_{gi})=\left[C_{gpi}(S_{gi}) \quad C_{gpi}\left(\sqrt{S_{gi2} \quad Q_{gi2}}\right)\right] K_{gi}, \quad (12)$$

where $C_{gpi}(P_{gi}) = aP_{gi2} + bP_{gi} + c$ is active power generation cost, Q_{gi} is reactive power output of ith generator, $S_{gi} = \sqrt{P_{gi}^2 + Q_{gi}^2}$ is apparent power of ith generator and K_{gi} is profit rate of active power, usually between $0.05 \sim 0.1$;

2. Static VAr compensators:
The second term in Equation 4 is the total production cost of static VAr compensators, which can be expressed as the following equation, for the device installed at bus j:

$$C_{shj}(Q_{shj}) = r_{shj} Q_{shj}, \quad (13)$$

where r_{shj} is the price of the reactive power per MVAr, which depends on different factors, such as capital investment of the compensator, its period of lifetime and average utilization factor. For example, a SVC with an investment cost of \$48,000/MVAr, a lifetime of 30 years and an average use of 2/3, has its r_{cj} as [4,5]:

$$r_{shj} = \frac{22000}{30 \times 365 \times 24 \times \frac{2}{3}} = 0.1225(\$\text{MVAr}). \quad (14)$$

REACTIVE POWER MARKET SOLUTION USING PSO

The following procedure can be used for obtaining the optimal solution to the proposed reactive power market.

Step 1

Initial populations of agents are generated randomly inside the searching space, specified by the upper and lower bands of control variables. In this problem, control variables consist of the output voltage of generators and the reactive power outputs of static VAr compensators. Initial velocity values are also assigned to each particle. For each agent, P_{best} is initialized with the current position.

Step 2

For each individual, Equations 5 and 6 are evaluated. If the obtained values for state variables satisfy Conditions 9 and 11, then, go to Step 3, otherwise, assign a high value to the objective function and, then, go to Step 5.

Step 3

The control variables are fixed and, then, Equations 7 and 8 are evaluated. if the obtained values for state variables satisfy Conditions 9 and 11, go to Step 4, otherwise, assign a high value to the objective function and, then, go to Step 5.

Step 4

Particles are divided into different groups. For each particle, P_{best} and L_{best} are calculated for each neighborhood. Finally, for each particle, L_{best} is selected, according to the method presented previously.

Step 5

New velocities are calculated using Equation 2.

Step 6

New positions are calculated using Equation 3.

Step 7

If the iteration number reaches the maximum, then, stop and print the final results, otherwise, go to Step 2.

SIMULATION AND RESULTS

In this section, the reactive power market is simulated using the IEEE 30-bus test system and the results obtained for optimal reactive power procurement are presented. The performance of the reactive power market is studied for two different cases. In the first case, it is assumed that all generators, which have been committed for active power production, are obliged to provide a sufficient reactive power reserve, according to the ISO requirement, however, they will be paid only for reactive energy production or absorption. In other words, all A_{gqi}^{vsm} are equal to zero. In the second case, the cost associated with the reactive power reserve takes into account using $A_{gqi}^{vsm} neq0$. In both cases, the feasible solution of the market is achieved by means of different methodologies, such as: GA [20], PSO, IPSO and GAMS software. In these studies, the performance of the IPSO solver is compared with other methods to demonstrate its feasibility in the optimal reactive power procurement problem. The network configuration of the IEEE 30-bus test system and its transmission lines data are given in [21]. The characteristics of generating units are tabulated in Table 1.

The second column in Table 1 represents the reactive power output of generators when they are required to fix their output voltage at the desired values indicated in column three. In this condition, the real power loss of the system is about 2.45 MW.

The reactive power costs of generating units are calculated by Equation 12, where the associated parameters used in this formula are presented in Table 2. Figures 3 to 5 show the capability curves of each generator. It is assumed that these curves are submitted to the ISO from each generating unit. These curves demonstrate the maximum reactive power that each participant is willing to produce.

The additional energy necessary to provide active power losses is procured from the slack generator (G_1),

Table 1. Generating unit characteristics.

	P_g (MW)	Q_g (MVAr)	V_G	$P_{g\,max}$ (MW)
GEN$_1$	25.97	-12.37	1.00	80
GEN$_2$	60.97	-13.36	1.00	80
GEN$_{13}$	37	-0.05	1.00	50
GEN$_{22}$	21.59	6.08	1.00	50
GEN$_{23}$	19.2	-3.28	1.00	30
GEN$_{27}$	26.91	-1.87	1.00	55

Table 2. Coefficient factors of Equation 12.

	a (\$/Mw^2)	b (\$/Mw)	c (\$)	K_g
GEN$_1$	0.02	2	0.0	0.1
GEN$_2$	0.0175	1.75	0.0	0.1
GEN$_{13}$	0.0625	1	0.0	0.1
GEN$_{22}$	0.0083	3.25	0.0	0.1
GEN$_{23}$	0.025	3	0.0	0.1
GEN$_{27}$	0.025	3	0.0	0.1

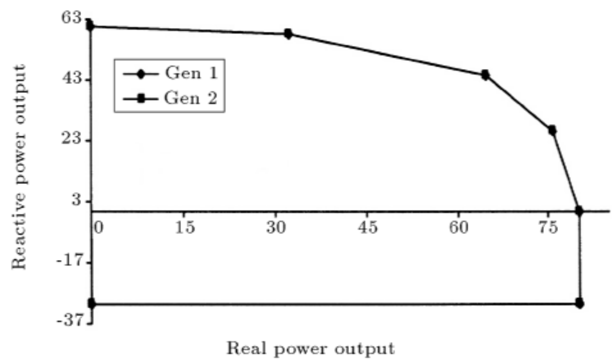

Figure 3. Capability curves of generators 1 and 2.

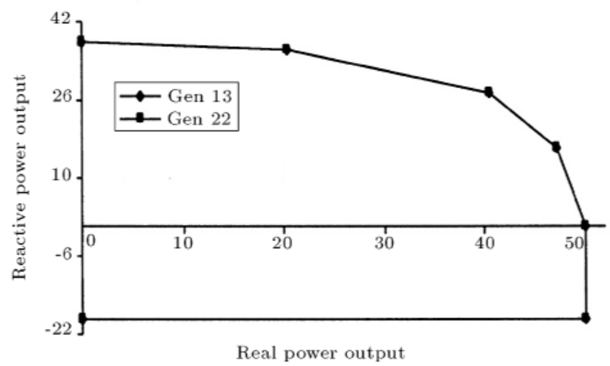

Figure 4. Capability curves of generators 13 and 22.

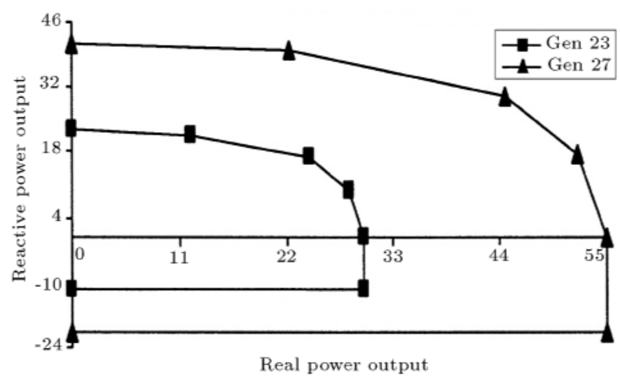

Figure 5. Capability curves of generators 23 and 27.

Table 3. Static compensators operating costs.

	C_6	C_{19}	C_{28}
Cost (\$/MVAr)	0.1	0.15	0.07

based on the electricity market clearing price. In all simulations, MCP is assumed to be 9.5 $/MW to figure out the cost of loss. The reactive power prices of electronic-based static VAr compensators are given in Table 3. These compensators are installed at buses 6, 19 and 28, respectively. Fixed or switched capacitors/inductors cannot be entitled to provide reactive power as an ancillary service, since they are not equipped with regulator devices and, hence, their reactive power output varies drastically with any voltage changes.

The voltage stability margin can be improved by efficient reactive power procurement. This is shown on a typical P V curve of a power system in Figure 6, where consumption loads are fixed. A feasible solution for reactive power dispatch may not exist if available reactive power support is not adequate to satisfy the voltage inequality constraints under heavy loading conditions. In this paper, it is assumed that the voltage stability margin, defined by the ISO, can be reached using available reactive support. This value is assigned to be 0.1 per unit for the secure operation of the power system under normal operation.

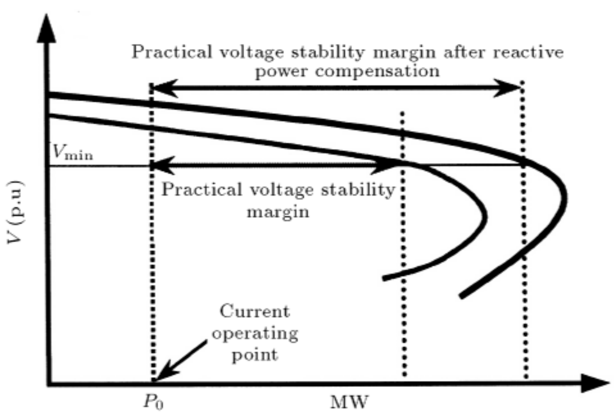

Figure 6. Typical PV curve of a power system.

Case 1

In this case, the optimum reactive energy is procured for the normal condition. Thus, all A_{gqi}^{vsm} are assigned to zero and, then, the optimum point of the market is obtained, using GA, the original PSO algorithm, the proposed IPSO algorithm and GAMS software [22], which have been tabulated in Table 4.

The MINOS5 solver in GAMS software is used for the reactive market computation. The GAMS software has only been used to justify the correctness of the solutions obtained by means of the population-based algorithm. However, a comparison between evaluated costs of each method highlights the fact that population based algorithms may reach a better solution than a gradient-based method. In heuristic methods, the probability of escaping from local minima usually increases with an increase in population size.

To provide a rational comparison between the applied methods, similar population size is generated in each algorithm. The number of agents in IPSO and PSO is considered to be 24. This means that 24 new positions will be explored in every epoch. A genetic algorithm is initialized with 300 agents first and, then, the best 24 individuals make a good population during the searching procedures. The results clearly show that the GA has a poor performance in finding the best solution among the alternatives considered. The IPSO present the best solution, in which the active power loss is 1.9705 (MW). This is about 19.6% less than before.

Doing numerous simulations have indicated that the PSO algorithm has a good operation, moving the particles all over the search space. The results obtained from a 100 times execution of different heuristic algo-

Table 4. Market simulation results.

	GA	PSO	IPSO	GAMS
P_{G1} (MW)	25.648	25.5	25.5	25.5
Q_{G1} (MVAr)	3.9591	3.5247	3.5256	3.4737
Q_{G2} (MVAr)	12.231	10.839	10.842	10.601
Q_{G13} (MVAr)	8.6942	7.8017	7.7976	9.4239
Q_{G22} (MVAr)	10.342	9.4003	9.3969	8.9853
Q_{G23} (MVAr)	6.5433	6.268	6.2683	6.2212
Q_{G27} (MVAr)	4.6327	4.3474	4.3314	4.9317
Q_{c6} (MVAr)	24.805	34.0670	34.2516	33.247
Q_{c19} (MVAr)	5.1774	4.1914	4.2209	4.171
Q_{c28} (MVAr)	22.536	17.0336	16.8361	16.371
Total Cost($)	27.557	26.1196	26.1194	26.1493

rithms over the reactive power market are summarized in Table 5, which can provide useful information about the performance of each methodology. As Table 5 indicates, among the other methods, IPSO has a robust characteristic and better performance.

In Table 6, the PSO sensitivity to the variation of C_1 and C_2 is presented. As mentioned before, selecting these factors can affect the performance of the optimization algorithm. From the results of this table, one can conclude that it is proper to set $C_1 = C_1 = 2.0$.

Figure 7 shows the typical convergence characteristics of GA, PSO and IPSO methodologies. It is clear from the figure that IPSO is converged to high quality solutions at the early iterations. The dynamic convergence curve of the PSO is similar to the IPSO and both of them can achieve a better solution, with low iteration, in respect to a genetic algorithm, where a decrease in the value of the objective function changes more slightly over iteration.

On the other hand, the convergence rate of the PSO depends on the factors w, C_1 and C_2. Since w is usually chosen less than one, the velocity of the agents decays to zero. This situation deteriorates the potential of the PSO not to be trapped in local optima. Referring to Equations 2 and 3, one can easily find out that particle position does not have any change when its associated velocity becomes zero. A typical variation of velocity for a particle is depicted in Figure 8. Figure 9 shows the same diagram for an improved particle swarm optimization algorithm. A comparison between these two figures reveals that IPSO can converge to global optima, even in the last iterations. It also shows that the value of a particle's velocity can only be used as a convergence criterion for the PSO algorithm.

The impact of population size generated in the searching space has been investigated and the obtained results are shown in Figure 10. This study indicates that the IPSO can achieve a better solution in earlier iterations, if the number of individuals increases. However, trials with different population size can converge to near optimal value. Table 7 has also indicated the objective value obtained by setting different population sizes.

Table 5. Statistical results.

	The Best Solution	The Worst Solution	Mean	Variance
IPSO	26.1194	27.0780	26.2403	0.175366
PSO	26.1196	28.1932	26.3442	0.3098
GA	26.7032	28.2218	27.374	0.51109

Table 6. PSO sensitivity to parameter variation.

C_1	0.5	1.0	3.0	2.0
C_2	2.0	2.0	2.0	2.0
Cost Value	30.019	29.013	28.552	28.193
C_1	2.0	2.0	2.0	2.0
C_2	0.5	1.0	3.0	2.0
Cost Value	32.066	30.547	28.703	28.193

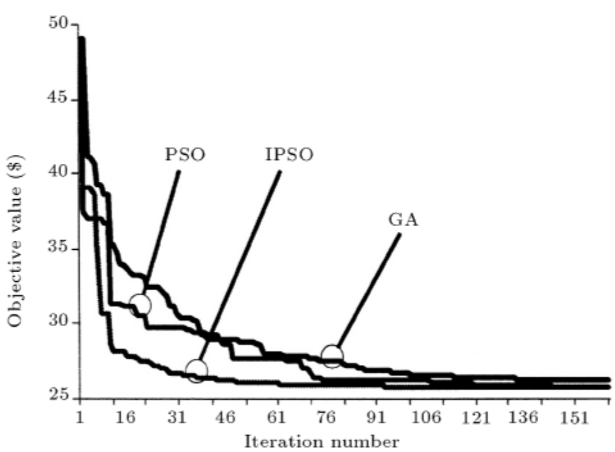

Figure 7. Dynamic behavior of GA, PSO and IPSO.

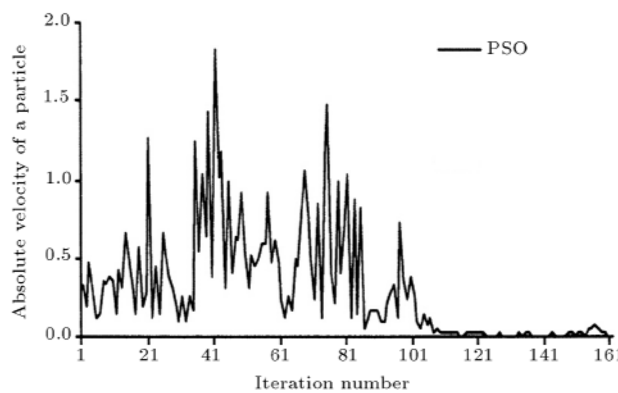

Figure 8. Variation of particle velocity by iteration for PSO.

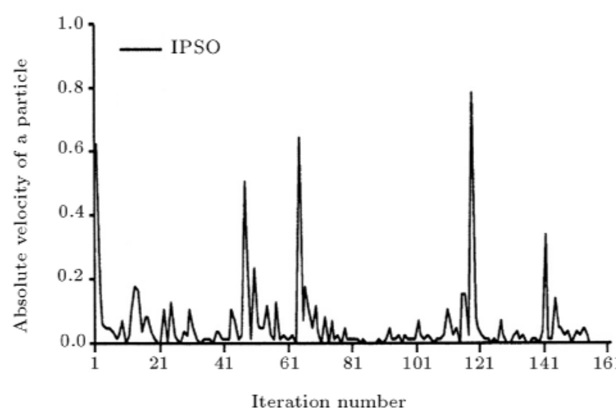

Figure 9. Variation of particle velocity by iteration for IPSO.

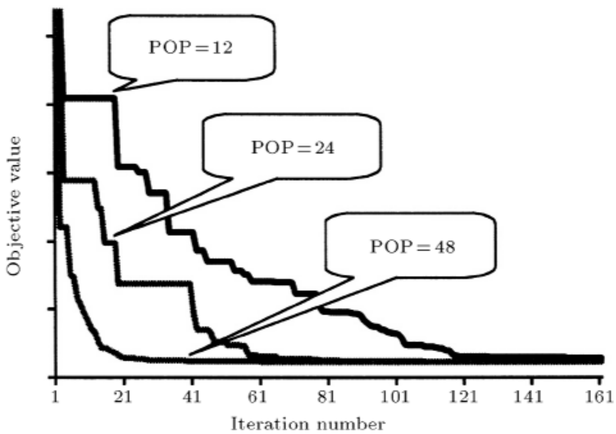

Figure 10. IPSO convergence characteristic for different population.

Table 7. Results obtained by different population size.

Iteration	pop = 12	pop = 24	pop = 48
1	53.380	53.380	53.380
10	45.438	31.479	29.106
20	40.426	31.479	26.454
30	38.512	28.067	26.252
40	35.656	28.064	26.204
50	33.529	27.407	26.174
60	32.029	27.407	26.156
70	31.939	27.245	26.126
80	29.823	26.945	26.123
90	29.556	26.776	26.121
100	28.214	26.717	26.121
110	27.247	26.199	26.120
120	26.533	26.135	26.120
130	26.489	26.129	26.120
140	26.482	26.123	26.119
150	26.438	26.120	26.119
160	26.402	26.120	26.119

Case 2

This case deals with a situation in which reactive energy and the reactive power reserve are determined simultaneously for security operation of the power system. In this paper, it is assumed that generators are allowed to enter into the reactive power reserve market, and other types of reactive power generation can only participate in the reactive energy market. As mentioned previously, it is intended to determine the efficient amount of reactive reserve needed to prevent voltage instability, happening due to perturbation in load values. Hence, it is assumed that all competitive suppliers could bid the cost of their reactive capacity proportional to the associated cost of reactive energy through the factors, A_{gqi}^{vsm}.

Assigning different values to the factors, A_{gqi}^{vsm}, can define various strategies for the reactive power suppliers who take part in the reactive reserve market. One strategy can be defined by setting all A_{gqi}^{vsm} to the same value. The results obtained for different values of A_{gqi}^{vsm} are presented in Table 8.

The simulation results in Table 8 show the impact of the reactive power costs on both reactive energy and reactive reserve dispatching. It also shows that static compensators have a good opportunity to provide the necessary reactive power when the prices of the reactive power reserve of the generators are extremely high. The results obtained for the two first generators show that they have a large opportunity to make market power in the system.

The amount of reactive power of the first generator does not change with increasing reactive costs. The revenue of the second generator notably changes in the situation expressed by $A_{gq}^{vsm} = 5.0$, in comparison with $A_{gq}^{vsm} = 1.0$. This is somehow true for the generator G_{13}. These signals can lead ISO to identify weak areas with less reactive power capacity and to reinforce these parts of the network.

CONCLUSION

Reactive power management can be carried out for different purposes, such as minimizing power losses, improving voltage profile and, also, providing a sufficient voltage stability margin. In this paper, first, a model to handle both reactive energy and reactive reserve has been presented, in which reactive power can be efficiently dispatched locally. Second, an improved version of the PSO has been presented, which increases the particle's endeavor to escape from the local minima. Then, the proposed method has been successfully implemented for the optimal procurement of reactive power in the open electricity market. In the proposed reactive market structure, the voltage stability constraint is implemented as a soft constraint to guarantee the security of the power system, due to some happenings in the power system, such as sudden load perturbation. Thus, reactive power management becomes a NLP problem with non-linearity in both the objective and its associated constraints. The simulation results carried out for the IEEE 30-bus system, demonstrate the excellent capability of the IPSO in obtaining the best solution, as well as convergence time,

Table 8. Results of Case 2.

MVAr	$A_{gq}^{vsm} = 0.5$		$A_{gq}^{vsm} = 1.0$		$A_{gq}^{vsm} = 5.0$	
	RE[a]	RR[b]	RE	RR	RE	RR
Q_{G1}	3.387	3.685	3.263	3.556	3.070	3.40
Q_{G2}	8.046	12.42	6.499	10.86	2.533	6.968
Q_{G13}	5.546	6.909	4.412	5.773	1.525	2.894
Q_{G22}	6.115	10.213	4.314	8.408	-0.275	3.740
Q_{G23}	5.116	6.0124	4.348	5.238	1.856	2.757
Q_{G27}	2.697	3.936	1.834	3.064	-0.163	1.069
Q_{c6}	40	-	40	-	39.98	-
Q_{c19}	5.140	-	6.1812	-	11.06	-
Q_{c28}	21.79	-	27.245	-	39.42	-
Loss (MW)	1.995		2.028		2.173	
Cost ($)	27.45		28.355		31.491	

a) RE: Reactive Energy; b) RR: Reactive Reserve

in comparison with those obtained from the GA and the PSO algorithms. It is clear from the results that the proposed PSO method can avoid the shortcoming of premature convergence found in the GA method.

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